Return Smoothing and Risk Sharing Elements in Life Insurance from a Client Perspective

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Abstract

In many countries, traditional participating life insurance (TPLI) contracts are typically equipped with a cliquet-style (year-to-year) guarantee. Life insurers pool the assets and liabilities of a heterogeneous portfolio of TPLI contracts. This allows for intergenerational risk sharing. Together with certain smoothing elements in the collective investment, it also results in rather stable returns for the policyholders. Despite the current low interest rate environment, TPLI contracts are still popular in the segment of retirement savings. Standard approaches which focus solely on the cash-flow at maturity cannot explain their popularity. In a recent paper, Ruß & Schelling (2018) have introduced a descriptive model of decision making which takes into account that potential future changes in the account value impact the decision of long-term investors at outset. Based on this, we illustrate how smoothing and risk sharing elements provided by a life insurer can significantly increase the subjective utility for such investors. Furthermore, we show that for these investors TPLI contracts are more attractive than common unit-linked (guaran-

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teed) products. Hence, our findings explain the popularity of TPLI contracts and provide helpful insights into decision making in the context of retirement savings.

Keywords: Cumulative Prospect Theory, Myopic Loss Aversion, Mental Accounting, Smoothing, Risk Sharing, Retirement Savings, Traditional Participating Life Insurance

JEL: D14, D81, G11, G22, G41, J26, J32

1 Introduction

Traditional participating life insurance (TPLI) contracts (also referred to as with-profit life insurance contracts) have been the core business of life insurers for many years. In contrast to individual retirement savings products, life insurers pool the assets and liabilities of a heterogeneous portfolio of TPLI contracts which allows for intergenerational risk sharing. In many countries, TPLI contracts are typically equipped with an cliquet-style (year-to-year) guarantee where a guaranteed return must be credited to the policyholder's individual account at the end of each year. Additionally, TPLI contracts receive a surplus participation which is based on the return of a collective investment which is subject to various smoothing elements. In particular, rather stable returns are achieved by building up collective reserves on both sides of the balance sheet in good years and dissolving these reserves to compensate for years with poor (or even negative) returns.[1] Goecke (2013) shows that such smoothing and risk sharing elements can (in absence of a guarantee) heavily reduce the short-term risk without significantly affecting the long-term risk-return-profile. In that sense, life insurers operate like a buffer between the capital market and the policyholders.

However, the current low interest rate environment has forced life insures to reduce guaranteed rates for new contracts. While smoothing elements can reduce the volatility of returns, they cannot compensate for a long-term decline in the capital market returns. Hence, also realized returns for TPLI contracts have decreased over the past years. Furthermore, due to insurance portfolios with long-term contracts and rather high guaranteed rates (especially in old contracts) in combination with rather restrictive solvency requirements, the insurer's asset allocation allows only for low risk taking. For long-term investors this very likely results in a suboptimal distribution of the terminal value. In addition, smoothing and intergenerational risk sharing mechanisms are opaque by nature. For all these reasons, TPLI contracts have been heavily criticized by consumer protection organizations.[2] Consequently, life insurers are cur-

[1] There is a broad literature on different aspects of TPLI contracts. Most of the literature focuses on the valuation and product design in the context of capital requirement issues. For an overview we refer to Goecke (2013) or Reuß *et al.* (2015).

[2] In this regard, a frequently cited criticism is that new contracts subsidize old contracts (with much higher guaranteed rates) and hence suffer from an ex-ante "collective malus". On the other hand, the new contracts

rently reinventing their business. In particular, they tend to develop capital efficient versions of TPLI contracts[3] with different types of guarantees or offer more (individualized) unit-linked contracts. Despite these tendencies, versions of TPLI contracts are still very popular in the segment of retirement savings (this is also true for slightly modified products[4] which also make use of the same collective smoothing and risk sharing elements). Moreover, a complete shift to individualized contracts leads to a loss of intergenerational risk sharing and questions the role of the life insurer in this context.

The aim of this paper is to explain the popularity of TPLI contracts and to shed light on how smoothing and risk sharing elements are perceived by long-term investors. Studies show that Expected Utility Theory (EUT) and even Cumulative Prospect Theory[5] (CPT) investors who focus solely on the terminal value would not buy products with cliquet-style guarantees, cf., e.g., Ebert *et al.* (2012). Gollier (2008) shows that an intergenerational risk transfer can be social welfare increasing and Goecke (2013) demonstrates advantages of collective over individual investments. In particular, Goecke (2013) suggests that investors reevaluate their investment regularly and that a volatile performance causes stress. Also, several other authors pointed out that investors show such a tendency, cf., e.g., Benartzi & Thaler (1995) and Gneezy & Potters (1997) as well as Koranda & Post (2014) with the focus on an index linked product. In a recent paper, Ruß & Schelling (2018) have argued that in particular long-term investors also get subjective utility and disutility from interim gains and losses in the account value. They argue that this already impacts the investment decision at outset and propose a modification of CPT that takes this into account. The so-called Multi Cumulative Prospect Theory (MCPT) is able to explain the demand for cliquet-style guarantees in a simple model framework (Black-Scholes

benefit from assets (particularly bonds with rather high coupons) that have been bought in the past, resulting in an ex ante "collective bonus". It is not intuitively clear which effect is larger. Recent research tries to shed light on these effects. Hieber *et al.* (2016) introduce conditions for a fair valuation of insurance contracts in the case of a heterogeneous insurance portfolio that ensure that new contracts are not exposed to an ex ante "collective malus" (and vice versa do not receive a ex ante "collective bonus"). Further, in a similar framework Eckert *et al.* (2018) propose a measure to quantify the "collective malus/bonus" of certain contracts.

[3] Capital efficiency can be interpreted as profitability in relation to capital requirement, cf. Reuß *et al.* (2015) for more details.

[4] E.g., certain index linked products, cf. Alexandrova *et al.* (2017) for more details.

[5] CPT introduced by Tversky & Kahneman (1992) is one of the most popular behavioral counterparts to EUT. Most importantly, it takes into account that actual decision making is often based on heuristics which can lead to systematic biases. Cf. Section 2 for more details.

market with constant risk-free rate, single premium, etc.). Further, Graf *et al.* (2018) show that MCPT is also able to explain the demand for life-cycle funds which decrease the risk exposure when approaching retirement. Both results suggest that MCPT is more accurate in predicting decision making of long-term investors than standard approaches like EUT and CPT. Based on these insights it seems natural that return smoothing and risk sharing elements provided by life insures are essential aspects for long-term investors when making the investment decision.

As we are particularly interested in analyzing the impact of smoothing and risk sharing elements we model these elements in detail by means of a stylized life insurance company based on the situation in Germany. We also consider a rather realistic model framework with respect to other aspects like stochastic interest rates, different types of charges, regular premium payments, etc. We will confirm that CPT in its standard form is not able to explain the popularity of TPLI contracts. Subsequently, we will show that MCPT-investors strongly prefer smoothed returns as well as TPLI contracts (compared to common unit-linked products). We will show that this is also true in the case of a (moderate) ex-ante collective malus and even if the subjective utility is only partly influenced by potential annual changes. Hence our findings offer a convincing explanation for observed decisions in retirement savings. Understanding the decision making is an important requirement to improve product design and ultimately help long-term investors to make the right choice to ensure a desired standard of living in old age.

The remainder of the paper is organized as follows: In Section 2, we briefly introduce the concept of MCPT. Section 3 describes the TPLI based on a stylized life insurance company. In particular, we model assets and liabilities and describe in detail the implemented smoothing and risk sharing elements. In Section 4 we specify the model parametrization and present the results of our analyses focusing on the impact of smoothing and risk sharing elements from a long-term investor's perspective. Subsequently, in Section 5 we compare TPLI contracts with various unit-linked products to analyze the popularity of TPLI contracts compared to other common investment choices in retirement savings. Finally, Section 6 concludes and provides an outlook for future research.

2 Modeling Decision Making of Long-term Investors[6]

Cumulative Prospect Theory (CPT) introduced by Tversky & Kahneman (1992) has been developed as a descriptive theory to model and predict how humans actually make decisions. It considers gains and losses with respect to a reference point and is based on an S-shaped value function v which assumes that investors are typically loss averse and a probability distortion function w which takes into account that investors tend to overweight tail events with small probabilities and underweight events with high probabilities. Although CPT explains actual human behavior that cannot be explained by Expected Utility Theory (EUT), even CPT frequently fails to explain typical behavior of long-term investors. In particular, there are many long-term investment products that are very popular which neither an EUT-investor nor a CPT-investor would buy (cf. Ebert *et al.* (2012), Ruß & Schelling (2018) or Graf *et al.* (2018)).

One reason is that CPT (like EUT) is typically applied such that investment products only generate subjective utility in connection with actual cash flows - thus, in case of long-term investments only at maturity. However, even for long-term investments, investors regularly evaluate their investment. If a reported value is lower than a previous value, this will be perceived as a loss which typically looms larger than a gain of similar amount, cf., e.g., Barberis *et al.* (2001).[7] This motivates that for long-term investors, the initial subjective utility of an investment is not only dependent on the distribution of the terminal wealth (relative to some reference point), but also on the possible future interim value changes. To capture this effect, Ruß & Schelling (2018) have introduced a modification of CPT, the so-called Multi Cumulative Prospect Theory (MCPT) which essentially uses CPT with multiple reference points and evaluation periods to measure the subjective utility of the potential interim value changes. Since the difference between CPT and MCPT typically becomes larger for an increasing investment horizon, MCPT is particularly useful to explain and predict actual behavior for long-term in-

[6]This Section is closely following Ruß & Schelling (2018) and Graf *et al.* (2018).

[7]Benartzi & Thaler (1995) propose the theory of myopic loss aversion, a combination of loss aversion and frequent investment evaluation, and provide an explanation for the equity premium puzzle and the preference of long-term investors for low-risk investments. Moreover, mental accounting, introduced by Thaler (1985), implies that investors tend to take into account potential future fluctuations of the contract's value already when making an investment decision.

vestment decisions.

MCPT considers an investor and an investment Ξ with time horizon $[0, T]$, $T \in \mathbb{N}$, at time $t = 0$. Throughout this paper we assume that premiums are paid annually in advance at time $t+$ for $t \in \{0, \ldots, T-1\}$. Moreover, we assume that future interim evaluations take place annually. We consider for all $t \in \{1, \cdots, T\}$ the annual gain or loss $X_t := A_t - \chi_t$, where A_t is the account value of the investment Ξ at time t (before premium payment) and χ_t is the reference point for time t. The natural reference point choice for each period is given by $\chi_t = \overline{A}_{(t-1)+}$, that is, the (reported) account value of the contract at time $t-1$ plus the premium P paid at time $(t-1)+$.[9]

We can evaluate the CPT value of each annual value change X_t by

$$CPT(X_t) = \int_{-\infty}^{0} v(x) d\left(w\left(F_t(x)\right)\right) + \int_{0}^{\infty} v(x) d\left(-w\left(1 - F_t(x)\right)\right),$$

where $F_t(x) = \mathbb{P}(X_t \leq x)$ and v is the investor's value-function which is defined as $v(x) := x^a \mathbb{1}\{x \geq 0\} - \lambda |x|^a \mathbb{1}\{x < 0\}$ where $\lambda > 0$ is the loss aversion parameter and $a \in \mathbb{R}_+$ controls the risk appetite. The probability distortion function is given by $w(p) := \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}}$ with $\gamma \in (0.28, 1]$, where the lower boundary for γ is chosen such that w is strictly monotonically increasing for $p \in [0, 1]$. The MCPT utility at time $t = 0$ is then defined by

$$MCPT(\Xi) := \sum_{t=1}^{T} \eta^t CPT(X_t) \tag{1}$$

with a discounting parameter $\eta \in \mathbb{R}_+$.

The MCPT utility reflects the subjective utility of the potential annual changes. Ruß & Schelling (2018) also suggest a combined model given by the weighted sum of the CPT and the MCPT utility. In doing so, the combined model captures also the subjective utility of the terminal value relative to a reference point χ, that is, $X = A_T - \chi$. The natural CPT reference

[9]Note that the premium P typically differs from the savings premium which is reduced by premium proportional charges, cf. Section 3.

point χ is given by the sum of all premiums ($\chi = T \cdot P$). The combined model is given by

$$CPT^{com}(\Xi) := s \cdot MCPT(\Xi) + (1-s) \cdot CPT(X) \tag{2}$$

where $s \in [0,1]$ controls the influence of the annual value changes on the subjective utility.

