When and How Framing Makes Annuitization Appealing: A Model-Based Analysis

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Abstract

Several experimental studies provide evidence that annuities are much more appealing under a consumption frame than under an investment frame. However, due to the complexity of the annuitization decision, the drivers of this result and their interaction are not yet fully clear. We consider a theoretical model to analyze the impact of various determinants. The results suggest that the main driver are the different reference points. Partial annuitization seems attractive under a consumption frame in most cases if the subjective life expectancy is not significantly shorter than the objective average life expectancy and if the aspired standard of living is not already covered by other sources of regular income. However, the impact of other behavioral aspects like loss aversion or subjective risk perception vary for different levels of wealth and regular incomes.

Keywords: Annuities, Framing, Mental Accounting, Prospect Theory, Reference Points, Retirement Savings

JEL: D14, G11, G22, G41, J26, J32

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1 Introduction

The idea of a life annuity is to provide a fixed stream of income for the rest of the life. Life annuities therefore protect against the risk of running out of money in old age, which is called longevity risk. Starting with Yaari (1965) and Fischer (1973), numerous authors incorporated longevity risk in life-cycle models of saving and consumption under standard economic assumptions. While early studies are based on several simplifying assumptions like no bequest motive, complete markets, actuarially fair annuities, etc., more recent studies like Davidoff *et al.* (2005) or Peijnenburg *et al.* (2016) examine the annuitization problem also under more realistic assumptions. The results of the vast majority of this literature shows that risk-averse utility maximizers prefer to annuitize significant fractions of their wealth at retirement age. Yet, in reality annuitization rates are often rather low and in particular only few individuals voluntarily purchase life annuities. The discrepancy between theoretically optimal and observed annuitization rates is known as the "Annuity Puzzle".

There is a wide stream of literature exploring possible explanations for this puzzle.[1] Explanations include the role of family risk sharing and the related bequest motive (cf. Brown & Poterba (1999), Post *et al.* (2006) or Lockwood (2012)), the role of preannuitized wealth, social security and real estate property (cf. Bernheim (1991), Mitchell *et al.* (1999) or Dushi & Webb (2004)) as well as incomplete markets, high loadings, asymmetric information and liquidity restrictions (cf., e.g., Friedman & Warshawsky (1990) or Gupta & Li (2007)). The literature shows that all these factors have an important impact on the demand for annuities and are able to explain the lack of full annuitization. Nevertheless, especially for individuals of middle wealth the lack of voluntary annuitzation remains puzzling (cf., e.g., Dushi & Webb (2004), Davidoff *et al.* (2005) or Benartzi *et al.* (2011)). Additionally, against the background of current challenges like the demographic change and the related consequences for social security systems, voluntary annuitization seems increasingly important for this group to maintain an aspired standard of living in old age (cf., e.g., Wilke (2009) for details in Germany).

[1] We refer to Brown (2007), Benartzi *et al.* (2011) and Milevsky (2013) for extensive reviews of the literature.

More recent literature suggests that behavioral aspects are crucial in order to understand the annuity puzzle. The concept of narrow framing[2] suggests that individuals tend to focus on specific investments without considering many other options. Moreover, individuals with this bias tend to lose sight of the big picture, evaluate the investment standalone and overlook for example interactions with other already existing investments and/or the impact of their investment on their future consumption. Studies confirm this by showing that annuities are often considered as a gamble on a long life rather than a protection against longevity risk. By doing so, the risk of an early death and therefore of losing the annuitized wealth dominates the longevity risk (cf. Gazzale & Walker (2009)). Furthermore, behavioral biases like loss aversion with respect to the annuitized wealth (which serves as reference point),[3] time preference, and overweightig of small probabilities suggest that annuitization becomes even less appealing. Under this so-called *investment frame* individuals focus solely on the investment risk and return characteristics and not on maintaining an aspired standard of living. As Hu & Scott (2007) point out, this investment frame provides an explanation for low voluntary annuitization rates.

Results from the field of psychology show that many decisions depend on how choices are presented – the so-called framing effect (cf. Tversky & Kahneman (1981) and Thaler (1985)). By means of an online survey Brown *et al.* (2008) find evidence that annuities are much more appealing when presented under a so-called *consumption frame*, where individuals focus on maintaining an aspired standard of living expressed through consumption. Also, Goedde-Menke *et al.* (2014), who have conducted and analyzed a representative survey among German consumers, as well as Nolte & Langer (2016), who used laboratory experiments, conclude that this framing effect has a strong impact on annuitization. However, Brown *et al.* (2013) provide evidence that even under the consumption frame individuals typically make suboptimal decisions according to standard life-cycle models. These findings raise the followig questions: "How do individuals evaluate annuitization under the consumption frame?" and "What are the impacts of typical behavioral biases like reference points, loss aversion, and subjective probabilities as

[2]Cf. Tversky & Kahneman (1981), Kahneman & Lovallo (1993) or also Benartzi & Thaler (1995).

[3]Loss aversion suggests that individuals are much more sensitive to losses than to gains measured to a reference point (cf. Kahneman & Tversky (1979)).

well as different levels of financial means?". As empirical data is thin, we contribute to this newly emerging literature by analyzing the impact of various determinants within a theoretical model framework. In doing so, we do not analyze whether partial annuitization is optimal from a specific point of view, but rather focus on how individuals actually perceive the annuitization decision under different frames.

The results illustrate that while under an investment frame annuitization is not appealing for most individuals, under a consumption frame partial annuitization is often preferred. We disentangle the impact of different drivers like loss aversion or probability distorion under the consumption frame dependent on the level of liquid wealth and regular income provided by social security. The presented insights improve our understanding of decision making in the context of old-age provision. Moreover, we find that a rather short subjective life expectancy reduces annuitization rates significantly. Therefore, to increase annuitization rates and protection against longevity risk, information on the life expectancy and longevity seems essential.[4]

The remainder of this article is organized as follows. Section 2 describes and motivates the framework and the framing perspectives which includes the investment frame as well as the consumption frame. Moreover, we specify the main model assumptions and the considered parametrization of the model for the numerical analysis. In Section 3 we present the results of the numerical analysis. Section 4 summarizes and gives an outlook for future research.

2 Methodology

The aim of this chapter is to propose a model that attempts to describe how individuals actually perceive and (possibly subconsciously) evaluate the annuitization decision under different frames. We consider an individual at retirement age x at time $t_0 = 0$ who deals with the

[4] This is also closely related to framing issues. For example Payne *et al.* (2013) give experimental evidence that framing strongly affects annuitization by comparing the subjective life expectancies in a "live-to" with a "die-by" setting. The findings show that the subjective life expectancy is significantly higher in the "live-to" frame than in the "die-by" frame.

question of annuitization which is assumed to be a one-time decision at retirement age.[5] The random subjective remaining lifetime is given by $\tau^{sub} \leq \omega$ (in full years) with ω denoting the maximal remaining lifetime. For simplicity, we assume that death is only considered at the end of the year.[6] For the individual's subjective probability of survival from time t_0 to time t, we use the common actuarial notation ${}_tp_x^{sub} := P\left(\tau^{sub} \geq t\right)$ for $t \in \{0, \cdots, \omega\}$.

Moreover, we denote the objective average remaining lifetime by $\tau^{obj} \leq \omega$ and the corresponding objective average probabilities of survival by ${}_tp_x^{obj}$. Differences between the subjective and the objective average probabilities can arise for objective reasons like a better or worse health condition or due to estimation errors caused by cognitive distortions. The actuarially fair annuity factor for a life annuity is then defined by $\ddot{a}_x^* := \sum_{k=0}^{\omega} {}_kp_x^{obj} \cdot P(0,k)$ with $P(s,d)$ denoting the fair price of a zero bond with duration d at time s.[7] We assume that effects of adverse selection and other market incompleteness are covered by an expense factor c^{ann}. Therefore, the applied annuity factor can be calculated by $\ddot{a}_x := (1 + c^{ann}) \cdot \ddot{a}_x^*$. Moreover, we denote the resulting constant annual annuity[8] payment by A^{ann}. We capture effects from inflation by considering an inflation-adjusted model framework.

Focusing on individuals of middle wealth, we assume that future consumption is only determined by income and liquid wealth and hence independent of illiquid assets (which are therefore not explicitly modeled in the framework).[9] More precisely, we denote with W_t the liquid wealth

[5]There is a large literature which analyzes annuitization under settings which allow individuals to adjust annuitization during the whole remaining life-span (cf., e.g., Dushi & Webb (2004), Hainaut & Devolder (2006) or Horneff *et al.* (2008b)). This line of research mainly focuses on optimal investment and annuitization strategies (from a normative perspective). However, in reality most individuals do not adjust their annuitization rate on a regular basis but rather deal with the question of annuitization when approaching the retirement age. As we focus on the question why individuals are attracted by annuities under a consumption frame, the one-time decision problem represents the typical decision problem most individuals are confronted with when approaching retirement age and seems therefore suitable from a descriptive perspective (cf. also Benartzi *et al.* (2011)).

[6]If the remaining lifetime is for example equal to 0, the individual dies at time $t = 1-$.

[7]We use fair prices of zero bonds which are in line with the financial market described in Section 2.3.

[8]If not stated otherwise, we use the term "annuity" for life annuity.

[9]Note that there is large empirical evidence that consumption is mainly driven by income and liquid assets and only minor by other rather illiquid assets, cf. for example Skinner (1996) or also Levin (1998). Also, the results of Venti & Wise (2004) suggest that elderly do typically not plan to use home equity to support general nonhousing consumption. Moreover, they also show that even in bad states, housing equity is largely preserved while other assets are consumed. This is also related to Shefrin & Thaler (1988) and their behavioral life-cycle hypothesis which assumes that due to mental accounting, not all assets are considered as fungible.

of the individual at time t and the annuitization rate at time t_0 by $\theta^{ann} \in [0,1]$. We assume that the remaining liquid wealth is invested in a balanced fund with constant stock ration $\theta^S \in [0,1]$. The fraction invested in the balanced fund is denoted by $\theta^{bal} = 1 - \theta^{ann}$ and the return in period $[t-1, t)$ by R_t^{bal}. Furthermore, we assume that the individual receives predefined regular constant livelong social security benefits A^{soc} at the beginning of each year. The total secure income of the individual at the beginning of the period $[t, t+1)$ is therefore $I_t := A^{soc} + A^{ann}$ for $t \in \left\{0,\ 1, \cdots, \tau^{sub}\right\}$.

