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Random Fields II
Exercise Sheet 7

for the Exercises on Januar 22 from 12.00 - 2.00pm in Room 220

Exercise 1

(a) Let $W = \{W(t),\, t \in [0,1]\}$ be the Wiener process. Show that there exist constants $\alpha, c > 0$ such that

$$\sqrt{\mathbf{E}(|W(s) - W(t)|^2)} \le \frac{c}{|\log|s-t||^{1/2+\alpha}}$$

for all $s, t : \; |s - t| < \varepsilon,\ \varepsilon > 0$.

(b) Let $W = \{W(t),\, t \in [0,1]^d\}$ be the Brownian sheet. Show that

$$\mathbf{E}(|W(s) - W(t)|^2) \le 2d|t - s|, \quad s, t \in [0,1]^d.$$

(c) How can one derive from a) (resp. b)) the continuity and boundedness (with probability one) of the underlying Wiener process (resp. Brownian sheet)?

Exercise 2 Prove the basic, but very important Gaussian inequality

$$\left(\frac{1}{x} - \frac{1}{x^3}\right)\varphi(x) < \Psi(x) < \frac{1}{x}\cdot\varphi(x),$$

where $\varphi(x)$ is the density function and $\Psi(x) = \int_x^\infty \varphi(u)\,du$ the tail distribution function of a standard Gaussian random variable. *Hint: For the lower bound use the substitution* $u \mapsto x + v/x$ *and the inequality* $e^{-z} > 1 - z,\ z > 0$.