Stochastics II | **Universität Ulm**
WS 2017/2018 | Prof. Dr. Evgeny Spodarev
November 23, 2017 | Dr. Vitalii Makogin

Exercise sheet 5 (total – 19 points) till November 29, 2017

Exercise 5-1 (4 points)

Let $\{N(t), t \in \mathbb{R}_+\}$ be the Poisson process with intensity function λ. Compute
a) $\mathbf{P}(N(1) = 1, N(3) = 2, N(4) = 4)$; b) $\mathbf{P}(N(4) \geq 3, N(2) = 2 | N(1) = 1)$;
c) $\mathbf{P}(N(t) = i | N(s) = j), t > s$; d) $\mathbf{E}\frac{1}{N(t)+1}$.

Exercise 5-2 (3 points)

Let $\{N(t), t \in \mathbb{R}_+\}$ be the inhomogeneous Poisson process with intensity $\lambda(t) = 2e^{-t}$. Compute

1. (1 point) $\mathbf{P}(N(2) > 7 | N(1) = 7)$,
2. (2 points) $\mathbf{P}(N(3) - N(1) = 1 | N(2) = 3)$.

Exercise 5-3 (3 points)

Let $\{N(t), t \in \mathbb{R}_+\}$ be the Poisson process with intensity $\lambda > 0$ and S_n be the time moment of its n-th jump, and

$$X(t) = \begin{cases} N(t), & t \in [S_{2n}, S_{2n+1}), \\ N(t) - 1, & t \in [S_{2n-1}, S_{2n}). \end{cases}$$

Draw the trajectories of the process X. Calculate $\mathbf{P}(X(3) = 2), \mathbf{P}(X(3) = 2, X(5) = 4)$. Is the process X a process with independent increments?

Exercise 5-4 (3 points)

A retailer has two stores A and B in two different cities. The number of customers entering into stores A and B during the time period $[0, t]$ are characterized by two independent Poisson processes with intensities λ_1 and λ_2, respectively. Find all multidimensional distributions of the total number of customers for both stores A and B during the time period $[0, t]$.

Exercise 5-5 (6 points)

Let $N = \{N(t), t \in [0, \infty)\}$ be a renewal process. Let $\chi(t) = S_{N(t)+1} - t$ be its excess time. Then $C(t) = t - S_{N(t)}$ is called the *current lifetime* and $D(t) = \chi(t) + C(t)$ the *lifetime* at time $t > 0$. Now let $N = \{N(t), t \in [0, \infty)\}$ be the Poisson process with intensity λ.

1. (2 points) Calculate the distribution of the time of excess $\chi(t)$.
2. (2 points) Show that the distribution of the current lifetime is given by
$$\mathbf{P}(C(t) \leq s) = e^{-\lambda t}\delta_t(s) + \int_0^s f_{C(t)|N(t)>0}(x)dx, s \in [0, t]$$
with $f_{C(t)|N(t)>0}(x) = \lambda e^{-\lambda x}\mathbb{I}\{x \leq t\}$.
3. (2 point) Show that $\mathbf{P}(D(t) \leq x) = (1 - (1 + \lambda \min\{t, x\})e^{-\lambda x})\mathbb{I}\{x \geq 0\}$.