As we aim to model and analyze actual decision making, we refrain from deriving consumption patterns that maximize preference functions, but rather restrict the analysis to several consumption plans based on typical recommendations (for example by financial advisers). Hence, we assume that the individual plans the future consumption c_t^{act} at time $t \in \left\{0, \cdots, \tau^{sub}\right\}$ for period $[t, t+1)$ dependent on income and liquid wealth according to the following rule:

$$c_t^{act} := \begin{cases} \min\left(I_t + W_t, c^{mg}\right), & I_t + R_t^{bal} < c^{mg} \text{ and } W_t < k_t\left(c^g - I_t\right)^+ \\ \max\left(c^{mg}, \min\left(I_t + \frac{W_t}{k_t}, c^{max}\right)\right), & \text{else} \end{cases} \quad (1)$$

where c^{mg} represents the minimal consumption goal per year needed to maintain a desired standard of living. This includes basic needs like housing, energy and food, which represents a minimal requirement, as well as expenses for comforts of everyday life (like for a car or for leisure activities). A consumption below this level leads to cuts in the standard of living. c^g represents the aspired consumption goal per year that is sufficient to additionally meet further aspirations (for example traveling during the retirement period). Further, we assume that the individual does not plan to spend more than c^{max} per year. The liquid wealth at time t is determined by $W_t := W_{t-1} + I_{t-1} - c_{t-1}^{act} + R_t^{bal}$ for $t \in \left\{1, \cdots, \tau^{sub}\right\}$. Moreover, k_t denotes an age-depending withdrawal rule which affects the planned spending of the liquid wealth. In the base case we consider the remaining life expectancy rule which is based on the remaining subjective life expectancy of the individual.[10] Consequently, the withdrawal

[10]We consider in Section 3 also the effect of other common withdrawal rules like a fixed rule and a limiting age rule.

rate increases over time with decreasing remaining subjective life expectancy. Additionally, we require that k_t is at least three, which serves as a safety cushion. More precisely, we set $k_t = \max\left(\mathbb{E}\left(\tau^{sub}\middle|\tau^{sub} \geq t\right) - t, 3\right)$. The individual applies the withdrawal rate k_t in case of adequate liquid financial means, that is, the income together with the return from the fund investment is larger than the minimal consumption goal c^{mg}, or the current wealth is sufficient to cover the aspired consumption goal c^g for at least k_t periods. Otherwise, that is, if the income together with the return from the fund investment is less than c^{mg} and the liquid wealth is not sufficient to cover c^g for at least k_t periods, the individual consumes c^{mg} as long as possible to avoid cuts in the standard of living.

2.1 Investment Frame

Under the investment frame we assume that the individual evaluates the annuitization decision solely on the investment risk and return characteristics of the outcome and isolated from implications on the future consumption. Under this frame the individual compares the total outcome of the investment with the lump sum that is invested (in our case W_0). Hence, to model the subjective utility under the investment frame, we assume that the individual considers W_0 as reference point. The return of the outcome with respect to this reference point is defined by[11] $X^i := (1-\theta^{ann}) \cdot W_0 + \sum_{t=0}^{\tau^{sub}} \left(A^{ann} + R_{t+1}^{bal}\right) - W_0 = \sum_{t=0}^{\tau^{sub}} \left(A^{ann} + R_{t+1}^{bal}\right) - \theta^{ann} \cdot W_0$. A positive value of the outcome X^i defines a gain and a negative value a loss.[12] We follow Tversky & Kahneman (1992) and assume that the individual's subjective utility under the investment frame is driven by an S-shaped value function $v(\cdot)$ defined by $v(X^i) := (X^i)^{\alpha} \cdot \mathbb{1}\{X^i > 0\} - \lambda \left|X^i\right|^{\alpha} \cdot \mathbb{1}\{X^i \leq 0\}$, where $\lambda > 0$ is the loss aversion parameter (loss aversion if $\lambda > 1$) and $\alpha > 0$ determines the risk appetite. Finally, we assume that, the individual's preference is based on the outcome X^i evaluated according to Cumulative Prospect Theory (CPT) by

$$V^{CPT}\left(X^i\right) := \int_{-\infty}^{0} v(x) d\left(w\left(F(x)\right)\right) + \int_{0}^{\infty} v(x) d\left(-w\left(1 - F(x)\right)\right) \tag{2}$$

[11] It is worth noting that one can think of various ways on how to define X^i in the investment frame. We refrain from considering discounting etc. by using the most simple definition. The results presented in the later sections qualitatively remain for reasonable other definitions of X^i.

[12] Note that focusing on the case $\theta^{ann} = 1$, can also be interpreted as an isolated evaluation of the annuity product under a narrow frame.

with $F(s) = \mathbb{P}(X \leq s) = \int_{-\infty}^{s} d\mu_X$ with μ_X the probability measure given by the random variable X and $w(\cdot)$ the probability distortion function $w(p) := \frac{p^\gamma}{(p^\gamma+(1-p)^\gamma)^{\frac{1}{\gamma}}}$ with $\gamma \in (0.28, 1]$, where the lower boundary for γ is chosen, such that $w(p)$ is strictly monotonically increasing for $p \in [0, 1]$.[13]

2.2 Consumption Frame

Several studies like Brown *et al.* (2008) suggest that explaining annuities in the context of consumption protection in old age changes subconsciously the reference points and the evaluation. This motivates that under the consumption frame the individual evaluates the subjective utility based on future consumption as described by formula (1), where the minimal and the aspired consumption goal serve as reference points. Several studies suggest that individuals consider multiple reference points like minimal requirements, the status quo, aspirations or goals (cf. Koop & Johnson (2012) or Knoller (2016)). Hence, we assume that the individual is also aware of his or her minimal consumption requirement c^{mr} which is needed to cover the basic needs (for example for housing, energy and food). A consumption below this level leads to harsh cuts in the standard of living. Consequently, following the Tri-Reference Point Theory introduced by Wang & Johnson (2012), we assume that the individual considers three reference points when making the annuitization decision: the minimal consumption requirement c^{mr}, the minimal consumption goal c^{mg} and the aspired consumption goal c^g.

Depended on the future consumption c_t^{act}, the outcome can therefore fall into four different functional regions determined by the three reference points:

- functional region 1 (**full success**: $c_t^{act} \geq c^g$): The individual considers the outcome as full success if future consumption is equal or above c^g. Since this is a region of gains the individual is risk averse.
- functional region 2 (**on target**: $c^{mg} \leq c_t^{act} < c^g$): The individual's consumption is on target if future consumption is between c^{mg} and c^g. However, not reaching c^g as well as

[13] Note that we refrain from a different treatment of gains and losses with respect to probability distortion and that $\gamma = 1$ represents the case without probability distortion.

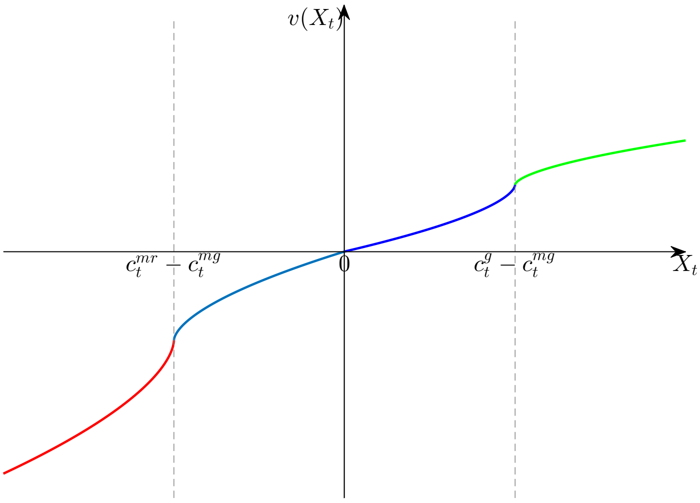


Figure 1: Illustration of a double S-shaped value function.

the cushion with respect to c^{mr} trigger a risk seeking behavior in this region. Moreover, because c^g is not reached, the positive subjective utility is assumed to be slightly reduced by loss aversion with respect to c^g.

- functional region 3 (**below target**: $c^{mr} \leq c_t^{act} < c^{mg}$): The individual considers the outcome as below target if future consumption is above c^{mr} but below c^{mg}. Contrary to region 2, the individual is risk averse in this region because of the small cushion to c^{mr}. Due to the shortfall with respect to c^{mg}, the subjective utility is reduced by loss aversion.

- functional region 4 (**total failure**: $c_t^{act} < c^{mr}$): The individual considers the outcome as a total failure if the future consumption is below c^{mr}. Therefore, the individual is risk-seeking in this region and the subjective utility is heavily reduced by loss aversion because of not reaching c^{mr}.