3 Traditional Participating Life Insurance

In this Section we model a traditional participating life insurance (TPLI) contract within a stylized insurance company. As insurance portfolios are heterogeneous, for example with respect to the guaranteed rates, TPLI contracts are influenced by intergenerational effects between different cohorts, cf. Hieber *et al.* (2016). In particular, policyholders of all cohorts participate in the returns of the same assets and these returns are subject to smoothing elements on both sides of the insurer's balance sheet (see below). Since a key question to be answered in this paper is the attractiveness these elements for a long-term investor, we model these aspects very detailed based on the situation in Germany.

We consider a TPLI contract with a duration of T years and a policyholder with initial age of x years. We assume an annual premium P paid in advance at time $t+$ for $t \in \{0, \dots, T-1\}$. The contract provides a cliquet-style annual guaranteed rate i_0^g on the account value. Additionally, the TPLI contract receives a surplus participation which is subject to regulation, but allows for some discretion by the insurance company. In particular, we assume that at the end of each year the insurance company specifies a total interest rate which is credited in the subsequent year to the account value of the policyholder (details below).[11] The premium can be derived by the actuarial principle of equivalence based on the guaranteed benefit G and annual charges c_t^p as a percentage of the premium, that is, $P = \frac{G}{\sum_{t=0}^{T-1}(1-c_t^p)(1+i_0^g)^{T-t}}$. In case of death the current account value is paid out.[12] Throughout the paper, we call $P \cdot (1-c_t^p)$ the savings premium. Further, we assume that first- and second-order mortality rates and charges coincide and no lapses,

[11] Note that this common practice for German life insurers.

[12] Hence, the death benefit does not impact the premium.

Assets		Liabilities		
BV_t^B	BV_t^S	IR_t	AR_t	$RfB_t = RfB_t^D + RfB_t^S$
BS_t		BS_t		

Table 1: Structure of the balance sheet at time t.

tax payments etc. are considered, such that the investment surplus is the only source of surplus.

The TPLI contract is based on a life insurance company described by a balance sheet and specific management rules.[13] The insurance portfolio at the initial date $t = 0$, has been built up over the previous T years and consists of $T - 1$ cohorts of contracts with remaining time to maturity 1 to $T - 1$ years. At the beginning of each year t a new cohort of $l_x^{(t)}$ policyholders joins. The number of policyholders of this cohort remaining in the portfolio at time $t + k$ is given by $l_{x+k}^{(t)} = l_{x+k-1}^{(t)} \cdot (1 - q_{x+k-1})$ for $k \in \{1, \dots, T\}$ with q_x denoting the mortality rate of an x-year old person. While premium, duration, initial age of the policyholder and charges (as percentage of the premium) are assumed to be equal for all cohorts, the guaranteed rates and hence the guaranteed benefits are modeled cohort specific.[14] We denote the guaranteed rate of a cohort with initial date t as i_t^g. For the new cohorts joining the company after $t = 0$ the guaranteed rate is calculated as 60% of the average return of zero bonds with maturity of 10 years over the last 5 years, where the result is rounded down to a tenth of a percentage point and zero representing the minimum.[15]

The balance sheet of the insurance company at the beginning of year t with balance sheet total BS_t is displayed in table 1. The assets are given by the book values of a bond portfolio BV_t^B and of a stock portfolio BV_t^S. The bond portfolio consists of coupon bonds (yielding at par) with initial maturity $T_B = 10$. We follow German local GAAP (HGB) accounting rules[16] and assume that bonds are recognized at acquisition costs and stocks at strict lower-of-cost-

[13]Similar models have been used by Reuß *et al.* (2016), Reuß *et al.* (2015), Burkhart *et al.* (2015) and Seyboth (2011).

[14]The guaranteed interest rate for the initial cohorts and for the cohort joining at time $t = 0$ are assumed to be given (see Section 4.1 for details).

[15]This is in line with EU-regulation on maximum allowed guaranteed interest rates, cf. EU (2002).

[16]Cf. Reuß *et al.* (2016) for details.

or-market principle. Differences in market and book values may result in unrealized gains and losses (UGL). According to local GAAP, unrealized losses on stocks have to be realized at the end of the year, that is, the ratio $d_{neg} = 100\%$ of the unrealized losses on stocks is realized annually. Furthermore, we assume that in case of unrealized gains on stocks the ratio d_{pos} is realized annually in order to stabilize the investment return.

The insurance company follows a strategic asset allocation by annually rebalancing the assets based on a stock ratio $q_t \in [q_{min}, q_{max}]$ (in terms of market values) at the end of the year. If necessary, bonds are sold proportionally to their market values.[17] Further, the insurer increases the stock ratio if the weighted average of the coupon rates (at the end of the year and before rebalancing) $\overline{cp}_t$ is rather low compared to the weighted average guaranteed rate of all contracts in the portfolio $\overline{i^g}$. More precisely, we define the stock ratio by $q_t = \min\left\{\max\left\{q_{min} \cdot \left(1 + \left(\frac{1+\overline{i^g}}{1+\pi^{\overline{cp}} \cdot \overline{cp}_t} - 1\right) \cdot 100\right), q_{min}\right\}, q_{max}\right\}$ with adjustment factor $\pi^{\overline{cp}} \geq 0$.

The rebalancing of the assets takes place at the end of each year and takes into account the cash flow CF_{t+} at the beginning of the year [18] which is invested in a riskless bank account earning the interest rate $r_t(1)$, and the cash flow at the end of the year.[19] The total (book value) investment return rate is then given by

$$i^*_{t+1} = \frac{CF_{t+} \cdot r_t(1) + CP_{t+1} + \mathrm{UGL}^{real}_{t+1}}{BV^S_t + BV^B_t + CF^+_t}$$

with $\mathrm{UGL}^{real}_{t+1}$ denoting the realized portion of the UGL.

The liabilities consist of the insurer's profit (loss) IR_t at the end of year $t-1$, the sum of the actuarial reserves of all contracts[20] AR_t, and the reserves for premium refunds RfB_t, sometimes also referred to as uncommitted provision for premium refunds which are instrumental in

[17] Cf. Burkhart *et al.* (2015) or Seyboth (2011) for further details.

[18] Given by the premium payments less expenses and the insurer's profits. Cf. appendix B for details on the insurer's future profits in the stochastic simulation.

[19] Given by coupon payments CP_{t+1} plus nominal repayments of bonds at maturity minus benefit payments to the policyholders, cf. Burkhart *et al.* (2015) for details.

[20] The actuarial reserve ${}_kAR_t$ of one contract at the end of the k-th year of its duration at time t can be calculated recursively by ${}_kAR_t = ({}_{k-1}AR_{t-1} + P \cdot (1 - c^p_{t-1})) \cdot (1 + i^g_{t-k})$ with ${}_0AR_t = 0$.

smoothing investment returns within TPLI's, cf. Alexandrova *et al.* (2017). The RfB is modeled by two parts: credited non revisable bonus reserve[21] RfB_t^D and a terminal bonus fund RfB_t^S which can be used by the company for smoothing returns and as a buffer to cover losses. It follows that the account value of one contract with remaining duration $T-k$ at time $t+$ is given by ${}_kA_{t+} = {}_kAR_t + {}_kRfB_t^D + P \cdot (1-c_t^p)$. In subsequent years, the guaranteed rate applies to the account value and hence also to the credited non revisable bonus reserve ${}_kRfB_t^D$.

Next, we describe the mechanisms of the surplus distribution. Based on the investment return rate i_{t+1}^*, we can determine the total investment return of the insurance company by $R_{t+1}^* = \left(RfB_t^S + A_{t+}^\Sigma\right) \cdot i_{t+1}^*$ where A_{t+}^Σ denotes the sum of all account values in the portfolio. The total investment surplus at the end of the year is given by $Sp_{t+1} = R_{t+1}^* - R_{t+1}^g$ where $R_{t+1}^g = \sum_{k=0}^{T-1} l_{x+k}^{(t-k)} \cdot {}_kA_{t+} \cdot i_{t-k}^g$ denotes the sum of the guaranteed interest credited to the policyholders. The part of the investment surplus that is distributed to the policyholders is given by $PS_{t+1} = \max\left\{0; \alpha^{Sp} R_{t+1}^* - R_{t+1}^g\right\}$ where α^{Sp} denotes the participation rate.[22] Ideally, this part of the investment surplus is taken to finance the part of the (cohort specific) total interest rates ${}_ki_t$ that exceeds the guaranteed rate, that is, $\Delta R_{t+1}^i := \sum_{k=0}^{T-1} l_{x+k}^{(t-k)} {}_kA_{t+}({}_ki_t - i_{t-k}^g)$.[23] However, it is not always the case that the investment surplus is sufficient to cover all total interest payments. In this case the insurer is allowed to dissolve reserves in the terminal bonus fund (and possibly also other unrealized gains) and, if necessary, the insurer covers the residual.[24] The remaining part of the investment surplus represents the insurer's profit or loss IR_{t+1}.[25]

Finally, we describe how the insurance company decides on the total interest rate which defines the annual return for the policyholder on the account value. We assume that the total interest rate is based on an adjusted investment return i_t which is subject to various smoothing

[21] ${}_kRfB_t^D$ denotes the part that has been credited to one contract with time to maturity $T-k$ at time t.

[22] According to the German MindZV (Mindestzuführungsverordnung) $\alpha^{Sp} \geq 0.9$.

[23] Note that the part exceeding the guaranteed rate is credited to the non revisable bonus reserve ${}_kRfB_t^D$.

[24] In detail, if $Sp_{t+1} \geq 0$ and $\Delta R_{t+1}^i \leq PS_{t+1}$, then the investment surplus suffices to cover all total interest payments and the remaining part of PS_{t+1} is credited to the terminal bonus fund. If $Sp_{t+1} \geq 0$ and $\Delta R_{t+1}^i > PS_{t+1}$ or if even $Sp_{t+1} < 0$ then the investment surplus is not sufficient to cover all interest payments to the policyholders. In this case, the residual is covered by the terminal bonus fund. If the terminal bonus fund is not sufficient to cover the residual, first remaining unrealized gains are realized before the insurer is liable.

[25] Appendix B provides details under the considered settings.

elements. Firstly, it is based on the average (book value) investment returns[26] of the last 3 years[27], that is, $\overline{i^*}_t = \frac{\sum_{j=0}^{2} i^*_{t-j}}{3}$. Secondly, we assume that the insurer reduces (increases) i_t in case of rather low (high) reserves. Additionally, for expiring contracts i_t is increased by a terminal bonus rate i_t^{term} depending on the current reserves. Based on this, the total interest rate of each cohort ${}_ki_t$ is defined by the maximum of the corresponding guaranteed rate i^g_{t-k} and the adjusted investment return i_t.

