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Dynamic Financial Analysis: Classification, Conception, and Implementation

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Abstract

Dynamic financial analysis (DFA) models an insurance company's cash flow in order to forecast assets, liabilities, and ruin probabilities, as well as full balance sheets for different scenarios. In the past years DFA has become an important tool for the analysis of an insurance company's financial situation. In particular, it is a valuable instrument for solvency control, which is now becoming important as regulators encourage insurance companies to determine risk-based capital using internal risk management models. This article considers three aspects: First, we discuss the reasons why DFA is of special importance today. Second, we classify DFA in the context of asset liability management and analyze its fundamental concepts. As a result, we identify several implementation problems that have not yet been adequately considered in the literature, and therefore our third aspect is a discussion of these areas. In particular we consider the generation of random numbers and the modeling of nonlinear dependences in a DFA framework.

Introduction: Why Is DFA of Special Importance Today?

Until the 1990s, the European insurance business was considered profitable but fairly static. Embedded as it was in a dense regulatory network, uniform products, tariffs, and limited competition, as well as stable market developments, had resulted in continuous growth and profits for many decades (for the pre-1990s situation, see Farny, 1999, pp. 146-152; Rees and Kessner, 1999, pp. 367-371). However, these basic conditions changed fundamentally in the mid 1990s. Three factors of this change are of special importance.

First, deregulation of the financial services market created increasing competition, which intensified managerial focus on profit. Increasing market transparency and the entrance of foreign competitors led to intensive price competition, margin erosion, and cost pressure (see Hussels et al., 2005). Some insurance companies responded by trying to position themselves as either global or niche players; other companies that had no clear competitive position ran the risk of being crowded out of the market (see Popielas, 2002).

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Second, substantial changes occurred in capital market conditions. Due to historical low interest rates, the previously common strategy of buying safe bonds with long-term maturity and high interest rates became problematic as long-term investments that came to term had to be replaced by bond issues carrying much lower interest rates. Against this background, the minimum interest rate warranties and further product options, which are especially prevalent in life insurance contracts, became difficult to maintain because an appropriate investment return could no longer be obtained by quasi-risk-free investing in government bonds. This calls attention to the need for well-founded investment strategies that optimize risk and return, as well as for more appropriate asset liability management techniques.

Third, changing supervisory and legal frameworks in the past years have resulted in the reregulation of the financial services market and made necessary the establishment of an integrated risk management system within insurance companies. There has been a recent, fundamental reorganization of the solvency rules in the European Union, which is discussed in Solvency II (see, e.g., von Bomhard, 2005; Eling et al., 2007). Under the new solvency rules, regulators encourage insurance companies to determine their risk-based capital by internal risk management models. In addition, recent modifications to the International Financial Reporting Standards (IFRS) and their consequences for valuation and the information that must be shown on balance sheets have become one of the central issues in the insurance industry (see Blommer, 2005; Meyer, 2005).

Against this background, the systematic, holistic analysis of an insurance company's assets and liabilities takes on a special relevance (see Liebenberg and Hoyt, 2003). This is just the sort of analysis that dynamic financial analysis (DFA) is designed to do. DFA is defined as a systematic approach to financial modeling in which financial results are projected under a variety of possible scenarios, showing how outcomes might be affected by changing internal and/or external conditions (see Casualty Actuarial Society, 1999). The entire insurance company is modeled from a macroperspective in order to simulate future development of its financial situation. DFA is characterized by an explicit multiperiod approach and a cash-flow orientation. It has become an important tool for analysis and decision making in the past years, especially in the nonlife and reinsurance business.

The literature contains several surveys and applications of DFA. Casualty Actuarial Society (1999) provides an overview of DFA and its usage in a property–casualty context. The DFA research committee of the Casualty Actuarial Society started developing DFA in the late 1990s. Their main results are reported in a DFA handbook. In another overview, Blum and Dacorogna (2004) present the value proposition, the elements, and examples of DFA use. Wiesner and Emma (2000) incorporate DFA into the strategic decision process of a workers' compensation carrier. D'Arcy et al. (1998) describe an application of the publicly available "Dynamo" DFA model to a property–liability insurer.

Lowe and Stanard (1997) and Kaufmann et al. (2001) both provide an introduction to this field by presenting a model framework, as well as an application of their models. Lowe and Stanard (1997) present a DFA model that is used by a property catastrophe reinsurer to handle the underwriting, investment, and capital management process. Kaufmann et al. (2001) give a model framework comprising the components most DFA models have in common and integrate these components in an up-and-running model. Blum et al. (2001) use DFA for modeling the impact of foreign exchange risks on reinsurance decisions, whereas D'Arcy and Gorvett (2004) use DFA to determine whether there is an optimal growth rate in the property–liability insurance business. Using data from a German nonlife insurance company, Schmeiser (2004) develops an internal risk management approach for property–liability insurers based on DFA.

However, implementing a DFA system for an insurance company involves several problems that have not been adequately considered in the DFA literature to date. On one hand, a very detailed model of the enterprise and its environment appears desirable. On the other hand, such an in-depth model is costly and difficult to maintain (e.g., because of the higher data requirements). Nevertheless, regardless of how complex it might be, an accurate model is essential for good model results. But creating a DFA model gives rise to numerous problems, such as the generation of random numbers, especially from claim distributions, or the mapping of the correct dependence structure between random variables. Further problems result from the time horizon employed or due to scarce data in the case of operational risks or extreme events. Thus, after classification of DFA and a presentation of its basic concept, these implementation problems are the central focus of this article.

