Participating Life Insurance Contracts under Risk Based Solvency Frameworks: How to increase Capital Efficiency by Product Design

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Abstract

Traditional participating life insurance contracts with year-to-year (cliquet-style) guarantees have come under pressure in the current situation of low interest rates and volatile capital markets, in particular when valued in a market-consistent valuation framework. Furthermore, such guarantees lead to rather high capital requirements under risk based solvency frameworks, e.g. Solvency II.

We present several alternative product designs and analyze their impact on the insurer's financial situation. We also introduce a measure for "Capital Efficiency" that considers both, profits and capital requirements, and compare the different product designs with respect to capital efficiency in a stochastic market consistent valuation model.

1 Introduction

Traditional participating life insurance products play a major role in old-age provision in Continental Europe and in many other countries. These products typically come with a guaranteed benefit at maturity, which is calculated using some guaranteed minimum interest rate. Furthermore the clients receive an annual surplus participation that depends on the performance of the insurer's assets. With so-called cliquet-style guarantees, once such surplus has been assigned to the policy at the end of the year, it increases the guaranteed benefit based on the same guaranteed minimum interest rate. This product design can create significant financial risk.

Briys and de Varenne [1997] were among the first to analyze the impact of interest rate guarantees on the insurer's risk exposure. However, they considered a simple point-to-point guarantee where surplus (if any) is credited at maturity only. The financial risks of cliquet-style guarantee products have later been investigated e.g. by Grosen and Jorgensen [2000]. They introduce the "average interest principle", where the insurer aims to smooth future bonus distributions by using a bonus reserve as an additional buffer besides the policy reserve (the customer's account). Besides valuing the contract they also calculate default probabilities (however under the risk-neutral probability measure $\mathbb{Q}$). Grosen et al. [2001] extend the model of Grosen and Jorgensen [2000], and introduce mortality risk. Grosen and Jorgensen [2002] modify the model used by Briys and de Varenne [1997] by incorporating a regulatory constraint for the insurer's assets and analyzing the consequences for the insurer's risk policy.

Mitersen and Persson [2003] analyze a different cliquet-style guarantee framework with so-called terminal bonuses whereas Bauer et al. [2006] specifically investigate the valuation of participating contracts under the German regulatory framework.

While all this work focuses on the risk-neutral valuation of life insurance contracts (sometimes referred to as "financial approach"), Kling et al. [2007a] and Kling et al. [2007b] concentrate on the risk a contract imposes on the insurer (sometimes referred to as "actuarial approach") by means of shortfall probabilities under the real-world probability measure $\mathbb{P}$.

Barbarin and Devolder [2005] introduce a methodology that allows for combining the financial and actuarial approach. They consider a contract similar to Briys and de Varenne [1997] with a point-to-point guarantee and terminal surplus participation. To integrate both approaches, they use a two-step method of pricing life insurance contracts: First, they determine a guaranteed interest rate such that certain regulatory requirements are satisfied, using value at risk and expected shortfall risk measures. Second, to obtain fair contracts, they use risk-neutral valuation and adjust the participation in terminal surplus accordingly. Based on this methodology, Gatzert and Kling [2007] investigate parameter combinations that yield fair contracts and analyze the risk implied by fair contracts for various insurance contract models. Gatzert [2008] extends this approach by introducing the concept of "risk pricing" using the "fair value of default" to determine contracts with the same risk exposure. Graf et al. [2011] (also building on Barbarin and Devolder [2005]) derive the risk minimizing asset allocation for fair contracts using different risk measures like the shortfall probability or the relative expected shortfall.

Under risk based solvency frameworks such as Solvency II and Swiss Solvency Test (SST), the risk analysis of interest rate guarantees becomes even more important.

Under these frameworks, the capital requirement is derived from a market-consistent valuation considering the risk an insurer takes. This risk is particularly high for long term contracts with a year-to-year guarantee based on a fixed (i.e. not path dependent) guaranteed interest rate. Measuring and analyzing the financial risk in relation to the required capital, and analyzing new risk figures such as the Time Value of Options and Guarantees (TVOG) is a relatively new aspect which gains importance with new solvability frameworks. E.g. the largest German insurance company (Allianz) announced in a press conference on June 25, 2013[1] the introduction of a new participating life insurance product that fundamentally modifies the type of interest rate guarantee (in a similar way as we do in the remainder of this paper). It was stressed that the TVOG is significantly reduced for the new product. Also, it was mentioned that the increase of the TVOG resulting from an interest rate shock (i.e. the solvency capital requirement for interest rate risk) is reduced by 80% when compared to the previous product. This is consistent with the findings of this paper.

The aim of this paper is a comprehensive risk analysis of different participating life insurance products. Currently, there is an ongoing discussion, whether and how models assessing the insurer's risk should be modified to reduce the capital requirements (i.e. by introducing an "ultimate forward rate" set by the regulator). We will in contrast analyze how (for a given model) the insurer's risk and hence capital requirement can be influenced by product design. Since traditional cliquet-style participating life insurance products lead to very high capital requirements, we will introduce products with modified types of guarantees which reduce the insurer's risk,

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[1] Cf. "Allianz Capital Markets Day" presentation on June 25, 2013: Allianz [2013], particularly slide D24.

profit volatility, as well as the capital requirements under risk based solvency frameworks. In order to compare different product designs from an insurer's perspective, we develop and discuss the concept of "Capital Efficiency" which relates profit to capital requirements. We identify the key drivers of capital efficiency which are then used in our analyses to assess different product designs.

The remainder of this paper is structured as follows:

In Section 2, we present the three considered products that all come with the same level of guaranteed maturity benefit but with different types of guarantee:

- Traditional product: a traditional contract with a cliquet-style guarantee based on a guaranteed interest rate >0,

- Alternative product 1: a contract with the same guaranteed maturity benefit that is valid only at maturity; additionally, there is a 0% year-to-year guarantee on the account value,

- Alternative product 2: a contract with the same guaranteed maturity benefit that is valid only at maturity but no other guarantee; this means that the account value may decrease in some years.

In spite of the different types of guarantees, all the products include a surplus participation depending on the insurer's return on assets. Our model is based on the surplus participation requirements given by German regulation. That means in particular that each year at least 90% of the (book value) investment return has to be distributed to the policyholders.

To illustrate the mechanics, we will illustrate the different products under different deterministic scenarios. This shows the differences in product design and how they affect the insurer's risk.

In Section 3, we introduce our stochastic model which is based on a standard financial market model: The stock return and short rate processes are modeled using a correlated Black-Scholes and Vasicek model.[2] We then describe how the evolution of the insurance portfolio and the insurer's balance sheet are simulated in our asset-liability-model. The considered asset allocation consists of bonds with different maturities and stocks. The model also reflects management rules.

Furthermore, we introduce a measure for capital efficiency based on currently discussed solvency regulations such as the Solvency II framework. We also propose a more tractable measure for an assessment of the key drivers of capital efficiency.

In Section 4, we present the numerical results. We show that the alternative products are significantly more capital efficient: financial risk and therefore also capital requirement is significantly reduced, although in most scenarios all products provide the same maturity benefit to the policyholder. The asymmetry, i.e. particularly the heavy left tail of the insurer's profit distribution is reduced by the modified products. This leads to a significant reduction of both, the TVOG and the solvency capital requirement for interest rate risk.

Section 5 concludes and provides an outlook on further research.

2 Considered products

In this section, we describe the three different participating life insurance products that we will analyze in what follows. Note that for the sake of simplicity, we assume for all three products that in case of death of the policyholder in year t, the current account value AV_t (defined below) is paid at the end of year t but no additional death

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benefit is provided. This allows us to ignore mortality for the calculation of premiums and actuarial reserves. Similarly, we assume that the surrender value is equal to the account value.

