The Benefits of Return Smoothing in Insurer's Cover Funds – Analyses From a Client's Perspective

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Abstract

Traditional life insurance typically uses some mechanism that is aimed at smoothing the returns of the (collective) assets in the insurer's so-called cover fund. We consider a generic smoothing mechanism and numerically analyze how it impacts the risk-return characteristics of a traditional life insurance contract distinguishing between pathwise volatility (of the annual returns) and the volatility of terminal wealth. We find that pathwise volatility is significantly reduced while the distribution of terminal wealth is hardly affected. We conclude that using multiple segregated cover funds (that come with different asset allocations) as building blocks for more complex products enables insurers to offer a variety of risk-return profiles of terminal wealth in combination with a rather low pathwise volatility (compared to investments without smoothing mechanism). This increases subjective attractiveness for a typical consumer. We consider a variety of such products (static and dynamic investment products) and compare them to similar purely market-based products that do not use an insurer's cover fund. Analyzing risk-return characteristics, (objective) utility, and (subjective) attractiveness under Cumulative Prospect Theory and extensions of it, we conclude that products that become possible by implementing multiple segregated cover funds can increase both, objective utility and subjective attractiveness.

Keywords: Retirement Savings, Collective Assets, Return Smoothing, Insurer's Cover Fund, Cumulative Prospect Theory

JEL: D14, G11, G22, G41, J26, J32

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1 Introduction

Due to the demographic change and the resulting challenges for pay-as-you-go pension systems, private, capital-funded retirement savings become increasingly important. In this segment, many consumers prefer products with relatively high guarantees and low interim fluctuations. In the current low-interest environment, such products can either no longer be offered at all or come with a very limited return potential, which results in a rather low objective utility for consumers. Since the attractiveness of products is crucial for the actual choice of the consumer, insurance companies and other providers of retirement savings products try to offer products that are (subjectively) attractive for consumers without limiting the upside potential too much.

The problems described above particularly apply to participating life insurance products (also referred to as with-profit life insurance products) which are often equipped with year-to-year guarantees.[1] The resulting protection against interim losses is important to many consumers and appears to be one main reason why these products have been very popular in many countries, cf., Ruß & Schelling (2018). Another key characteristic of participating life insurance products is the participation in the return of a "collective investment" of the insurer (which we refer to as the insurer's "cover fund"). The return of this fund is stabilized by various return smoothing mechanisms. The results in Goecke (2013) and Ruß & Schelling (2021) show that collective return smoothing mechanisms alone (that is, without additional guarantees) could already heavily reduce short-term fluctuations while preserving the long-term risk-return characteristics of the underlying fund. Hence, Ruß & Schelling (2021) conclude that products with collective smoothing mechanisms (but without guarantee) might be (subjectively) similarly attractive to long-term investors as products with formal guarantees. Since they can be offered in any interest rate environment without the burdens imposed to the insurer by formal guarantees (e.g., high capital requirement), they might play an important role in overcoming the above mentioned problems. Further, there is a considerable variety of literature dealing with smoothing mechanisms: Hansen & Miltersen (2002) discuss the effect of a smoothed sur-

[1] Participating life insurance contracts have been discussed extensively in the literature, cf., Bauer *et al.* (2006), Gatzert & Kling (2007), and Bohnert *et al.* (2014).

plus participation on the minimum guarantee rate in Danish pension plans. They analyze fair contract settings between consumers and the insurer and consider the consequences of pooling for a group of inhomogeneous consumers. Guillén *et al.* (2006) define a smoothing mechanism where contingent claims pricing techniques can be used to compute fair market values of contracts. Løchte Jørgensen (2007) considers a specific return smoothing mechanism and shows that the resulting smoothed returns follow approximately a lognormal distribution. Moreover, Maurer *et al.* (2016) analyze the actuarial techniques to smooth reporting of firm assets and liabilities and find that smoothing adds value to both the policyholder and the insurer. Also, Lichtenstern & Zagst (2021) consider smoothing in the post-retirement phase by means of a specific pension adjustment mechanism as well as by a buffer portfolio. They determine the optimal investment strategies during the decumulation phase to maximize the client's expected accumulated utility from the stochastic future pension cash flows and conclude that smoothing can provide remarkable benefits to clients.

In reality, insurers' cover funds come with rather low equity exposures (mainly due to high guarantees offered in the past in combination with solvency requirements). Moreover, insurers in many countries typically do not offer different cover funds with different risk-return characteristics. Hence, there is no possibility for the policyholder to choose a cover fund that best meets individual preferences. In this paper, we add to the literature by analyzing a setting where an insurer offers several segregated cover funds with different asset allocation which make use of collective smoothing mechanisms but come without guarantees. These cover funds can be offered standalone or as a "building block" within more complex products.

The main goal of this paper is an analysis from the client's perspective of the resulting products. We consider both, the risk of short-term fluctuations, as well as the distribution of the terminal benefit. We show that the existence of multiple, segregated cover funds enables insurers to offer different risk-return profiles of the terminal benefit. Each of these come with reduced (when compared to investments without smoothing mechanisms) pathwise volatility, which appears highly relevant for the subjective attractiveness.

We also analyze various static and dynamic investment strategies (without formal guarantees but with protection levels) using different collective cover funds as low-risk asset and compare them with products which use market-based safe assets. We find that dynamic products based on cover funds have a significantly higher return potential compared to market-based products with similar short-term risk characteristics. This particularly holds for conservative products (which are relevant for risk averse investors). In addition to risk-return characteristics, we investigate how collective cover funds affect both, the objective utility (measured in an Expected Utility Theory framework) and the subjective attractiveness (measured in several frameworks based on Cumulative Prospect Theory) of these products. We find that in most cases, static products based on cover funds with rather high equity ratios outperform all other considered products in terms of objective utility as well as subjective attractiveness. Sensitivity analyses show that this also holds in an environment of higher interest rates and other choices of the smoothing mechanism. Therefore, our results indicate that the simultaneous offering of multiple, segregated cover funds with different asset allocations would enable insurers to develop a range of retirement savings products including solutions that are at the same time subjectively attractive and objectively preferable for different types of consumers.

The remainder of this paper is organized as follows. In Section 2, we introduce the capital market model, the insurer's cover fund, as well as the market-based investment strategies. In Section 3, we analyze the impact of smoothing focusing on static investment products. Next, in Section 4, we analyze static as well as dynamic investment products which make use of an insurer's cover fund and compare them with similar market-based products. In particular, we analyze the expected utility and the subjective attractiveness. We also discuss a wide range of sensitivity analyses. Finally, Section 5 concludes and provides an outlook.

2 Model Framework

In this section, we describe our capital market model with stochastic interest rates and stocks. Moreover, we define the insurer's cover fund and the considered market-based investment strategies.