The remainder of this article is organized as follows. In the next section, various asset liability management techniques are discussed, with the intention of demonstrating the key characteristics of DFA. With this overarching perspective in mind, we then present specifics of DFA, including the key elements required and the process of implementation. In the last section, we discuss various critical implementation issues associated with DFA, such as development of the model, selection of assumptions, assuring appropriate random number generation, and ultimately the modeling of linear and nonlinear dependences. We conclude and summarize both what we know and what areas are in need of future research in the final section.

Classification of Dynamic Financial Analysis in the Context of Asset Liability Management

In insurance companies, decision making traditionally proceeds independently in different organizational units. For example, actuarial risks are managed in isolation from investments and associated market risks (see Philbrick and Painter, 2001, p. 103). However, this isolated management of assets and liabilities is suboptimal because it neglects diversification effects at the total enterprise level. In contrast, asset liability management considers assets and liabilities simultaneously in order to optimize the liquidity and balance structure of the entire enterprise. DFA can be integrated into this context.

Asset liability management techniques can be organized into three different groups, as is illustrated in Figure 1. Depending on the time horizon, the consideration of uncertainty, and the planning goal, the three groups are (1) deterministic immunization techniques, which aim at managing liquidity and interest rate risks; (2) optimization techniques for the determination of an efficient risk return structure; and (3) DFA models, which allow for a multiperiod investigation of the financial situation with consideration of stochastic variables. The rest of this section is devoted to presenting the key characteristics and the differences between these models in order to point out the special focus and benefit of DFA.

FIGURE 1
Three Groups of Asset Liability Management Models

Model Group / Criteria	Immunization Techniques: Cash-Flow Matching Duration Matching	Optimization Techniques: Markowitz Kahane/Nye	Dynamic Financial Analysis (DFA)
Time Horizon	Multi Period	Single Period	Multi Period
Consideration of Uncertainty	No	Variances and Co-Variances	Distribution Functions and Stochastic Processes
Goal	Management of Liquidity- and Interest Rate Risks	Simultaneous Optimization of Risk and Return	Analysis of the Financial Situation Over Time

Group 1 comprises immunization techniques. Liquidity risks are handled by cash-flow matching and interest rate risks by duration matching. Cash-flow matching is a deterministic analysis of the cash-flow stream over a specific period of time. In the first step, the cash-flow profiles of assets and liabilities are examined. Matching takes place in the second step, which involves a reconciliation of payments from the assets with payouts for liabilities. Using this procedure, liquidity squeezes should be promptly recognized and eliminated (for more on cash-flow matching, see, e.g., Feldblum, 1989). In duration matching, the duration of liabilities and corresponding investments are accurately coordinated and the balance is immunized against interest rate changes at a specific date. However, complete immunization also eliminates any chances that may accompany changes in interest rates, a problem that can be avoided by a partial hedging in the context of conditional immunization (see, e.g., Elton and Gruber, 1992).

Immunization techniques are most frequently used as planning methods for fixed-income securities. They are less suitable for assets and liabilities with a more stochastic character, such as shares or liabilities from the property insurance business. Assets and liabilities are more commonly managed with classical techniques of risk return optimization instead of immunization methods. Markowitz (1952) and Kahane and Nye (1975) present examples of risk return optimization approaches. These techniques comprise Group 2 of the asset liability management models. The Markowitz approach utilizes classical portfolio optimization of the investments; however, liabilities are not considered in this approach. In contrast, the risk return models of Kahane and Nye (1975), Kahane (1977), Chen (1977), and Leibowitz and Henriksson (1988) incorporate liabilities as an individual investment group and thus the correlations between assets and liabilities enter the model framework.

Group 3 of the asset liability management models contains DFA. In contrast to the other two groups, this type of model is capable of multiperiod planning on the basis of stochastic influence factors, enabling the user to survey possible future paths of assets and liabilities. This is done by modeling the interest rates and the stock markets as well as by mapping the development of uncertain liabilities. DFA allows an integrated modeling of the insurance company and its environmental factors (e.g., competition, capital market, regulation). A special form of DFA considered in Schmeiser (2004) is scenario testing. In scenario testing, the future prospects of the insurance company, especially its tendency to shortfall (e.g., measured by the ruin probability), are tested under different predetermined scenarios. Examples of such scenarios are unfavorable changes in interest rates or heavy increases of the firm's loss ratios. This so-called stress testing is of special relevance when determining risk-based capital following Solvency II.

In the insurance industry, asset liability management depends on the particular line of business because actuarial obligations and the structure of investments differ for each class of insurance in respect to their maturity, risk exposure, and risk-affecting factors. Thus life insurers and nonlife insurers practice different forms of asset liability management.

DFA is more frequently used in the nonlife insurance business than in the life insurance business due to the higher uncertainty of liabilities in the former. The nonlife insurance business—in contrast to the life insurance business—has rather a short-term character (see Farny, 1997, p. 71). The business is characterized by very volatile claim distributions, for example, due to the existence of large losses, making it very difficult to forecast liability cash flows (see Kaufmann et al., 2001, p. 214). Therefore, nonlife insurers rarely use immunization techniques for asset liability management; stochastic models like DFA are far more common.