More precisely, the adjusted investment return is defined by

$$i_t = \pi^{i^*}\overline{i^*}_t + \pi^{\rho}(\rho_t - \rho_{target}) + \pi^{\tilde{\rho}}(\tilde{\rho}_t - \tilde{\rho}_{target}) \text{ with } \rho_t = \frac{RfB_t^S + UG_t}{BS_t} \text{ and } \tilde{\rho}_t = \frac{RfB_t^S}{BS_t},$$

where ρ_t defines the current reserve ratio[28] and $\tilde{\rho}_t$ the current terminal bonus reserve ratio. Further, ρ_{target} and $\tilde{\rho}_{target}$ denote the target reserve ratios and ρ_{min} and $\tilde{\rho}_{min}$ the corresponding minimal values. Additionally, $\pi^{i^*} \geq 0$, $\pi^{\rho} \geq 0$ and $\pi^{\tilde{\rho}} \geq 0$ denote adjustment factors to control the impacts of the different aspects. The total interest rate at time t for the cohort with initial date $t-k$ applied in the period $[t, t+1)$ is then defined as

$${}_ki_t = i^g_{t-k} + \max\left\{i_t - i^g_{t-k}, 0\right\} \cdot \mathbb{1}_{\{\tilde{\rho}_t \geq \tilde{\rho}_{min} \wedge \rho_t \geq \rho_{min}\}} \tag{3}$$

and at maturity (or in case of death) as ${}_ki_t = \max\left(i_t + i_t^{term}, i^g_{t-k}\right)$. We define $i_t^{term} = \tau_t \cdot \frac{RfB_t^S}{A_{t+}}$ with adjustment factor $\tau_t \in [\tau_{min}, \tau_{max}]$ which controls that the terminal bonus rate is higher (lower) in case of higher (lower) terminal reserves.[29]

4 Analyzing Smoothing and Risk Sharing Elements

In this Section we will analyze the effect of smoothing and risk sharing elements from a long term investor's perspective. First, in Section 4.1 we specify the parameter setting and the

[26] Note that the (book value) investment return depend on realized gains and losses.
[27] This is in accordance to the key figure C10 published by GDV (2016).
[28] This definition is in line with the key figure D10 catalog for German life insurers given by GDV (2016).
[29] $\tau_t = \tau_{min} + (\tau_{max} - \tau_{min}) \cdot \frac{\tilde{\rho}_t - \tilde{\rho}_{min}}{\tilde{\rho}_{target} - \tilde{\rho}_{min}}$ for $\tilde{\rho}_{min} \leq \tilde{\rho}_t \leq \tilde{\rho}_{target}$ and $\tau_t = \tau_{min} \cdot \mathbb{1}_{\{\tilde{\rho}_t < \tilde{\rho}_{min}\}} + \tau_{max} \cdot \mathbb{1}_{\{\tilde{\rho}_{target} < \tilde{\rho}_t\}}$, else.

considered TPLI contract types. Then, Section 4.2 presents the results.

4.1 Specification and Starting Conditions

The assets are based on a financial market model which is given by a stock process S following a geometric Brownian motion and a short rate process r described by a Vasicek model, cf. Vasicek (1977). The parameters have been chosen in accordance with the European money market and recent literature. A detailed description is given in appendix A.

We analyze the performance of one contract starting in $t = 0$ with guaranteed rate $i_0^g = 1.25\%$ (chosen to be in line with the maximum rate allowed by the German regulation in 2016, cf. DAV (2017)) and annual premium $P = 1$ €. We denote the account value at time t of this contract as A_t. Further, for the insurance portfolio we assume that all policyholders are 40 years old at inception of their contract. All contracts have an initial duration of $T = 20$ years. Annual charges c_t^p consist of annual administration charges $\beta = 5\%$ (as percentage of the premium) and initial acquisition charges $\alpha = 2.5\%$ (as percentage of the premium sum), which are equally deducted over the first five years.[30] Hence, $c_t^p = \beta + \frac{\alpha T}{5}\mathbf{1}_{t\in\{0,\dots,4\}}$. Mortality is based on the German standard mortality table (DAV 2008 T) and we do not consider surrender.[31] Moreover, at the beginning of each year t, a new cohort of $l_x^{(t)} = 1000$ policyholders joins the insurance portfolio. The initial portfolio[32] at time $t = 0$ is derived by a projection based on a deterministic (past) scenario with the first cohort joining in 1988 ($t = -28$). The guaranteed rates for the initial cohorts are assumed to coincide with the maximum rate allowed by the German regulation between 1988 and 2015, cf. DAV (2017). All values are given in appendix B in table 7.

At time $t = 0$, the book value of the assets coincides with the book value of the liabilities. As a management rule, we assume that the stock ratio[33] is between 7.5% and 15% in the deter-

[30] The value has been chosen according to the German Life Insurance Reform Act (LVRG) from 2015.

[31] Note that we consider mortality only for the purpose of risk sharing and smoothing effects in the insurance portfolio. We assume that the investor focuses solely on the case of survival until maturity.

[32] That is, the initial cohort sizes, the corresponding actuarial reserves and the reserves for premium refunds.

[33] The average ratio of German life insurance companies invested in stocks and comparable assets in 2015 was 10,4%, cf. GDV (2016).

q_{min} (%)	q_{max} (%)	T_B (years)	d_{pos} (%)	d_{neg} (%)	α^{Sp} (%)	π^{i*}	τ_{min}	τ_{max}
7.5 (10)	15 (17.5)	10	20	100	90	0.9	0	0.3
$\tilde{\rho}_{target}$ (%)	$\tilde{\rho}_{min}$ (%)	ρ_{target} (%)	ρ_{min} (%)	π^{ρ}	$\pi^{\tilde{\rho}}$	$\pi^{\overline{cp}}$		
6	2	12	4	0.1	0.1	0.75		

Table 2: Parameter setting for the management rules in the base case.

ministic (past) scenario and between 10% and 17.5% in the stochastic (future) projection.[34] The coupon bond portfolio is split in bonds with time to maturities between 1 and $T_B = 10$ years, whereby the proportions result from the deterministic scenario. For the deterministic scenario we use coupon and spot rates based on the historical annual average yields of German government coupon bonds with maturity between 1 and 10 years.[35] The annual stock returns are based on the historical returns of the German stock index DAX between 1988 and 2015 provided by the Deutsche Bundesbank (2016), cf. appendix B table 7. Further, we set $\rho_{target} = 12\%$, $\tilde{\rho}_{target} = 6\%$, $\rho_{min} = 4\%$ and $\tilde{\rho}_{min} = 2\%$.[36] The adjustment factors are set to $\pi^{i^*} = 0.9$, $\pi^{\rho} = 0.1$, $\pi^{\tilde{\rho}} = 0.1$ and $\tau_{min} = 0$, $\tau_{max} = 0.3$, $\pi^{\overline{cp}} = 0.75$. This ensures that the total interest rate is primarily affected by the average investment return rate of the last three years. Nevertheless, the higher the gap between the target (terminal) reserve ratio and the current (terminal) reserve ratio, the larger the adjustment of the total interest rate. The parameter for the management rules are summarized in table 2.

The deterministic scenario results in the initial balance sheet ($t = 0$) displayed in table 3. Further initial key values resulting from the deterministic scenario are summarized in table 4.

[34] The higher corridor for the stock ratio in the future projection is motivated by the sustained trend of insurers to reset their risk limits and to increase their appetite for higher risk investments (including a shift from public to private assets). E.g., an annual international survey conducted by BlackRock in 2018 finds that almost half (47%) of insurers surveyed plan to increase portfolio risk exposure over the next 1-2 years, while only 4% plan to reduce risk exposure, cf. BlackRock (2018). Moreover, in the last four surveys (since 2015) at most 12% of the surveyed insurers planned to reduce risk exposure, while in 2015 and 2016, 57% and 47% planned to increase risk exposure, respectively. However, we also provide a sensitivity analysis with respect to the stock ratio in Section 5.

[35] Data from Deutsche Bundesbank (2016). Note that for the sake of a smooth shift from historical to model based yield curves, we calculate the yield curves for $t \in \{-3, -2, -1\}$ based on zero bond prices in the stochastic financial market and the average three-month EURIBOR rates of the last six months of the respective year.

[36] The target reserve ratio is set approximately to the average of the corresponding ratio D10 of the key figure catalog for German life insurance companies given by GDV (2016) between 2007 and 2015 (data available since 2007) reduced by roughly 3% because we do not consider any further equity in our model.

BV_0^B	BV_0^S	IR_0	AR_0	RfB_0	RfB_0^D	RfB_0^S	BS_0
206,455	20,914	894	211,058	15,416	7,127	8,289	227,369

Table 3: Initial values of the balance sheet in the base case in €.

ρ_0 (%)	$\tilde{\rho}_0$ (%)	q_0 (%)	MV_0^B (€)	MV_0^S (€)	${}_0i_k$	i_0^{term} (%)	τ_0^{term}
9.5	3.65	11.58	212,821	$27,865$	$\max(3.2\%, i^g_{-k})$	0.54	0.1234

Table 4: Selected initial values resulting from the deterministic scenario in the base case.

The initial (terminal) reserve ratio is given by $\rho_0 = 9.5\%$ ($\tilde{\rho}_0 = 3.65\%$) and is therefore below the target. The stock ratio is $q_0 = 11.58\%$ and the total interest rate for the first year is given by $\max(3.2\%, i^g_{-k})$, that is, all policyholders earn at least 3.2% on their account value. The additional terminal bonus rate in the first year amounts to 0.54%. Hence, the total interest rate for expiring contracts in the first year is given by 3.74%.[37]

The contract which is based on these assumptions is considered as the base case and is denoted as contract A. In this case the initial (terminal) reserve ratio equals 79% (60%) of the target. Further, the guaranteed rates of most contracts in the initial insurance portfolio are significantly higher than the guaranteed rate of contract A, cf. table 7 in appendix B. This causes on average a disadvantage for contract A, that is, contract A is expected to suffer more than profit from intergenerational effects.[38] Eckert *et al.* (2018) try to formalize this and define that a contract receives an "ex ante collective bonus"[39] if on average it will earn more than an investment in a reference portfolio that replicates the market values of the assets of the insurance company, that is, $CB = \mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\left(A_T - A_T^{ref}\right)\right] > 0$ with $A_T^{ref} = \sum_{t=0}^{T-1} P(1-c_t^p)\prod_{k=t}^{T-1} Perf_{t+1}^{MVA}$ denoting the terminal value of an investment in a reference portfolio with annual return $Perf_t^{MVA}$ and $\mathbb{Q}$ the risk-neutral measure. For the sake of better comparability, we consider the ex ante collective bonus in relation to the fair value of the alternative investment, that is, $CB^{\%} = \frac{CB}{FV} - 1$ with $FV = \mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^T r_u du} A_T^{ref}\right]$. If $CB < 0$ and hence $CB^{\%} < 0$, we say that the contract is exposed to an "ex ante collective malus". This is the case for contract A where

[37]These values are similar to the values of most life insurers in Germany in 2015, cf. ASSEKURATA (2015).
[38]Cf. also Hieber *et al.* (2016) for more details on these effects.
[39]Note that this also includes payments to or from the insurer (insurer's profit), cf. appendix B.

$CB^{\%} = -6.12\%$. To separate the impact of smoothing and risk sharing from the impact of systematic intergenerational effects at some point in time, we consider the following three additional contract settings:

B: We assume the same initial setting as in case A, but with adjusted initial (terminal) reserve ratio of 100%.[40] $CB^{\%}$ is in this case -5.08%.

C: We additionally assume that all contracts in the insurance portfolio have the same guaranteed rate of 1.25%. We generate the initial portfolio based on this assumption and adjust the initial (terminal) reserve ratio to 100%. This results in $CB^{\%} = -2.31\%$.

D: We consider the same setting as in case C. Additionally, we increase the surplus participation rate to $\alpha^{Sp} = 97\%$ for all policyholders in order to obtain $CB^{\%} \approx 0\%$.

It is worthwhile noting that in all considered cases the present value of the insurer's future profit (PVFP) is positive. Details and further key figures are described in appendix B.

Furthermore, to analyze the asset smoothing elements which are based on a collective investment, we consider also two fictitious contracts:

E: Contract E invests the savings premium $P(1 - c_t^p)$ in the reference portfolio replicating the market value of the assets of the insurance company under the setting of case D.[41] This contract represents the case of an investment without smoothing and risk sharing. Obviously, $CB^{\%} = 0\%$ in this case.

F: Contract F is assumed to invest the savings pemium in an investment that earns the average investment return $\overline{i^*}_t$ of the insurance company under the setting of case D. This contract represents the case with asset smoothing but without further risk sharing effects. The asset smoothing results in an ex ante collective malus $CB^{\%} = -1.38\%$.

For the sake of comparability we assume that contracts E and F come with the same premium and annual charges c_t^p as the other contracts.

[40] In order to meet the balance equation assets are increased proportionally.

[41] Note that the returns of the reference portfolio depend on the insurance portfolio structure. However, the differences between the considered cases are negligible for our analysis.