The four functional regions can be translated in a function of $X_t := (c_t^{act} - c^{mg}) \cdot \mathbb{1}\{t \leq \tau^{sub}\}$ and modeled by the following double S-shaped value function (cf. Figure 1 for an illustration):[14]

$$v^{TRP}(X_t) := \begin{cases} (X_t - (c^g - c^{mg}))^{\alpha} + \lambda_1 (c^g - c^{mg})^{\alpha}, & X_t > c^g - c^{mg}, \\ -\lambda_1 \left(|X_t - (c^g - c^{mg})|^{\alpha} - (c^g - c^{mg})^{\alpha}\right), & 0 < X_t \le c^g - c^{mg} \\ \lambda_2 \left(|X_t + (c^{mg} - c^{mr})|^{\alpha} - |c^{mg} - c^{mr}|^{\alpha}\right), & c^{mr} - c^{mg} < X_t \le 0 \\ -\lambda_3 \, |X_t + (c^{mg} - c^{mr})|^{\alpha} - \lambda_2 \, |c^{mg} - c^{mr}|^{\alpha}, & X_t \le c^{mr} - c^{mg} \end{cases} \quad (3)$$

for $t \le \omega$ and with $\lambda_1,\ \lambda_2$ and $\lambda_3 > 0$ denoting the loss aversion parameters with respect to the corresponding consumption levels and $\alpha > 0$ the risk appetite parameter. It is worth noting that the three loss aversion parameters capture the different impact of the loss aversion in the different functional regions. We assume that the impact of the loss aversion increases from region 2 to region 4, that is, we require $\lambda_1 \le \lambda_2 \le \lambda_3$.

Similar to the different consumption levels, we assume that the individual considers also three levels of bequest: b^g denotes the aspired bequest goal, b^{mg} the minimal bequest goal, and b^{mr} the minimal bequest requirement. The subjective utility of bequest is assumed to be based on the liquid wealth at time of death,[15] that is, $W_{(\tau^{sub}+1)-} := W_{\tau^{sub}} + I_{\tau^{sub}} - c^{act}_{\tau^{sub}} + R^{bal}_{\tau^{sub}}$. Again, we model the subjective utility by a double S-shaped value function $v^{TRP}(\cdot)$ with corresponding reference points. The considered bequest outcome is defined by $X_t^{beq} := \min\left(W_{(\tau^{sub}+1)-} - b^{mg}, b^{max}\right) \cdot \mathbb{1}\{t = \tau^{sub}\}$. We assume that bequest generates only additional subjective utility until a certain threshold b^{max} which represents the maximal planned amount of bequest. This is motivated by studies which indicate that many (and in particular high liquid) bequests are not on purpose (cf. Hurd (1989) or Benartzi *et al.* (2011)).

In total, based on the double S-shaped value functions the preference function for the annuitization decision with outcome $X^{con} = \left\{X_0,\ \cdots,\ X_{\omega},\ X_0^{beq},\ \cdots,\ X_{\omega}^{beq}\right\}$ is then determined

[14]This value function is also closely related to the value function proposed by Tversky & Kahneman (1992). Note that by setting $c^g = c^{mg} = c^{mr}$ the value function reduces to the typical S-shaped value function used in CPT with loss aversion λ_3.

[15]Note that we have assumed that death occurs at the end of the year.

by

$$V^{con}\left(X^{con}\right) := \sum_{t=0}^{\omega} \rho^{t} \cdot \left((1-s) \cdot V^{TRP}\left(X_t\right) + s \cdot \rho \cdot V^{TRP}\left(X_t^{beq}\right) \right) \tag{4}$$

with

$$V^{TRP}\left(X\right) := \int_{-\infty}^{0} v^{TRP}(x) d\left(w\left(F(x)\right)\right) + \int_{0}^{\infty} v^{TRP}(x) d\left(-w\left(1-F(x)\right)\right),$$

where $s \in [0, 0.5]$ controls the impact of the bequest motive.[16] We assume that the probability distortion function $w(\cdot)$ is the same as in the CPT case. Moreover, ρ denotes a time preference discounting factor which captures the subjective time preference of the individual.

2.3 Model Assumptions and Parametrization

Financial Market Model

The financial market model is based on a stock process S described by a geometric Brownian motion and a short rate process r described by the Vasicek model (cf. Black & Scholes (1973) and Vasicek (1977)). The parameters have been chosen in accordance with the European money market and recent literature (cf. appendix A for more details). In the presented base case we restrict our analysis to a balanced fund[17] with a stock ration $\theta^S = 60\%$. The fraction of the balanced fund is chosen to be in line with typical "rules of thumb" often recommended by financial advisers (cf. for example Polyak (2005) or Whitaker (2005)). It therefore appears reasonable that many individuals consider stock ratios in this magnitude when comparing an annuity product with a withdrawal plan based on an investment fund.[18] The expected inflation-adjusted return of the considered balanced fund is 3.3% with standard deviation 12.5% (in the

[16] Note that we only consider values of s up to 0.5. Values above 0.5 imply that the bequest motive dominates the consumption motive and seem therefore not reasonable under a consumption frame. In fact, under a consumption frame rather small values of s seem appropriate for most individuals. Further, note that our analysis is restricted to liquid wealth and that other illiquid assets can also meet the bequest motive.

[17] The balanced fund invests a constant fraction θ^S in a stock investment and the remaining part in a pension fund based on a "rolling" zero bond investment. We apply daily rebalancing.

[18] Moreover, behavioral heuristics like mental accounting (Thaler (1985)) indicate that individuals do not compare the whole variety of possible investment choices but rather focus on small samples.

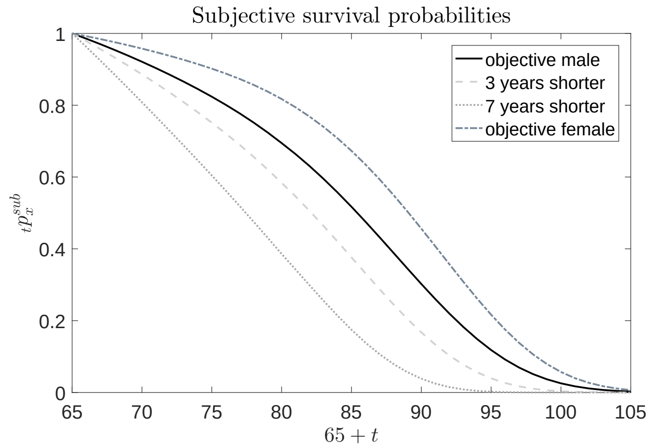


Figure 2: Subjective survival probabilities $_tp_x^{sub}$ for an individual aged 65 years.

long run).[19]

Survival Probabilities and Annuity Factor

We use objective average survival probabilities $_tp_x^{obj}$ with $x = 65$ based on the German Federal Statistical Office's cohort mortality tables with trend (V2) for males[20] born in the year 1952 (cf. Federal Statistical Office (2017)). For the simulation we consider $\omega = 120$. The mortality tables of the Federal Statistical Office have a cut-off age of 100 years. Hence, we extrapolated the mortality tables until an age of 120 years using a Kannisto model approach starting at an age of 80 years (cf. Wilmoth *et al.* (2007)).

To analyze the question of how the subjective survival probabilities influence the annuitization decision, we consider different specifications of $_tp_x^{sub}$ (cf. Figure 2). In the base case, we assume that the subjective survival probabilities correspond with the objective average survival probabilities for males, that is, $_tp_x^{sub} = {_tp_x^{obj}}$. Additionally, we consider lower and higher subjective survival probabilities. This can be due to better (or worse) than average health conditions or other objective reasons, but also due to behavioral biases like the anchor effect. In this context

[19]It is noted that the results presented in Section 3 depend on the parametrizations of the financial market and the choice of the balanced fund. However, numerous sensitivity analyses show that under reasonable assumptions, the structure of the results and the described impacts remain very similar (cf. also Section 3).

[20]On average male individuals have a shorter remaining life expectancy than females, that is, the longevity risk is lower. In the European Union unisex annuity rates are applied for females and males as a result of the European Union Gender Directive (cf. Council Directive 2004/113/EC described in European Union (2004)) prohibiting any gender-based discrimination. This implies that under otherwise identical conditions annuities are on average more attractive for females than for males.

the anchor effect suggests that many individuals use the age at death of the generation of their parents or grandparents as an anchor when estimating their own remaining lifetime. By doing so, the individuals do not account for the fact that the life expectancy has increased steadily in the last century (cf., e.g., Oeppen & Vaupel (2002)) and tend to underestimate their life expectancy (cf. Bucher-Koenen & Kluth (2012)).[21] We reduce subjective survival probabilities by multiplying the objective average probabilities of death of the corresponding cohort by a factor, such that the subjective life expectancy at age 65 is exactly 3 years (respectively 7) shorter than the objective average life expectancy (which is 19 years or age 84).[22] To model individuals with a longer subjective life expectancy (for example particularly healthy or female individuals), we consider the case where the subjective survival probabilities correspond to the objective average survival probabilities for females. The subjective life expectancy of these individuals is 22.4 years (age 87.4).

To calculate the fair annuity factors $\ddot{a}_x$, we use the objective average survival probabilities for males and fair prices of zero bonds which are in line with the financial market model. Further, we assume that the applied annuity factors are reduced by the expense factor $c^{ann} = 15\%$ which captures also adverse selection effects.[23] Note that based on these assumptions the fair annuity factor for a life annuity for a male individual results in $\ddot{a}_x^* = 18.61$ and the applied annuity factor in $\ddot{a}_x = 21.41$. That is, the yearly fixed nominal annuity payout at the beginning of each year equals 4.67 € per 100 € premium.[24]

[21] Note that while lower annuitization rates are rational if lower probabilities are due to objective reasons, this is not true if lower probabilities are due to behavioral biases.

[22] Cf. Vaupel *et al.* (1979) for a precise description.

[23] This value has been chosen such that the annuity payments are in line with (unisex) annuitiy rates in the German annuity market in 2017.

[24] In reality, many annuity products are surplus participating. Since we consider an inflation-adjusted model, we refrain from considering any effects from surplus participation or similar mechanisms, which in reality can be used to compensate (at least partially) losses of purchasing power due to inflation.

Consumption and Subjective Utility

We consider individuals with the following annual consumption characteristics:[25] The minimal requirement is assumed to be $c^{mr} = 12{,}000$ €, the minimal goal is $c^{mg} = 18{,}000$ €, the aspired goal $c^{g} = 24{,}000$ €, and the maximal consumption is set to $c^{max} = 36{,}000$ €.[26] Based on these assumptions, we consider individuals who have different financial means, which is expressed through the social security benefits and the initial liquid wealth.