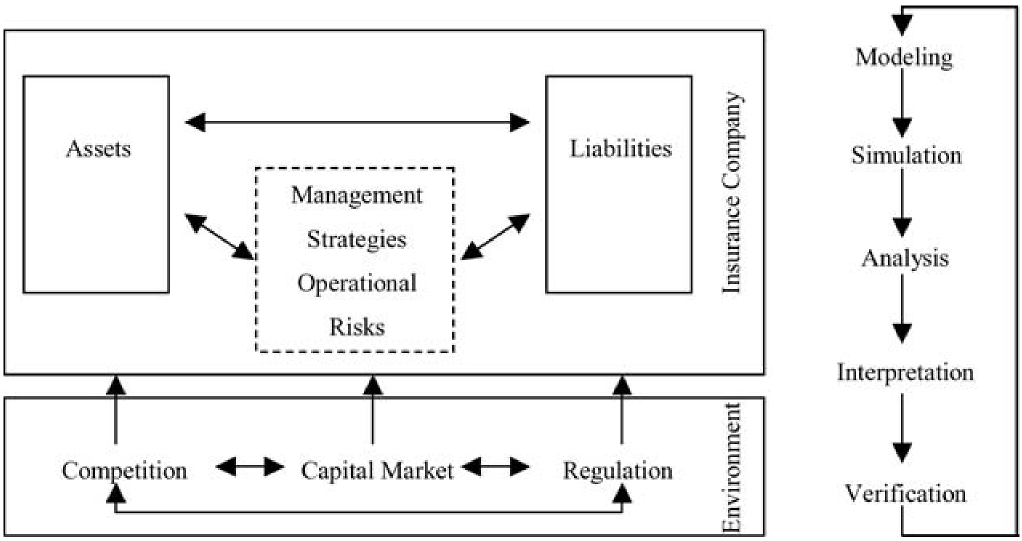
In contrast, the life insurance business is characterized by its long-term nature. The occurrence of a loss and the size of the claims are less stochastic than in the nonlife insurance business. In particular, when modeling liabilities, the (stochastic) trend of mortality must be taken into consideration. Also, asset modeling for life insurance is fundamentally different from that for nonlife insurers because the investments are marked by a very long planning horizon. Therefore, in the life insurance business there are fewer liquidity risks, but more market risks (in particular, risks of interest rate changes). This segment of the insurance industry thus mostly uses immunization and optimization techniques for asset liability management, although there are a few older instances of DFA applications to be found (see D'Arcy and Gorvett, 2004, pp. 584-585).

Conception of Dynamic Financial Analysis

DFA originated from the field of operations research and mainly uses simulation techniques for problem analysis. For this approach, the insurance company is modeled and a large number of possible scenarios are computer simulated. Different variables can be assessed for their influence on company's profit and on other important events, such as insolvency. Compared to other asset liability management methods, DFA provides additional information. In addition to the mean and the standard deviation of the variables, information about the probabilities of certain states, like illiquidity, are available, as are full balance sheet estimates. Based on this information, long- and short-term planning, benchmarking, and solvency testing, as required under Solvency II, are feasible.

Thus, DFA is a valuable tool that can help management test its strategies and learn from the results in a theoretical environment. Possible paths for the different variables are open, instead of being predetermined; there is no need to fit the insurance company into predetermined scenarios and thus make it vulnerable to other, untested possibilities. A further benefit of DFA is the capability of multiperiod analysis, which is difficult,

FIGURE 2
Conception of DFA



if not impossible, using analytic modeling or dynamic programming (see Blum and Dacorogna, 2004, p. 510). Marginal analysis is also feasible, which can show the effects of adding or closing certain business lines or the influence of single contracts on the company's profits.

As mentioned, DFA is an attempt to model the insurance company and the surrounding environment. Figure 2 presents a general conception of DFA, including the key elements that need to be considered and the different phases of DFA development. In the following we describe these elements and the DFA development process. We do not provide a complete overview of all DFA modeling details; however, this short survey covers the key elements and concepts and will be sufficient as background to our main purpose, which is to focus on problems resulting from these concepts and the modeling of the key elements, to be covered in the following section. For the details not considered here, the reader is referred to Casualty Actuarial Society (1999), Blum and Dacorogna (2004), Lowe and Stanard (1997), and Kaufmann et al. (2001).

In the DFA framework, it is convenient to reduce the complex insurance business to a few important elements. Such key elements include assets, liabilities, management behavior, and operational risks. In modeling the firm's environment, factors such as competitors, regulation (e.g., asset allocation rules, tax system, accounting rules), and the capital market (e.g., interest rates, foreign exchange rates) should be considered.

As the core business of an insurance company is providing insurance coverage, one crucial point in developing a DFA system is modeling liabilities in respect to loss distributions and expected payout streams. In this context, the company's reinsurance program and its impact on liabilities can be shown as well.

Following the idea of cash-flow matching, assets should be allocated such that the income stream matches the payout for liabilities. Besides maintenance of liquidity, there are other aims for asset allocation, such as maximization of earnings and safety, targets that naturally compete with each other.

