Stochastics II
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Universität Ulm
Prof. Dr. Evgeny Spodarev
Dr. Vitalii Makogin

Exercise sheet 5 (total – 16 points) till November 23, 2016

Exercise 5-1 (3 points)

Let $\{N(t), t \in \mathbb{R}_+\}$ be the Poisson process with intensity λ. Compute

1. $\mathbf{P}(N(1) = 1, N(2) = 2, N(3) = 4)$,
2. $\mathbf{P}(N(1) \leq 1, N(2) = 2, N(3) \geq 4)$,
3. $\mathbf{P}(N(t) = 2k + 1), k \in \mathbb{N}$.

Exercise 5-2 (3 points)

Let $\{N(t), t \in \mathbb{R}_+\}$ be the Poisson process with intensity λ. Compute

1. $\mathbf{P}(N(3) \geq 4, N(2) = 2 | N(1) = 1)$,
2. $\mathbf{P}(N(t) = i | N(s) = j), t > s$.
3. $\mathbf{E}\frac{1}{N(t)+1}$.

Exercise 5-3 (3 points)

Let τ_n be the time moment of the nth jump for the Poisson process. Prove that the distribution density of τ_n equals

$$\frac{\lambda^n x^{n-1}}{(n-1)!} e^{-\lambda x}, x \geq 0,$$

i.e., $\tau_n \sim Erlang(\lambda, n)$.

Exercise 5-4 (5 points)

Let $N^{(1)} = \{N^{(1)}(t), t \in \mathbb{R}_+\}$ and $N^{(2)} = \{N^{(2)}(t), t \in \mathbb{R}_+\}$ be independent Poisson processes with intensities λ_1 and λ_2 built on the independent sequences $T_1^{(1)}, T_2^{(1)}, \dots$ and $T_1^{(2)}, T_2^{(2)}, \dots$ Show that $N = \{N(t) := N^{(1)}(t) + N^{(2)}(t), t \in [0, \infty)\}$ is a Poisson process with intensity $\lambda_1 + \lambda_2$.

Exercise 5-5 (2 points)

A battery has a lifetime distributed uniformly over the interval (30, 60) (in units of hours). Let $N(t)$ be the number of batteries that have failed after t hours. What is $\lim_{\to\infty} \frac{N(t)}{t}$?