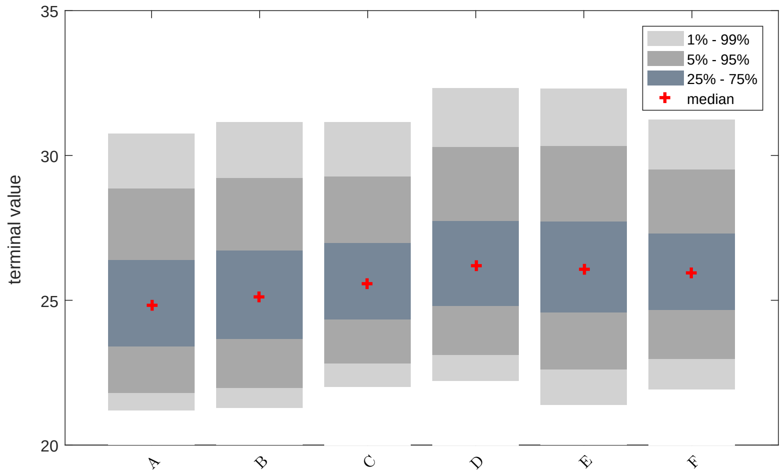


Figure 1: Percentiles of the terminal value of the TPLI contracts A–D and of the contracts E and F.

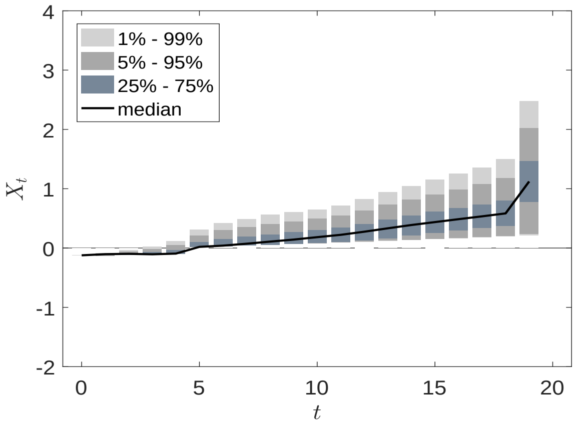
4.2 Results

Due to the complexity of the model, all results are based on Monte Carlo simulations with 20,000 trajectories. The numerical analysis is based on a stochastic simulation of the financial market under the real-world measure $\mathbb{P}$ (as well as under the risk-neutral measure $\mathbb{Q}$ for the purpose of fair valuation) which is done on a daily basis assuming 252 trading days per year.

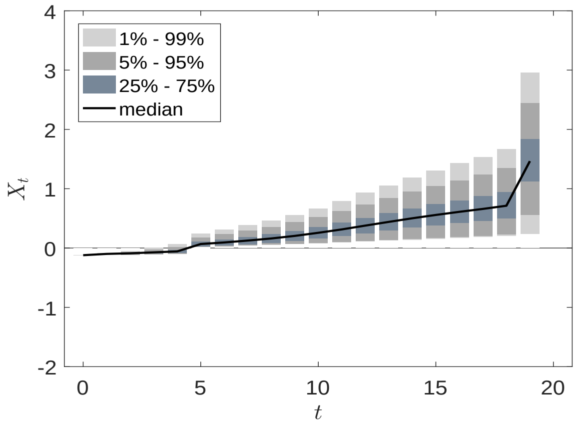
4.2.1 Key Figures

First, we investigate the distribution of the terminal value and the annual changes in the account value since these distributions are main drivers of the further results.

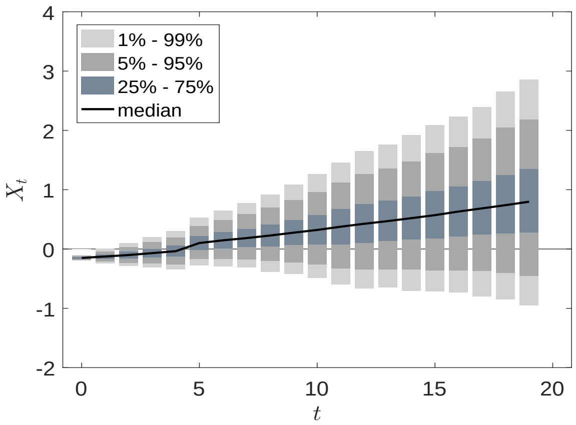
Figure 1 displays the percentiles of the terminal value of the different TPLI contracts A–D, as well as of the fictitious contracts E and F. We find that the distributions of the terminal value are very similar. All distributions are slightly right skewed and have a median between approximately 24.8 € (A) and 26.2 € (D). The displayed percentiles are all in the range of 21.2 € and 32.3 € and hence always above the accumulated premiums. The terminal value of the TPLI contracts increases slightly for a lower ex ante collective malus. The percentiles of product E and F show that the asset smoothing elements implemented by the insurance company reduce the variability of the terminal value without significantly reducing its expected value.



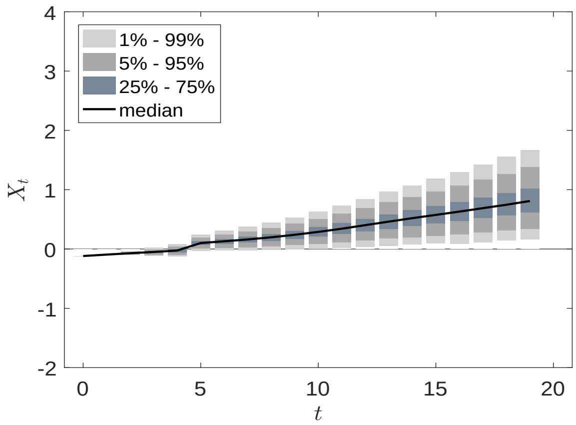
(a) Contract A



(b) Contract D



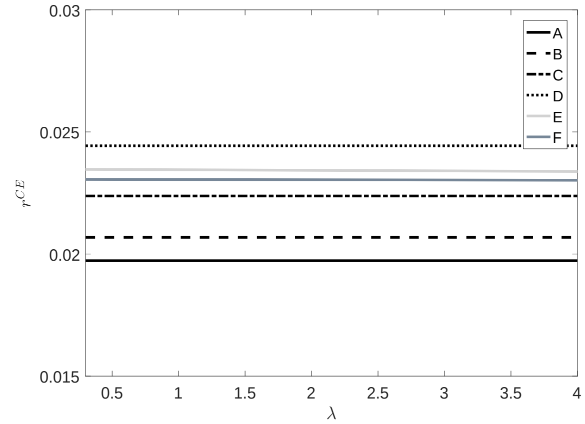
(c) Contract E



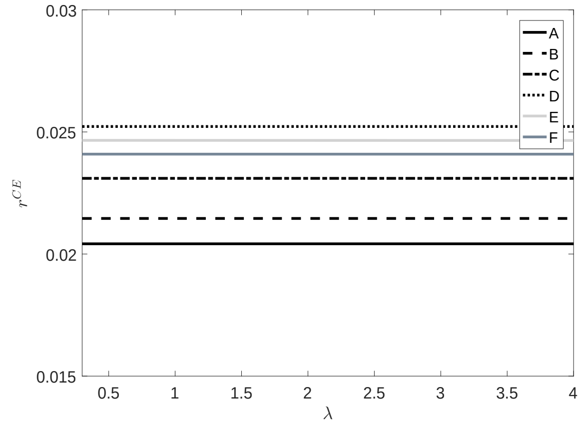
(d) Contract F

Figure 2: Percentiles of the annual changes of TPLI contracts A and D as well as of the contracts E and F.

Figure 2 shows the percentiles of the annual changes in the account value of the considered TPLI contracts A and D, as well as of the fictitious contracts E and F. The changes in the account value are defined as $X_t = A_t - \overline{A}_{(t-1)+}$ for $t \in \{1, \ldots, T\}$ with $\overline{A}_{(t-1)+}$ denoting the account value at time $t-1$ plus the premium P paid at time $(t-1)+$. The upper panels show that the patterns of the annual changes of the TPLI contracts do not significantly differ (thus, we refrain from displaying the annual changes for contract B and C). In the first five years they are slightly negative due to the acquisition charges which are deducted over the first five years. Subsequently, the annual changes are in almost all cases positive and (on average) increasing from year to year due to the higher account value. The annual change in the last year is (on average) significantly higher due to the additional terminal bonus. The lower left panel displays the annual changes of the unsmoothed fictitious contract E. The percentiles show that the dis-



(a) $a = 0.88$ and $\gamma = 0.65$



(b) $a = 0.88$ and $\gamma = 1$

Figure 3: r^{CE} in the CPT case depended on λ with (left panel) and without (right panel) probability distortion for the TPLI contracts A–D, as well as for the fictitious contracts E and F.

tribution of the annual changes of contract E are much wider compared to the other contracts and include in particular a significant risk of annual losses. The annual changes of contract F illustrate that the implemented asset smoothing elements result in much tighter distributions of the annual changes (lower right panel). While the median values are very similar as for contract E, the asset smoothing elements heavily reduce the risk of annual losses (and also the potential for high annual gains). Moreover, the results show that asset smoothing elements based on a collective investment alone (without an embedded guarantee) already dramatically reduces the probability for annual losses.

In combination with the results displayed in Figure 1 this shows that the smoothing elements based on the collective investment of a life insurer can heavily reduce the variability of annual returns without significantly changing the risk-return characteristics of the terminal value.

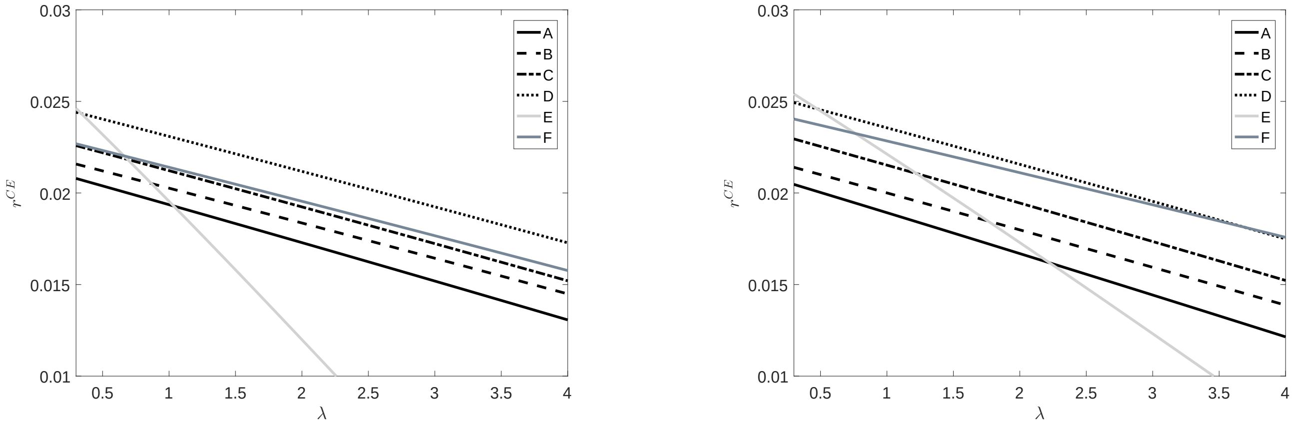
4.2.2 CPT and MCPT Analysis

As described in Section 2 we consider CPT, MCPT and a combined model to analyze investor preferences. We use MCPT to analyze the influence of the annual changes in the account value on the subjective utility. If not stated otherwise, we fix $a = 0.88$ and $\gamma = 0.65$ as suggested by Tversky & Kahneman (1992) and perform analyses for different values of λ. Moreover, we focus on the case without discounting, that is, $\eta = 1$. As Ruß & Schelling (2018), we derive certainty equivalent contracts. We solve the following equation numerically for each contract to obtain the corresponding fixed annual return r^{CE} that an investor (CPT-investor for $s = 0$ and MCPT-investor for $s = 1$) would regard equally desirable as the considered contract Ξ.

$$CPT^{com}(\Xi) = \begin{cases} s \cdot \sum_{t=0}^{T-1} \left(\sum_{k=0}^{t} \left(P(1-c_t^p) e^{r^{CE}(t-k)} - P \right) \right)^a + \\ (1-s) \cdot \left(\sum_{t=0}^{T-1} \left(P(1-c_t^p) e^{r^{CE}(T-t)} - P \right) \right)^a, & CPT^{com}(\Xi) \geq 0 \\ -\lambda \cdot s \cdot \sum_{t=0}^{T-1} \left| \sum_{k=0}^{t} \left(P(1-c_t^p) e^{r^{CE}(t-k)} - P \right) \right|^a + \\ -\lambda \cdot (1-s) \cdot \left| \sum_{t=0}^{T-1} \left(P(1-c_t^p) e^{r^{CE}(T-t)} - P \right) \right|^a, & CPT^{com}(\Xi) < 0 \end{cases}$$

Figure 3 shows the certainty equivalent returns as a function of loss aversion (λ) for a CPT-investor who does not value annual changes ($s = 0$) with (left panel) and without (right panel) probability distortion. We find that probability distortion only slightly reduces the certainty equivalents without changing the pattern of the result. For a CPT-investor the results for E and F show that asset smoothing elements slightly reduce the subjective utility. Hence, a pure CPT-investor would prefer the unsmoothed contract E since smoothing mainly reduces interim fluctuations which are not considered under CPT. While the TPLI contracts A, B, and C are less attractive than the fictitious contracts, contract D is the most appealing contract. Not surprisingly, the results illustrate that an ex ante collective malus makes the TPLI contracts A, B, and C less appealing. However, the results for contract D show that the embedded guarantee can also increase the subjective utility if the smoothing and risk sharing elements do not result in an ex ante collective malus for the contract.