2.1   The traditional product

First, we consider a traditional participating life insurance contract with a cliquet-style guarantee. It provides a guaranteed benefit G at maturity T based on annual premium payments P. The pricing is based on a constant interest rate i and reflects annual charges c_t. Based on the actuarial equivalence principle[3], the relation between the annual premium and the guaranteed benefit is given by

$$\sum_{t=0}^{T-1}(P - c_t)^t \cdot (1 + i)^{T-t} = G$$

During the lifetime of the contract, the insurer has to build up sufficient actuarial reserves AR_t for the guaranteed benefit at maturity. For the traditional product, at any time t, the same constant interest rate i is applied to calculate the (prospective) actuarial reserve, i.e.

$$AR_t = G \cdot \left(\frac{1}{1+i}\right)^{T-t} - \sum_{k=t}^{T-1}(P - c_k) \cdot \left(\frac{1}{1+i}\right)^{k-t}.$$

The development of the actuarial reserves is then given by:

$$AR_t = (AR_{t-1} + P - c_{t-1}) \cdot (1 + i).$$

Traditional participating life insurance contracts typically include an annual surplus participation that depends on the performance of the insurer's assets. For example, German regulation requires that at least $p = 90\%$ of the (local GAAP book value) earnings on the insurer's assets have to be credited to the policyholders' accounts. For the traditional product, any surplus assigned to a contract immediately increases

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[3] For the concept of equivalence principle, see e.g. Saxer [1955], Wolthuis [1994].

the guaranteed benefit based on the same interest rate i. More precisely, the surplus s_t is credited to a bonus reserve account BR_t (where $BR_0 = 0$) and the interest rate i will also apply each year on the bonus reserve:

$$BR_t = BR_{t-1} \cdot (1 + i) + s_t.$$

The client's account value AV_t consists of the sum of the actuarial reserve AR_t and the bonus reserve BR_t; the maturity benefit is equal to AV_T.

In this setting, i is a year-to-year minimum guaranteed interest rate, i.e. (in book value terms) at least this rate has to be earned each year on the assets backing the account value. Therefore, such contracts contain an optionality that is often referred to as asymmetry: If the asset return is above i, a large part (e.g. 90%) of the return is credited to the client as a surplus and the shareholders receive only a small portion (e.g. 10%) of the return. If, on the other hand, the asset returns are below i, then 100% of the shortfall has to be compensated by the shareholder. Additionally, if the insurer earns a high return on assets and hence distributes a high surplus, this increases the insurer's future risk since the rate i has to be earned on this surplus amount in subsequent years. Such products constitute a significant financial risk to the insurance company, in particular in a framework of low interest rates and volatile capital markets.[4]

The mechanics of this year-to-year guarantee is illustrated in Figure 2-1 for two exemplary scenarios. We consider a traditional policy with term to maturity $T = 20$ years and a guaranteed benefit of $G = 20,000$ €. Following the current situation in Germany, we let $i = 1.75\%$ and assume a surplus participation rate of $p = 90\%$ on the asset returns.

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[4] This was also a key result of the QIS5 final report preparing for Solvency II, cf. EIOPA [2013a] and BaFin [2013].

The first scenario is not critical for the insurer. The asset return (which is arbitrarily assumed for illustrative purposes) starts at 3%, then over time drops to 2% and increases back to 3%. The chart shows this asset return, the "client's yield" (i.e. the interest credited to the client's account including surplus), the "required yield" (which is defined as the minimum rate that has to be credited to the client's account), and the insurer's yield (which is the portion of the surplus that goes to the shareholder). Obviously, in this simple example, the client's yield always amounts to 90% of the asset return and the insurer's yield always amounts to 10% of the asset return. By definition, for this product design, the required yield is constant and always coincides with $i = 1.75\%$.

In the second scenario, we let the asset return drop all the way down to 1%. Whenever 90% of the asset return would be less than the required yield, the insurer has to credit the required yield to the account value. This happens at the shareholder's expense, i.e. the insurer's yield is reduced and even becomes negative. This means that a shortfall occurs and the insurer has to provide additional funds.
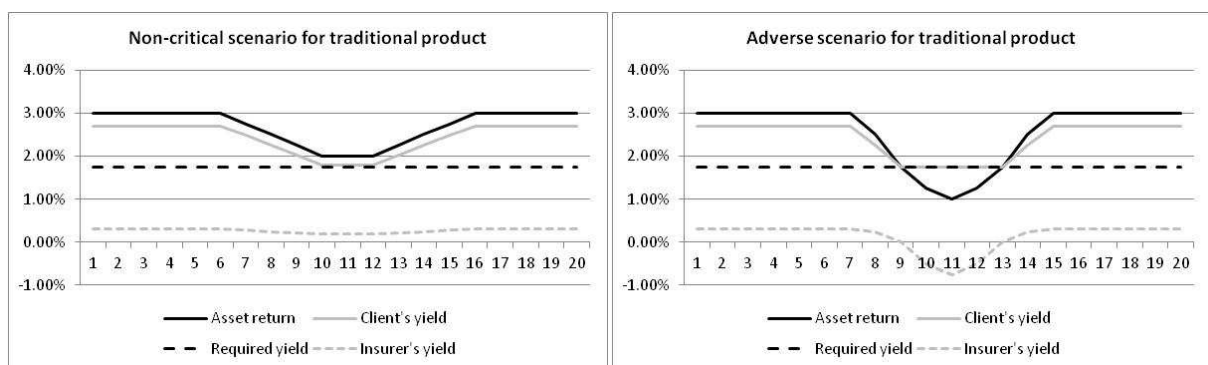

Figure 2-1: Two exemplary scenarios for the traditional product: asset returns and yield distribution

It is worthwhile noting that in this traditional product design, the interest rate i plays three different roles:

- pricing interest rate i_p used for determining the ratio between the premium and the guaranteed maturity benefit,

- reserving interest rate i_r, i.e. technical interest rate used for the calculation of the prospective actuarial reserves,

- year-to-year minimum guaranteed interest rate i_g, i.e. a minimum return on the account value.

2.2 Alternative products

We will now introduce two alternative product designs which are based on the idea to allow different values for the pricing rate, the reserving rate and the year-to-year minimum guaranteed interest rate on the account value. This leads to the following formulae for the relation between the annual premium and the guaranteed benefit and the actuarial reserves:

$$\sum_{t=0}^{T-1}(P - c_t)^t \cdot \left(1 + i_p\right)^{T-t} = G$$

$$AR_t = G \cdot \left(\frac{1}{1+i_r}\right)^{T-t} - \sum_{k=t}^{T-1}(P - c_k) \cdot \left(\frac{1}{1+i_r}\right)^{k-t}$$

Note, that in the first years of the contract, negative values for AR_t are possible in case of $i_p < i_r$, which implies a "financial buffer" at the beginning of the contract.

The year-to-year minimum guaranteed interest rate i_g is not relevant for the formulae above, but it is simply a restriction for the development of the client's account, i.e.

$$AV_t \geq (AV_{t-1} + P - c_{t-1}) \cdot \left(1 + i_g\right),$$

where $AV_0 = max\{AR_0, 0\}$ is the initial account value of the contract.

The crucial difference between such new participating products and traditional participating products is that the guaranteed maturity benefit is not increased during the lifetime of the contract. Note, however, that the prospective reserve AR_t in this

setting is only a minimum reserve for the guaranteed maturity benefit: If at any time this minimum reserve exceeds the account value, then the account value has to be increased to the minimum reserve, i.e. it always holds that $AV_t \geq AR_t$. Under "normal" circumstances (i.e. whenever the account value exceeds the minimum reserve), the technical reserve (under local GAAP) coincides with the account value and exceeds the minimum reserve; note that the surrender value is equal to the account value and the technical reserve may not be below the surrender value.

The required yield on the account value in year t is equal to

$$z_t = max\left\{\frac{max\{AR_t,0\}}{(AV_{t-1}+P-c_{t-1})} - 1, i_g\right\}. \tag{1}$$

Within the maximum function in (1), the first condition assures that the account value is non-negative and never lower than the actuarial reserve. The required yield decreases if the bonus reserve (which is included in AV_{t-1}) increases; in this sense, the bonus reserve acts as a "financial buffer" which reduces future yield requirements. The second condition simply makes sure that the year-to-year minimum guaranteed interest rate is always credited. Depending on the choice of i_g, the account value may increase with a lower rate than the technical rate i_r, or even decrease if $i_g < 0$.

The surplus participation rules remain unchanged: the policyholder's share p (e.g. 90%) of the asset return is credited to the policyholders (but not less than z_t). Hence, as long as the policyholder's share is above the technical interest rate used in the traditional product, there is no difference between the traditional and the alternative product designs. Only in years with an adverse capital market development, the yield on the account value may be reduced if the required yield z_t allows for it.

