Markov Chains and Monte Carlo Simulation
Exercise Sheet 6
for the Exercises on July 3, 2012 from 12 - 14 in Room 120

Exercise 1 (Linear congruential generator (LCG)) (12 points)

(a) Determine the periodicity $m_0$ of the LCG considering the seed $z = 1$ and the parameters
   i) $m = 512$, $a = 51$, $c = 0$,
   ii) $m = 131$, $a = 5$, $c = 0$,

(b) Write an implementation (e.g. in Java) of the LCGs given in (a.i) and (a.ii) and print
    out the first 10 pseudo-random numbers generated by your LCGs.

(c) To investigate if pseudo-random numbers are of good quality, one plots the random
    numbers $u_i = z_i/m$ as points $(u_1, u_2), \ldots, (u_{m_0-1}, u_{m_0})$ into the unit square $[0, 1]^2$. If
    the numbers are of good quality, they fill the unit square in a uniform way. Check this
    property for our two LCGs.

Exercise 2 (12 points)

Consider the linear congruence generator from Exercise 1, but now with $m = 2^{31} - 1$, $a = 16,807$, $c = 0$, and $z_0 = 1$. Write an implementation (e.g. in Java) of Pearson’s $\chi^2$-
    test for the uniform distribution on $[0, 1]$ of the pseudo-random variables $u_i = z_i/m$. Choose
    a class number $R = 10$, sample size $n = 100,000$ and significance level $\alpha = 0.05$. Check by
    repeated simulations of the above scenario how often the hypothesis of a uniform distribution
    on $[0, 1]$ is rejected.

Exercise 3 (4 points)

Calculate the generalized inverse of the Cauchy distribution $F : \mathbb{R} \rightarrow [0, 1]$ given by

$$F(x) = \frac{1}{2} + \frac{1}{\pi} \arctan x$$

and write a short algorithm how to simulate samples of a Cauchy distributed random variable
    by the inversion method using uniformly on $(0, 1)$ distributed i.i.d. random variables.