An insurance company's management is an important factor in the firm's development. Through its daily actions and strategy setting, management influences the structure of assets and liabilities for long time periods. An example is the underwriting policy, which has a heavy impact on the market position.

Another important factor is operational risks. This type of risk is defined by the Basel Committee on Banking Supervision (BIS, 2004, p. 137) as "the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but excludes strategic and reputational risk." Although this definition applies to the banking sector, operational risk is equally relevant in the insurance industry and most likely will be covered in the Solvency II rules.

There are many exogenous variables that affect the insurance business. One major factor is regulation, which, for example, puts heavy constraints on asset allocation in many countries. This type of regulation, which is primarily aimed at protecting insureds, constricts the efficiency of asset allocation. Further, market competition is a field with complex interdependences. For example, superior asset allocation in comparison to competitors can give an advantage in product pricing. The positioning of competitors has a direct influence on management decisions and contracts written and, therefore, on liabilities.

The process of implementing and using a DFA system can be described as follows. In the *modeling* stage, the key factors, as discussed above, must be identified by the developers of the DFA system. These factors and their dependences then must be incorporated into the DFA model and calibrated to meaningful historical data (see, e.g., Kaufmann et al., 2001, p. 245). After that, in the *simulation* stage, the modeled company is run through various possible paths, dependent on the modeled stochastic variables. Here, increasing the number of simulation cycles can improve the quality of results, in the sense of reaching full outcome distributions instead of point estimates, for the following *analysis*. In this phase, the results are analyzed and possible prosperous or dangerous scenarios are identified. Based on this information, in the *interpretation* stage, the strategies can be adapted so as to avoid the dangerous scenarios and achieve the prosperous ones. In other words, management can use the model to support its decisions.

After decisions are made and a certain time period has elapsed, the real-world outcomes of the decisions can be used as a benchmark for the DFA results. Thus, by *verification*, the real-world outcomes are compared to the simulated results of the DFA model. Subsequently, using this feedback, the DFA model is adjusted so as to provide more accurate simulations in the future, which closes the DFA development loop by bringing us back to the *modeling* stage.

IMPLEMENTATION OF DYNAMIC FINANCIAL ANALYSIS

As stated in Lowe and Stanard (1997), it makes a substantial difference whether a DFA model is implemented for one single line of insurance business or for an entire multiline insurance company. It is difficult enough to identify all relevant variables, measure their dependences, and proceed to implementation for a single line of business; doing the same for an entire insurance company with many lines of business is, obviously, far more difficult. Thus, because the modeling and implementation stages are so crucial to success, they are the main focus of this work.

The remainder of this section consists of two subsections. The first is an overview of strategic decisions that need to be made in setting up the DFA model, such as the make or buy decision, the scope of the system, and the time horizon to be employed, topics of most concern to management. The second subsection deals with operational decisions important to realization of the DFA model. Of these, we first discuss modeling the key elements, and then proceed to the technical details related to the design of these elements—the generation of random numbers and the implementation of dependence structures.

Strategic Decisions

One of the first strategic considerations after deciding to implement DFA is whether to make or to buy a DFA model,[1] a decision that must be made on a case-by-case basis by each individual insurance company. The decision will depend on in-house know-how and capacities, the availability of suitable products on the market, and, if it is determined to buy the DFA model, the extent to which the model can be customized. Finally, and possibly most important, is a cost–benefit analysis. This list of considerations is not exhaustive but it serves to illustrate the complexity of decisions necessary even at the very early stage of DFA implementation. Once the initial decision about whether to make or buy has been made, things grow even more complex. Management will need to decide, for example, how complex or comprehensive the system should be, what time horizon will be employed, and which variables will be implemented.

One of the most critical of these decisions will be the scope of the DFA system. Which elements should be included and how detailed should these be? On one hand, in-depth representation of reality is desirable. On the other hand, more detail means more expense and greater difficulty to maintain. It is questionable whether all interdependences are ascertainable and really necessary for good model results. A model incorporating only the most significant variables makes it easier to identify the interrelations between inputs and outputs; higher transparency eases interpretation of results and decision making (see Kaufmann et al., 2001, p. 246; D'Arcy et al., 1998, p. 79).

Regardless of which elements and interdependences are finally incorporated, accurate modeling is essential. Related to accuracy is the question of the most suitable structure of the model—should it be a one-piece structure or should it consist of a modular environment? In most cases, the modular environment is the best and obvious choice due to easier administration and the ability to add or remove elements (see Blum and Dacorogna, 2004, p. 516). Another thing that must be considered is computing power. This is, perhaps, not the bottleneck it used to be. But enhanced computing power has been accompanied by a demand for more accurate DFA models and a larger number of simulation runs. Thus, computing power is once again an issue that cannot be overlooked.

[1] Examples are Igloo by Paratus Consulting, TAS P/C by Tillinghast, and Dynamo by MHL Consulting, which is a freeware Excel-based DFA model (see Blum and Dacorogna, 2004, p. 516).

Also of great importance in interpreting DFA results is the time horizon to be employed. The results may not be relevant to strategic decision making if the regarded time period is short. However, there are also problems concerning longer time periods, for example, data uncertainty and the variability of outputs. The longer the time period considered, the more uncertain is the input data, leading to greater variability of results. Less exact results are a questionable basis for decision making (see Kaufmann et al., 2001, p. 214).