Further, the results show that under CPT loss aversion plays no role for these types of products. The TPLI contracts come with an embedded guarantee which prevents losses and the fictitious products are based on a rather conservative investment (insurers asset stock ratio is between 10% and 20%) which makes losses in case of a long-term investment very unlikely. Hence,

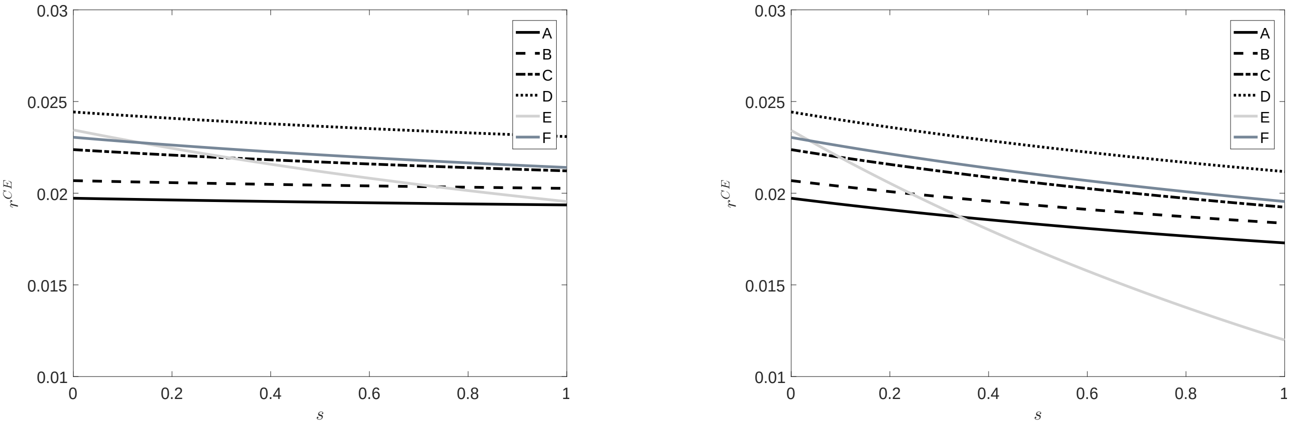


(a) $a = 0.88$ and $\gamma = 0.65$ (b) $a = 0.88$ and $\gamma = 1$

Figure 4: r^{CE} in the $MCPT$ case depended on λ with (left panel) and without (right panel) probability distortion for the TPLI contracts A–D, as well as for the fictitious contracts E and F.

applying CPT to describe actual human preferences in such cases assumes that the investor's degree of loss aversion does not impact the decision at all (at least in the common status quo case where the reference point is given by the accumulated premiums). This (obvious) result casts further doubts that CPT in its standard form is appropriate to describe actual decision making in the context of long-term investments.

Figure 4 shows the results for an MCPT-investor who only values annual changes and does not assign any weight to the terminal value ($s = 1$). We find that the patterns for the TPLI contracts A–D differ only slighly in the case with (left panel) and without (right panel) probability distortion. In contrast to the CPT case, the r^{CE} decreases in λ, that is, loss aversion with respect to annual changes reduces the attractiveness of the considered contracts. This is mainly caused by the acquisition charges which generate losses in the first years. Again, we find that an ex ante collective malus makes the TPLI contract less appealing. More interestingly, the results for contracts E and F show the huge impact of the asset smoothing elements on the attractiveness of the TPLI contracts. Without asset smoothing elements the r^{CE} declines heavily with increasing loss aversion. We find that loss averse MCPT-investors ($\lambda > 1$) prefer in all cases contract F over contract E. The left panel shows that probability distortion, particularly the overweighting of the small probabilities of rather high annual losses, makes contract E



(a) $\lambda = 1$

(b) $\lambda = 2$

Figure 5: r^{CE} in the CPT^{com} case depended on $s \in [0, 1]$ without (left panel) and with typical loss aversion $\lambda = 2$ (right panel) for the TPLI contracts A–D, as well as of the fictitious contracts E and F.

even less appealing for loss averse investors. Conversely, probability distortion makes contract F more appealing due to the overweighting of rather high gains and the absence of high losses. Comparing the TPLI contracts A–D with contract F shows that the asset smoothing elements based on a collective investment are the main reason why TPLI contracts are attractive for loss averse MCPT-investors. This explains why TPLI contracts are even appealing in the case of low guaranteed rates.

Finally, we analyze investors who consider both, annual changes and the terminal value. Figure 5 shows the r^{CE} in the combined model as a functions of the weight $s \in [0, 1]$ that is assigned to annual changes. The results show that even without loss aversion (left panel), a very small value of s ($\approx$ 15%) is sufficient to make contract F more appealing than contract E. For investors with a typical loss aversion of $\lambda = 2$ (right panel), a value of $s \approx 5\%$ is already sufficient. This shows that the subjective utility of annual changes explains the attractiveness of asset smoothing elements even if annual changes only partly influence the investor's subjective utility.

Summarizing, this Section shows that for (loss averse) investors who gain subjective utility and disutility from potential annual changes, return smoothing elements based on a collective investment heavily increase the attractiveness of TPLI contracts. This is even true in case of a significant ex ante collective malus and without guarantee. So far, however, we have analyzed the TPLI contracts in isolation from other common investments choices for retirement savings. Thus, to understand the popularity of TPLI contracts, we additionally need to analyze the preferences of long-term investors under the consideration of other common investment choices. This will be done in the next Section.

5 Explaining the Popularity of TPLI Contracts

In this Section we compare TPLI contracts with common unit-linked products. Note that we do not aim to find the "optimal" investment choice but rather analyze the typical decision problem between a small number of choices which long-term investors are often confronted with (e.g., when consulting a financial advisor for retirement savings).[42] In Section 5.1 we define the unit-linked products. Then, Section 5.2 presents the results under different preference assumptions.

5.1 Unit-linked Product Specification

For all products, we assume an annual premium P paid in advance and a contract duration of T years. Again, A_t denotes the account value at time t and c_t^p the percentage of the premium proportional charges reducing the invested premium which are assumed to be equal as for the TPLI contracts. For unit-linked products additional account proportional charges γ^a are deducted on an annual basis from the account value. These consist of fund charges γ^F and, if applicable, guarantee fees γ^g. For all unit-linked products we set $\gamma^F = 1\%$. Moreover, denote with $Perf_{t,t+1}$ the performance of the underlying investment from t to $t+1$. At the beginning, the account value is $A_0 = P(1 - c_0^p)$. The account value at the end of the year is then derived in two steps: First, all account proportional charges, denoted as γ^a, are deducted from the projected value, that is, $A_{t-} = A_{(t-1)+} Perf_{t,t+1}(1 - \gamma^a)$. Second, if applicable, an annual guarantee or terminal guarantee is taken into account to derive A_t and A_T, respectively. While $t < T$ the account value at the beginning of the next year (after payment of the premium) is

[42] This is in line with studies which show that investors tend to consider rather small samples of investment choices isolated from other choices or future opportunities, cf. Kahneman & Lovallo (1993).

given by $A_{t+} = A_t + P(1 - c_t^p)$.

Unit-linked products without guarantee

The case without guarantee is represented by a *balanced fund* investing a fixed part $\theta \in [0,1]$ in a risky asset and $(1-\theta)$ in a less risky asset. The risky asset is modeled by the stock investment S and the less risky asset by a rolling bond investment R (cf. appendix A for details). We assume daily rebalancing to achieve the desired equity portions.

Unit-linked products with guarantee

In addition to the simple product without guarantee, we consider also different products equipped with a guarantee which ensure that the policyholder receives at maturity at least the accumulated savings premiums[43], that is, $G_T := P\left(\sum_{t=0}^{T-1}(1-c_t^p)\right)$.

Firstly, we consider common types of guarantees offered in the segment of variable annuities (VA). For the sake of simplicity we assume that the VA products implement a suitable hedging strategy to generate the guaranteed amount.[44] To finance the hedging, an account proportional guarantee fee γ^g is charged. The remaining part is invested in an underlying balanced fund with stock ratio $\theta \in [0,1]$. The payoff at maturity of the VA product is given by $A_T = \max(A_{T-}, G_T)$. We only consider fair contracts, that is, we derive the fair guarantee fee[45] γ^g numerically such that the fair value of the embedded option coincides with the present value of the future guarantee fees.[46] Besides the pure money-back VA product we also consider a VA product with an additional annual protection in form of a cliquet-style (year-to-year) guarantee $G_t = d^{pl} \cdot A_{(t-1)+}$ with protection level d^{pl}. We consider products with a protection level d^{pl} of 90% and 98%, that is, the account value cannot decrease by more than 10% or 2%, respectively, within one year. Similar as for the pure-money back guarantee we can derive fair

[43]This is motivated by unit-linked products actually offered in the market, cf. also Graf *et al.* (2012).

[44]In the market there are various different variants of VA products. A complete consideration of all variants would exceed the scope of this paper. Hence, we restrict the analyses to VA products with some basic guarantee features. We refer to Bauer *et al.* (2008) for a detailed description and a framework for valuation of VA products and to `www.annuityfyi.com` for information on types of VA products currently offered in the US market.

[45]Cf. table 5 in appendix A for the fair guarantee fees depending on the underlying balanced fund.

[46]That is, $\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_s ds}\max(G_T - A_T, 0)\right] - \sum_{t=1}^{T}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^t r_s ds}\gamma^g A_{(t-1)+} Perf_{t-1,t}\right] = 0$.

contract fees γ^g. We restrict the analysis to products with reasonable guarantee fees $\gamma^g \leq 1\%$.[47]

Secondly, we consider constant proportion portfolio insurance (CPPI) products which achieve a certain target amount by dynamically investing in riskless and risky assets, cf. Black & Perold (1992). Since continuous rebalancing is not possible in practice, we assume a daily reallocation of the underlying asset structure. In our case we assume that a CPPI product invests at time t a fraction x_t in the risky stock S and the remaining part $y_t = A_{t+} - x_t$ in zero bonds. Moreover, we assume that leveraging more than the current account value is not possible. Each day, the asset allocation for the client's account is determined by $x_t = \max(0, \min(A_{t+}, m(A_{t+} - F_t)))$ and $y_t = A_{t+} - x_t$, where m denotes the multiplier and $(A_{t+} - F_t)$ the cushion with floor F_t. Note that CPPI products without further protection are exposed to shortfall risks, that is, the probability that the account value falls below the target amount exceeds zero.[48] In reality most providers (at least partially) hedge this risk. As we analyze the products from a clients perspective, we refrain from implementing hedging strategies and assume an additional account proportional charge $\gamma^{g,CPPI}$.