2.1 The Capital Market Model

We assume that the short rate for $t \in [0, T]$ and $T \in \mathbb{N}$ follows a two-factor Hull-White model as described in Brigo & Mercurio (2007). This model is frequently used to model the current low-interest environment, cf., Korn *et al.* (2018). It contains two correlated mean reverting processes and allows a perfect fit to observed bond prices. The dynamics of the stock market is modeled by a generalized Black-Scholes model with a risk premium λ_S and volatility σ_S. A more detailed description can be found in Appendix A.

2.2 Insurer's Cover Fund

Smoothing mechanisms as described in Chapter 1 are often complex, can include elements of management discretion and can differ from country to country. Therefore, inspired by Korn *et al.* (2018), we generically model the impact of smoothing as follows: We denote the insurer's cover fund with a stock ratio $\phi \in [0, 1]$ by ICF^{ϕ} and the smoothing period by $sp \geq 2$ years. The yearly smoothed return ($\tilde{r}_{ICF^{\phi}}(t)$) based on the applied smoothing procedure between time $t-1$ and t (for $t = 1, 2, \dots$) under the real world measure P is given by

$$\tilde{r}_{ICF^{\phi}}(t) = \left(\prod_{k=1}^{sp} \left(\phi(1 + r_s(t-k)) + (1-\phi)(1 + r_{RB}(t-k))\right)\right)^{1/sp} - 1, \tag{1}$$

where $r_s(t)$ is the yearly return of the stock market between time $t-1$ and t, cf., Appendix A. Further, $r_{RB}(t)$ denotes the yearly return of a rolling bond investment (RB) between time $t-1$ and t (for $t = 1, 2, \dots$), where the bond is bought and sold $\frac{1}{\Delta t}$ times in a year for $\Delta t \in (0, 1]$, e.g., monthly ($\frac{1}{\Delta t} = 12$) or daily ($\frac{1}{\Delta t} = 252$). Thus, the return is given by

$$r_{RB}(t) = \prod_{i=1}^{1/\Delta t} (1 + \hat{r}_{RB}(t+i)) - 1 \text{ with } \hat{r}_{RB}(t) = \frac{P(t, t - \Delta t + d)}{P(t - \Delta t, t - \Delta t + d)} - 1, \tag{2}$$

where $d \geq 0$ denotes the term to maturity of the bond investment and $P(t_0, t_1)$ denotes the price of a zero coupon bond at time t_0 with maturity t_1.[2] Note that throughout the paper the annual return of an asset (Ξ) is denoted by r_Ξ and the return in a time interval of Δt is denoted by $\hat{r}_\Xi$. Finally, for the yearly return of the insurer's cover fund ($r_{ICF^\phi}(t, \alpha)$), the smoothed return is adjusted by the surplus participation rate $\alpha \in [0, \infty)$, i.e.,

$$r_{ICF^\phi}(t, \alpha) = \tilde{r}_{ICF^\phi}(t) - (1 - \alpha) \max(\tilde{r}_{ICF^\phi}(t), 0) \text{ for } t = 1, 2, \ldots. \tag{3}$$

Hence, α determines the proportion of the positive return which will be credited to the insurer's cover fund. The corresponding return between time $t - \Delta t$ and t can be computed by

$$\hat{r}_{ICF^\phi}(t, \alpha) = \big(1 + r_{ICF^\phi}(t, \alpha)\big)^{\Delta t} - 1. \tag{4}$$

Note that $\hat{r}_{ICF}(t, \alpha)$ is constant during the year.

The participation rate (α) is chosen such that the insurer's cover fund is initially fair if the cover fund is held until maturity (T),[3] i.e.,

$$\mathbb{E}_Q\left[\exp\left(-\int_0^T r^*(s)ds\right) \prod_{t=1}^{T} (1 + r^*_{ICF}(t, \alpha))\right] = 1, \tag{5}$$

where Q denotes the risk-neutral measure, $r^*(s)$ the short rate under Q and r^*_{ICF} the yearly return of the ICF under Q, cf., Appendix A for more details. The fair α usually differs from 1, e.g., since the returns of the past influence $r_{ICF}(t)$ (at least for $t = 1, \ldots, sp$), and can also be

[2] We refer to Brigo & Mercurio (2007) for details on pricing zero coupon bonds in this model.

[3] For dynamic products, which may invest in and deinvest from the ICF several times during the investment horizon, this can lead to small deviations compared to fairly priced products. In the following analyses, we neglect this effect. Also note that in a sensitivity analysis below, we consider different values of (initially fair) alphas and the results are qualitatively similar.

greater than one.[4]

2.3 Assets and Products

We analyze different smoothed products based on insurer's cover funds and compare them with products using only market-based assets. For all products we assume a single upfront premium $A_0 > 0$, a term to maturity of T, and no charges. We use different low-risk assets and use stocks as risky assets. In addition to the insurer's cover funds, we consider as low-risk assets a rolling bond investment with a return of $r_{RB}(t)$ as described in the previous subsection as well as an investment in a T-year zero coupon bond (TB). The return from time 0 to time T of a (pure) TB product is given by $1/P(0,T)$ and the return between time $t-\Delta t$ and t by

$$\hat{r}_{TB}(t) = \frac{P(t,T)}{P(t-\Delta t,T)} - 1. \tag{6}$$

We consider two different product groups: Firstly, we examine *static investment*[5] products where the low-risk investments $\Xi \in \{ICF, RB, TB\}$ is mixed with a fixed fraction of stocks ($\theta \in [0,1]$). We denote them as static-ICF, static-RB, and static-TB, respectively and refer to θ as direct stock ratio. The return of a static investment between two rebalancing points in time t and $t-\Delta t$ is given by

$$\hat{r}^{\theta}_{\Xi}(t) = \Big(\theta(1+\hat{r}_s(t)) + (1-\theta)(1+\hat{r}_{\Xi}(t))\Big), \tag{7}$$

where $\hat{r}_s(t)$ denotes the stock return between time $t-\Delta t$ and t. Note that for static-ICF products with $\phi > 0$ the product's total fraction of stocks is given by $\theta^{tot} = \theta + (1-\theta)\phi \geq \theta$.

Secondly, we analyze *dynamic investment* products in the form of CPPI-strategies with a protection level of $PL \leq 1$ in terms of the initial investment $A_0 > 0$. The part of the investment which is not invested in the stock is invested in a low risk investment $\Xi \in \{ICF, RB, TB\}$.

[4]In reality, products based on the insurer's cover fund usually come with an annual guarantee. For these products a participation rate of less than 1 primarily compensates for the guarantee. Note that in this paper, we explicitly consider products based on an insurer's cover fund without guarantee as we are interested in the effects of smoothing in collective investments.

[5]Note that we assume a rebalancing at all Δt time points for the static investment products.