Operational Decisions

Modeling the Key Elements of DFA. The key elements of a DFA system were identified in "Conception of Dynamic Financial Analysis." In this subsection, we discuss the modeling of those key elements.

For the liabilities, different dimensions need consideration. On one hand, the shape of distributions differs because in some insurance branches distributions are influenced by relatively homogeneous claims (e.g., collision claims) and in others by extreme losses (e.g., natural disaster). On the other hand, there is the change of distribution functions over time. Due to inflation and the concentration of wealth in highly industrialized countries, distributions—the shape, mean values, standard deviations, and extreme values—can change considerably. Integrating reinsurance adds even more complexity; both proportional and nonproportional reinsurance must be considered. This brings to light another advantage of DFA—it can be used to evaluate the adequacy and cost efficiency of a reinsurance program. In this context, it is possible to simulate cumulative claim distribution values or to generate individual claim data, which means simulating the claim number (e.g., poisson random numbers) and associated claim size for every claim separately. Simulating individual claims is favored because it allows implementation of certain nonproportional reinsurance contracts.

The asset side also presents modeling complications. Manifold constraints make it difficult to model this element properly. For one thing, in many countries regulation constrains the ability of insurance companies to invest freely or as would be recommended by an optimization model. Another complication is that an insurance company needs sufficient liquidity to pay claims at any time, without affecting the earnings. A superior return–risk portfolio can be created with the help of the Markowitz approach or another optimization model. Additionally, changes in asset prices, interest rates, and currency rates at different points of time can be regarded within an intertemporal asset management.

As mentioned, DFA can help managers test their strategies and be used as support for their decisions. In addition to this valuable function, management behavior can be modeled so that multiperiod analysis is feasible. Management behavior and management-induced strategies can be implemented in the model by setting decision rules, constraints, or objectives. Consider liquidity development, for example. If, in the simulation, liquidity drops below a certain level, predetermined rules can affect a sale of long-term assets, reflecting actual management behavior in this situation.

Operational risk is hard to identify and measure, mainly because such events are very scarce and thus it is difficult to determine suitable loss distributions. One example of this rare and hard to predict type of risk is the case of large business fraud. However, even this type of risk must be incorporated into a comprehensive DFA system to be useful in risk management decision making.

Asset allocation regulations and other rules (such as the restriction on issuing bond capital by direct insurers) must be incorporated in the model if they have an impact on the insurer's business. Multinational insurers have to keep track of different national regulations and, possibly, special prescriptions for foreign insurers. Underwriting cycles can be used to model the competition, another important variable. By modeling a stochastic process, hard and soft market phases are educible and so the firm can thus make appropriate decisions about whether to charge higher prices for insurance or prepare for worsened economic conditions (see D'Arcy et al., 1997). The model does this by way of response functions. These response functions contain certain rules, for example, for management behavior, as discussed above, which reflect the insurance company's reaction to market changes (see Daykin et al., 1994).

The main variables of the capital market, such as interest rates, asset prices, and currency exchange rates, are important for building a realistic DFA model. These variables are mostly generated in the framework of an economic scenario generator. An approach for dealing with the complex interdependences of market variables is the stepwise simulation of economic variables in a cascade style, which leads to a hierarchical dependence between the variables. In a first step, a primary economic driver, e.g., the short rate, is simulated. This variable influences other economic parameters generated in a next step, which themselves then can affect variables simulated in the following steps (see, e.g., Kaufmann et al., 2001). As mentioned, one of the most important variables is the short-term interest rate because it is, for example, the basis for deriving the term structure and, thus, for valuing the bond portfolio (for more on interest rate and term structure modeling, see, e.g., Cairns, 2004; James and Weber, 2000). The interest rate, as well as other economic parameters, can be modeled with the help of stochastic processes, which can show different features suitable for mapping real-world behavior. For instance, there is the mean-reverting property, which can be applied to model the short rate. Another example is the movement of stocks, which can be modeled using diffusion (random walk) or jump-diffusion processes. The latter are random walk processes that additionally make jumps in market prices and thus allow the replication of market shocks (for more on stochastic processes, see, e.g., Hull, 2003; Rolski et al., 1999).

Generation of Random Numbers. The generation of random numbers is essential to a successfully operating DFA system because the generation process can be a considerable source of error or bias for the DFA results. As most advanced commercial software programs available for DFA have already solved this problem quite well, this point is mostly of interest for those users who want to implement their own DFA solutions. In this case, the random numbers and their statistical properties must be checked by the developers of the DFA model or verified by a third party (see Seila, 1995). In this section, we will first discuss some of the problems that can arise with the generation of uniform random numbers. Then we will demonstrate the creation of random numbers from loss distributions, after which we will address specific problems associated with modeling of extreme events.