Again, we consider two types of guarantees: The first type (pure money-back guarantee) applies a dynamic strategy to pay at least G_T at maturity and invests y_t in zero bonds with maturity T and price $p_t(T)$. The floor is given by $F_t = G_{[t]} \cdot \frac{p_t(T)}{(1-\gamma^a)^{T-[t]}}$ with $G_t = P(1-c_t^p) + \mathbb{1}_{\{1 \leq t \leq T\}} \cdot G_{t-1}$. The second type has an embedded cliquet-style guarantee with an annual guarantee G_t^{Cli} and therefore in each period $[t, t+1)$ for $t \in \{0, \ldots, T-1\}$ invests the amount y_t in zero bonds with maturity $t+1$ and price $p_t([t]+1)$. The floor is then given by $F_t = G_{[t]}^{Cli} \cdot \frac{p_t([t]+1)}{(1-\gamma^a)}$ with $G_t^{Cli} = P(1-c_t^p) + \mathbb{1}_{\{1 \leq t \leq T\}} \cdot A_{[t]}$. We analyze products with multiplier $m = 3$ and additional fee $\gamma^{g,CPPI} = 0.1\%$ as well as $m = 4$ and $\gamma^{g,CPPI} = 0.2\%$.

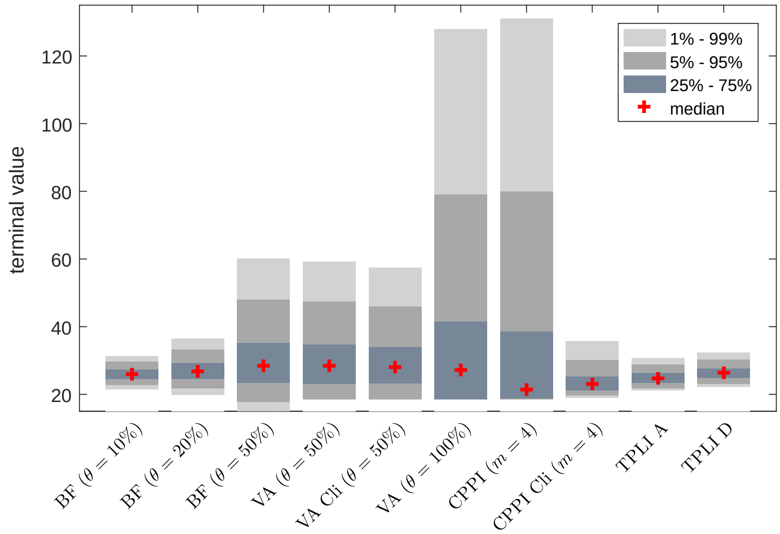


Figure 6: Percentiles of the terminal value of selected unit-linked products and TPLI product A and D.

5.2 Results

Again, we illustrate at first the distributions of the terminal value and the annual changes in the account value of the different product types before analyzing preferences of different investors.

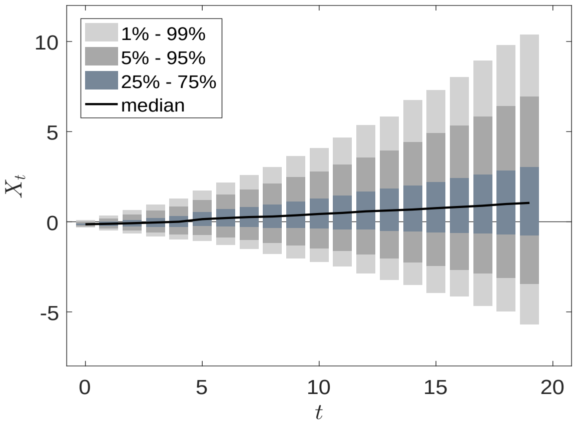
5.2.1 Key Figures

Figure 6 displays the percentiles of the terminal value of selected unit-linked products and the TPLI contracts A and D. The displayed specifications (stock ratio and risk multiplier) have been chosen to illustrate exemplarily the distributions of the terminal value for different unit-linked products.[49] The results show that compared to most unit-linked products the upside potential of the TPLI contracts is very limited (comparable with the upside potential of unit-linked products investing in a low-risk balanced fund with stock ratio $\approx 10\%$). However, products with a higher upside potential perform significantly worse in bad scenarios. Especially for the CPPI products it can be observed that the distributions are very right-skewed and there is a rather large probability that the terminal value is only the guarantee, cf. Graf *et al.* (2012).

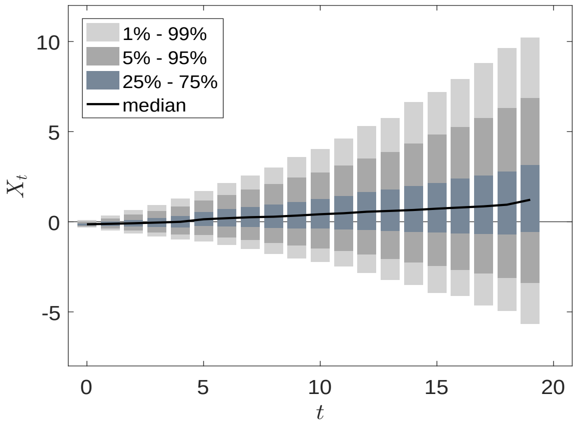
[47]Consequently, we only allow stock ratios $\theta \in [0, 0.6]$ for $d^{pl} = 90\%$ and $\theta \in [0, 0.1]$ for $d^{pl} = 98\%$. All fair guarantee fees are provided in table 6 in appendix B.

[48]This includes overnight risk, that is, the risky asset loses more than $\frac{1}{m}$ during one period, as well as the risk of a changing floor due to interest rate changes.

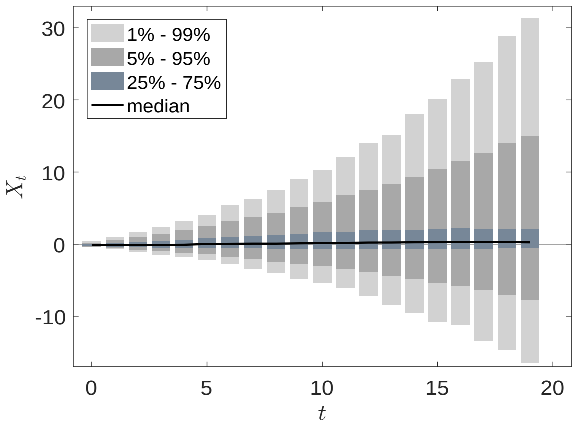
[49]Note that Figure 6 displays only a small sample of the analyzed product specifications to illustrate the most important differences between the distributions of the terminal value of the different product types.



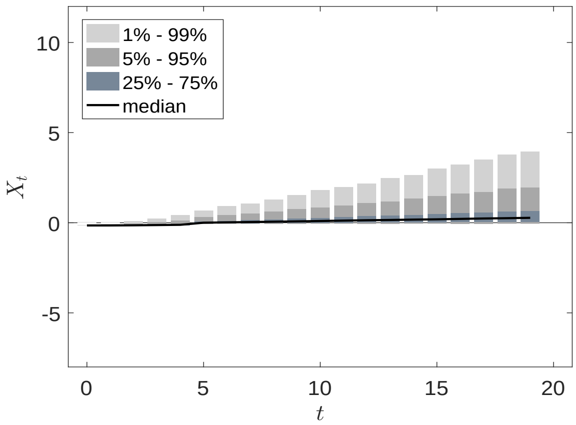
(a) Balanced fund ($\theta = 50\%$)



(b) VA money-back ($\theta = 50\%$)



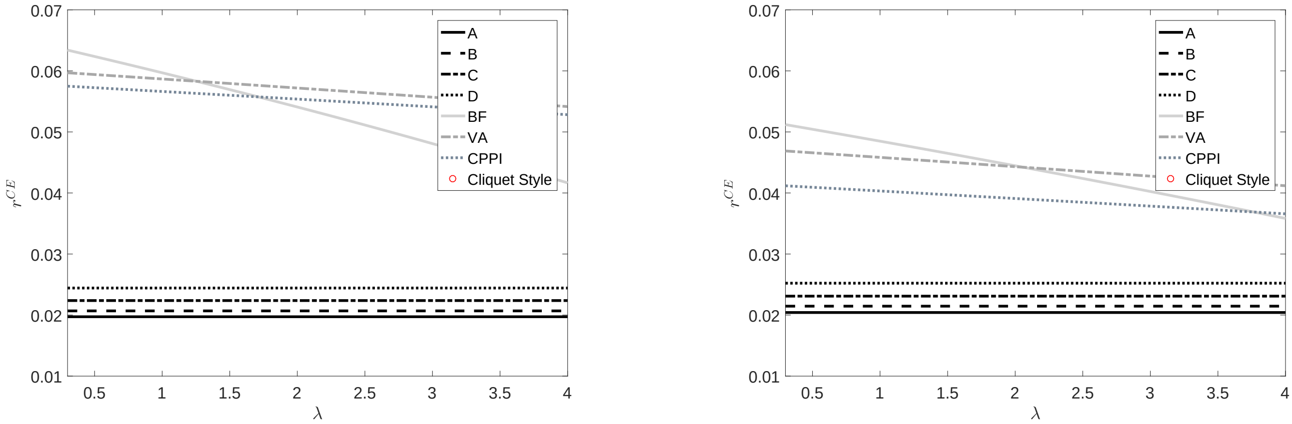
(c) CPPI money-back ($m = 4$)



(d) CPPI Cliquet ($m = 4$)

Figure 7: Percentiles of the annual changes of selected unit-linked products (different scale in panel (c)).

Figure 7 shows the percentiles of the annual changes in the account value of selected unit-linked products. The upper left panel illustrates that balanced funds have a significant risk for rather high annual losses (the higher the stock ratio the higher the risk). The upper right panel shows that the VA money-back guarantee only slightly changes the distributions of the annual changes. In particular, the risk for rather high annual losses is virtually identical as for the underlying balanced fund. Further analyses show that also the VA Cliquet products have similar distributions but with significantly lighter tails. The CPPI money-back product shows very extreme annual changes with high up- and downside potential (lower left panel). The CPPI Cliquet product generates almost no annual losses. In contrast to the TPLI contracts, the distribution of the annual changes of the CPPI Cliquet product are more right-skewed (higher upside potential, but also significantly lower median which is very close to zero).



(a) $a = 0.88$ and $\gamma = 0.65$

(b) $a = 0.88$ and $\gamma = 1$

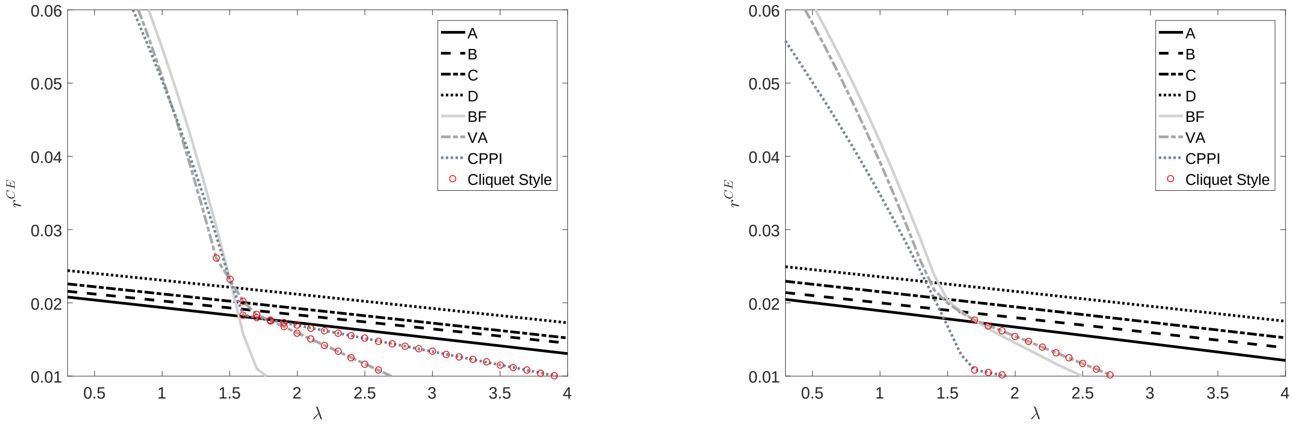
Figure 8: r^{CE} in the CPT case depended on λ with (left panel) and without (right panel) probability distortion. Plot displays for unit-linked products the maximum r^{CE} of all considered parameter settings (stock ratio and risk multiplier).

5.2.2 CPT and MCPT Analysis

Similar to Section 4.2 we analyze the preferences of different investors under the same parameter setting assumptions to compare common unit-linked products with the TPLI contracts A–D.