The social security benefit is varied between 6,000 €, 12,000 € and 18,000 € per year. The considered values describe three fundamentally different initial situations: An individual with low social security benefits of only 6,000 € per year faces a risk of harsh cuts in the standard of living (not reaching the minimal requirement c^{mr}). For individuals with a medium social security benefit of 12,000 € per year, the minimal requirement consumption c^{mr} is already fully covered by the social security benefit. However, for consumption beyond the minimal requirement, in particular to reach the aspired goal, additional resources (either from a withdrawal plan or an annuity) are needed. In the light of current demographic trends and the fact that in most countries social security benefits are implemented as a layer of the old-age provision system which provides only basic income, these two initial situation seem particularly relevant when considering individuals of middle wealth.[27] Nevertheless, we also consider individuals, whose minimal consumption goal is already covered by social security benefits, that is, $A^{soc} = 18{,}000$ €.

For the initial liquid wealth W_0 we consider 50,000 €, 100,000 €, 200,000 €, and 500,000 €. Figure 3 shows exemplarily the fundamentally different structures of the future consumption in the case of a social security benefit of 12,000 € without annuitization ($\theta^{ann} = 0$) for the different levels of initial liquid wealth. Individuals with a rather a low initial liquid wealth of

[25]The values have been chosen to represent typical individuals of middle wealth at retirement age in Germany and are based on empirical data from Germany between 2013 and 2017 (cf. for example Federal Statistical Office (2013) and Deutsche Rentenversicherung - German Statutory Pension Insurance Scheme (2017)).

[26]The maximal consumption has been chosen to be reasonable for individuals of middle wealth and in relation to the other consumption goals. Moreover, it is noted that the structure of the presented results remains even without the restriction of a maximal consumption.

[27]For example the German federal ministry of labor and social affairs notes in a current report that in future the German statutory pension insurance will in general not be sufficient to maintain the accustomed standard of living, cf. German federal ministry of labor and social affairs (2017), p. 12.

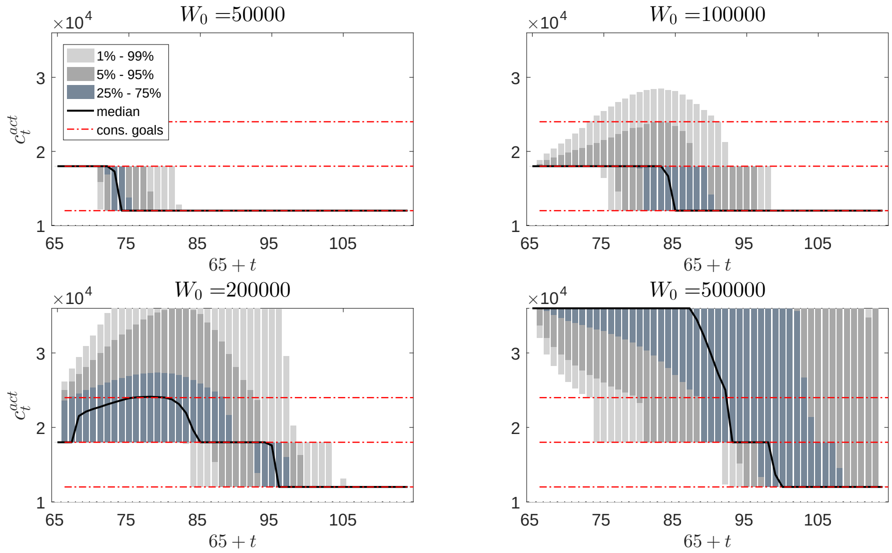


Figure 3: Percentiles of the future consumption in case of a social security benefit of 12,000 € and different levels of initial liquid wealth without annuitization.

50,000 € (100,000 €) are only able to maintain the minimal consumption goal c^{mg} until age 70-75 (80-85). A higher initial liquid wealth of 200,000 € (500,000 €) is sufficient to cover c^{mg} (c^{max}) until the age of roughly 95 (90) in most cases.

Further, we assume that all individuals require a non negative bequest, that is, $b^{mr} = 0$ €.[28] The bequest motive is further driven by the minimal bequest goal, the aspired bequest goal, and the maximal planned bequest, which are, $b^{mg} = 0.1 \cdot W_0$, $b^g = 0.2 \cdot W_0$, and $b^{max} = 0.3 \cdot W_0$.

In the base case, we assume that the individual's subjective utility is based on the following parametrization: The risk appetite parameter is set to $\alpha = 0.88$ and the probability distortion is set to $\gamma = 0.65$. This parametrization is chosen to be in line with Tversky & Kahneman (1992). For the investment frame we set the loss aversion parameter in the base case to $\lambda = 2.4$. In the consumption frame, the parameters for the loss aversion reflect the different impact of the loss aversion depending on the functional region (cf. Section 2.2). If not stated otherwise, we use $\lambda_1 = 1.2$, $\lambda_2 = 2.4$, and $\lambda_3 = 4.8$. Therefore, λ_1 only slightly reduces the subjective utility in case of on target compared to the case of full success. λ_2 and λ_3 applied in the the region of below target and total failure have been chosen such that the loss aversion ratio $\lambda_r := \frac{\lambda_2}{\lambda_1} = \frac{\lambda_3}{\lambda_2}$

[28] Note that under the considered framework, this is always fullfilled.

equals to two.[29] A total failure which leads to harsh cuts in the standard of living is punished by a significantly higher loss aversion. Last, in the base case we refrain from considering subjective time preference, that is, $\rho = 1$.[30] We investigate the impact of subjective time preference as well as other behavioral biases and model assumptions in Section 3.2.

3 Results

This Section presents the results of the numerical analysis based on Monte Carlo Simulations with 500,000 trajectories. The main goal of the analyses is to improve our understanding of the impact of framing and other behavioral biases when making the annuitization decision. Therefore, we focus on the structure of the results and note that precise numbers of course depend on the respective assumptions.

3.1 Comparing the Impact of the Frames

At first, we compare the results under the different frames for $A^{soc} = 12{,}000$ €. The impact of the social security benefit is analyzed subsequently in Section 3.2.

Investment Frame

The left panel of Figure 4 displays the certainty equivalents (CE) (in % of the initial wealth) that an individual under the investment frame would consider as equally desirable as the outcome of the corresponding annuitization decision.[31] If the value is below one the outcome is considered as not attractive. For $\theta^{ann} = 1$ the left panel of Figure 4 displays the case where the investor evaluates the annuity isolated from other products. In this case we find that the annuity product results in a $CE < 1$ (0.86), that is, 14% lower than the price of the annuity product. Hence, the annuity product is considered as an immediate loss. One main reason is the risk of high losses in case of an early death. Loss aversion and probability distortion (overweighting the small probability of an early death) intensify this result. Further analyses show that even in

[29]Consequently, the negative utility generated by a moderate loss amounts to twice the positve utility generated by a moderate gain in the same magnitude, which is in line with Tversky & Kahneman (1992).

[30]Note, in this regard, that we consider an inflation-adjusted framework.

[31]For the calculation of the certainty equivalent under CPT we refer to Ebert *et al.* (2012).

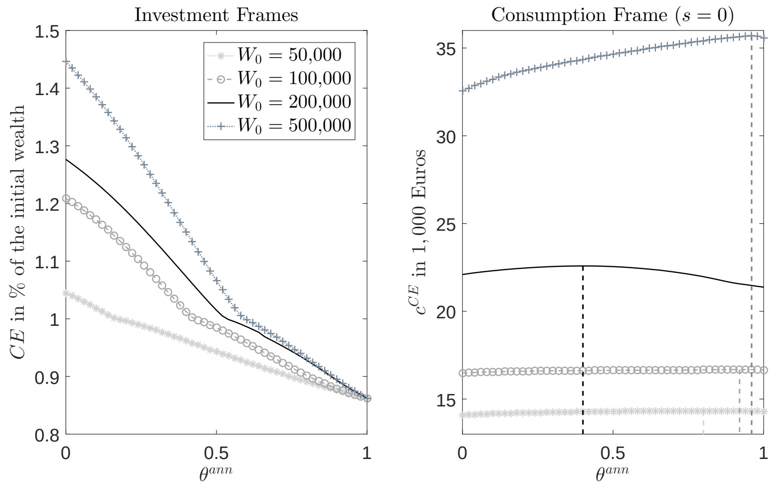


Figure 4: Left panel: Certainty equivalent (CE) in % of the initial wealth under the investment frame depending on the annuitization rate θ^{ann} for different levels of initial wealth. Right panel: Certainty equivalent consumption (c^{CE}) under the consumption frame in the case without bequest motive ($s = 0$) depending on the annuitization rate θ^{ann} for different levels of initial wealth (vertical lines indicate the preferred annuitization rates).

the case without loss aversion ($\lambda = 1$) and without probability distortion ($\gamma = 1$), the annuity product has a $CE < 1$ (0.95). In case of a shorter subjective life expectancy (3 or even 7 years shorter), the annuity product has only a CE of 0.79 or 0.67, respectively. One explanation for this result is the expense factor of 15% which increases the applied annuity factor and limits the chances of positive or even high investment returns of the annuity product significantly. Nevertheless, even without expenses ($c^{ann} = 0$), the annuity product has a $CE > 1$ only in the case of a (very) low loss aversion ($\lambda < 1.25$ with probability distortion and $\lambda < 1.45$ without probability distortion). For an individual with a longer subjective life expectancy, we find that the annuity product has a $CE > 1$, only in cases without probability distortion and with a rather low loss aversion ($\lambda < 1.75$) as well as with probability distortion ($\gamma = 0.65$) and a very low loss aversion ($\lambda < 1.3$).