The basis for the generation of any specific random number is the creation of uniform random numbers U, which are uniformly distributed in the interval [0,1]. When generating these uniform distributed values of U, it is important that the generators create random numbers with good statistical properties. Random generators produce pseudo-random numbers, which means that they generate random numbers by way of an arithmetic function. The problem with these generators is that after a certain quantity, the numbers are iterated and thus the required independence in a vector of random numbers cannot be ensured when many random numbers are produced. Also, in some arithmetic functions there can be a considerable dependence between a random value and its predecessor. These are two analytical tractable criteria, called cycle length and one-step serial correlation, which are suitable tests of the quality of a random number generator. The user of DFA should be aware of these criteria and apply a generator with a suitable cycle length and without serial correlation problems (see Bratley et al., 1987; Seila, 1995).

Also of importance is the required independence between different random vectors, which means that the generator should not start with the same sequence for every random vector. Furthermore, the random generator should be portable to other computer systems so that it creates identical results on different systems. A useful feature for DFA is the ability to fix the "seed" of the random numbers. This enables the user to modify parameters in the simulation study and to compare the results of the change for the same random numbers (see Fishman, 1996). To avoid the pitfalls connected with the creation of uniform random numbers, the generator should be tested for the presence of these problems and discarded for another generator in the event of unfavorable results. Commonly used generators are the linear "mixed congruential approach" or quadratic and inverse generators from the group of nonlinear generators, which overcome some limitations of the linear methods (see Fishman, 1996, p. 670; Daykin et al., 1994, p. 468).

In the context of DFA, random values from claim distributions are more desirable than pure uniform random numbers. However, the just-described random numbers U are the basis for generation of the claim distribution values. Together with the inverse of the distribution function, and based on the inverse transformation method,

$$x = F_X^{-1}(U), \tag{1}$$

one can determine these desired random number values x (see, e.g., Rubinstein, 1981). For the inverse distribution function, it is necessary to estimate the distribution function first. One way to do this is to fit the available empirical loss data to theoretical parametric distributions, for example, with the help of maximum likelihood estimation. This procedure would determine the best parameter set of the given data set for different types of distributions. Then, using goodness of fit (e.g., with the chi-square or the Kolmogorov–Smirnov goodness-of-fit test), the most suitable distribution can be chosen. Another way to determine the distribution function, is to use nonparametric or semiparametric methods. An analytic solution is only rarely available for the inverted distribution function in the case of a parametric distribution. If this analytic solution is not available, which is usually the case, the developer must choose an approximation of the inverse function or use recursive assignment, that is, quantile transformation, to calculate these values. For this approach, the random values U, lying in the interval [0,1], are assumed to be random distribution values of the desired distribution. The required values x can be later inferred from these distribution values using the available assignments of the distribution function from x to $F(x)$ in reverse (see Frey and Nießen, 2001, p. 102). For non- or semiparametric fitting without an analytical solution, the same approach can be applied but there must be a high density of assignments between the distribution values and the x-values in order to achieve suitable results.

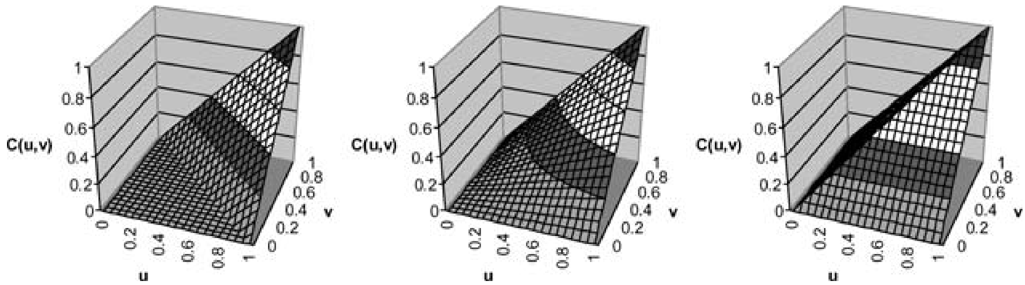
A problematic issue in generating random numbers from loss distributions is the case of extreme events. If the empirical data were fitted to a parametric distribution, it is possible that even despite a general goodness of fit, the parametrical distributions will result in a bad match in the tails of the data. This can happen because of the up to a certain degree predetermined shape of the parametrical distribution. As it can be crucial to an insurance company's survival to correctly estimate the tail behavior of its claims, this problem cannot be ignored. One way to integrate tail behavior is to use special tail estimators like the Hill estimator. The Hill estimator neglects the body of the distribution and considers only the tails. It fits the parameters of a distribution (e.g., the pareto distribution) by applying maximum likelihood technique to the data of the tails (see Hill, 1975). Another possible solution is the application of non- or semiparametric fitting methods, which are more flexible in the detection and display of the real nature of data. However, a disadvantage of these approaches is that with the "widening" lack of data in the tail region, the fit can also worsen, which casts doubt on the accuracy (for the issue of modeling extreme events, see Embrechts et al., 2003).

Implementation of Dependence Structures. Of concern in the context of random number generation is the correct mapping of dependence structures, for example, between macroeconomic factors and the dependent variables in the insurance company. In this section, implementation of the multivariate normal dependence structure and the copula concept are presented.