Figure 8 shows the certainty equivalent returns r^{CE} as a function of loss aversion (λ) for a CPT-investor with (left panel) and without (right panel) probability distortion. The results show that all unit-linked products dominate the TPLI contracts. Hence, investors who solely focus on the terminal value prefer unit-linked products in all cases. The most attractive type is either a balanced fund or a VA product with pure money-back guarantee. In particular, typical CPT-investors (with loss aversion $\lambda \approx 2$ and probability distortion $\gamma = 0.65$ as displayed in the left panel) prefer the pure money back VA product. This confirms existing results for CPT-investors, cf., e.g., Ebert *et al.* (2012), who show that CPT cannot explain the popularity of products with cliquet-style guarantees as in the case of TPLI contracts.

Figure 9 shows the results for an MCPT-investor. The upper left panel shows that for a typical MCPT investor (with loss aversion $\lambda \approx 2$ and probability distortion $\gamma = 0.65$ as displayed in the left panel), all considered TPLI contracts are preferred over all other products. The TPLI



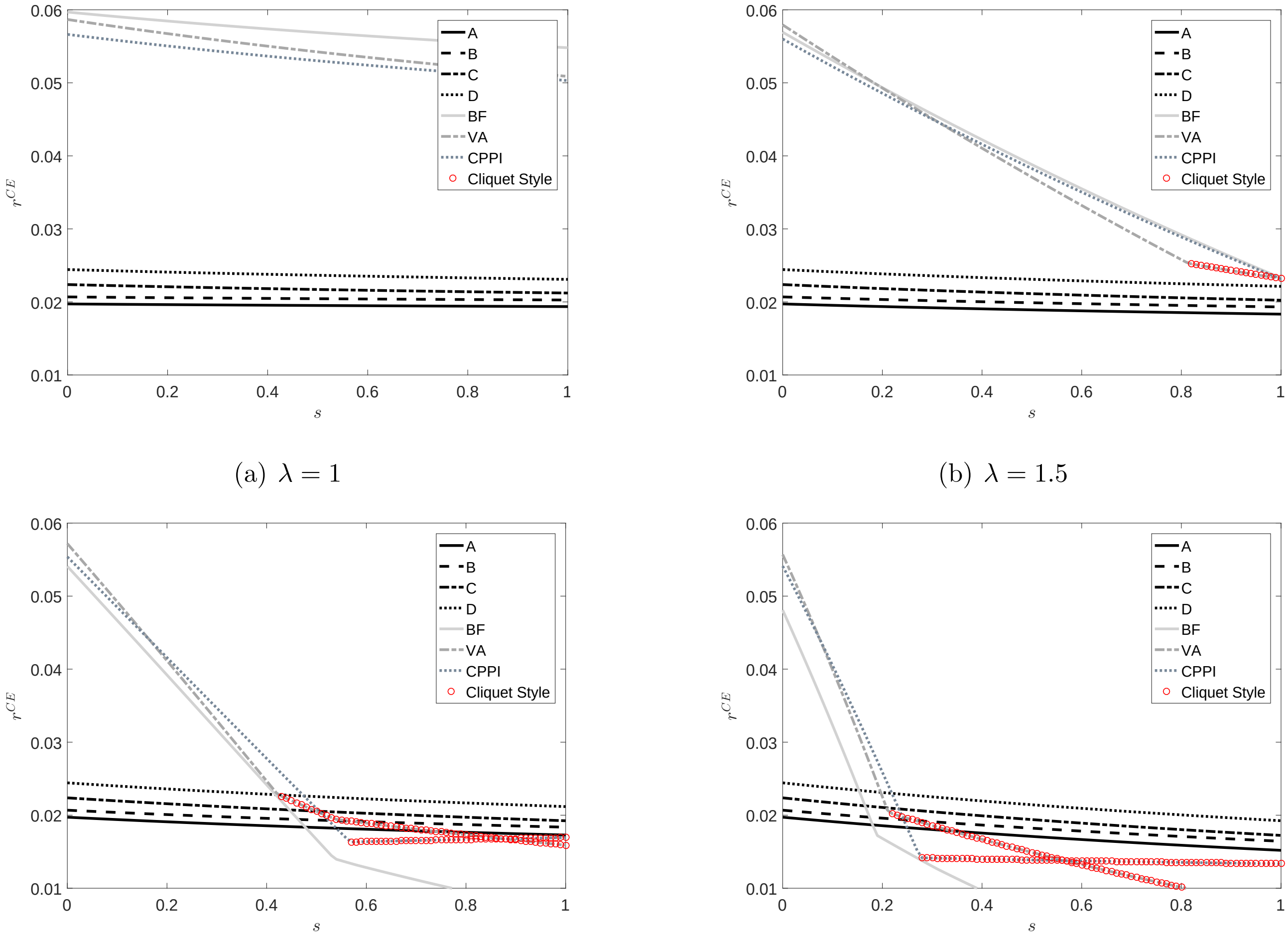
(a) $a = 0.88$ and $\gamma = 0.65$ (b) $a = 0.88$ and $\gamma = 1$

Figure 9: r^{CE} in the $MCPT$ case depended on λ with (left panel) and without (right panel) probability distortion. Plot displays for unit-linked products the maximum r^{CE} of all considered parameter settings (stock ratio and risk multiplier).

contract is even preferred in the case of a significant collective malus (contract A). Comparing the two panels illustrates the different impact of the probability distortion on the r^{CE} for the different product types. While for the TPLI contracts probability distortion has almost no impact (almost no annual losses and only moderate upside potential), for the balanced fund and the VA products we find that the r^{CE} is heavily reduced in combination with loss aversion. This is due to overweighting of small probabilities of rather high annual losses. Interestingly, for the CPPI products we find that probability distortion significantly increases the r^{CE} due to the very right-skewed distributions (very low probability events with very high gains). The effect is particularly strong in case of a cliquet-style guarantee due to the limited losses. Overall, the results show that the consideration of the subjective utility of the annual changes in the form of the MCPT is able to explain the popularity of TPLI contracts.[50]

Figure 10 shows the r^{CE} in the combined model as a functions of the weight s that is assigned to annual changes. The results show that typical loss averse MCPT investors ($\lambda = 2$) prefer TPLI contracts with a moderate collective malus over other products if the weight assigned to the annual changes is above roughly 50% (lower left panel). For $s > 80\%$ all TPLI contracts,

[50]It is worth noting that for unit-linked products we can also confirm the result of Ruß & Schelling (2018) within this more realistic framework, that is, for most loss-averse MCPT-investors products with a cliquet-style (year-to-year) guarantee are more attractive than products without or with a terminal guarantee only.



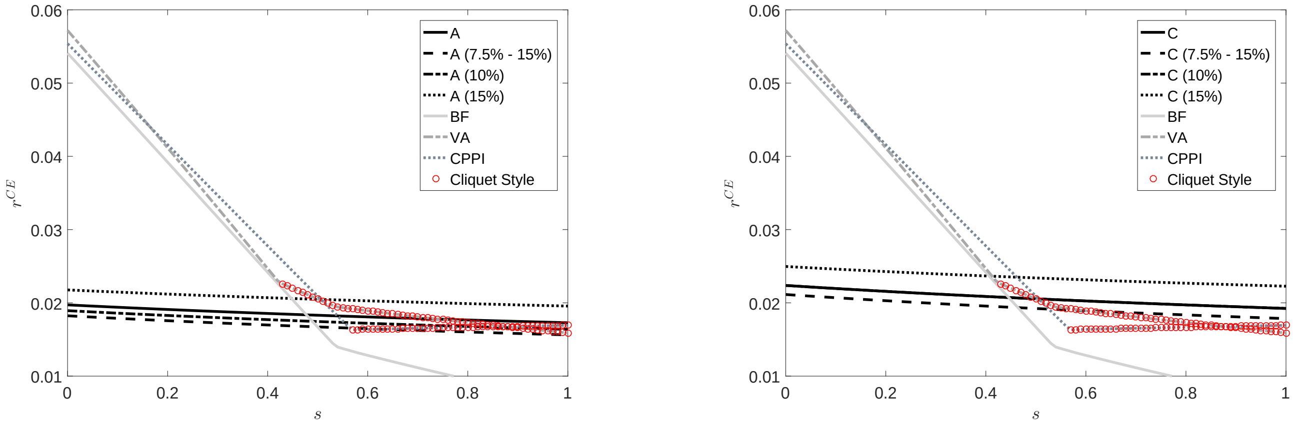
(a) $\lambda = 1$ (b) $\lambda = 1.5$

(c) $\lambda = 2$ (d) $\lambda = 3$

Figure 10: r^{CE} in the CPT^{com} case depended on s for different settings of λ and $\gamma = 0.65$ and $a = 0.88$. Plot displays for unit-linked products the maximum r^{CE} of all considered parameter settings (stock ratio and risk multiplier).

that is, even with a significant bonus malus, are preferred over all other products. The other panels illustrate the impact of loss aversion. Without loss aversion (upper left panel) and even in case of a low loss aversion of $\lambda = 1.5$ the results show that TPLI contracts are less appealing than alternative products. Conversely, for MCPT investors with a rather high loss aversion of $\lambda = 3$ a rather small weight $s \approx 35\%$ is sufficient to make the TPLI contracts more appealing than the alternative products.

Last, we discuss some of the assumptions. We have tried to chose all parameters carefully such that the analyzed products are modeled as realistic as possible. However, in particular the numerous management rules of the life insurer for TPLI contracts allow for a large degree



(a) TPLI A case (b) TPLI C case

Figure 11: r^{CE} in the CPT^{com} case depended on s for $\lambda = 2,\ \gamma = 0.65$ and $a = 0.88$ and different settings of the stock ratio in case of TPLI contract A (left panel) and C (right panel). Plot displays for unit-linked products the maximum r^{CE} of all considered parameter settings (stock ratio and risk multiplier).

of freedom. The consideration of TPLI contracts A–D as well as the fictitious contracts E and F in Section 4 provide insights into the impact of some of the key aspects (reserve ratio, guaranteed rates of the insurance portfolio, smoothing and risk sharing elements). Another important aspect is the asset allocation of the insurer. In all cases we have assumed that the corridor for the stock ratio for the stochastic (future) projection is given by $[10\%, 17.5\%]$.[51] To analyze the impact of the asset allocation we also consider the results for lower corridor with $q_{min} = 7.5\%$ and $q_{max} = 15\%$, that is, we assume that the corridor is equal to the corridor used for the deterministic (past) scenario.[52] Moreover, we also consider the cases that the stock ratio in the stochastic (future) projection is held constant at 10% and 15%, respectively.[53] Figure 11 displays the results in the combined model with loss aversion $\lambda = 2$ for TPLI contract A (left panel) and contract C (right panel) subject to the adjusted stock ratios (all other assumptions being equal) as well as for the unit-linked products. The results illustrate that a slighly higher (lower) stock ratio of the underlying collective investment is perceived as more (less) attractive by a long-term investor. Moreover, we find that in the case of a lower stock ratio in combination

[51] In case of contract A (C) the average stock ratio of the insurer (at the beginning of the year) over all scenarios is 11.08% (10%).

[52] In case of contract A (C), the average stock ratio of the insurer (at the beginning of the year) over all scenarios is 8.61% (7.5%).

[53] Note that the PVFP of the insurer remains positive in all cases.

with a significant bonus malus (TPLI contract A)[54], unit-linked products with a clique-style guarantee are slightly preferred. In case of a higher stock ratio the TPLI contact A is again preferred for $s \geq 0.5$. Moreover, if the contract is only exposed to a moderate collective malus (TPLI contract C), we find that even in the case of a lower stock ratio the TPLI contract is preferred if the weight assigned to the annual changes is above 50% - 60%.

Summarizing, we have shown that MCPT can explain the popularity of TPLI contracts and that this remains true even if annual changes only partly impact the investor's subjective utility.

6 Conclusion and Outlook

In this paper, we have analyzed smoothing and risk sharing elements provided by life insurers from a long-term investor's perspective. We have also considered various unit-linked products to analyze the popularity of TPLI contracts compared to other common investment choices.

