Prof. Dr. V. Schmidt
Dipl.-Math. Ole Stenzel

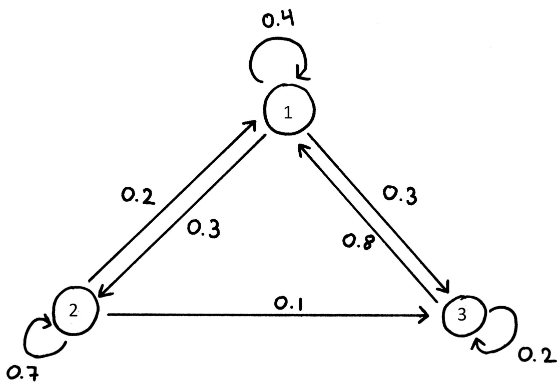
Summer Term 2012

Markov Chains and Monte Carlo Simulation
Exercise Sheet 1

for the Exercises on April 24, 2012 from 12 - 14 in Room 120

Exercise 1 (6 points)

(a) Set up a transition matrix $\mathbf{P}$ corresponding to the state diagram



(b) Draw a state diagram corresponding to the following transition matrix:

$$\mathbf{P} = \begin{pmatrix} 0.25 & 0.15 & 0.2 & 0.4 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0.3 & 0.4 & 0.1 & 0.2 \end{pmatrix}$$

(c) Look closely at state 3 in your diagram of part b). What do you notice is strange about the way information flows in state 3? What effect do you think this might have on the long-range behaviour of this system?

Exercise 2 (4 points)

Prove the following statement: If $\mathbf{P}$ is a stochastic $\ell \times \ell$ matrix, then $\mathbf{P}^k$ is a stochastic matrix for every $k \in \mathbb{N}$.

Exercise 3 (8 points)

Two players A and B gamble. Before the game starts, player A has a Euro and player B has b Euro (where a and b are some positive integers). A fair coin is thrown. If the coin shows heads, then player A pays 1 Euro to player B and vice versa. The game ends (i.e. no money is exchanged any more) as soon as one of the players has no money left. Let X_n represent the amount of money that player A has after the nth throw, $n \geq 0$.

(a) Define an appropriate state space E for the random variables $X_0, X_1, \ldots$ and find the initial distribution $\boldsymbol{\alpha}$. Next, give a recursive representation $X_n = \phi(X_{n-1}, \xi_n)$ (with iid random variables ξ_n and $X_0 \sim \boldsymbol{\alpha}$, where X_0 is independent of $\{\xi_n\}$) of the sequence $\{X_n\}$, i.e. determine ϕ and $\{\xi_n\}$ appropriately.

(b) Show that any sequence $\{X_n\}$ defined recursively by $X_n = \phi(X_{n-1}, \xi_n)$ (as in part a)) fulfills the Markov property

$$\mathbb{P}(X_n = i_n | X_{n-1} = i_{n-1}, \ldots, X_0 = i_0) = \mathbb{P}(X_n = i_n | X_{n-1} = i_{n-1})$$

for any $i_0, \ldots, i_n \in E$, $n \geq 1$ such that $\mathbb{P}(X_{n-1} = i_{n-1}, \ldots, X_0 = i_0) > 0$.

(c) Find the transition matrix of the Markov chain defined in part a).