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Summer Term 2012

Markov Chains and Monte Carlo Simulation
Exercise Sheet 6

for the Exercises on July 3, 2012 from 12 - 14 in Room 120

Exercise 1 (Linear congruential generator (LCG)) (12 points)

(a) Determine the periodicity m_0 of the LCG considering the seed $z = 1$ and the parameters

 i) $m = 512$, $a = 51$, $c = 0$,

 ii) $m = 131$, $a = 5$, $c = 0$,

(b) Write an implementation (e.g. in Java) of the LCGs given in (a.i) and (a.ii) and print out the first 10 pseudo-random numbers generated by your LCGs.

(c) To investigate if pseudo-random numbers are of good quality, one plots the random numbers $u_i = z_i/m$ as points $(u_1, u_2), \ldots, (u_{m_0-1}, u_{m_0})$ into the unit square $[0,1]^2$. If the numbers are of good quality, they fill the unit square in a uniform way. Check this property for our two LCGs.

Exercise 2 (12 points)

Consider the linear congruence generator from Exercise 1, but now with $m = 2^{31} - 1$, $a = 16,807$, $c = 0$, and $z_0 = 1$. Write an implementation (e.g. in Java) of Pearson's χ^2-test for the uniform distribution on $[0,1]$ of the pseudo-random variables $u_i = z_i/m$. Choose a class number $R = 10$, sample size $n = 100,000$ and significance level $\alpha = 0.05$. Check by repeated simulations of the above scenario how often the hypothesis of a uniform distribution on $[0,1]$ is rejected.

Exercise 3 (4 points)

Calculate the generalized inverse of the Cauchy distribution $F : \mathbb{R} \to [0,1]$ given by

$$F(x) = \frac{1}{2} + \frac{1}{\pi} \arctan x$$

and write a short algorithm how to simulate samples of a Cauchy distributed random variable by the inversion method using uniformly on $(0,1)$ distributed i.i.d. random variables.