If we consider all values of θ^{ann}, we find that the CE decreases strictly monotonically in the annuitization rate. Hence, no annuitization is preferred. This holds for all considered levels of initial wealth. The reasons are the same as in the case $\theta^{ann} = 1$. The results remain also true without loss aversion and probability distortion, other (reasonable) financial market conditions and balanced funds, in the case without expenses, longer and shorter subjective life expectancies as well as for all levels of social security benefits.

In total, we can conclude that under an investment frame annuitization (full or partial) is not appealing for most individuals. Moreover, the results suggest that this is mainly driven by the nature of the reference point considered under the investment frame.

Consumption Frame

The right panel of Figure 4 displays the certainty equivalent consumption (c^{CE}) under the consumption frame in the case without bequest ($s = 0$) depending on the annuitization rate θ^{ann}, where the vertical lines indicate the preferred annuitization rates. The certainty equivalent consumption (c^{CE}) is defined as the fixed future consumption that is equally desirable under the consumption frame as the corresponding future consumption c^{act}.[32]

Under the consumption frame without bequest, we find that annuitization of a significant fraction of the initial wealth is optimal for all considered individuals. For an initial wealth of 50,000 €, 100,000 € and 500,000 € the preferred annuitization rates are above 80%. For an initial wealth of 200,000 € the preferred annuitization rates equal 40%. Interestingly, the reasons for the high annuitization rates differ.

For an initial wealth of 50,000 €, consumption is always below or equal to c^{mg}. Without annuitization consumption for the first roughly 10 years is equal or at least very close to this goal. However, at an age of around 75 the individual runs out of liquid wealth. Therefore, the individual very likely faces annual "losses" in the magnitude of $c^{mg} - c^{mr} = 6,000$ € thereafter. A higher annuitization rate results in a higher regular income – which is, however, still significantly lower than c^{mg} – and therefore reduces the amount of the annual losses in later years. But, in many cases it reduces consumption in early years. Hence, independent of the annuitization rate, the individual faces a high risk of running out of liquid wealth. However, due to the small cushion to c^{mr}, the individual is assumed to be risk averse in functional region

[32] The value can be derived numerically, for example, by means of a regula falsi method.

3 and therefore prefers a high annuitization rate (certain but smaller losses with respect to c^{mg}).

Also an initial wealth of 100,000 € is not sufficient to provide an annuity such that the regular income covers c^{mg}. However, with full annuitization the annual losses with respect to c^{mg} can be reduced significantly when running out of liquid wealth. Hence, due to loss aversion the individual prefers a high annuitization rate.

An initial wealth of 200,000 € is sufficient to provide an annuity such that the regular income fully covers c^{mg}. But, a high annuitization rate significantly reduces the potential for even higher consumption. Moreover, in this case also a self-managed withdrawal plan is able to cover the consumption goal for many years (in case without annuitization the future consumption until age 85 is at least equal to c^{mg}). In contrast to individuals with lower initial wealth, the individual has a considerable financial cushion and is therefore less affected by single years with negative returns of the balanced fund. Nevertheless, without any annuitization the individual is still facing a significant risk of running out of liquid wealth at an age of 95 (probability of more than 10%). In this case, without annuitization, the individual has to reduce consumption to the level of the social security benefits. Hence, due to loss aversion, the individual is attracted by the annuity. Nevertheless, as we assume that the individual is risk seeking in functional region 2, the individual prefers only a moderate annuitization rate of 40%.

An initial wealth of 500,000 € can afford an annuity such that the regular income is only slightly below c^{max}. Without any annuitization even these individuals face the risk of running out of liquid wealth at an age of around 100, and even more important, with a moderate annuitization rate, for example of 40%, the future consumption drops below c^{g} at an age around 95 in more than 50% of the cases. Also in younger ages (starting with age 75) consumption is significantly below c^{max} with a probability of roughly 5%-25%. Since individuals are assumed to be risk averse for positive outcomes above c^{g} (functional region 1) and since consumption is limited by c^{max}, high annuitization rates are also preferred by these individuals.

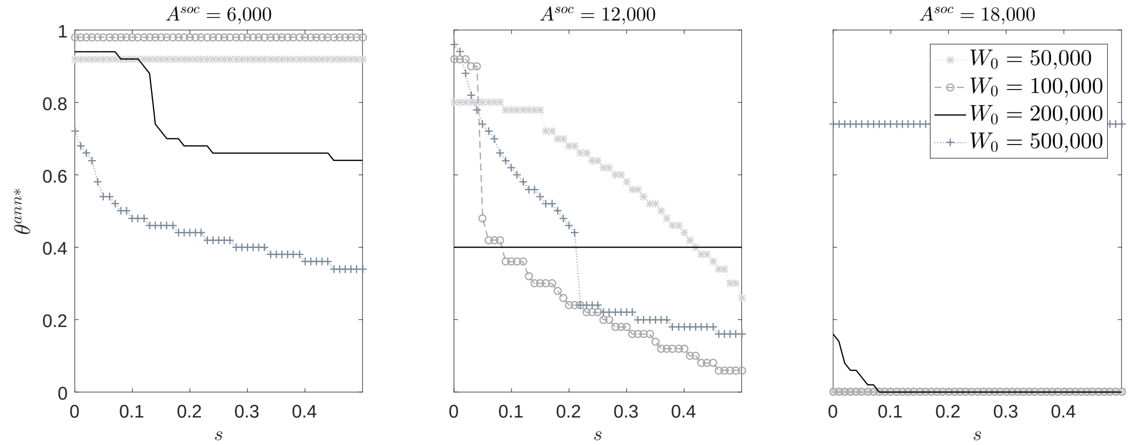


Figure 5: Preferred annuitization rates depending on the social security A^{soc}, the initial wealth W_0 and the bequest motive s. Left panel: $A^{soc} = 6{,}000$ €. Middle panel: $A^{soc} = 12{,}000$ €. Right panel: $A^{soc} = 18{,}000$ €.

3.2 Detailed Analyses under the Consumption Frame

While the consideration of consumption goals as reference points is the main driver of the higher annuitization rates under the consumption frame, the impacts of other factors depend on the characteristics of the individual. Next, we analyze impacts of different factors in detail.

The Impact of Social Security Benefits and the Bequest Motive

Figure 5 displays the preferred annuitization rates under the consumption frame for $A^{soc} = 6{,}000$ €, 12,000 € and 18,000 € depending on the bequest motive $s \in [0, 0.5]$.

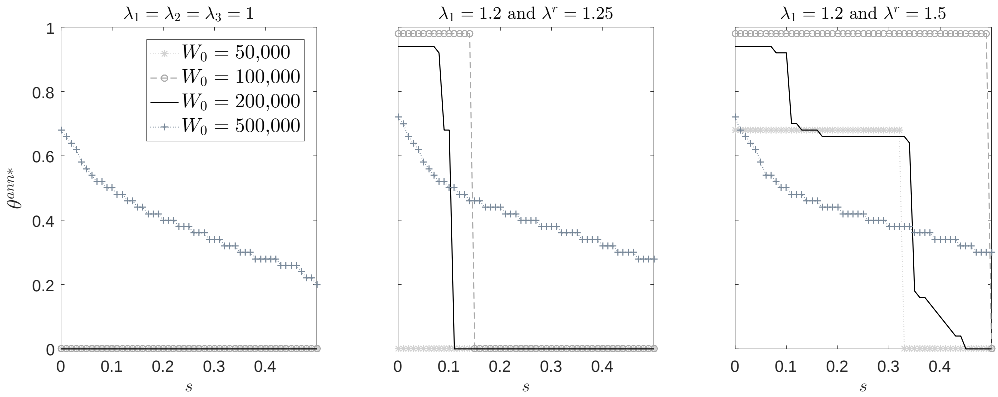
In case of $A^{soc} = 6{,}000$ €, very high annuitization rates are preferred by all considered individuals. Annuitization of a significant fraction of the initial liquid wealth is also preferred in case of a moderate bequest motive (cf. Figure 5 with $s > 0$). This is mainly due to loss aversion and the risk of not reaching the consumption goals (particularly c^{mr}). For an initial liquid wealth of 50,000 € and 100,000 €, the bequest motive has only a minor impact as financial means are needed to reach the minimal consumption requirement.[33] For individuals with initial liquid wealth of 200,000 € and 500,000 € preferred annuitization rates decrease in s. Nevertheless, even in case of a rather strong bequest motive ($s \approx 0.5$) high annuitization rates remain at-

[33]For these individuals the subjective utility from additional consumption is higher than from leaving a bequest. Therefore, these individuals leave in most scenarios no bequest.

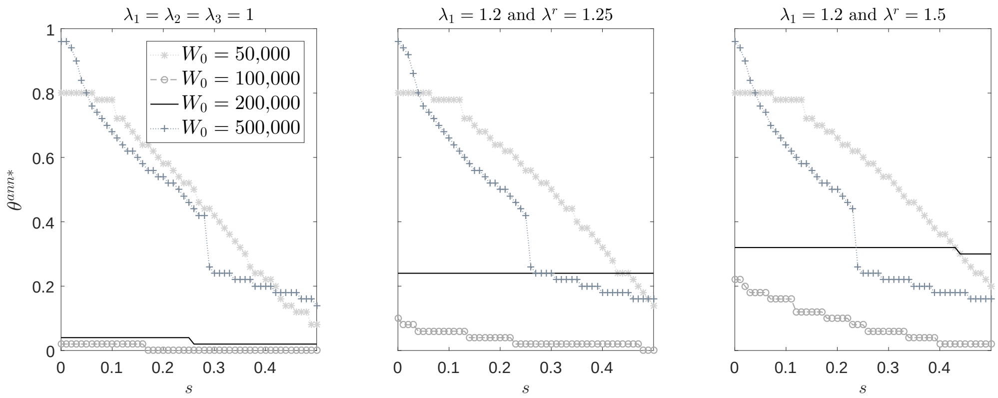
tractive.