The CPPI strategy is implemented as follows:

1. At time t, compute the floor $F_t = PL \cdot A_0 \cdot P(t,T)$, the cushion $C_t = A_t - F_t$, the exposure to the stock investment $E_t = \min(mC_t, A_t)$, where $m \in \{1,2,3,4,5\}$ is the multiplier, and the low risk investment $R_t = A_t - E_t$.

2. Then, $A_{t+\Delta t} = E_t(1+\hat{r}_s(t+\Delta t)) + R_t(1+\hat{r}_\Xi(t+\Delta t))$. We repeat that for $t = 0, \ldots, T-\Delta t$.

The average direct stock ratio of a CPPI strategy is given by $\bar{\theta} = \frac{\Delta t}{T}\mathbb{E}\left[\sum_{k=0}^{\frac{T}{\Delta t}-1} E_{k\cdot\Delta t}\right]$ and the total fraction of stocks by $\bar{\theta}^{tot} = \bar{\theta} + (1-\bar{\theta})\phi$, where $\phi = 0$ for market-based CPPI strategies. Note that the probability of achieving a terminal value below the protection level is greater than 0 and typically increasing in the multiplier and the protection level.[6]

3 Impact of Return Smoothing on Risk-Return Characteristics

In this section, we compare static-ICF products using $\phi \in \{10\%, 30\%, 50\%\}$[7] and a smoothing period of 3 (=sp) years,[8] with static-RB products as introduced in Section 2. Note that in this section, we only consider these two product types, which are both based on a rolling bond investment, in order to separate the impact of smoothing from other product features. For the market-based products, we assume daily rebalancing and 252 trading days per year and consequently set $\Delta t = 1/252$. Further, we investigate 41 (direct) stock ratios θ between 0% and 100% in steps of 2.5%. To compare the different products, we generate 50,000 Monte Carlo simulations paths and analyze the expected return, the standard deviation of annual returns within a path as well as the standard deviation of the annualized return from 0 to T. For a given asset evolution $(A_t)_{\{t=0,\ldots,T\}}$ (which depends on the low-risk asset and stock ratio), we define the

[6] More details can be found in Table 12 in Appendix D.

[7] We refrain from analyzing higher stock ratios within the ICF since in practice, rather high stock ratios would lead to several risks and issues from the provider's perspective, cf., e.g., our comments on market value adjustments in Section 5. Hence, in practice providers would either refrain from offering such products, or they would use smoothing mechanisms that are specifically tailored to deal with such risks and issues which would make the use of a generic smoothing algorithm as a proxy increasingly unrealistic.

[8] See, Korn *et al.* (2018). Also, in the sensitivity analysis, we show that the results also hold for longer smoothing periods.

	λ_S	σ_S	d	ϕ	T	Simulations (n)	
	4%	20%	10Y	10%, 30%, 50%	30	50,000	
History	2017/18	2018/19	2019/20	History	2017/18	2018/19	2019/20
Stock	1.28%	5.12%	-5.05%	Rolling Bond	2.33%	7.40%	1.81%

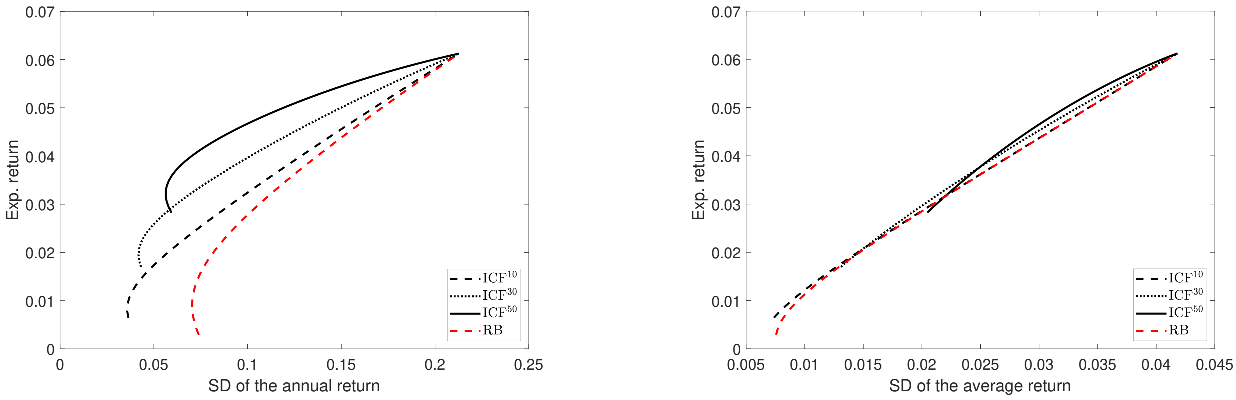
Table 1: The base case parameters of the stock market (λ_S, σ_S), the parameters of the ICF (d, ϕ, T), the number of simulations (n) and the historical stock and rolling bond returns.

standard deviation of annual returns (pathwise volatility $\sigma[r_t]$) as $SD\left[\left(\frac{A_{t+1}-A_t}{A_t}\right)_{\{t=0,\ldots,T-1\}}\right]$, where the standard deviation (SD) is taken over all $t = \{1, \ldots, T\}$. The standard deviation of the average return ($\sigma[\bar{r}]$) is given by $SD\left[\left(\frac{A_T}{A_0}\right)^{1/T} - 1\right]$. The following results are based on the base case parameter setup presented in Table 1. The parametrization of the short rate model and the stock market are taken from Graf *et al.* (2021) and can be found in Table 9 in Appendix A. To be in line with Graf *et al.* (2021), we model the historical rolling bond returns with the historical Nelson-Siegel-Svensson (NSS) parameters which are given by the Deutsche Bundesbank[9] and for the historical returns of the stock market, we take the returns of the EURO STOXX 50 performance index of the same time intervals.[10] The corresponding returns can also be found in Table 1.

The fair participation rate α is 0.865 for a stock ratio of 10%, 0.924 for $\phi = 30\%$, and increases to 0.972 for $\phi = 50\%$. It is below 1 because the historical return is above the average return implied by the model under the risk-neutral measure Q. Due to the asymmetric participation in the surpluses and losses, cf., Equation (3), an ICF with a higher stock ratio has a higher participation rate due to the lower volatility of the rolling bond return. This effect is strengthened by the lower historical stock return compared to the historical rolling bond return.

The resulting risk-return characteristics of the different products are shown in Figure 1. In both panels the expected return is shown on the y-axis. Further, on the x-axis the standard deviation of annual returns ("pathwise volatility") is shown in panel (a) and the standard deviation of the annualized return in panel (b).

[9] The NSS-parameters can be found in Deutsche Bundesbank (2021).
[10] The historical performance of the EURO STOXX 50 is taken from Deutsche Börse AG (2021).