Obviously, only combinations fulfilling $i_g \leq i_p \leq i_r$ result in suitable products. If the first inequality is violated, then the year-to-year minimum guaranteed interest rate results in a higher guaranteed maturity benefit than intended by the pricing rate. If the second inequality is violated then at outset of the contract, additional reserves (exceeding the first premium) are required.

In what follows, we will consider two concrete alternative product designs. Obviously, the choice of i_g fundamentally changes the mechanics of the guarantee embedded in the product (or the "type" of guarantee), whereas the choice of i_p changes the level of the guarantee. Since we want to focus on the effect of the different guarantee mechanisms, we use a pricing rate that coincides with the technical rate from the traditional product. Hence the guaranteed maturity benefit remains unchanged. Since the legally prescribed maximum value for the reserving rate also coincides with the technical rate from the traditional product, we get $i_p = i_r = 1.75\%$.

In our alternative product 1, we set $i_g = 0\%$ (0% year-to-year guarantee) and for alternative 2 we set $i_g = -100\%$ (no year-to-year guarantee). In order to illustrate the mechanics of the alternative products, Figure 2-2 and Figure 2-3 show the two scenarios from Figure 2-1 for both alternative products. In the first scenario (shown on the left) the required yield z_t on the account value gradually decreases to zero for both alternative product designs because buffers (in the sense described above) are created. For alternative 1, the required yield decreases to zero and stays there, while for the alternative 2 it even becomes negative after some years.

The adverse scenario on the right shows that the required yield rises again after years with low asset returns since the buffer is reduced. However, contrary to the traditional product, the asset return stays above the required level and no shortfall occurs.

From a policyholder's perspective, both alternative product designs provide the same maturity benefit as the traditional product design in the first scenario. This is due to the fact that the asset return (and therefore the client's yield) is always above 1.75%. In the second scenario, however, the maturity benefit is slightly lower for both alternative product designs since (part of) the buffer built up in years 1 to 8 can be used to avoid a shortfall. In this scenario, there is no difference between the two alternative products for the policyholder, because the asset return (and therefore the client's yield) is always positive.
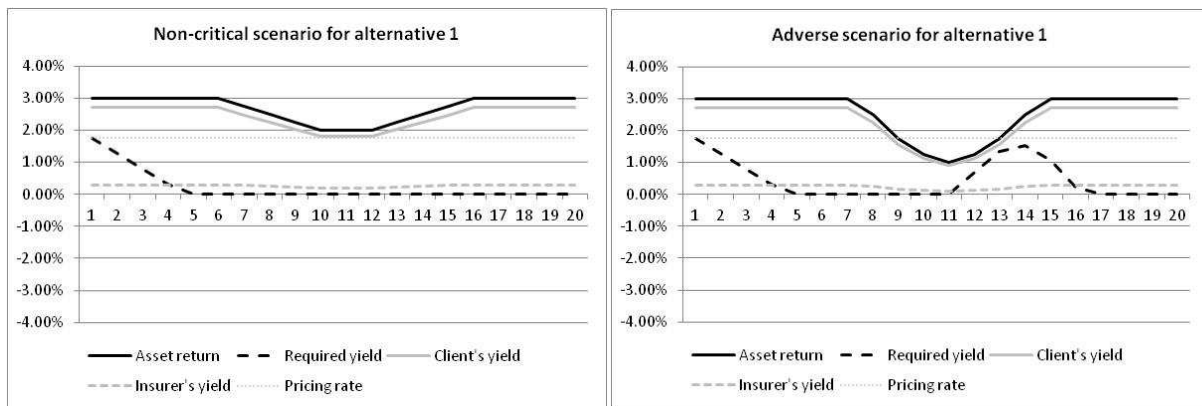

Figure 2-2: Two exemplary scenarios for alternative 1: asset returns and yield distribution.
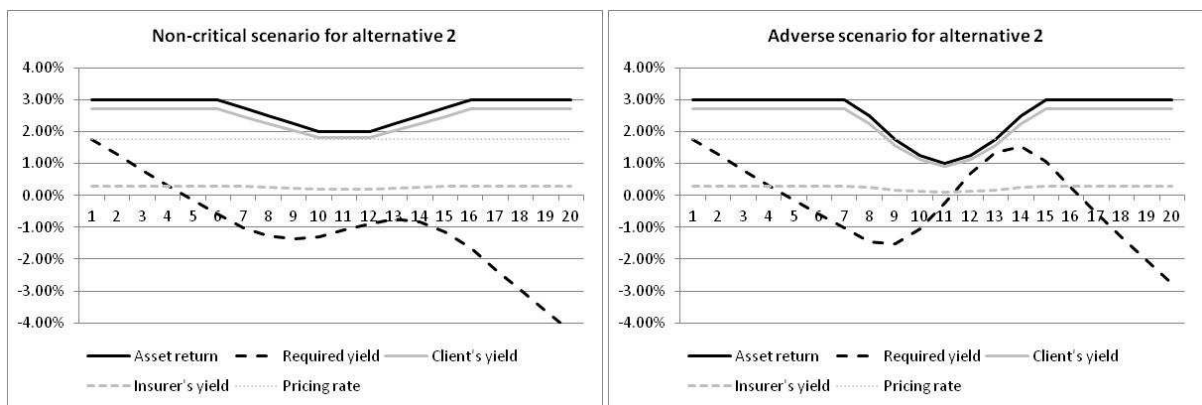

Figure 2-3: Two exemplary scenarios for alternative 2: asset returns and yield distribution.

Even if scenarios where the policyholder's benefit for the alternative product designs is smaller than for the traditional design appear unlikely, the difference in the product design has a significant impact on the insurer's solvency requirements since the financial risks particularly in adverse scenarios are a key driver for the solvency

capital requirement. This will be considered in a stochastic framework in the following sections.

3 Stochastic modeling and analyzed key figures

Since surplus participation is typically based on local GAAP book values (in particular in Continental Europe), we use a stochastic balance sheet and cash flow projection model for the analysis of the alternative product designs presented in the previous section. The model includes management rules concerning asset allocation, reinvestment strategy, handling of unrealized gains and losses and surplus distribution. It allows us to derive key figures for the comparison of the considered products with respect to profitability and capital requirements. Since the focus of the paper is on the valuation of future profits and capital requirements we will introduce the model under a risk-neutral measure. Similar models have been used (also in a real-world framework) in Kling et al. [2007a], Kling et al. [2007b] and Graf et al. [2011].

3.1 The financial market model

We apply a financial market model where the insurer's assets are invested in a portfolio consisting of coupon bonds and stocks. We treat both assets as risky assets in a risk-neutral, frictionless and continuous financial market. Additionally, cash flows during the year are invested in a riskless bank account (until assets are reallocated). We let the short rate process r_t follow a Vasicek model, and the stock price S_t follows a geometric Brownian motion described by

$$dr_t = \kappa(\theta - r_t)dt + \sigma_r dW_t^{(1)} \text{ and}$$

$$\frac{dS_t}{S_t} = r_t dt + \rho \sigma_S dW_t^{(1)} + \sqrt{1 - \rho^2} \sigma_S dW_t^{(2)},$$

where $W_t^{(1)}$ and $W_t^{(2)}$ each denote a Wiener process on some probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ with a risk-neutral measure $\mathbb{Q}$ and a filtration $\mathcal{F}$, to which $W_t^{(1)}$ and $W_t^{(2)}$ are adapted. The parameters κ, θ, σ_r, σ_S and ρ are deterministic and constant. For the purpose of performing Monte Carlo simulations the stochastic differential equations can be solved to

$$S_t = S_{t-1} \cdot exp\left(\int_{t-1}^{t} r_u du - \frac{\sigma_S^2}{2} + \int_{t-1}^{t} \rho\sigma_S dW_u^{(1)} + \int_{t-1}^{t} \sqrt{1-\rho^2}\sigma_S dW_u^{(2)} \right) \text{ and}$$

$$r_t = e^{-\kappa} \cdot r_{t-1} + \theta(1 - e^{-\kappa}) + \int_{t-1}^{t} \sigma_r \cdot e^{-\kappa(t-u)} dW_u^{(1)},$$

where $S_0 = 1$ and the initial short rate r_0 is a deterministic parameter. Then, the bank account is given by $B_t = \exp\left(\int_0^t r_u du \right)$. It can be shown that the four (stochastic) integrals in the formulae above follow a joint normal distribution[5]. Monte Carlo paths are calculated using random realizations of this multidimensional distribution. The yield curve is then given by[6]

$$r_t(s) = exp\left[\frac{1}{s}\left(\frac{1-e^{-\kappa s}}{\kappa} r_t + \left(s - \frac{1-e^{-\kappa s}}{\kappa} \right) \cdot \left(\theta - \frac{\sigma_r^2}{2\kappa^2} \right) + \left(\frac{1-e^{-\kappa s}}{\kappa} \right)^2 \frac{\sigma_r^2}{4\kappa} \right) \right] - 1$$

for any time t and term $s > t$. Based on the yield curves, we calculate par yields that determine the coupon rates of the considered coupon bonds.

