Stochastics II | **Universität Ulm**
WS 2017/2018 | Prof. Dr. Evgeny Spodarev
November 7, 2017 | Dr. Vitalii Makogin

Exercise sheet 3 (total – 16 points) till November 15, 2017

Exercise 3-1 (3 points)

Prove that if $\{X(t), t \in \mathbb{R}_+\}$ is a mean square continuous process with càdlàg trajectories then the process $Y(t) = \int_0^t X(s)ds, t \geq 0$ is mean square differentiable and its derivative in mean square sense $Y'(t) = X(t), t \in \mathbb{R}_+$. Find mean value and covariance functions of Y.

Exercise 3-2 (3 points)

A radiation measuring device accumulates radiation with the rate that equals a Röntgen per hour, right up to the failing moment. Let $X(t)$ be the reading at point of time $t \geq 0$. Find the mean and covariance functions for the process X if $X(0) = 0$, the failing moment has distribution function F, and after the failure the measuring device is fixed (a) at zero point; (b) at the last reading.

Exercise 3-3 (3 points)

Let $\{X(t), t \geq 0\}$ be a stochastic process with independent increments for all $t \in \mathbb{R}_+$ $\mathbf{E}|X(t)|^2 < \infty$. Prove that its covariance function is equal to $K_X(t,s) = F(t \wedge s), t, s \in \mathbb{R}_+$, where F is some non-decreasing function.

Exercise 3-4 (4 points)

Let $\{X(t), t \in \mathbb{R}_+\}$ be a real-valued process with stationary and independent increments, and $\mathbf{Var}(X(1) - X(0)) > 0$. Prove that the processes X and $Y(t) := X(t+1) - X(t), t \geq 0$ are mean square continuous but not mean square differentiable.

Hint: You may use the solution of Cauchy's functional equation $f(x+y) = f(x) + f(y)$. If f is bounded on any interval then the solution has a form $f(x) = cx$, where $c \in \mathbb{R}$.

Exercise 3-5 (3 points)

Let the mean and covariance functions of the process $X = \{X(t), t \geq 0\}$ be equal to $\mu_X(t) = \frac{t}{1+t}, t \geq 0$, $K_X(t,s) = e^{-(t-s)^2}, t, s \geq 0$. Prove that X is mean square differentiable. Let X' be its derivative defined in L_2 sense. Please find:

1. (1 points) its variaogram $\gamma(t,s), t, s \geq 0$.
2. (2 points) $\mathbf{E}[X(t)X'(s)], t, s \geq 0$.