(a) Average SD of annual returns

(b) The SD of the average return

Figure 1: The risk-return characteristics for different static-ICF products and a static-RB product based on the standard deviation of annual returns (left panel) and on the standard deviation of the average return (right panel).

In both panels, the outer right point of a line corresponds to products with a direct stock ratio of 100% and consequently all lines meet at the same point. The other end of the line corresponds to a direct stock ratio of 0%. We typically observe that static-ICF products with a higher embedded stock ratio (ϕ) have a higher expected return but typically also a higher volatility. As direct stock ratios approach zero, the decreasing expected return comes with an increase in risk (at least pathwise volatility) since diversification effects are no longer used. Most importantly, we find in panel (b) that for a given expected return the standard deviation of the average return is very similar for all four investment types while in panel (a) for a given expected return, the standard deviation of the annual return of static-ICF products are significantly lower compared to static-RB products. In particular, panel (a) shows that the static-RB products are clearly dominated by static-ICF products when considering the annual fluctuations of the product. This already illustrates the potential of ICF based products to reduce pathwise volatility with little effect on the distribution of terminal wealth.

4 Analysis of Static and Dynamic Products

In this section, we analyze static and dynamic investment products introduced in Section 2. For both, we consider five different options for the low-risk asset: ICF^{10}, ICF^{30}, ICF^{50}, a T-year zero coupon bond (TB), and a rolling-bond investment (RB).

4.1 Base Case

Again, we assume daily rebalancing and use 50,000 simulation paths. For the static products we investigate 41 (direct) stock ratios θ between 0% and 100% in steps of 2.5%. For the dynamic products we consider different protection levels $PL \in \{50\%, 70\%, 80\%, 90\%\}$ as well as different multipliers $m = \{1, 2, 3, 4, 5\}$.[11]

4.1.1 Characteristics of the Dynamic Investment Products

Firstly, we analyze the characteristics of the dynamic investment products which have not been discussed in the previous analysis. In Table 2 we display the expected return ($\bar{r}$), the standard deviation of the average return ($\sigma[\bar{r}]$), the standard deviation of annual returns ($\sigma[r_t]$), the corresponding average (direct as well as total) stock ratio over the entire duration ($\bar{\theta}$ and $\bar{\theta}^{tot}$) and the shortfall probability with respect to the protection level ($P_{PL} := P(A_T < PL)$) for the different dynamic products.

As expected, for fixed m, a decreasing protection level results in a higher average stock ratio and consequently in a higher average return (as well as higher standard deviations). The same holds true for a fixed protection level and an increasing multiplier. Further, the shortfall probability is lower for lower protection levels or lower multipliers, except for the dynamic-TB (which always has a shortfall probability very close to 0).

It is noteworthy that for given m and PL, all considered dynamic-ICF products have a higher average direct stock ratio than other dynamic products (and the difference is even larger when considering the ICF product's total stock ratio $\bar{\theta}^{tot}$) which is mainly due to the smoothing effect which reduces fluctuations of the direct stock ratio. This results in higher expected returns for the dynamic-ICF products compared to other dynamic products. The return difference is par-

[11] Note that in this setting, the maximum possible protection level is given by $1/P(0,T) \approx 0.95$. We examine only protection levels which are significantly below 95% so that significant stock ratios are possible.

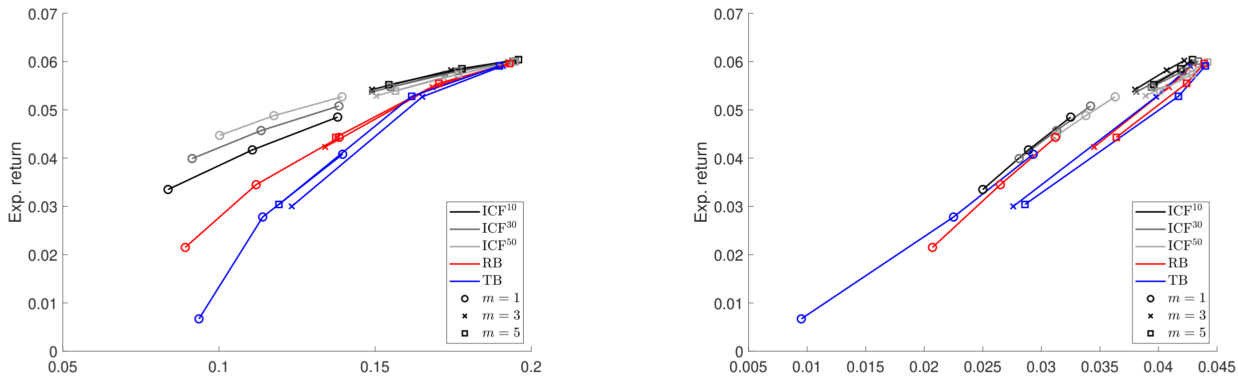
	$m = 1$ and $PL = 90\%$					$m = 3$ and $PL = 90\%$					$m = 5$ and $PL = 90\%$				
	ICF with $\phi =$			RB	TB	ICF with $\phi =$			RB	TB	ICF with $\phi =$			RB	TB
	10%	30%	50%			10%	30%	50%			10%	30%	50%		
$\bar{r}$	3.35	3.99	4.47	2.15	0.67	5.42	5.37	5.29	4.23	3	5.52	5.47	5.39	4.43	3.04
$\sigma[\bar{r}]$	2.5	2.81	3.13	2.07	0.95	3.8	3.81	3.89	3.45	2.76	3.96	3.94	4.01	3.64	2.86
$\sigma[r_t]$	8.37	9.14	10.02	8.92	9.36	14.9	14.89	15.04	13.4	12.33	15.44	15.5	15.65	13.74	11.92
$\bar{\theta}$	33.27	35.28	36.27	23.90	12.75	60.39	59.17	57.58	45.27	31.95	61.91	61.04	59.66	45.33	25.22
$\bar{\theta}^{tot}$	39.94	54.70	68.14	23.90	12.75	64.35	71.42	78.79	45.27	31.95	65.72	72.73	79.83	45.33	25.22
P_{PL}	11.0	12.1	14.5	18.7	0	26.0	25.5	26.5	34.7	0	33.4	30.7	30.6	43.3	0
	$m = 1$ and $PL = 80\%$					$m = 3$ and $PL = 80\%$					$m = 5$ and $PL = 80\%$				
$\bar{r}$	3.78	4.29	4.68	2.84	1.88	5.65	5.58	5.49	4.99	4.54	5.71	5.65	5.58	5.13	4.56
$\sigma[\bar{r}]$	2.70	2.97	3.26	2.38	1.76	3.95	3.98	4.06	3.84	3.6	4.09	4.09	4.16	4.03	3.76
$\sigma[r_t]$	9.71	10.2	10.83	9.96	10.24	16.23	16.09	16.1	15.26	14.74	16.66	16.6	16.67	15.54	14.29
$\bar{\theta}$	41.3	42.2	42.3	34.5	29.0	68.6	66.8	64.8	58.8	52.7	69.5	68.2	66.6	58.1	45.0
$\bar{\theta}^{tot}$	47.17	59.54	71.15	34.5	29.0	71.74	76.76	82.40	58.8	52.7	72.55	77.74	83.3	58.1	45.0
P_{PL}	6.1	8.7	11.5	9.1	0	21.6	22.1	23.6	26.9	0	28.8	27.1	27.3	35.4	0
	$m = 1$ and $PL = 50\%$					$m = 3$ and $PL = 50\%$					$m = 5$ and $PL = 50\%$				
$\bar{r}$	4.85	5.08	5.27	4.43	4.08	6.02	5.99	5.95	5.95	5.91	6.04	6.01	5.99	5.96	5.91
$\sigma[\bar{r}]$	3.25	3.42	3.63	3.12	2.93	4.22	4.29	4.38	4.3	4.27	4.29	4.34	4.42	4.39	4.40
$\sigma[r_t]$	13.8	13.84	13.94	13.85	13.96	19.4	19.25	19.17	19.21	19.09	19.59	19.49	19.46	19.31	18.98
$\bar{\theta}$	63.6	63.2	62.3	61.6	60.5	87.9	86.5	85.1	85.6	84.5	88.4	87.3	86.2	85.3	82.3
$\bar{\theta}^{tot}$	67.24	74.24	81.15	61.6	60.5	89.11	90.55	92.55	85.6	84.5	89.56	91.11	93.1	85.3	82.3
P_{PL}	0.9	2.4	4.5	0.8	0	10.7	11.9	13.3	11.3	0	14.4	14.4	15.0	15.7	0