3.2   The asset-liability model

The insurer's simplified balance sheet at time t is given by Table 3-1.

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[5] Cf. Zaglauer and Bauer [2008]. A comprehensive explanation of this property is included in Bergmann [2011].

[6] See Seyboth [2011] as well as Branger and Schlag [2004].

Assets	Liabilities
BV_t^S	X_t
BV_t^B	AV_t

Table 3-1: Balance sheet at time t.

On the liability side, we consider the account value of the insurance contracts AV_t as defined in section 2. Since the analysis is performed for a specific portfolio of insurance contracts on a stand-alone basis, there is no explicit allowance for shareholder's equity or other reserves on the liability side. Further, we do not consider new business. X_t denotes the profit or loss of the shareholders in year t, with corresponding cash flow at the beginning of the next year.

In our projection of the assets and insurance contracts, incoming cash flows (premium payments at the beginning of the year, coupon payments and repayment of nominal at the end of the year) and outgoing cash flows (expenses at the beginning of the year and benefit payments at the end of the year) occur. In each simulation path, cash flows occurring at the beginning of the year are invested in a bank account. At the end of the year, the market values of the stocks and coupon bonds are derived and the asset allocation is readjusted according to a rebalancing strategy with a constant stock ratio of q based on market values. Conversely, $(1 - q)$ is invested in bonds and any money on the bank account is withdrawn and invested in the portfolio consisting of stocks and bonds.

If in the process of rebalancing bonds need to be bought, the amount is fully invested into newly issued coupon bonds yielding at par with a given term M. However, towards the end of the projection, when the insurance contracts' remaining term is less than M years, we invest in bonds with a term that coincides with the longest remaining term of the insurance contracts. If bonds need to be sold, they are sold

proportionally to the market values of the different remaining terms in the existing portfolio.

With respect to accounting, we use book-value accounting rules following German GAAP: Coupon bonds are considered as held to maturity and their book value BV_t^B is always given by their nominal amounts (irrespective if the market value is higher or lower). In contrast, for the book value of the stocks BV_t^S, the insurer has some discretion. This may result in unrealized gains or losses (UGL).

Of course, interest rate movements as well as the rebalancing will cause fluctuations with respect to the UGL of bonds. Also, the rebalancing may lead to the realization of UGL of stocks. In addition, we assume an additional management rule with respect to UGL of stocks: We assume that the insurer wants to create rather stable book value returns (and hence surplus distributions) in order to signal stability to the market. We therefore assume that a ratio d_{pos} of the UGL of stocks is realized annually if unrealized gains exist and a ratio d_{neg} of the UGL is realized annually if unrealized losses exist. In particular, $d_{neg} = 100\%$ has to be chosen in a legal framework where unrealized losses on stocks are not possible.

Based on this model, the total asset return on a book value basis can be calculated in each simulation path each year as the sum of coupon payments from bonds, interest payments on the bank account and the realization of UGL. The split between policyholders and shareholders is driven by the parameter p explained in section 2 which describes the minimum portion of the asset return that has to be credited to the policyholders. If the cumulative required yield on the account values of all policyholders is larger than this share, there is no surplus for the policyholders, and exactly the required yield z_t is credited to every account. Otherwise, surplus is credited to the policies which amounts to the difference between the policyholders'

share of the asset return and the cumulative required yield. As this is the typical practice e.g. in Germany, we assume that this surplus is distributed among the policyholders such that all policyholders receive the same total yield (defined by the required yield plus surplus rate), if possible. To achieve that, we apply an algorithm that sorts the accounts by required yield, i.e. $\left(z_t^{(1)}, \dots, z_t^{(k)} \right), k \in \mathbb{N}$ in ascending order. First, all contracts receive their respective required yield. Then the available surplus is distributed: Starting with the contract(s) with the lowest required yield $z_t^{(1)}$, the algorithm distributes the available surplus to all these contracts until the gap to the next required yield $z_t^{(2)}$ is filled. Then all the contracts with a required yield lower or equal to $z_t^{(2)}$ receive an equal amount of (relative) surplus until the gap to $z_t^{(3)}$ is filled, etc. This is continued until the entire surplus is distributed. The result is that all contracts receive the same total yield if this unique total yield exceeds the required yield of all contracts. Otherwise, all contracts with a required yield above some threshold receive exactly their required yield and all contracts with a required yield below that threshold receive an identical total yield that exceeds the respective contracts' required yield (i.e. they receive some surplus) but is lower than the threshold value.

From this, the insurer's profit X_t results as the difference between the total asset return and the amount credited to all policyholder accounts. If the profit is negative, a shortfall has occurred which we assume to be compensated by a corresponding capital inflow (e.g. from the insurer's shareholders) at the beginning of the next year.[7]

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[7] We do not consider the shareholders' default put option resulting from their limited liability, which is in line with both Solvency II valuation standards and the market consistent embedded value framework (MCEV), cf. e.g. Bauer et al. [2012] or DAV [2011], section 5.3.4.

Balance sheet and cash flows are projected over τ years until the run-off of the in-force portfolio at time zero.

3.3 Key drivers for capital efficiency

The term capital efficiency is frequently used in an intuitive sense, in particular amongst practitioners, to describe the feasibility, profitability, capital requirement, and riskiness of products under risk based solvency frameworks such as Solvency II (to be introduced in the European Union) or Swiss Solvency Test (SST, already in place in Switzerland). However, to the best of our knowledge, no formal definition of this term exists. Also, there is probably no unique correct definition of this term. Nevertheless, it seems obvious, that capital requirement alone is not a suitable figure for managing a product portfolio from an insurer's perspective. Rather, capital requirement and the resulting cost of capital should be considered in relation to profitability.

Therefore, a suitable measure of "Capital Efficiency" could be some ratio of profitability and capital requirement, e.g. based on the distribution of the random variable

$$\frac{\sum_{t=1}^{\tau} \frac{X_t}{B_t}}{\sum_{t=1}^{\tau} \frac{RC_{t-1} \cdot CoC_t}{B_t}}$$

The numerator represents the present value of the insurer's future profits, whereas the denominator is equal to the present value of future cost of capital. RC_t denotes the required capital at time t under some risk based solvency framework, i.e. the amount of shareholder's equity needed to support the business in force. The cost of capital is derived by applying the cost of capital rate CoC_t for year t on the required

capital at the beginning of the year.[8] In practical applications, however, the distribution of this ratio might not be easy to calculate. Therefore, moments of this distribution, a separate analysis of (moments of) the numerator and the denominator or even just an analysis of key drivers on that ratio might have to do as a proxy.

In this spirit, we will use a Monte Carlo framework to calculate the following key figures using the model described above:

A typical market consistent measure for the insurer's profitability is the expected present value of future profits (PVFP)[9], which corresponds to the expected value of the numerator. The PVFP is estimated based on Monte Carlo simulations:

$$PVFP = \frac{1}{N}\sum_{n=1}^{N}\sum_{t=1}^{\tau}\frac{X_t^{(n)}}{B_t^{(n)}},$$

where N is the number of scenarios, $X_t^{(n)}$ denotes the insurer's profit/loss in year t in scenario n, and $B_t^{(n)}$ is the value of the bank account after t years in scenario n.

In addition, the degree of asymmetry of the shareholder's cash flows can be characterized by the distribution of the PVFP over all scenarios[10] and by the time value of options and guarantees (TVOG). Under the MCEV framework[11], the latter is defined by

$$TVOG = PVFP - PVFP_{CE}$$

where $PVFP_{CE} = \sum_{t=1}^{\tau}\frac{X_t^{(CE)}}{B_t^{(CE)}}$ is the present value of future profits in the so-called "certainty equivalent (CE)" scenario. This deterministic scenario reflects the expected development of the capital market under the risk-neutral measure. It can be

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[8] This approach is similar to the calculation of the cost of residual nonhedgeable risk (CRNHR) as introduced in the MCEV Principles in CFO-Forum [2009], although RC_t reflects the total capital requirement including hedgeable risks.