For A^{soc} = 12,000 €, that is the basic needs are fully covered by social security benefits, the bequest motive has a stronger impact on the preferred annuitization rates. For an initial liquid wealth of 50,000 € a moderate bequest motive ($s < 0.2$) reduces the preferred annuitization rates only slightly because maintaining consumption close to c^{mg} as long as possible generates more subjective utility than leaving a bequest in case of an early death. For an initial liquid wealth of 100,000 €, the preferred annuitization rate is below 40% for $s > 0.1$. Without annuitization, these individuals have a high probability to meet their bequest goals if they die before age 85 (note that the probability of death before 85 is around 50%). A higher annuitization rate reduces this probability significantly. For an initial liquid wealth of 200,000 €, the bequest motive has almost no impact on the preferred annuitization rate. One reason is that individuals draw no subjective utility from leaving liquid wealth above b^{max}. Without annuitization these individuals meet b^{max} in most cases until age 87. However, in bad states (worst 5%) the bequest can already fall below b^g at an age of around 80. In particular, in states with negative investment returns, annuitization (which is not exposed to capital market risk) can even result in a higher bequest. Furthermore, the higher annuity reduces the withdrawal required to maintain the aspired standard of living. Therefore, these individuals prefer partial annuitization. Also, for an initial liquid wealth of 500,000 € annuitization rates of over 50% are preferred for $s < 0.2$. Only if the bequest motive becomes stronger ($s > 0.2$), preferred annuitiation rates drop down to around 20%. Again, the results are based on the assumption that individuals draw no subjective utility from leaving liquid wealth above b^{max} which is achieved in most scenarios even in the case of partial annuitization.

In the case of $A^{soc} = 18,000$, that is, individuals are not exposed to any losses, annuitization is avoided by most individuals (initial liquid wealth between 50,000 € and 200,000 €) as they are risk seeking in the functional region 3. Contrary, individuals with an initial liquid wealth of 500,000 € prefer high annuitization rates (risk averse in the functional region 1). For these individuals a high annuitization rate eliminates the risk of not reaching the consumption goals



(a) $A^{soc} = 6{,}000$ €



(b) $A^{soc} = 12{,}000$ €

Figure 6: Preferred annuitization rate for $A^{soc} = 6{,}000$ € (a) and 12,000 € (b) depending on the bequest motive s for different levels of initial wealth under various specifications of the loss aversion. Left panel: no loss aversion. Middle panel: $\lambda_1 = 1.2$ and $\lambda^r = 1.25$. Right panel: $\lambda_1 = 1.2$ and $\lambda^r = 1.5$.

at old age (without reducing the consumption in early years due to the maximal consumption goal c^{max}). As the remaining liquid wealth is still sufficient to cover the bequest motive in almost all cases, preferred annuitzation rates remain high also in the case of a bequest motive.

In total, we can conclude that the level of the social security benefits (in relation to the consumption goals) has a significant impact. Individuals with social security benefits below the minimal consumption goal prefer high annuitization. Moreover, for these individuals annuitization of a significant fraction of the initial liquid wealth remains attractive even with a moderate bequest motive.

The Impact of Loss Aversion

Figure 6 displays the impact of loss aversion on the preferred annuitization rates under the consumption frame in the case of $A^{soc} = 6{,}000$ € (a) and $A^{soc} = 12{,}000$ € (b). The left panels show the case without loss aversion, that is, all loss aversion parameters are set equal to one. The middle panel and the right panels display the results for lower loss aversion ratios λ^r (1.25 and 1.5, respectively) with $\lambda_1 = 1.2$.

In the case of low social security benefits and initial liquid wealth of up to 200,000 €, we find that loss aversion has an important impact. Without loss aversion no annuitization is preferred. One reason is that individuals are risk seeking if the future consumption is below the minimal requirement c^{mr} (functional region 4). However, the results also show that a rather low level of loss aversion (middle and right panels) is sufficient to make annuitization attractive in many cases. For individuals with initial liquid wealth of 500,000 €, loss aversion has almost no impact on the annuitization rate because the consumption of these individuals is mainly in the region of gains above c^g. We find similar results for $A^{soc} = 12{,}000$ €. In these cases, loss aversion has only a noticeable impact for an initial liquid wealth of $100,000$ € and 200,000 €. For these individuals, the consumption is fluctuating between positive outcomes (functional region 1 and 2) and negative outcomes (functional region 3 and 4), while for individuals with initial liquid wealth of $50,000$ € and 500,000 €, the consumption is almost only in the region of negative and positive outcomes, respectively.

The Impact of Probability Distortion

To analyze the impact of probability distortion, we set $\gamma = 1$. We find that switching off probability distortion has a significant impact on the annuitization rate. Even in the case without bequest motive the preferred annuitization rates reduce to 10% to 20% for individuals with social security $A^{soc} = 12{,}000$ € and initial liquid wealth of up to 200,000 €. The reason is that with probability distortion, the probability of becoming very old and running out of liquid wealth in old age is overweighted. Moreover, to increase the consumption in old age, these

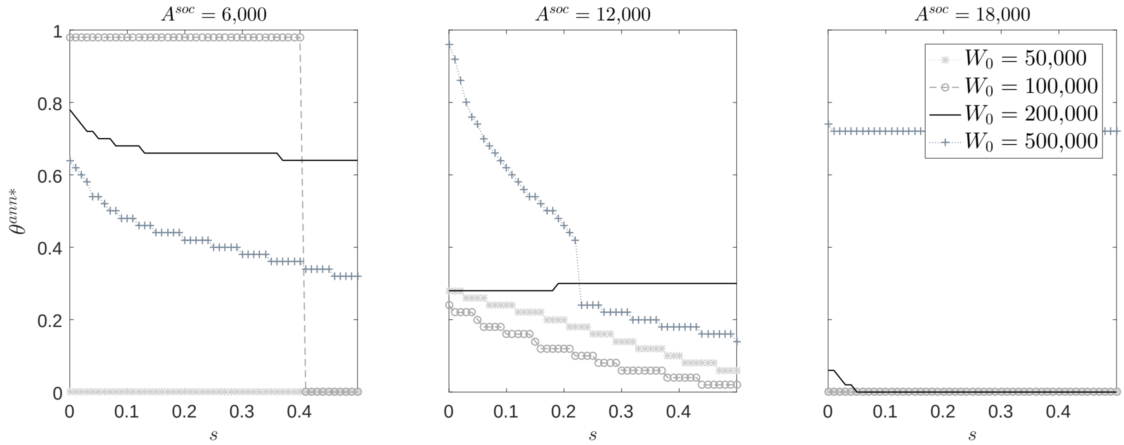
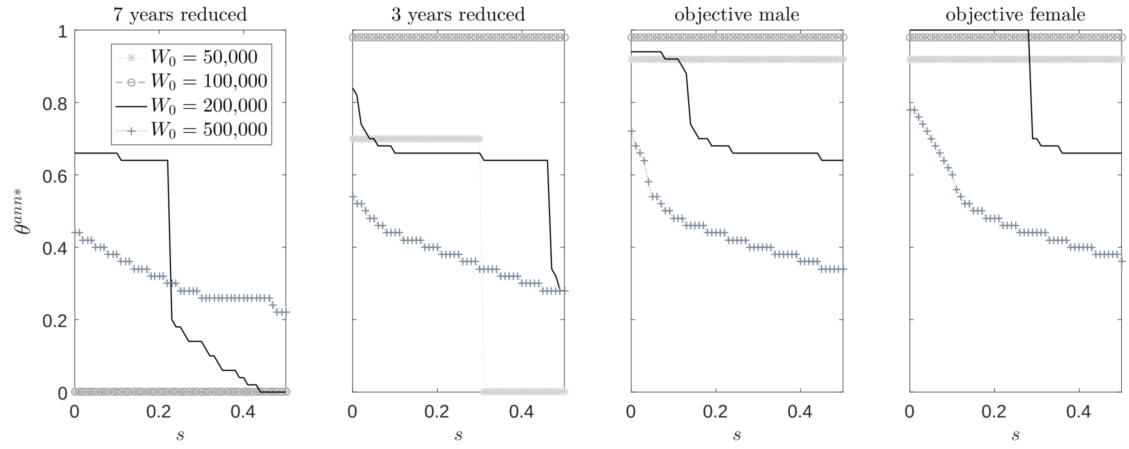


Figure 7: Preferred annuitization rate under the consumption frame depending on the bequest motive s for different levels of initial wealth and time preference $\rho = 0.98$.

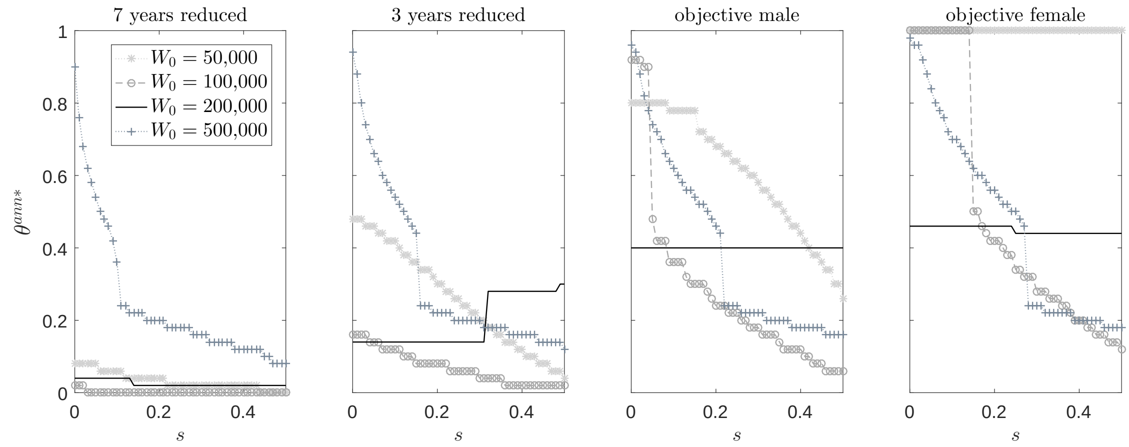
individuals have to reduce their consumption in early years.[34] This is not the case for individuals with an initial liquid wealth of 500,000 €. For these individuals, annuitization only reduces the risk of running out of liquid wealth in old age without reducing consumption in early years (due to c^{max}). Hence, annuitization rates are less affected by probability distortion. However, for individuals whose basic needs are not covered by social security benefits ($A^{soc} = 6{,}000$ €) the impact is weaker. Due to loss aversion with respect to c^{mr}, preferred annuitization rates remain rather high (in the magnitude of 40% - 50%).