Table 2: Different risk measures for dynamic products with different multipliers (m) and different protection levels (PL) based on different low-risk investments (ICF, RB,TB). All results are given in percent.

ticularly pronounced for higher protection levels and lower multipliers (i.e., products probably chosen by conservative, risk averse consumers). But this comes with a higher volatility of the terminal benefit, however, still significantly lower than for comparable (same expected return) market-based products, cf. Figure 2 (b). Beyond that, the standard deviations of the annual returns are similar for all considered dynamic products.[12] Although the average total stock ratios of the dynamic-ICF products are significantly higher than for the dynamic-RB products, the shortfall probabilities are mostly very similar for both (and in many cases the shortfall probabilities of the dynamic-ICF products are even lower than for the dynamic-RB products).[13]

[12]Note that the dynamic-TB is the only considered product with no shortfall risk. However, the results show that this comes with a significantly lower average stock ratio and consequently average return compared to other dynamic products (for m and PL).

[13]Note that this may also be partly driven by the ICF pricing approach and the choice of α, cf., page 5.



(a) Average SD of annual returns

(b) The SD of the average return

Figure 2: The risk-return characteristics for different dynamic-ICF, dynamic-RB and dynamic-TB products based on the standard deviation of annual returns (left panels) and based on the standard deviation of the average return (right panel). Note that one line corresponds to a group of dynamic products of a fixed type (specified by the color), fixed multiplier (specified by the symbol) and different protection levels.

Figure 2 displays the risk-return characteristics of the dynamic investment strategies from Table 2, where risk is measured by pathwise fluctuations in the left panel and of the terminal wealth in the right panel (as in Figure 1 in Section 3). We observe in the left panel that for a given and rather low pathwise volatility the dynamic-ICFs have a significantly higher expected return. Further, we observe that for a high protection level (lower volatility), the dynamic-ICF^{50} products dominate while for lower protection levels (higher volatility) dynamic-ICF^{10} products dominate.[14] However, with increasing volatility, the ICF and market-based dynamic investment products (and hence also their Risk-Return characteristics) become more similar due to the increasing direct stock ratios. Another interesting effect can be seen in the results for the dynamic-TB products. There, products with a multiplier of $m = 3$ are dominated by products with a multiplier of 1 and 5. This can be explained by the impact of the market value changes of the terminal bond on the stock exposure for different multipliers which results in average total stock ratios which first increase in m and then decrease again, cf., Table 2. When risk is measured by variability of terminal wealth (right panel of Figure 2), all products are much closer to the "efficient frontier". However, in particular, for higher standard deviations, market-based products are clearly dominated by ICF-based products. Overall, these results show that dynamic-ICF products have a significant potential to earn a higher return than a

[14]This can also be seen in Table 2, where for dynamic-ICF products with fixed multiplier $m \in \{3, 5\}$ and protection level, the returns decrease (slightly) for a higher ICF stock ratio (ϕ), while the pathwise volatility increases (slightly) for a high protection level $PL = 90\%$ and decreases for a low protection level $PL = 50\%$.

comparable dynamic product with market-based low risk assets and similar risk characteristics.

4.1.2 Preferences under EUT, CPT, MCPT and PMCPT

In this section, we analyze the objective expected utility and the subjective attractiveness of the different products. For the former, we apply Expected Utility Theory (EUT) with power utility $u(x) = x^{1-\gamma}/(1-\gamma)$, where $\gamma \geq 0$ ($\neq 1$) is the risk aversion parameter. To measure the subjective attractiveness, we use Cumulative Prospect Theory (CPT) as described in Kahneman & Tversky (1979) and Multi Cumulative Prospect Theory (MCPT) proposed by Ruß & Schelling (2018) which can better explain observed long term investment decisions than CPT by taking annual fluctuations into account.[15] Also, we include Partial Multi Cumulative Prospect Theory (PMCPT) in our analysis, which is a weighted average of CPT and MCPT, i.e., $PMCPT(X) = \omega CPT(X) + (1-\omega) MCPT(X)$, where X refers to the investment product and $\omega \in [0,1]$ to the weight. Note that for these subjective theories the policyholder is typically assumed to be risk averse for gains and risk seeking for losses with risk attitude $a \in \mathbb{R}_+$. Further, policyholders are assumed to be loss averse, described by a loss aversion parameter $\lambda \geq 1$. Also, we assume that probabilities are distorted based on the distortion function proposed by Prelec (1998), which distorts the probabilities for losses (α^-, β^-) and gains (α^+, β^+) differently. To compare the results of the different products, we compute the corresponding certainty equivalent (CE) returns, cf., Ruß & Schelling (2018) cf., Appendix C. The preference parameters used in our analysis can be found in Table 3. The choice of the subjective parameters (except for ω) is based on l'Haridon & Vieider (2019) and is the same for CPT, MCPT and PMCPT. The risk aversion parameter for EUT is in line with Chiappori & Paiella (2011).