[9] The concept of PVFP is introduced as part of the MCEV Principles in CFO-Forum [2009].

[10] Note that this is a distribution under the risk-neutral measure and has to be interpreted carefully. However, it can be useful for explaining differences between products regarding PVFP and TVOG.

[11] Cf. CFO-Forum [2009].

derived from the initial yield curve $r_0(s)$[12] based on the assumption that all assets earn the forward rate implied by the initial yield curve. The TVOG is also used as an indicator for capital requirement under risk-based solvency frameworks.

Comparing the expected PVFP between two different interest rate levels – one that we call basic level and a significant lower one that we call stress level – provides another important key figure for interest rate risk and capital requirements. As a key driver for the denominator, we therefore consider

$$\Delta PVFP = PVFP(basis) - PVFP(stress)$$

which can be interpreted as a proxy for the solvency capital requirement (SCR) for interest rate risk in the standard formula[13] of the Solvency II framework.

4   Results

4.1   Assumptions

The stochastic valuation model described in the previous section is applied to a portfolio of participating contracts with product parameters as given in Table 4-1. For simplicity, we assume that all policyholders are 40 years old at inception of the contract and mortality is based on the German standard mortality table (DAV 2008 T). Furthermore, we assume annual charges c_t that are typical in the German market consisting of annual administration charges $\beta \cdot P$ throughout the contract's lifetime, and acquisition charges $\alpha \cdot T \cdot P$ which are equally distributed over the first 5 years of the contract. Hence, the charge function is given by $c_t = \beta \cdot P + \alpha \cdot \frac{T \cdot P}{5} \mathbb{1}_{t \in \{0, \ldots, 4\}}$. Other product parameters are given in Table 4-1.

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[12] Cf. Oechslin et al. [2007].

[13] A description of the current version of the standard formula can be found in EIOPA [2013b].

i_p / i_r / i_g			
Traditional Product: 1.75% / 1.75% / 1.75%	Alternative 1: 1.75% / 1.75% / 0%		Alternative 2: 1.75% / 1.75% / -100%
G	T	β	α
20,000 €	20 years	3%	4%

Table 4-1: Product parameters

Stochastic projections are performed for a portfolio that was built up in the past (i.e. before $t = 0$) over 20 years based on 1,000 new policies per year. Hence, we have a homogeneous portfolio at the beginning of the projections with time to maturity between 1 year and 19 years (i.e. the projection horizon is $\tau = 19$ years). For each contract, the account value at $t = 0$ is derived from a projection in a (historic) certainty equivalent scenario (using parameters described below). In line with the valuation approach under Solvency II and MCEV, we do not consider new business.

The book value of the asset portfolio at $t = 0$ coincides with the book value of liabilities. We assume a stock ratio of $q = 5\%$ with unrealized gains on stocks at $t = 0$ equal to 10% of the book value of stocks. The coupon bond portfolio consists of bonds with a uniform coupon of 3.0% where the time to maturity is equally split between 1 year and $M = 10$ years.

Capital market parameters for the basic and stress projections are shown in Table 4-2. The parameters κ, σ_r, σ_S and ρ are directly adopted from Graf et al. [2011]. The parameters θ and r_0 are chosen such that they are more in line with the current low interest rate level. The capital market stress corresponds to an immediate drop of interest rates by 100 basis points.

	r_0	θ	κ	σ_r	σ_S	ρ
Basis	2.5%	3.0%	30.0%	2.0%	20.0%	15.0%
Stress	1.5%	2.0%				

Table 4-2: Capital market parameters

The parameters for the management rules are given in Table 4-3 and are consistent

with the current practice of German insurance companies.

q	M	d_{pos}	d_{neg}	p
5%	10 years	20%	100%	90%

Table 4-3: Parameters for management rules

For all projections, the number of scenarios is $N = 5,000$. Further analyses showed

that this allows for a precise estimation of the relevant figures.[14]

4.2 Comparison of Product Designs

For each of the product designs introduced in section 2 (traditional, alternative 1,

alternative 2), the key figures with respect to profitability and capital requirement

described in section 3 are derived for the portfolio described in the previous

subsection. All results are displayed as a percentage of the present value of future

premium income from the portfolio.

In Table 4-4, the PVFP and the TVOG for the base case are compared. For

alternative 1, the PVFP increases by 0.61 percentage points (from 3.63% to 4.24%)

compared to the traditional product design (which corresponds to a 17% increase of

profitability). This means that a product with a "maturity only" guarantee and a

guarantee that the account value will not decrease (alternative 1) is more profitable

than the product with a traditional year-to-year (cliquet-style) guarantee. This

difference is mainly caused by the different degree of asymmetry of the shareholder's

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[14] In order to reduce variance in the sample an antithetic path selection of the random numbers is applied, cf. e.g. Glasserman [2004].

cash flows which is characterized by the time value of options and guarantees. Since the $PVFP_{CE}$ (from the "certainty equivalent" scenario) is equal to 4.26% for all products in the base case, the difference of TVOG between the traditional product and alternative 1 is also 0.61 percentage points. This corresponds to a TVOG reduction of approx. 90% for alternative 1, which shows that the risk resulting from the interest rate guarantee is much lower for the modified product.

Compared to this, the differences between alternative 1 and alternative 2 are almost negligible in the considered setup. The additional increase of the expected PVFP is only 0.01 percentage points which is due to a slightly lower TVOG compared to alternative 1. This shows that the fact that the account value may decrease in some years in product alternative 2 does not provide a material additional risk reduction.

	Traditional product	Alternative 1	Alternative 2
$PVFP$	3.63%	4.24%	4.25%
$TVOG$	0.63%	0.02%	0.01%

Table 4-4: PVFP and TVOG for base case (as percentage of the present value of premium income).

Additional insights can be obtained by analyzing the distribution of the PVFP (see Figure 4-1)[15]: For the traditional product design, the distribution is highly asymmetric with a strong left tail and a significant risk of negative shareholder cash flows (on a present value basis). In contrast, both alternative product designs exhibit an almost symmetric distribution of shareholder cash flows which explains the low TVOG. Hence, the new products result in a significantly more stable profit perspective for the shareholders, while for the traditional product the shareholder is exposed to significantly higher shortfall risk.

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[15] Note that this is a distribution under the risk-neutral measure and has to be interpreted carefully. However, it is useful for explaining differences between products regarding PVFP and TVOG.
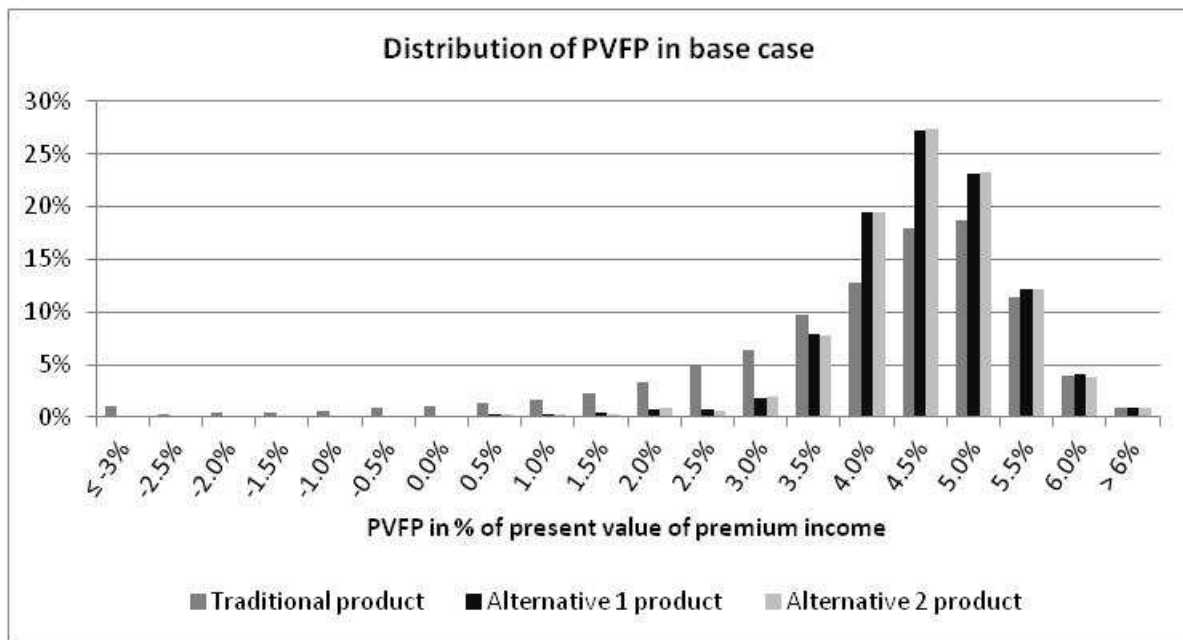
Figure 4-1: Histogram of PVFP in base case.