The Impact of Time Preference

Figure 7 displays the preferred annuitization rate under the consumption frame with time discounting $\rho = 0.98$. In particular individuals with a lower initial liquid wealth are much less attracted by annuitization. These individuals prefer to maintain c^{mg} as long as possible. A high annuitization rate, however, reduces the consumption already in young ages to a level below c^{mg} in order to finance consumption in old age which is now valued lower. If more initial wealth is available, the impact becomes smaller, since individuals can maintain a higher consumption for a longer period.



(a) $A^{soc} = 6{,}000$ €



(b) $A^{soc} = 12{,}000$ €

Figure 8: Preferred annuitization rate depending on the bequest motive s for different levels of initial wealth and $A^{soc} = 12{,}000$ € (a) and 6,000 € (b). Cases: Subjective life expectancy 7 and 3 years shorter (in expectation) than objective male (left panels) as well as objective male and female (right panels).

The Impact of the Subjective Life Expectancy

Figure 8 shows the preferred annuitization rates for individuals with different subjective life expectancies. All other parameters are equal to the base case. The results show that a reduction of 7 years dramatically reduces the annuitization rates. For $A^{soc} = 12{,}000$ € (lower panels) and initial liquid wealth of 50,000 €, 100,000 €, and 200,000 €, the annuitzation rates are below 10% even in the case without bequest. For $A^{soc} = 6{,}000$ € (upper panels), we find the same results for an initial liquid wealth of 50,000 € and 100,000 €. The simple reason is that these individuals perceive a much smaller subjective risk of not reaching the consumption goals in

[34] If we additionally switch off loss aversion, even partial annuitization becomes unfavorable.

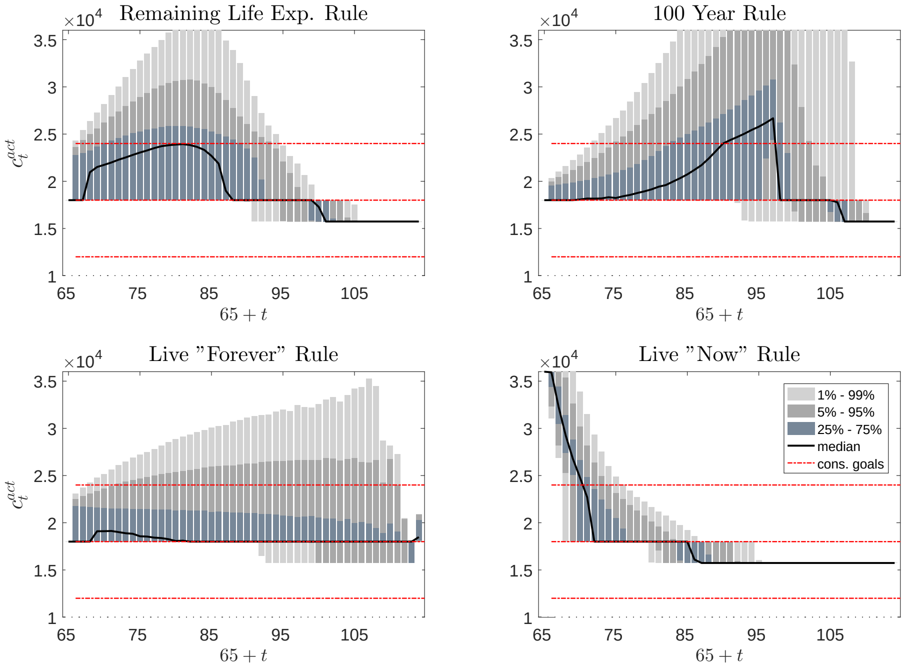


Figure 9: Percentiles of the future consumption in case of a social security benefit of 12,000 € and initial liquid wealth of 200,000 € with $\theta^{ann} = 0.4$ for different withdrawal rules.

older ages. However, if the subjective life expectancy is only 3 years shorter, annuitization of a significant fraction of the initial liquid wealth is preferred in many cases. For individuals with a longer subjective life expectancy (right panels), high annuitization rates are typically preferred.

The Impact of the Consumption Rule

So far, all presented results are based on the assumption that individuals plan consumption according to the remaining life expectancy rule. This rule leads in many cases to a hump-shaped consumption pattern over time (upper left panel of Figure 9). To investigate the impact of the consumption rule, we also consider other often recommended withdrawal rules:[35]

Firstly, we consider a limiting age rule, which defines the age-depending withdrawal by the difference between a limiting age (in our case set to 100 years) and the age at time t. Formally, the *100 year rule* is defined by $k_t = \max(100 - 65 - t, 3)$. Using a limiting age of 100 years leads to a consumption pattern which typically increases when approaching the age of 100. Since most of the liquid wealth is depleted at an age of 100, for older ages than 100 consumption

[35] The considered rules are, e.g., in line with Horneff *et al.* (2008a).

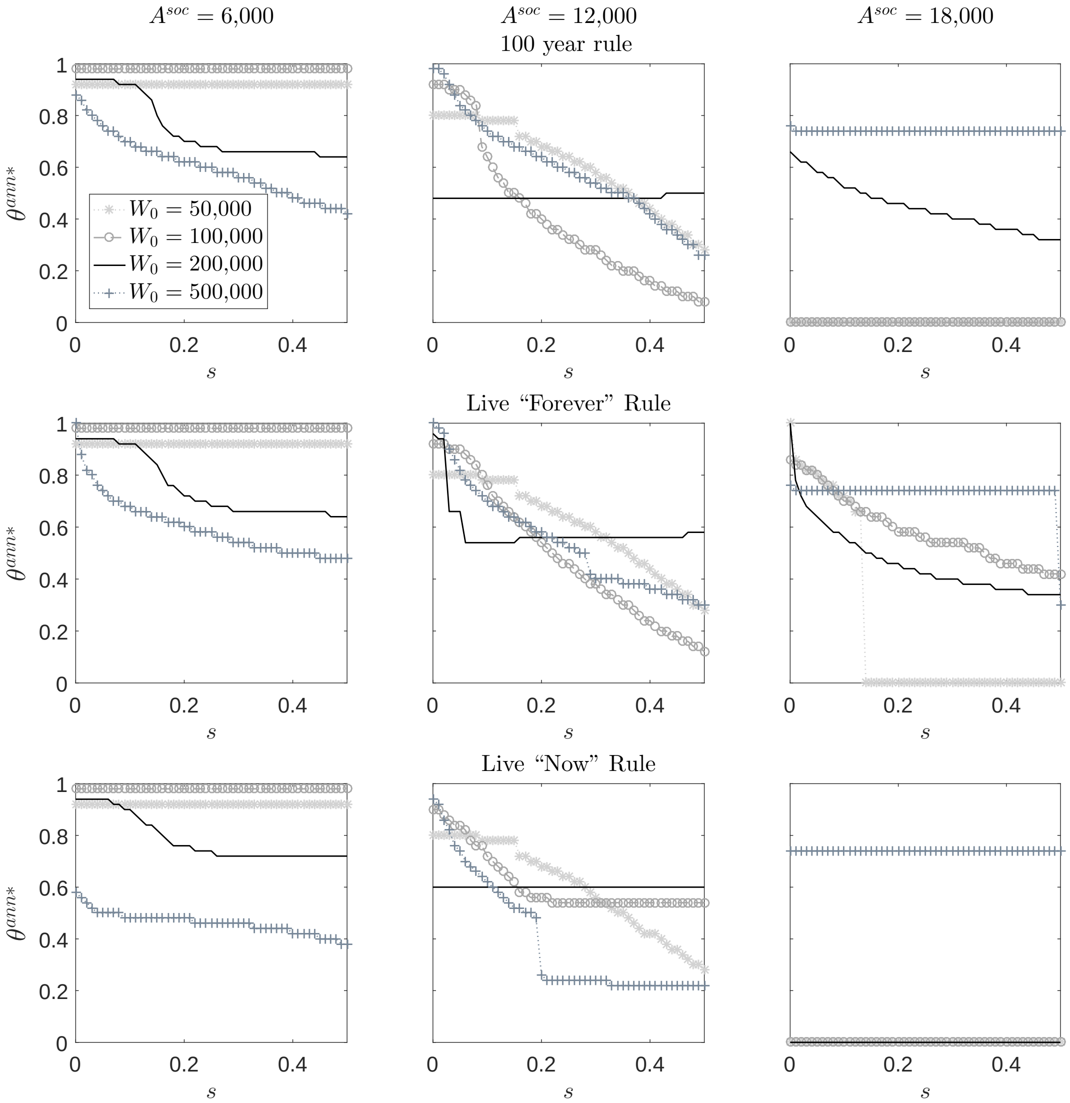


Figure 10: Preferred annuitization rate depending on the social security A^{soc}, the initial wealth W_0, the bequest motive s and the consumption rule. Left, middle and right panels: $A^{soc} = 6{,}000$ €, $12{,}000$ €, and $18{,}000$ €.

decreases rapidly (cf. upper right panel of Figure 9).