Linear dependence is the best-known dependence concept and can be measured using the Bravais–Pearson correlation coefficient R. Along with the linear relationship, nonlinear dependence structures also should be incorporated in the model because linear dependence is only sufficient for spherical or elliptical (e.g., multinormal) distributions. We will not get good results for other distribution forms by applying the Bravais–Pearson correlation coefficient to map dependence structures. Another disadvantage of linear correlation for the generation of random numbers is that these methods disregard joint major claims or tail dependence. If tail dependence is modeled incorrectly in the DFA system, "unexpected" joint major claims could cause severe "real-world" distress for the insurance company. Such flawed modeling arises because the dependence structure induced by a linear correlation structure leads to asymptotic tail independence for $R < 1$ (see Embrechts et al., 2002; Nelsen, 1999).

Despite these disadvantages, we can use, for the sake of convenience, a method of mapping linear dependence in the form of the linear correlation of ranks, described by Iman and Conover (1982). The idea is to transfer the desired Spearman's rank correlation coefficient R_S from a multivariate normal distribution to the required marginal distributions, without influencing these predetermined distribution shapes. Starting from independent multivariate standard normal distributed data $Z = N_{(0,I)}$, only indirect implementation of the rank correlation coefficient R_S is possible using the Bravais–Pearson correlation coefficient R. To obtain the correct rank correlation coefficients for the target data, the desired values R_S of the rank correlation matrix must be transformed to R by $R = 2\sin(\pi R_S/6)$, which are elements of the correlation matrix Ω. Then—using the results from the Cholesky decomposition ($\Omega = PP'$), that is, the lower triangular

FIGURE 3
Fréchet–Hoeffding Lower Bound, Product Copula, and Fréchet–Hoeffding Upper Bound



matrix P—the desired dependence structure can be implemented by matrix multiplication of P with the standard normal distributed data Z ($Y = P \times Z$).[2] Subsequently, as suggested by Embrechts et al. (2002), the distribution values of these accordingly correlated multivariate normal values $\Phi\,(Y)$ can be applied to derive the random numbers from the desired marginal distribution functions using the inverse transformation $(X_1, \ldots, X_n) = (F_{X_1}^{-1}(\Phi(Y_1)), \ldots, F_{X_n}^{-1}(\Phi(Y_n)))$. Thus the required multinormal rank ordering and the desired rank correlation R_S is implemented for the target data (see Iman and Conover, 1982; Embrechts et al., 2002).

The approach, which covers also nonlinear dependence, is the copula concept. Copulas are functions that join the marginal distribution functions to the joint multivariate distribution. Thereby, the copula C contains the whole dependence structure and the marginal distributions can be regarded independently from the dependence structure. This result from Sklars Theorem (see Sklar, 1959) can be formalized for two variables with continuous distributions by

$$F_{1,2}(x_1, x_2) = C(F_1(x_1), F_2(x_2)). \tag{2}$$

Copulas are defined on the unit space, which means that all values of $F_{1,2}$, F_1, and F_2, respectively, lie in the interval [0,1].[3] An important example of the bivariate copula class is the product copula: $C(u, v) = uv$ implying independence; i.e., the two marginals $F_1(x_1)$ and $F_2(x_2)$ are joined by the copula $uv = F_1(x_1)F_2(x_2)$. The result of this product is the value of the bivariate cumulative distribution function $F_{1,2}(x_1, x_2)$. Other examples are the Fréchet–Hoeffding lower bound: $C(u, v) = \max\,(u + v - 1, 0)$ standing for the strongest negative form of dependence (countermonotonicity) and the Fréchet–Hoeffding upper bound: $C(u, v) = \min(u, v)$ representing the strongest positive form of dependence (comonotonicity) (see, e.g., Nelsen, 1999). Figure 3 is a graphical representation of these copulas and shows their different three-dimensional shapes. The

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[2] Alternatively, and leading to the same results, one can use the result from the spectral decomposition and multiply by $P = \Omega^{1/2} = e \times diag\,(\lambda^{1/2}) \times e'$ (with λ the eigenvalue, e the eigenvector of the correlation matrix, and *diag* the diagonal matrix).

[3] The bivariate copula theory that is presented here can also be extended to the multivariate case by using a generalization of Equation (2): $F_{1,\ldots,n}(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n))$. See, e.g., Joe (1997).

values of the marginal distribution u and v are displayed in the horizontal plane. The joint dependence structure (*C(u,v)*, or the copula) is displayed in the third dimension.

The Bravais–Pearson correlation coefficient is also not the best for measuring dependence. It is possible that by using this coefficient, a correlation of zero can be measured for a specific data set even though there exists a form of dependence. Furthermore, the Bravais–Pearson correlation coefficient can exhibit values above −1 for countermonotonicity (i.e., $-1 \leq R < 0$) and below 1 for comonotonicity (i.e., $0 < R \leq 1$). Because the correlation values for the strongest form of dependence (positive or negative) can be near zero, the explanatory power of the Bravais–Pearson correlation coefficient is heavily constrained (see Embrechts et al., 2002, pp. 205-207). More suitable measures for dependence are Spearman's R_S and Kendall's τ because, in contrast to the Bravais–Pearson coefficient, they cover the full range of dependence between countermonotonicity (R_S, $\tau = -1$—Fréchet–Hoeffding lower bound) and comonotonicity (R_S, $\tau = 1$—Fréchet–Hoeffding upper bound) (see, Joe, 1997, p. 32).