γ	a	λ	β^+	α^+	β^-	α^-	ω
2.5	0.88	1.616	1.052	0.767	0.934	0.863	0.5

Table 3: Preference parameters used in the base case.

We now analyze all previously mentioned static and dynamic (CPPI) investment products.

[15] See also Graf *et al.* (2019) and Ruß & Schelling (2021). A more detailed description and definitions can be found in Appendix B.

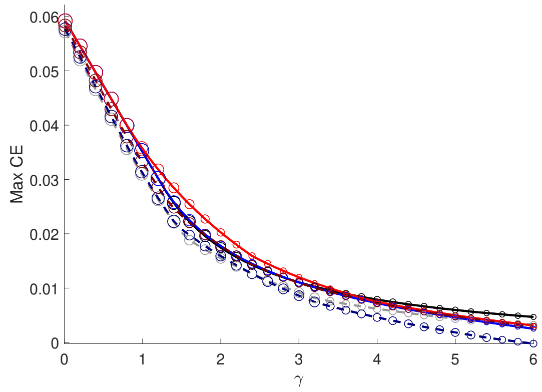
In Figure 3, we display certainty equivalent values of different product types under different preference theories depending on the risk aversion (panel (a)) and loss aversion (panel (b) and (c)). Due to the very large number of products considered, we only display the version of each product that comes with the highest CE-value. E.g., the value for the static-ICF product displayed at $\gamma = 2.5$ in panel (a) of Figure 3 is from the product with $\phi = 30\%$ and $\theta = 35\%$, since all other combinations of ϕ and θ would result in a lower CE value.[16] Note that for different parameter values of γ, respectively λ, the product with the highest CE of a product type can vary. The size of the circles represents the (average) direct stock ratio of the product with the highest CE. Further, Table 4 shows the product characteristics of the products with the highest CE (of all considered products) under different preference theories and for selected levels of risk and loss aversion, respectively.

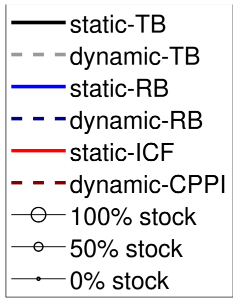
	$\gamma = 0.5$	$\gamma = 1.5$	$\gamma = 2.5$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 6$
EUT	$\theta = 100\%$	static-ICF^{50} $\theta = 57.5\%$	static-ICF^{30} $\theta = 35\%$	static-ICF^{30} $\theta = 25\%$	static-ICF^{30} $\theta = 17.5\%$	static-TB $\theta = 20\%$
	$\lambda = 1.25$	$\lambda = 1.616$	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 3$	
CPT	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	
MCPT	$\theta = 100\%$	static-ICF^{50} $\theta = 87.5\%$	static-ICF^{50} $\theta = 35\%$	static-ICF^{50} $\theta = 25\%$	static-ICF^{50} $\theta = 20\%$	
PMCPT	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	static-ICF^{50} $\theta = 45\%$	static-ICF^{50} $\theta = 32.5\%$	

Table 4: The product with the highest CE value for different preference theories and for selected levels of risk and loss aversion, respectively.

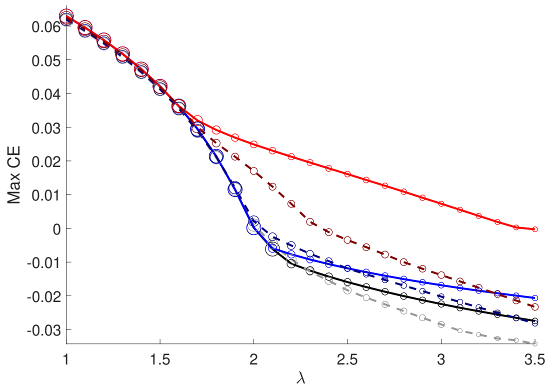
For a very low risk aversion under EUT ($\gamma < 1$), the CE-maximizing product variants within each product type are very similar and they have a stock ratio of 100% or a high multiplier and a low protection level. If risk aversion increases, the CE for all products decreases and the optimal product has a lower direct and total stock ratio, since a risk averse consumer prefers a more stable terminal value. This holds for all product types. E.g., for a medium risk aversion ($\gamma = 2.5$), the static-ICF^{30} and a direct stock ratio of 35% ($\theta^{tot} = 54.5\%$) has a higher CE

[16] Analogously for the dynamic products only the optimal products with respect to ϕ, PL and m are displayed.

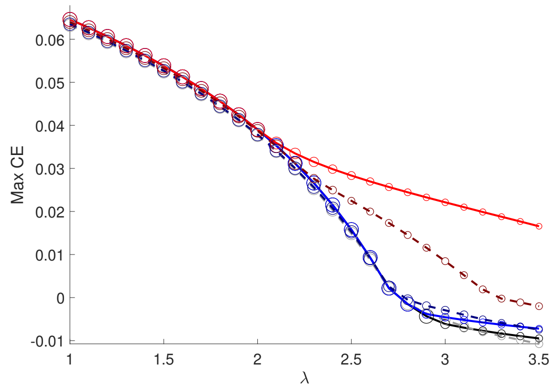




(a) EUT



(b) MCPT



(c) PMCPT

Figure 3: The maximum CE of different product types under different preference functions and for different levels of risk or loss aversion. The size of the circle corresponds to the (average) direct stock ratio of the product with the highest CE.

than other product types, while the optimal product for $\gamma = 3.5$ has a direct stock ratio of 17.5% ($\theta^{tot} = 42.25\%$), cf., Table 4. Further, for medium levels of risk aversion the respective CE-maximizing static-ICF has a higher CE than any product of any type. For a very high risk aversion ($\gamma > 4$) the static-TB with a low stock ratio has the highest CE, since a TB comes with a deterministic maturity value.

Under CPT (not displayed in Figure 3), a pure stock investment always yields the highest CE (cf., Table 4) as the probability for large gains is overweighted while the probability of a loss after 30 years is rather low. Note that all considered dynamic products have an average direct and total stock ratio of below 100% and therefore have a lower CPT CE value.[17]

[17] A lower protection level would lead to a higher stock ratio and thus to a higher CPT CE. In particular, a dynamic with protection level of 0% would lead to a stock ratio of 100% and thus the highest CPT CE.

