Stochastic Simulation
WiSe - 2022-23
Date–October 18, 2022

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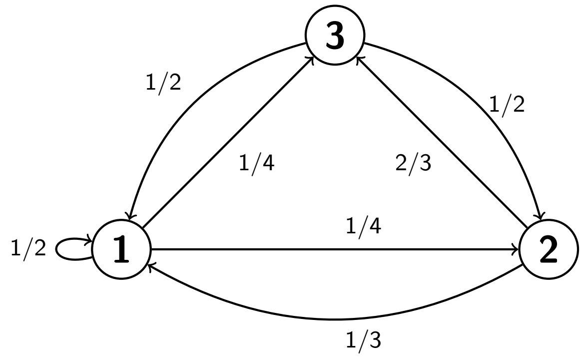
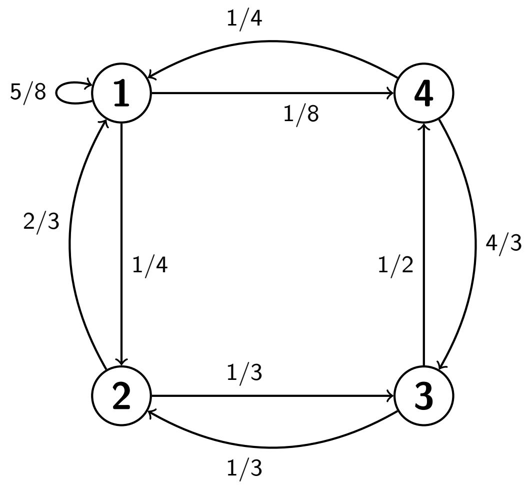
Exercise Sheet - 1
Submission Deadline : 27.10.2022, 12:00 hrs

Total Points: 17

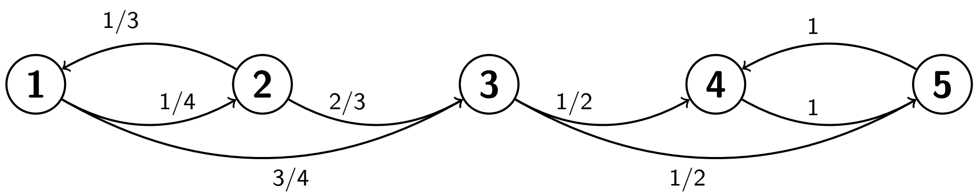
Exercise-1[1+1+1]. Find the corresponding transition matrix of the following graphs of Markov chains.



(a)



(b)



(c)

Exercise-2[1+1]. A certain calculating machine uses only the digits 0 and 1. It is supposed to transmit one of these digits through several steps. However, at every step, there is a probability p that the digit that enters this step will be changed when it leaves and a probability $q = 1 - p$ that it won't change. Form a Markov chain to represent the process of transmission by taking 0 and 1 as its state space. What is the matrix of transition probabilities?

Exercise-3[1+1]. Find the condition (if any) such that the following matrices are the transition matrices of a Markov chain.

(a) $\begin{pmatrix} (1-p_1) & p_1 & 0 & 0 & \dots \\ (1-p_2) & 0 & p_2 & 0 & \dots \\ (1-p_3) & 0 & 0 & p_3 & \dots \end{pmatrix}$ (b) $\begin{pmatrix} p_1 & p_2 & \dots & p_n \\ 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & 0 \end{pmatrix}$

Exercise-4[2+2]. Show that the Markov property is equivalent to each of the following condition:

1. $\forall n, m \geq 1$ and all $s, x_0, x_1, \dots, x_k \in \mathcal{X}$

$$P\left(X_{n+m} = s | X_1 = x_1, \dots, X_n = x_n\right) = P\left(X_{n+m} = s | X_n = x_n\right) \tag{1}$$

2. $\forall 0 \leq n_1 < \cdots < n_k \leq n$, and all $m > 1, s, x_1, \dots, x_k \in \mathcal{X}$

$$P\left(X_{n+m} = s | X_{n_1} = x_1, \dots, X_{n_k} = x_k\right) = P\left(X_{n+m} = s | X_{n_k} = x_k\right) \tag{2}$$

Exercise-5[1.5+1.5]. Consider a phone which can be in two states: "free"$= 0$ and "busy"$= 1$. The set of the states of the phone is $E = \{0, 1\}$. We assume that the phone can randomly change its state in time (which is assumed to be discrete) according to the following rules.

1. If at some time n the phone is free, then at time $n+1$ it becomes busy with probability p or it stays free with probability $1-p$.
2. If at some time n the phone is busy, then at time $n+1$ it becomes free with probability q or it stays busy with probability $1-q$.

Suppose that at time 0 the phone was free. What is the probability that the phone will be free at times $1, 2$ and then becomes busy at time 3?

Exercise-6[1+1+1]. Suppose that coin 1 has probability 0.6 of coming head whereas coin 2 has probability 0.7 of coming up head. We toss coin 2 and do the following

1. If coin flipped at the time n comes heads, then we select coin 1 to flip at the time $n+1$.
2. If coin flipped at the time n comes up tail, then we select coin 2 to flip at the time $n+1$.

Let $\{X_n, n \geq 0\}$ be a process where X_n represent the numbers of the coins we select at the time n.

1. Is $\{X_n, n \geq 0\}$ a Markov chain?
2. Find the transition matrix of this process.
3. Find the random mapping representation of this process.