Once the copula is determined by fitting the empirical copula to analytic tractable forms with the help of maximum likelihood technique (see, e.g., Klugman and Parsa, 1999), one can generate random numbers with the desired dependence structure. A possibility for the data generation is the conditional distribution function for v given u by $c_u(v) = \frac{\partial}{\partial u} C(u, v)$, which is the partial derivative of $C(u, v)$ with respect to u. In the bivariate case, one first must simulate two uniform distributed variables u and t. Then, the inverse of $c_u(v)$ with the values of t can be used to generate the variable $v = c_u^{(-1)}(t)$. Hence one has generated pairs of the random variables (u, v) with the desired dependence structure. In the past step, to obtain the values (x_1, x_2) from the joint multivariate distribution, the values of u and v must be inserted in the inverse functions of the marginal distributions $F_1^{(-1)}(\cdot)$ and $F_2^{(-1)}(\cdot)$. For more details and a full example, see Nelsen (1999), pp. 35-37. This approach makes feasible dependence structures beyond the possibilities of the method suggested by Iman and Conover (1982). However, the method is not without its problems. The convenience of an analytical tractable solution may lead to using a copula that fits to the empirical copula only to a small degree. In addition, the multivariate case is not very manageable and, in this case, implementation of a multivariate normal dependence structure is more convenient.

As mentioned previously, tail dependence is of special importance for DFA analysis. The problem of determining tail dependence or tail copulas arises from the lack of data comparable to the estimation of the marginal tail distributions (for an overview of possible tail dependence estimators, see Coles, 2001). However, frequently, pure parametric or nonparametric methods are inefficient estimators. Newer approaches to overcoming this weakness use semiparametric techniques. Heffernan and Tawn (2004), for example, describe a conditional approach that uses parametric regression to determine the parameters of the margins and a nonparametric method for the multivariate structure. Another example is Klüppelberg et al. (2005), which, in contrast, estimate the margins of the tails nonparametrically and the copulas using a parametric approach.

Summary

There are three main reasons why, within the past few years, DFA has evolved as an important tool for analyzing an insurance company's financial situation. First, deregulation of the financial services market created increasing competition and an intensified managerial focus on profit. Second, historical low interest rates have called attention to the need for well-founded investment strategies and for appropriate asset liability management techniques. Third, changing supervisory and legal frameworks resulted in the establishment of an integrated risk management system.

DFA can be used for this risk management purpose. The model's high flexibility and various analysis options (e.g., solvency analysis or the estimation of full balance sheets) represent DFA's most important characteristics. Within asset liability management, DFA can be classified as a group of multiperiod stochastic models. Other groups are deterministic immunization techniques and optimization approaches for the determination of an efficient risk return structure.

DFA maps the enterprise from a macroperspective on the basis of a simulation model. It follows an explicit multiperiod approach and a cash-flow orientation. In DFA, the complex insurance business is reduced to its most important elements—assets, liabilities, management behavior, and operational risks. The model also takes into consideration environmental factors, such as competitors, regulation, capital markets, and the dependence structures between these macroeconomic factors and the dependent variables in the insurance company. Implementation and use of DFA is a five-step process: modeling, simulation, analysis, interpretation, and verification. The final step, verification, creates a feedback loop for parameters that change over time and newly discovered modeling errors so that the first step, modeling, is reengaged, making DFA ever more sensitive, accurate, and valuable to its users.

Summarizing our results, what is the precise contribution of our article? After giving the overview on classification and conception of DFA, we have focused on several implementation aspects of DFA. Two of them are of special importance, since they were not analyzed in the DFA literature so far.

The first issue concerns the generation of random numbers. Problems arise from the creation of uniform random numbers, the generation of random claim distribution values, and, especially, from modeling extreme events and operational risks. It is essential to evaluate the quality of the random number generator. It is possible to fit the empirical claim distribution to parametric distributions using maximum likelihood estimations. However, if the goodness of fit is unacceptable, several non- or semiparametric methods or the use of tail estimators might be more appropriate. These methods have the added attractiveness of also being capable of improved modeling of extreme events and operational risks.

The second issue we have addressed is the correct mapping of dependence structures, which can be implemented using the multivariate normal dependence structure or the copula concept. The multivariate normal dependence structure fails to correctly model dependence, especially in tails, but it is convenient to implement in the multivariate case. The copula concept is the best-known method for modeling dependence structures and it also allows exact modeling of tail dependences. However, multivariate cases are very difficult to handle using the copula concept.

It is important to note that DFA models cannot predict the future. Rather, DFA is a very useful tool for risk analysis and decision support in that it can recommend possible course of action. For these recommendations to be of any use, however, the input parameters and the modeling must be of very high quality as a poor estimation of the input parameters and bad mapping will lead to an inferior prognosis.

The future will bring further refinement of DFA. At present, fundamental changes in supervisory and legal frameworks have created a need for holistic enterprise risk management tools. These changes have created the present climate of discussion surrounding DFA, both in science and in practice. Many technical improvements in DFA have already occurred, but a model specifically designed for the life insurance industry should be a high priority for future research. Moreover, using DFA to calculate risk-based capital standards following Solvency II raises many questions for future research, such as the appropriate risk and performance measurement in DFA. For example, a risk measure is needed that works on a multiperiod analysis. However, because the benefits of its use are so great, we expect that DFA will evolve into a common tool of risk management for all insurance companies.

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