Although annual value changes are punished under MCPT (Figure 3 (b)), for a low loss aversion $\lambda < 1.6$, the product with the highest CE is still a pure stock investment. For higher loss aversion a static-ICF product provides the highest CE (due to smoothed annual returns and hence reduced fluctuations). For $\lambda > 2.3$, static-ICF products with very low direct stock ratios are even the only products with a positive CE. Also, the stock ratio of the product with the highest CE decreases for an increasing loss aversion. Further, for $\lambda > 1.6$, dynamic-ICF products have a higher CE than purely market-based dynamic products. Note also that the most conservative dynamic-ICF still has a total average stock ratio of roughly 40% (for $m = 1$ and $PL = 90\%$, cf., Table 2), which is close to the total stock ratio of the most attractive static-ICF product for rather high loss aversions, cf., Table 4. Nevertheless, dynamic-ICF products typically have a higher fluctuation of the annual value changes than comparable static-ICF products which results in a lower MCPT-CE.[18]

Lastly, we consider PMCPT (cf., Figure 3 (c)), where we take into account annual values changes (MCPT) as well as the total value change (CPT). Overall, the effects are similar as for MCPT. But a pure stock investment now yields the highest CE for λ below 2.1, cf., Table 4. For higher loss aversion a static-ICF product yields (significantly) higher CE values than the other product types. Note that under pure CPT the difference between ICF based products with smoothed returns and comparable other products is only small as return smoothing only slightly impacts the characteristics of the terminal benefit. Therefore, the results under PM-CPT are mainly driven by the results under MCPT.

Overall, our results show that under all considered preference theories the static-ICF is in most cases preferred over the market-based products. Also, dynamic-ICF products dominate market-based dynamic investment strategies in most cases (expect for EUT in case of a very high level of risk aversion $\gamma > 5.5$). Further, we observe that in none of the considered settings

[18] Note that in the ICF we do not consider stock ratios above 50%, cf., Footnote 7 in Section 3. However, additional analyses show that for certain combinations of risk attitude (a) and loss aversion (λ) a higher stock ratio in the ICF would result in a subjectively even more attractive product. In practice that means that consumers would demand a product based on an ICF with the highest stock ratio that can feasibly be implemented.

a dynamic product outperforms all other products (although, under specific settings, e.g., for very high risk aversion or very low risk or loss aversion, we can find dynamic products with a CE that is close to highest CE). Moreover, we note that while for less conservative consumers (low γ respectively λ) a pure stock investment is the objective utility maximizing as well the subjectively preferred product, for more conservative consumers the products with the highest objective utility and the highest subjective attractiveness differ.

4.2 Sensitivity Analysis

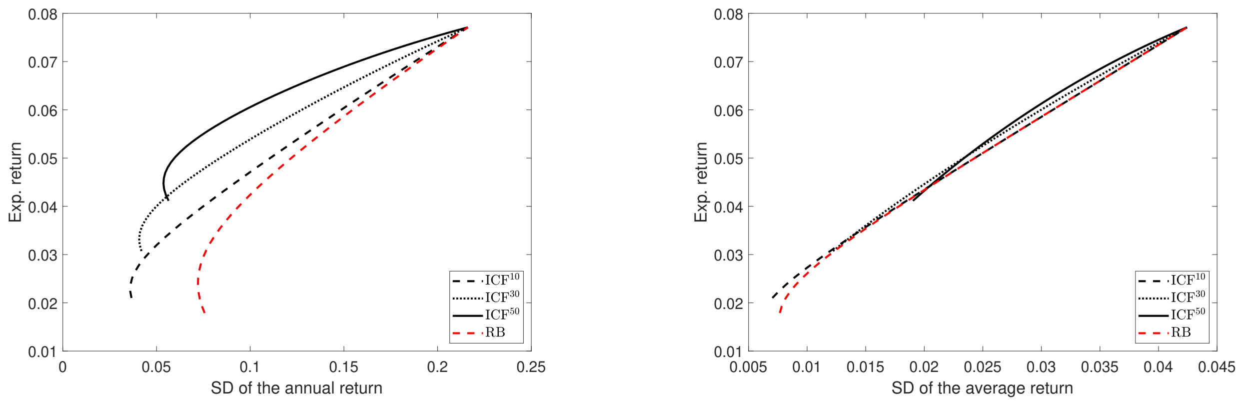
Now, we perform various sensitivity analyses with respect to the interest rates, the probability weighting, the risk attitude, as well as the length of the smoothing period.

4.2.1 Interest Rate Environment and Historical Returns

Firstly, we consider a different initial interest rate environment. In particular, we assume a higher initial market forward rate curve which is given by the historical NSS-parameters of the end of 2014 and the three previous years. The parameters as well as the historical stock return and the return of a rolling bond investment are given in Table 10 in Appendix A. The other parameters of the capital market model remain unchanged. Note that with this change, we increase the 20 year spot rate from -0.165% p.a. to 1.318% p.a. Also, the fair participation rate (α) is 0.8161 for an ICF with 10% stock, 0.8312 for an ICF with 30% stock and 0.8595 for an ICF with 50% stock. The fair participation rates are lower than in the base case because the historical stock and bond returns are significantly higher.

Effects of Smoothing

As in the base case we observe that the cover fund can significantly reduce the volatility of the annual returns while hardly affecting the volatility of the overall return, cf., Figure 4. This indicates that the effects of the return smoothing are qualitatively independent of the initial interest rate environment and the historical returns.



(a) Average SD of annual returns

(b) The SD of the average return

Figure 4: The efficient frontier for different static-ICF and a static-RB investment based on the standard deviation of annual returns and based on the standard deviation of the average return in the interest rate sensitivity scenario.

The Dynamic Investment Products

Again, Table 5 shows results for dynamic products based on the different low-risk assets and for different protection levels and multipliers. Due to the higher interest rates and stock returns the average direct and total stock ratio is higher for all considered products and the shortfall probability is significantly lower than in the base case. As a consequence of the higher average direct stock ratios, we observe higher expected returns and higher standard deviations. However, as in the base case, the dynamic products based on an ICF with a stock ratio of 10% yield similar or higher returns compared to the dynamic products based on a RB. Also, the shortfall probabilities are very similar in this case.

Due to the higher interest rate level, we can also analyze protection levels of 100% and 120%.[19] The results are shown in Table 6. As in the base case the differences between the dynamic-ICF and dynamic-RB are larger for conservative combinations of protection level and multiplier, e.g., $PL = 120\%$ and $m = 1$. In this case the expected return of the dynamic products based on the ICF^{10} is higher than for the dynamic-RB while the shortfall probability is higher for the dynamic-RB. Again, the standard deviations of the annual return are very similar. Only the standard deviation of the overall return is slightly higher for the dynamic-ICF^{10}.

[19] Note that the maximum possible protection level in this case is $PL \approx 148\%$.