Ultimately, the results described above can be traced back to differences in the required yield. A percentile plot of the required yield is shown in Figure 4-2. While for the traditional product, by definition, the required yield always amounts to 1.75%, it is equal to 0% in most scenarios for the alternative 1 product. Only in the most adverse scenarios, the required yield rises towards 1.75%. The required yield in the first projection year reflects the financial buffer available for the considered portfolio of existing contracts at $t = 0$. [16]
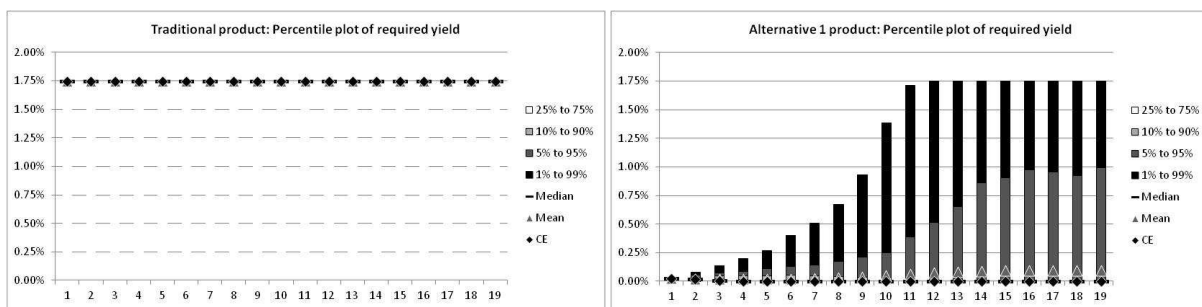

Figure 4-2: Percentile plots of required yield for traditional and alternative 1 product.

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[16] This is different from the illustrations in section 2 which consider individual contracts from inception to maturity.

Apart from the higher profitability, the alternative product designs also imply a lower capital requirement for interest rate risk. This is illustrated in Table 4-5 which displays the expected PVFP under the interest rate stress and the difference to the basic level. Compared to the basic level, the PVFP for the traditional product decreases by 75% which corresponds to an SCR for interest rate risk of 2.73% of the present value of future premiums. In contrast, the PVFP decreases by only around 40% for the alternative product designs and thus the capital requirement is only 1.66% and 1.65%, respectively.

	Traditional product	Alternative 1	Alternative 2
$PVFP(basis)$	3.63%	4.24%	4.25%
$PVFP(stress)$	0.90%	2.58%	2.60%
$\Delta PVFP$	2.73%	1.66%	1.65%

Table 4-5: PVFP for stress setup and PVFP difference between basic and stress level.

We have seen that a change in the type of guarantee results in a significant increase of the PVFP. This raises the question by what amount the level of guarantee in the traditional product setting needs to be reduced in order to achieve the same PVFP. Our analyses show that a traditional product with guaranteed interest rate $i = 0.9\%$ instead of 1.75% has the same expected PVFP (i.e. 4.25%) as the alternative product designs with $i_p = 1.75\%$. Hence, although changing only the type of guarantee and leaving the level of guarantee intact might be perceived as a rather small product modification by the policyholder, it has the same effect on the insurer's profitability as reducing the level of guarantee by a significant amount.

Furthermore, our results indicate that even in an adverse capital market situation the alternative product designs may still provide an acceptable level of profitability: The profitability of the modified products if interest rates are reduced by 50 bps roughly coincides with the profitability of the traditional product in the base case. This again

shows that the alternative product designs are more capital efficient than the traditional product design.

4.3 Sensitivity Analyses

In order to assess the robustness of the results presented in the previous section, we investigate three different sensitivities:

1. Lower interest rate level (-100 bps): The long term average θ and initial rate r_0 in Table 4-2 are replaced by $\theta = 2.0\%$, $r_0 = 1.5\%$ for the basic level, and $\theta = 1.0\%$, $r_0 = 0.5\%$ for the stress level.

2. More risky asset allocation: The stock ratio is set to $q = 10\%$ instead of 5%.

3. Higher initial buffer: The initial bonus reserve $BR_t = AV_t - AR_t$ is doubled for all contracts[17].

The results are given in Table 4-6.

If the assumed basic interest rate level is lowered by 100 bps, the PVFP decreases and the TVOG increases significantly for all products. In particular, the alternative product designs now also exhibit a significant TVOG. This shows that in an adverse capital market situation, also the guarantees embedded in the alternative product designs can lead to a significant shortfall risk for the shareholder and an asymmetric distribution of profits as illustrated in Figure 4-3. Nevertheless, the alternative product designs are still much more profitable and less volatile than the traditional product design and the changes in PVFP / TVOG are much less pronounced than for the traditional product: while the PVFP is reduced from 3.63% to 0.90%, i.e. by 2.73 percentage points for the traditional product, it is only reduced by 1.66 percentage points (from 4.24% to 2.58%) for alternative 1.

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[17] The initial book values of the assets are increased proportionally to cover this additional reserve.

Base case	Traditional	Alternative 1	Alternative 2
$PVFP$	3.63%	4.24%	4.25%
$TVOG$	0.63%	0.02%	0.01%
$PVFP(stress)$	0.90%	2.58%	2.60%
$\Delta PVFP$	2.73%	1.66%	1.65%
Interest rate sensitivity			
$PVFP$	0.90%	2.58%	2.60%
$TVOG$	2.13%	0.78%	0.76%
$PVFP(stress)$	-4.66%	-1.81%	-1.76%
$\Delta PVFP$	5.56%	4.39%	4.36%
Stock ratio sensitivity			
$PVFP$	1.80%	3.83%	3.99%
$TVOG$	2.45%	0.43%	0.26%
$PVFP(stress)$	-1.43%	1.65%	1.92%
$\Delta PVFP$	3.23%	2.18%	2.07%
Initial buffer sensitivity			
$PVFP$	3.74%	4.39%	4.39%
$TVOG$	0.64%	<0.01%	<0.01%
$PVFP(stress)$	1.02%	2.87%	2.91%
$\Delta PVFP$	2.72%	1.52%	1.48%

Table 4-6: PVFP, TVOG, PVFP under interest rate stress and $\Delta PVFP$ for base case and all sensitivities.