We have shown that return smoothing elements based on a collective investment of a life insurer can heavily stabilize annual returns without significantly changing the risk-return characteristics of the terminal value compared to an unsmoothed investment in the same assets. However, the results under CPT show that investors who focus solely on the terminal value prefer the unsmoothed investment. This and other existing results cast doubt that CPT applied in its standard form describes actual decision making of long-term investors sufficiently. In contrast to CPT-investors, MCPT-investors also gain utility from potential annual changes in the account value. For these investors products with smoothed returns are highly attractive. Moreover, for MCPT-investors TPLI contracts with smoothing and risk sharing elements are typically more attractive than common unit-linked products (with and without embedded guarantees) and this is also true in the case of a (moderate) ex-ante collective malus and even if the subjective utility is only partly influenced by potential annual changes. Hence, in contrast to

[54]Note that the collective malus depends on the stock ratio. $CB^{\%}$ is for contract A between -5.7% (15% stock ratio) and -6.7% (10% stock ratio) and for contract C between -1.4% (15% stock ratio) and -2.6% (stock ratio between 7.5% and 15%).

standard approaches, MCPT is able to explain the preference of many long-term investors for smoothed returns and the popularity of TPLI contracts. Combined with the results from Ruß & Schelling (2018) and Graf *et al.* (2018), this gives strong evidence that long-term investors consider potential annual changes already when making the investment decision and that this has an important impact on long-term investment choices, in particular, in the segment of retirement savings.

Understanding the drivers of actual decision making is essential to design products which fit the needs and are at the same time attractive for costumers. The results in this paper show that smoothing and risk sharing elements provided by life insurers are highly attractive for long-term investors while at the same time provide the investor with a terminal value that is very similar to an unsmoothed investment in the same assets. However, high year-to-year guaranteed rates force life insurers to invest in low-risk investments which is rather suboptimal for long-term investors with regard to the terminal value. The findings presented in this paper strongly indicate that participating products which make use of smoothing and risk sharing elements of a collective investment without or with rather low guaranteed rates (e.g., applied at maturity only) seem very promising in providing an objectively superior distribution of terminal value while at the same subjectively being attractive for the customer (as well as for the insurer due to the reduced risk, cf. Reuß *et al.* (2015)).[55]

While MCPT provides an explanation for the popularity of many long-term investment products, the decision making process of long-term investors is not yet fully understood. MCPT is based on the assumption that long-term investors (consciously or subconsciously) already consider future utility or disutility stemming from interim changes when making the investment decision. Future experimental and empirical studies are necessary to improve our understanding of this assumption. Further, future research should address how we can help long-term investors to make better decisions to improve their retirement savings and to ensure a desired

[55]It is noteworthy that providing costumers with appropriate information on these elements is essential. Participating products should therefore ideally be based on more transparent management rules for smoothing and risk sharing which are more easily to communicate to costumers.

standard of living in old age.

A Appendix - Financial Market Model

For the purpose of pricing, we consider a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})$ on a finite time horizon $[0, T]$, $T < \infty$ under the risk-neutral measure $\mathbb{Q}$ satisfying the usual conditions with $\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq T}$ and $\mathcal{F}_t$ the σ-algebra containing the available information at time t. The financial market model is given by a stock process S following a geometric Brownian motion and a short rate process r described by the Vasicek model, cf. Vasicek (1977). More precisely, $dS_t^{\mathbb{Q}} = S_t^{\mathbb{Q}} \left(r_t^{\mathbb{Q}} dt + \sigma_S d\widetilde{W}_t^S \right)$ and $dr_t^{\mathbb{Q}} = \kappa(\xi^{\mathbb{Q}} - r_t^{\mathbb{Q}})dt + \sigma_r d\widetilde{W}_t^r$ with $\sigma_S,\ \kappa,\ \xi^{\mathbb{Q}},\ \sigma_r > 0$ and $d\widetilde{W}_t^S d\widetilde{W}_t^r = \rho \in [-1, 1]$. Furthermore, we define a rolling bond investment R based on zero bonds with term to maturity $T_B < \infty$ years. The dynamic is given by $dR_t^{\mathbb{Q}} = R_t^{\mathbb{Q}} \left(r_t^{\mathbb{Q}} dt - \sigma_r B(t, t + T_B) d\widetilde{W}_t^r \right)$ with $B(t, t + T_B) = \frac{1}{\kappa} \left(1 - e^{-\kappa T_B} \right)$.[56]

The dynamics under the real word measure $\mathbb{P}$ are given by $dS_t^{\mathbb{P}} = S_t^{\mathbb{P}} \left((r_t^{\mathbb{P}} + \lambda_S)dt + \sigma_S dW_t^S \right)$ with constant risk premium $\lambda_S > 0$, $dr_t^{\mathbb{P}} = \kappa(\xi^{\mathbb{P}} - r_t^{\mathbb{P}})dt + \sigma_r dW_t^r$ with $\xi^{\mathbb{P}} = \xi^{\mathbb{Q}} + \frac{\sigma_r \lambda_r}{\kappa}$ and λ_r the price of the interest risk, and $dR_t^{\mathbb{P}} = R_t^{\mathbb{P}} \left((r_t^{\mathbb{P}} - \lambda_r \sigma_r B(t, t + T_B))dt - \sigma_r B(t, t + T_B) dW_t^r \right)$. Moreover, $dW_t^S = d\widetilde{W}_t^S - \frac{\lambda_S}{\sigma_S} dt$ and $dW_t^r = d\widetilde{W}_t^r - \lambda_r dt$ and therefore $dW_t^S dW_t^r = \rho$. $e^{-\int_0^r r_u du}$ is used as numeraire.

The parameters have been chosen in accordance with the European money market and recent literature (cf. Graf *et al.* (2011) or Hieber *et al.* (2016)). More precisely, we assume $\sigma_S = 20\%$, $\sigma_r = 1.5\%$, $\lambda_r = -23\%$, $\kappa = 30\%$, $\rho = 15\%$ and mean-reversion level $\xi^{\mathbb{Q}} = 4.2\%$ (and therefore $\xi^{\mathbb{P}} = 3.05\%$). Moreover, the risk premium is $\lambda_S = 4\%$.[57] Due to the current low interest rate environment we use a negative initial short rate $r_0 = -0.06\%$.[58] Further, we use $T_B = 10$ for the rolling bond investment. Additionally, Table 5 and 6 display the fair

[56] We can derive closed formulas of the processes and the zero bond prices, cf., e.g., Brigo & Mercurio (2007).

[57] This value is also used by the German product contact point for old-age provision (Produktinformationsstelle Altersvorsorge) to generate legally prescribed risk-return profiles for old-age provision products, cf. PIA (2016).

[58] The value of r_0 has been chosen to match the average value of the three-month EURIBOR rates of the last 6 months of 2015, cf. Deutsche Bundesbank (2016).

θ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
γ^g (%)	0.000	0.000	0.001	0.015	0.050	0.107	0.182	0.269	0.365	0.467	0.573

Table 5: Pure money back VA fair guarantee fees γ^g rounded to three decimals for different stock ratios θ.

θ	0	0.1	0.2	0.3	0.4	0.5	0.6
γ^g (%) for $d^{pl} = 90\%$	0.001	0.007	0.006	0.052	0.199	0.491	0.941
γ^g (%) for $d^{pl} = 98\%$	0.334	0.491	-	-	-	-	-

Table 6: VA cliquet-style fair guarantee fees γ^g rounded to three decimals for protection levels d^{pl} and different stock ratios θ.

guarantee fees γ^g for the VA products derived under the risk-neutral measure $\mathbb{Q}$.

B Appendix - Life Insurance Figures

Table 7 shows selected values of the deterministic scenario based on historical data from Deutsche Bundesbank (2016) and DAV (2017) which was used to derive the initial insurance portfolio of the insurance company.

Table 8 gives an overview of key figures of the profits of the ficticious insurance company resulting from the stochastic simulation in the different settings (A–D). In the base case A the present value of the insurer's future profits[59] amounts to $PVFP_0 = \sum_{t=1}^{20} \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_0^t r_u du} IR_t \right] = 3{,}358$ € and the average insurer's future profit per year is given by $\overline{IR}_t = 462$ €. This indicates that the future business of the insurance company is on average profitable. The Value-at-Risk (99.5%) of the insurer's future profits IR_t is -7,948 € which is -4.03% of the corresponding balance sheet total BS_t. The maximal loss amounts to -19,246 € which is -10.31% of the corresponding balance sheet total BS_t. Further, we can observe the asymmetry of the surplus distribution: the average loss (-3,477 €) that has to be borne by the insurer is higher than the average gain (703 €). However, the probability that IR_t becomes negative is only 5.77%.

[59] Cf., e.g., Burkhart *et al.* (2015) for details.

year t	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20
1Y spot rate	4.07	4.64	7.09	8.68	8.90	8.63	6.27	5.48	4.60	3.36
10Y spot rate	6.48	6.82	7.02	8.8	8.31	7.62	6.48	7.21	7.12	6.57
coupon rate		6.61	7.03	8.82	8.40	7.71	6.40	7.06	6.90	6.30
i_t^g		3.50	3.50	3.50	3.50	3.50	3.50	3.50	4.00	4.00
stock perf.		32.79	34.83	-21.90	-12.86	-2.09	46.71	-7.06	6.99	28.17
year t	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
1Y spot rate	3.51	3.58	3.10	4.69	3.98	3.39	2.26	2.22	2.27	3.34
10Y spot rate	5.9	4.73	4.76	5.35	4.98	4.92	4.29	4.23	3.45	3.87
coupon rate	5.71	4.65	4.65	5.31	4.90	4.82	4.16	4.11	3.39	3.84
i_t^g	4.00	4.00	4.00	4.00	3.25	3.25	3.25	2.75	2.75	2.75
stock perf.	47.11	17.71	39.10	-7.54	-19.79	-43.94	37.08	7.33	27.07	21.98
year t	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
1Y spot rate	4.12	3.47	0.88	0.53	0.86	0.01	0.05	0.02	-0.28	-
10Y spot rate	4.31	4.16	3.59	2.97	2.84	1.65	1.69	1.23	0.53	-
coupon rate	4.29	4.10	3.46	2.86	2.76	1.59	1.63	1.20	0.52	-
i_t^g	2.25	2.25	2.25	2.25	2.25	1.75	1.75	1.75	1.25	1.25
stock perf.	22.29	-40.37	23.85	16.06	-14.69	29.06	25.48	2.65	9.56	-

Table 7: Selected values (% p.a.) of the deterministic scenario based on historical data from Deutsche Bundesbank (2016) and DAV (2017).

A higher (terminal) reserve ratio (case B) can be used to offset moderate losses which is reflected e.g., in a lower probability for losses (3.93%) and a higher $PVFP_0 = 5{,}281$ €. If additionally the average guaranteed rates of the insurance portfolio are lower (case C) then there are almost no losses which have to be borne by the insurer (only 0.11% of the profits are negative). The $PVFP_0$ is in this case significantly higher (10,405 €). Increasing the surplus participation in this setting to $\alpha^{Sp} = 97\%$ (case D) reduces the $PVFP_0$ to 3,178 € but (virtually) without increasing the probability of losses nor the size of losses. In total, the results show that the fictitious life insurer is in none of the considered cases A–D exposed to excessive (unrealistic) losses and that the future business is in all cases on average profitable.

case	$\min(IR_t)$	$VaR_{99.5\%}(IR_t)$	$\overline{IR}_t$	$\max(IR_t)$	$PVFP_0$
A	-19,246 €	-7,948 €	462 €	4,249 €	3.358 €
B	-19,690 €	-7,181 €	546 €	4,314 €	5,281 €
C	-7,295 €	0 €	740 €	4,335 €	10,405 €
D	-7,384 €	0 €	226 €	1,349 €	3,178 €
case	$\min\left(\frac{IR_t}{BS_t}\right)$	$VaR_{99.5\%}\left(\frac{IR_t}{BS_t}\right)$	$\mathbb{P}\left(IR_t < 0\right)$	$\mathbb{E}\left[IR_t \vert IR_t < 0\right]$	$\mathbb{E}\left[IR_t \vert IR_t > 0\right]$
A	-10.31%	-4.03%	5.77%	-3.477 €	703 €
B	-10.64%	-3.34%	3.93%	-3.287 €	702 €
C	-3.93%	0%	0.11%	-1.494 €	742 €
D	-3.94%	0%	0.08%	-1.455 €	228 €

Table 8: Key figures of insurer's future profits in the cases A–D.

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