Secondly, we implement a simple (age-independent) fixed rule, that is, the withdrawal is determined by setting k_t to a fixed value $k \in \mathbb{R}_+$. We consider two versions: (a) The *Live "Forever" Rule* by setting k to the actual applied annuity factor, that is, $k = \ddot{a}_x = 21.41$. Consequently, the initial withdrawal equals the payout of a life annuity that can be afforded by the initial liquid wealth. This is a very cautious rule since the wealth at time t is always budgeted for another roughly twenty years (cf. lower left panel of Figure 9). Nevertheless, this rule is closely related to the very common $\approx 4\%$ rule which is often recommended for drawing income from

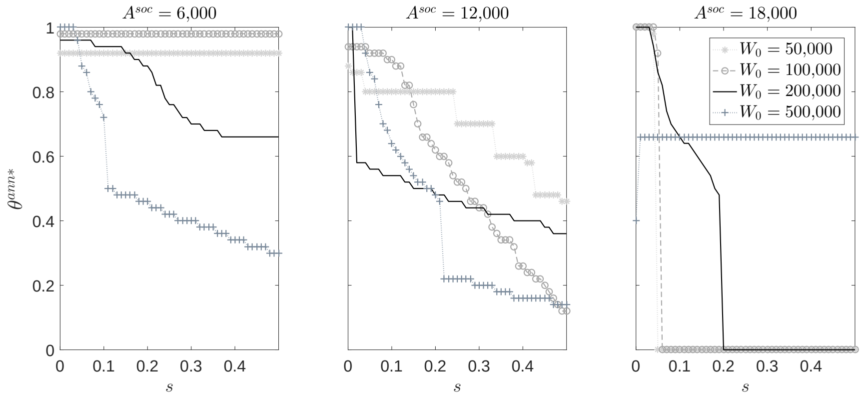


Figure 11: Preferred annuitization rates depending on the social security A^{soc}, the initial wealth W_0 and the bequest motive s if alternative investment is given by a pension fund. Left panel: A^{soc} = 6,000 €. Middle panel: A^{soc} = 12,000 €. Right panel: A^{soc} = 18,000 €.

a self-managed retirement portfolio. (b) The *Live "Now" Rule* by setting $k = 5$, which results in a rapidly decreasing withdrawal plan. Individuals following this rule can be interpreted as having a high preference for consumption in early years (cf. lower right panel of Figure 9).

Figure 10 displays the preferred annuitization rates under the different consumption rules for various levels of initial liquid wealth and social security benefits, while the other parameters are set equal to the base case. In total, we find that the structure of the results is rather similar for individuals with social security benefits of 6.000 € and 12.000 €. For these individuals the annuititzation of a significant fraction of the initial liquid wealth is preferred under all considered consumption rules. For individuals with a social security benefit of $18,000$ €, annuitization is preferred for an initial liquid wealth of 500,000 €. Moreover, for the 100 year rule and live "forever" rule, annuitization is also preferred by some individuals with an initial liquid wealth below 500,000 €. Of course, the impacts of the different behavioral biases can vary, for example, the impact of time preference in case of the live "now" rule is stronger. Nevertheless, most results hold qualitatively for all considered consumption rules.

The Impact of the Investment Alternative

Last, we investigate the impact of a different investment alternative, namely a pension fund with an inflation-adjusted long-term return of roughly 2% and a very low standard deviation of

3,7%.[36] There are two main reasons why we consider this very conservative alternative: Firstly, numerous studies show that most individuals prefer rather save investments. Secondly, there are several studies which show that especially individuals with lower financial literacy have a tendency to underestimate possible returns from funds, cf. Lusardi & Mitchelli (2007), Lusardi & Mitchell (2011) or Jappelli & Padula (2013). Hence, it seems reasonable that many individuals compare the annuity with such an investment. Figure 11 shows that the structure of the results is very similar to the base case. However, in most cases, the preferred annuitization rates are at least slightly higher. Preferred annuitization rates increase heavily for individuals with a high social security benefits of $A^{soc} = 18{,}000$ and initial liquid wealth between 50,000 € and 200,000 €. This is due to the lower investment return and the resulting significantly higher probability of running out of liquid wealth in old age.

Further analyses show that the structure of the results is also very similar if we assume that the individual considers all possible balanced funds, that is, all levels for the stock ratio between 0% and 100%. Individual with an initial liquid wealth of up to 200,000 € prefer in most cases a rather high annuitzation rate combined with a pure stock investment. Only individuals whose basic needs are not covered by social security benefits and with a rather low initial liquid wealth of up to 100,000 € prefer high annuitization rates in combination with a pension fund. One main reason is the strong loss aversion below c^{mr}.

Summarizing, we can conclude that the structure of the results and the described impacts remain very similar in case of other (simple) investments alternatives.

4 Conclusion and Outlook

In this paper, we have modeled the annuitzation decision under an investment frame as well as under a consumption frame. Furthermore, we have analyzed the impact of various determinants on the annuitization decision under both frames focusing on behavioral aspects.

[36]Note that the results are qualitatively remain for balanced funds with a rather low stock ratio (below 30%).

Under the investment frame we have shown that partial annuitization is only appealing for individuals with a significantly higher subjective life expectancy than the average objective life expectancy and only in combination with a rather low level of loss aversion. The results show that annuitzation rates will remain low as long as individuals evaluate the annuitization under the investment frame. Other behavioral biases like loss aversion additionally intensify the annuity aversion. These results are in line with previous studies (cf. Hu & Scott (2007)).

The main contribution of this work is that we are able to model and to disentangle impacts of various determinants on the annuitization decision under the consumption frame. The results show that under the consumption frame most individuals are attracted by partial annuitization if their subjective life expectancy is not significantly shorter than the objective average life expectancy. However, due to the fact that most individuals tend to underestimate their subjective life expectancy, framing the annuity as protection for consumption seems only promising in combination with education programs and trustworthy information on longevity (which is a non-trivial task, cf., e.g., Teppa *et al.* (2015)). Furthermore, while the main driver of the higher annuitization rates is the consideration of consumption goals as reference points, we show, that other determinants can play a crucial role. We find in almost all cases that especially individuals with social security benefits below the minimal consumption goal prefer high annuitization rates. Moreover, already a low level of loss aversion increases annuitization rates significantly in most cases. The impact of loss aversion is particularly strong for individuals whose basic needs are not fully covered by social security. The overweighting of the small probabilities of reaching old ages (probability distortion) increases annuitization rates particularly for individuals with lower levels of initial liquid wealth and whose basic needs are covered by social security. Individuals equipped with a rather high subjective time preference prefer significantly lower annuitization rates – particularly, in case of lower initial liquid wealth.

In total, the main results are in line with experimental and empirical findings (cf. Brown *et al.* (2008), Goedde-Menke *et al.* (2014) or Brown *et al.* (2013)) and suggest that framing can significantly increase voluntary annuitization. Due to the simplifying model assumptions and the complexity of the annuitization decision, we do not claim that the model captures all aspects of decision making in this context. However, the results show that if individuals (subconciously) consider multiple reference points when making the annuitization decision – as suggested by several studies – then behavioral aspects can have very diverse impacts on the decision. This paper provides helpful insights on these impacts and their interactions from a theoretical point of view. By doing so, it points out promising directions for future research to improve our knowledge on actual decision making in the context of old-age provision.

A Appendix - Financial Market Model

We consider a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ on a finite time horizon $[0, T]$, $T \in \mathbb{N}$, under the real-world measure $\mathbb{P}$ satisfying the usual conditions. $\mathbb{F} = (\mathcal{F}_t)_{0 \leq t \leq T}$ with σ-algebra $\mathcal{F}_t$ containing the available information at time t. The financial market model is based on a stock process S described by a geometric Brownian motion and a short rate process r described by the Vasicek model (cf. Black & Scholes (1973) and Vasicek (1977)). The dynamics are given by $dS_t = S_t\left((r_t + \lambda_S)dt + \sigma_S dW_t^S\right)$ with $S_0 > 0$ and $dr_t = \kappa(\xi - r_t)dt + \sigma_r dW_t^r$ with $r_0 \in \mathbb{R}$, σ_S, κ, ξ, $\sigma_r > 0$ and $dW_t^S dW_t^r = \eta \in [-1, 1]$, that is, $W_t^S = \eta W_t^r + \sqrt{1-\eta^2} W_t^*$ with W^* and W^r independent Brownian motions[37] under $\mathbb{P}$. Moreover, $\lambda_S > 0$ denotes the constant equity risk premium. Furthermore, we assume that the considered balanced fund invests the constant fraction θ^S in a stock investment and the remaining part in a “rolling” bond investment (pension fund) based on zero bonds with term to maturity $T_B = 5$ years.[38]

The simulation of the financial market is done on a daily basis assuming 252 days per year.[39] The parameters have been chosen in accordance with the European money market and recent literature (cf. Graf *et al.* (2011) or Hieber *et al.* (2016)), that is, $\sigma_S = 20\%$, $\sigma_r = 1.5\%, \kappa = 30\%$, $\eta = 15\%$. For the sake of simplicity, we consider a model without inflation and adjust the

[37]Note that the random remaining lifetime is assumed to be independent of the financial market.

[38]Note that we can derive closed formulas for the dynamics of the described processes and the price of zero bonds $P(0, t)$ in the Vasicek model, cf. Brigo & Mercurio (2007).

[39]We also apply a daily rebalancing between stock and bonds of the considered balanced funds.

the mean-reversion level[40] by 2% and set it to $\xi = 1.05\%$. Moreover, the stock risk premium is assumed to be $\lambda_S = 3\%$ and due to the current low interest rate environment we use a negative initial short rate $r_0 = -0.33\%$.[41]

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[40] Note that under the assumption that the price of the interest risk λ_r equals -23%, this corresponds to a mean-reversion level (with inflation) under the risk-neutral measure of $\xi^{\mathbb{Q}} = 4.2\%$ and is therefore also in line with the ultimate forward rate under Solvency II in 2017 (cf., e.g., Eiopa (2017)).

[41] The value of r_0 has been chosen to match the average value of the three-month EURIBOR rates of the first 6 months of 2017 (cf. Deutsche Bundesbank (2017)).

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