As expected, in the interest rate level sensitivity, an additional interest rate stress results in higher changes of PVFP. For all product designs, the PVFP after stress is negative and the capital requirement increases significantly. However, as in the base case (cf. Table 4-5), the SCR for the traditional product is more than one percentage point larger than for the new products.
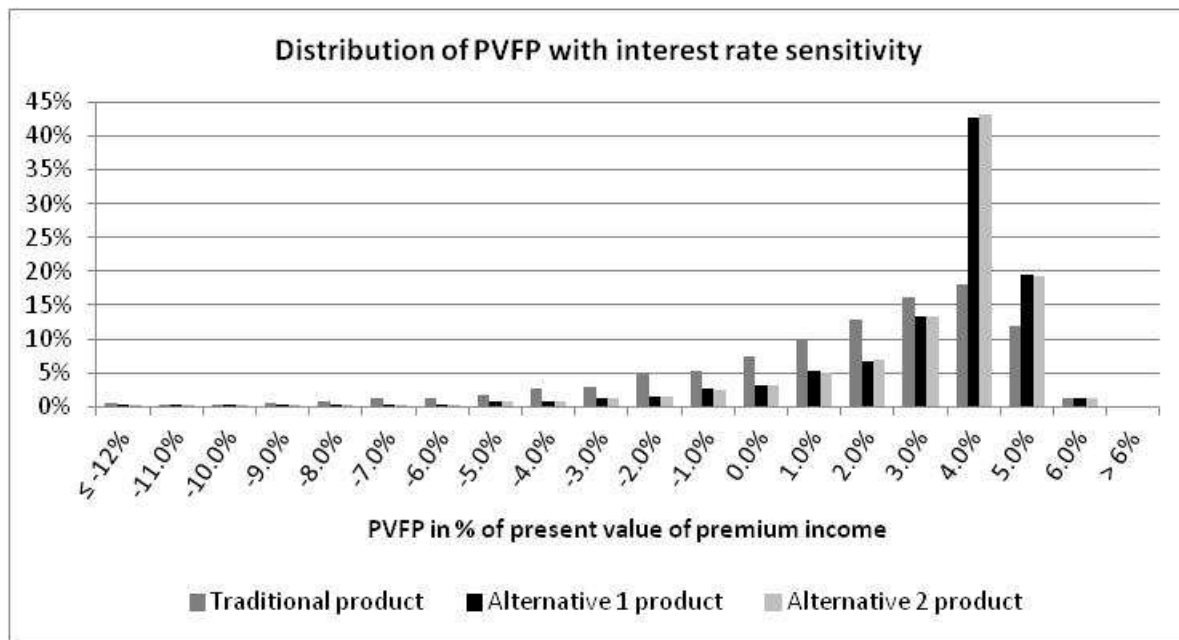
Figure 4-3: Histogram of PVFP for projection with interest rate sensitivity (-100bps).

The stock ratio sensitivity also leads to a decrease of PVFP and an increase of TVOG for all products. Again, the effect on the PVFP of the traditional product is much stronger: The profit is about cut in half (from 3.63% to 1.80%), while for the alternative 1 product the reduction is much smaller (from 4.24% to 3.83%), and even smaller for alternative 2 (from 4.25% to 3.99%). It is noteworthy that with a larger stock ratio of $q = 10\%$ the difference between the two alternative products becomes more pronounced which is reflected by the difference in terms of TVOG. Alternative 2 has a lower shortfall risk than alternative 1 since the account value may decrease in some years as long as the guaranteed benefit at maturity is preserved. Hence, we can conclude that the guarantee that the account value may not decrease becomes more risky if asset returns exhibit a higher volatility.

The results for the stressed PVFPs under the stock ratio sensitivity are in line with these results: First, the traditional product requires even more solvency capital: The SCR is half a percentage point larger than in the base case (3.23% compared to 2.73%), and it is also more than one percentage point larger than for the alternative

products with 10% stocks (2.18%/2.07%). Second, the interest rate stress shows a more substantial difference between the two different alternative products. While the difference of the SCR between alternative 1 and 2 was 0.01% in the base case, it is now 0.11%.

For the final sensitivity, an increase of the initial buffer, we observe a slight increase of the PVFP for all products. However, there are remarkable differences for the effect on TVOG between the traditional and the alternative products: While for the traditional product the TVOG remains approximately the same, for the alternative products it is essentially reduced to zero. This strongly supports our product motivation in section 2: For the alternative products, larger surpluses from previous years create a "buffer" reducing risk in future years.[18] Furthermore, the stressed PVFPs imply that the decrease of capital requirements is significantly larger with the alternative products: 0.14% reduction (from 1.66% to 1.52%) for alternative 1 and 0.17% reduction (from 1.65% to 1.48%) for alternative 2, compared to just 0.01% reduction for the traditional product.

Finally, we analyze the percentile plots for the required yields in the base case and in the different sensitivities. The results for alternative 1 are shown in Figure 4-4, and for alternative 2 in Figure 4-5. For all sensitivities, the required yield for alternative 1 product starts at a level of 0% since at the beginning of the projection the portfolio experienced a history with returns that were sufficient to create a "buffer". With the stochastic projection the required yield increases only for the most adverse scenarios such that only for a small share of scenarios the required yield rises to above 1.0% at any time. In the interest rate sensitivity, this ratio is a lot higher, and also the average

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[18] From this, we can conclude that if such alternative products had been sold in the past, the risk situation of the insurance industry would be significantly better today in spite of the rather high nominal maturity value guarantees for products sold in the past.

required yield significantly exceeds 0%. Similarly, for a larger stock ratio, the probability for a higher required yield increases when compared to the base case, however the effect is much less pronounced than in the interest rate sensitivity, and it is slightly reduced towards the end of the projection horizon. If the initial buffer is doubled, the increase of the required yield in adverse scenarios is essentially delayed by few years (until buffers are used up).
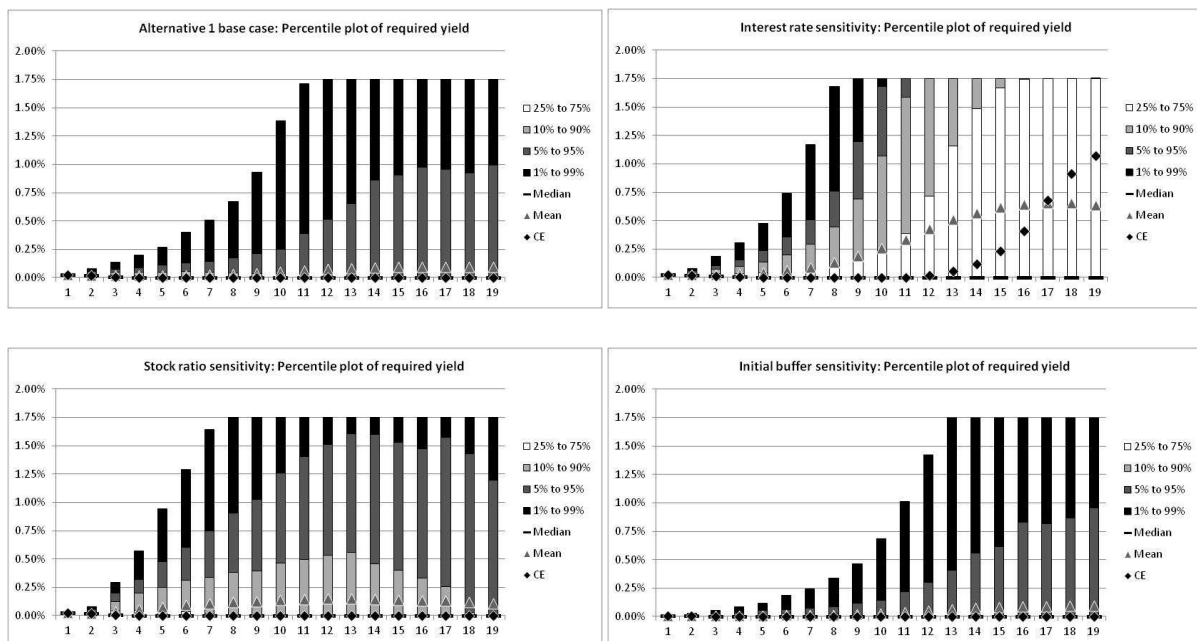

Figure 4-4: Percentile plots of the required yield in the base case and the sensitivities for alternative 1 product.

For product alternative 2 the required yield starts below zero at about –4.0% in the base case which means that the insurer could – in case of a negative asset return – even credit a negative surplus for the next year (and may avoid a shortfall). Over the analyzed time horizon, the range of possible values of the required yield becomes gradually larger: the 99th percentile increases to 1.75% (the maximum possible due to product design) and the 1st percentile decreases to below –20%. The median and the mean stay close together indicating a rather symmetric distribution of the required yield and decrease gradually to about –7.5%. For the interest rate sensitivity, the required yield starts at the same level, but then strongly increases in most scenarios.

