Stochastics II **Universität Ulm**
WS 2017/2018 Prof. Dr. Evgeny Spodarev
October 24, 2017 Dr. Vitalii Makogin

Exercise sheet 2 (total – 20 points) till November 08, 2017

Exercise 2-1 (2 points)

An insurance company has the initial capital K and receives premiums with constant rate c per time unit. The insurance portfolio consists of simple contracts with constant claim values $a > 0$. The claims appear independently and the time between two consecutive claims is exponentially distributed with mean $\gamma > 0$. Propose and describe the adequate stochastic model for the insurer's capital. Draw the trajectories of this process. Find the probability of the bankruptcy after the 2nd claim.

Exercise 2-2 (4 points)

Write an R program for simulation of Poisson process $\{N(t), t \geq 0\}$ on interval $[0, T]$ with intensity $\lambda > 0$. Simulate $n = 10000$ trajectories with $\lambda = 1$, $T = 10$. Plot one of the simulated sample paths and the histogram of $N(T)$. Formulate a hypothesis about the distribution of $N(T)$. Provide the appropriate statistical test for goodness of fit.
Please send the R code to vitalii.makogin@uni-ulm.de till 7.11.2017

Exercise 2-3 (4 points)

Let $\xi_k, k \in \mathbb{N}$ be independent random variables defined on some probability space $(\Omega, \mathcal{F}, \mathbf{P})$ and having geometric distribution with parameter $p \in (0,1) : \mathbf{P}[\xi_i = k] = p(1-p)^{k-1}, k \in \mathbb{N}, i \in \mathbb{N}$.

Let $S_n = \xi_1 + \ldots + \xi_n$ and $X_i = \begin{cases} 1, & \text{if there is } n \in \mathbb{N} \text{ such that } S_n = i, \\ 0, & \text{otherwise.} \end{cases}$

Let also $Y_k, k \in \mathbb{N}$ be independent Bernoulli random variables with $\mathbf{P}[Y_i = 1] = p$, $\mathbf{P}[Y_i = 0] = 1 - p$ defined on some other probability space $(\Omega^0, \mathcal{F}^0, \mathbf{P}^0)$. Show that the stochastic processes $\{X_i, i \in \mathbb{N}\}$ and $\{Y_i, i \in \mathbb{N}\}$ have the same finite-dimensional distributions.

Exercise 2-4 (2 points)

Suppose that all random variables $X(t)$ of a stochastic process $\{X(t), t \in \mathbb{R}_+\}$ are independent and uniformly distributed on $[0, 1]$. Prove that the process is not continuous in probability.

Exercise 2-5 (4 points)

Suppose that all trajectories of the process $\{X(t), t \in [0,1]\}$ are right continuous and have left-hand side limits. Prove that for any $\omega \in \Omega$ the trajectory $X(\cdot, \omega)$ is integrable on $[0, 1]$ and $\int_0^1 X(t)dt$ is a random variable.
Hint: Theorem. (Lebesgue's Criterion for integrablility) Let $f : [a, b] \to \mathbb{R}$. Then, f is Riemann integrable if and only if f is bounded and the set of discontinuities of f has measure 0.

Exercise 2-6 (4 points)

Prove that stochastic process $\{X(t), t \in \mathbb{R}_+\}$ is measurable assuming its trajectories are: (a) right continuous; (b) left continuous.