In the last years of the projection lifetime the required yield is at 1.75% for more than 25% of the scenarios. Hence, the interest rate stress is particularly dangerous in the long run after financial buffers have been absorbed by systematically low returns. For the higher stock ratio, the 1 to 99% percentile range is higher than in the base case: between −25% and 1.75% in the last year, i.e. the higher volatility of stocks is reflected in the distribution of the required yield. With the initial bonus reserve doubled, the required yield at the beginning is as low as −9.3%. Then it increases slightly as the additional buffer is consumed gradually in cases of an adverse capital market. At the end of the time horizon, the percentiles are almost on the same level as in the base case. This results from the very low bonus reserve at $t=0$ of those policies which have been in the portfolio for only a few years when the projection starts. Of course, these are the only policies remaining in the last years of the projection. For those policies doubling the initial bonus reserve has no material effect, and if adverse scenarios materialize close to the end of the time horizon, the policies with the larger additional reserves have already left.
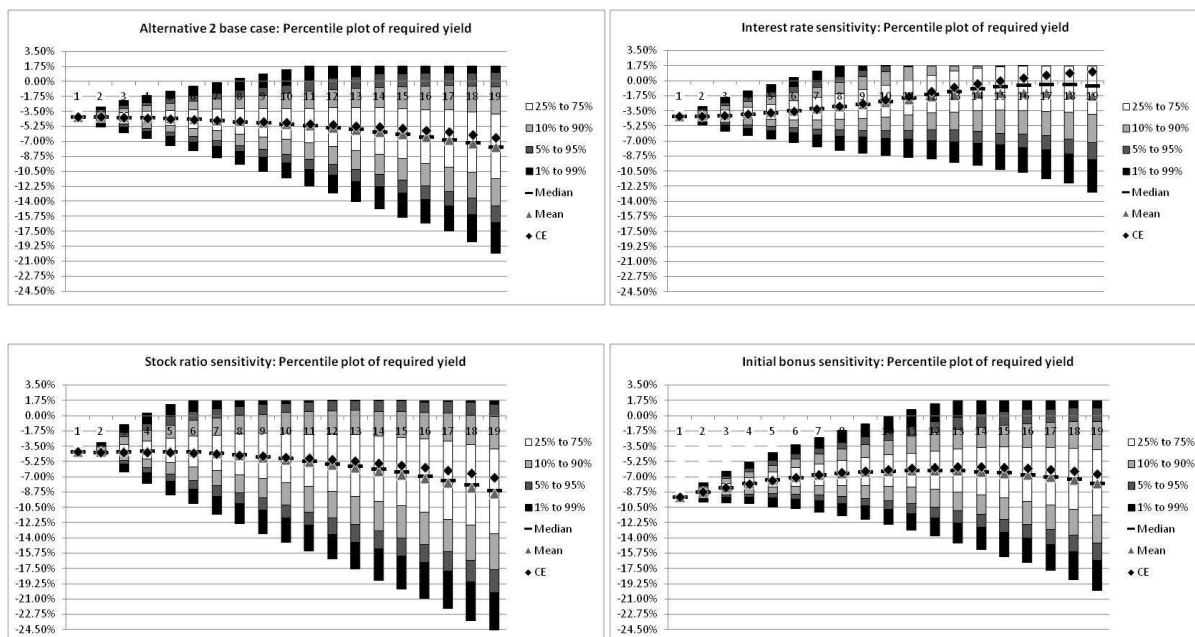

Figure 4-5: Percentile plots of the required yield in the base case and the sensitivities for alternative 2 product.

4.4   Reduction in the level of guarantee

So far we have only considered contracts with a different type of guarantee. We will now consider contracts with a lower level of guarantee, i.e. products where the pricing interest rate is below the reserving rate for the alternative products, i.e. $i_p < i_r$. If we apply $i_p = 1.25\%$ instead of 1.75%, the annual premium required to achieve the same guaranteed maturity benefit rises by approx. 5.4% which is essentially the additional initial buffer of this contract design. We also calculate the results for the traditional product with a lower interest rate $i = 1.25\%$, (which is applied for pricing, reserving and as minimum year-to-year guarantee). Homogeneous sample portfolios at $t = 0$ are derived using the assumptions described in section 4.1.

The results are presented in Table 4-7. We can see that the PVFP is further increased and the TVOG is very close to 0 for the modified alternative products which implies an almost symmetric distribution of the PVFP. The TVOG can even become slightly negative due to the additional buffer in all scenarios. Although the risk situation for the traditional product is also improved significantly due to the lower guarantee, the alternative products can still preserve their advantages. Particularly, a more remarkable effect can be seen for the SCR which amounts to 1.03% and 0.99% for the alternative products 1 and 2, respectively, compared to 1.69% for the traditional product. Hence, the buffer leads to a significant additional SCR reduction for the alternative products meaning that these are less affected by interest rate risk.

	Tradi-tional	Alter-native 1	Alter-native 2	Traditional $i = 1.25\%$	Altern. 1 w/ $i_p = 1.25\%$	Altern. 2 w/ $i_p = 1.25\%$
$PVFP$	3.63%	4.24%	4.25%	4.12%	4.31%	4.31%
$TVOG$	0.63%	0.02%	0.01%	0.14%	-0.05%	-0.05%
$PVFP(stress)$	0.90%	2.58%	2.60%	2.43%	3.28%	3.32%
$\Delta PVFP$	2.73%	1.66%	1.65%	1.69%	1.03%	0.99%

Table 4-7: PVFP, TVOG, PVFP under interest rate stress and $\Delta PVFP$ for the alternative products with lower pricing rate.

5 Conclusion and Outlook

In this paper, we have analyzed different product designs of traditional participating life insurance contracts with a guaranteed benefit at maturity. A particular focus of our analysis was on the impact of product design on capital requirement under risk based solvency frameworks such as Solvency II and on the insurer's profitability.

We have performed a consistent valuation of the different products, and have analyzed the key drivers of Capital Efficiency, particularly the value of the embedded options and guarantees and the insurer's profitability.

As expected, our results confirm that products with a typical year-to-year guarantee are rather risky for the insurer and hence result in a rather high capital requirement. Our proposed product modifications significantly enhance Capital Efficiency, reduce the insurer's risk, and increase profitability. Although the design of the modified products makes sure that the policyholder receives less than with the traditional product only in extreme scenarios, these products still provide a massive relief for the insurer since extreme scenarios drive the capital requirements under Solvency II and SST.

It is particularly noteworthy that starting from a standard product where the guaranteed maturity benefit is based on an interest rate of 1.75%, changing the type of the guarantee to our modified products (but leaving the level of guarantee intact)

has the same impact on profitability as reducing the level of guarantee to an interest rate of 0.9% and not modifying the type of guarantee.

We would like to stress that the product design approach presented in this paper is not model arbitrage (hiding risks in "places the model cannot see"), but a real reduction of economic risks. In our opinion, such concepts can be highly relevant in practice if modified products keep the product features that are perceived and desired by the policyholder, and do away with those options and guarantees of which policyholders often do not even know they exist. Similar modifications are also possible for many other old age provision products like dynamic hybrid products[19] or annuity payout products. Therefore, we expect that the importance of "risk management by product design" will increase. This is particularly the case since – whenever the same pool of assets is used to back new and old products – new capital efficient products might even help reduce the risk resulting from an "old" book of business by reducing the required yield of the pool of assets.

We therefore feel that there is room for additional research: It would be interesting to analyze similar product modifications for the annuity payout phase. Also – since many insurers have sold the traditional product variant in the past – an analysis of a change in new business strategy might be worthwhile: How would an insurer's risk and profitability change and how would the modified products interact with the existing business if the insurer has an existing (traditional) book of business in place and starts selling modified products today.

Another interesting question is how the insurer's optimal strategic asset allocation changes if modified products are being sold: If typical criteria for determining an

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[19] Cf. Kochanski and Karnarski [2011].

optimal asset allocation are given (e.g. maximizing profitability under the restriction that some shortfall probability or expected shortfall is not exceeded), then the c.p. lower risk of the modified products might allow for a more risky asset allocation and hence also higher expected profitability for the insurer and higher expected surplus for the policyholder. So, if this dimension is also considered, the policyholder would somehow be compensated for the fact the he receives a weaker type of guarantee. Furthermore, the insurer might share the reduced cost of capital with the policyholder, also resulting in higher expected benefits in the alternative product designs.

Finally, a direct analysis of a capital efficiency ratio and not only its key drivers might be worthwhile.

Since traditional participating life insurance products play a major role in old-age provision in many countries and since in the current interest framework and under risk based solvency frameworks these products have come under strong pressure, the concept of "Capital Efficiency" and the analysis of different product designs should be of high significance for insurers, researchers, and regulators to identify sustainable life insurance products. In particular, we would hope that legislators and regulators would embrace sustainable product designs where the insurer's risk is significantly reduced, but key product features as perceived and requested by policyholders are still present.

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