	$m=1$ and $PL=80\%$					$m=3$ and $PL=80\%$					$m=5$ and $PL=80\%$				
	ICF with $\phi=$			RB	TB	ICF with $\phi=$			RB	TB	ICF with $\phi=$			RB	TB
	10%	30%	50%			10%	30%	50%			10%	30%	50%		
$\bar{r}$	6.47	6.73	6.95	5.94	5.57	7.59	7.56	7.52	7.51	7.47	7.6	7.58	7.55	7.53	7.47
$\sigma[\bar{r}]$	3.33	3.51	3.72	3.14	2.94	4.32	4.39	4.49	4.36	4.33	4.4	4.45	4.53	4.45	4.46
$\sigma[r_t]$	13.98	14.06	14.21	13.93	14.04	19.52	19.36	19.28	19.4	19.26	19.71	19.6	19.57	19.5	19.14
$\bar{\theta}$	63.39	63.28	62.85	60.61	59.36	86.82	85.37	83.98	84.77	83.56	87.26	86.15	85.09	84.38	81.14
θ^{tot}	67.05	74.30	81.43	60.61	59.36	88.14	89.76	91.99	84.77	83.56	88.53	90.31	92.55	84.38	81.14
P_{PL}	1.2	2.7	4.8	0.9	0	12.7	13.9	15.1	11.8	0	16.5	16.6	16.9	16.4	0
	$m=1$ and $PL=50\%$					$m=3$ and $PL=50\%$					$m=5$ and $PL=50\%$				
$\bar{r}$	6.98	7.13	7.25	6.7	6.52	7.69	7.68	7.67	7.68	7.67	7.69	7.68	7.68	7.68	7.67
$\sigma[\bar{r}]$	3.65	3.77	3.91	3.54	3.45	4.32	4.38	4.45	4.34	4.33	4.34	4.39	4.44	4.38	4.38
$\sigma[r_t]$	16.61	16.6	16.6	16.62	16.67	20.91	20.84	20.8	20.89	20.86	21	20.95	20.93	20.95	20.85
$\bar{\theta}$	76.47	76.23	75.78	75.5	75.11	95.53	94.93	94.35	95.09	94.86	95.81	95.34	94.92	95.04	94.33
θ^{tot}	78.82	83.36	87.89	75.5	75.11	95.98	96.45	97.18	95.09	94.86	96.23	96.74	97.46	95.04	94.33
P_{PL}	0.2	0.8	1.8	0.1	0	5.2	6.0	6.8	4.6	0	6.8	7.1	7.5	6.5	0

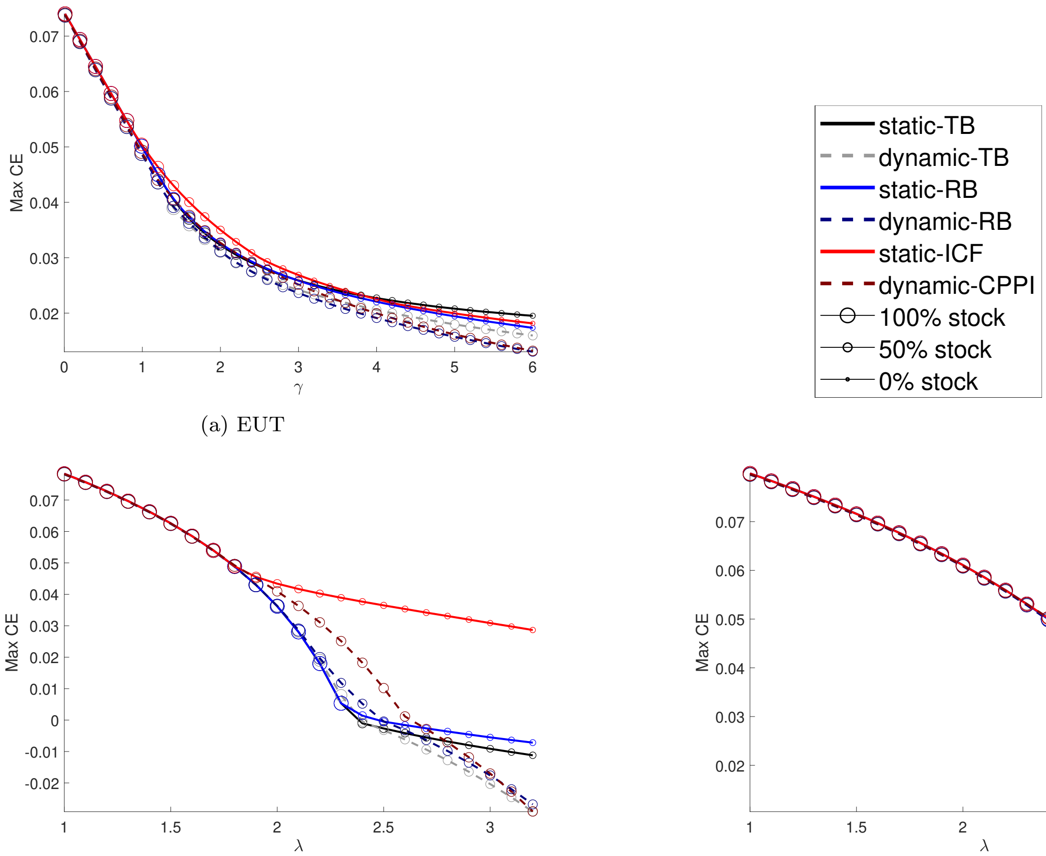
Table 5: Different risk measures for dynamic products with different multipliers (m) and different protection levels (PL) based on different low-risk investments (ICF, RB,TB) in the interest rate sensitivity scenario. All results are given in percent.

	$m=1$ and $PL=100\%$					$m=3$ and $PL=100\%$					$m=5$ and $PL=100\%$				
	ICF with $\phi=$			RB	TB	ICF with $\phi=$			RB	TB	ICF with $\phi=$			RB	TB
	10%	30%	50%			10%	30%	50%			10%	30%	50%		
$\bar{r}$	6.08	6.44	6.74	5.32	4.76	7.49	7.44	7.39	7.24	7.11	7.5	7.46	7.41	7.29	7.12
$\sigma[\bar{r}]$	3.11	3.34	3.58	2.84	2.52	4.25	4.31	4.4	4.25	4.18	4.36	4.39	4.48	4.38	4.36
$\sigma[r_t]$	12.27	12.5	12.82	12.19	12.36	18.3	18.11	18.05	17.93	17.67	18.56	18.43	18.42	18.09	17.39
$\bar{\theta}$	54.58	54.86	54.75	49.75	47.39	79.42	77.72	76.08	74.91	72.23	79.87	78.48	77.16	74.19	67.49
θ^{tot}	59.12	68.4	77.38	49.75	47.39	81.48	84.40	88.04	74.91	72.23	81.88	84.94	88.58	74.19	67.49
P_{PL}	2.6	5.0	7.6	2.6	0	17.9	18.9	20.1	17.9	0	23.2	22.8	23.0	24.4	0
	$m=1$ and $PL=120\%$					$m=3$ and $PL=120\%$					$m=5$ and $PL=120\%$				
$\bar{r}$	5.64	6.13	6.52	4.58	3.71	7.36	7.32	7.27	6.73	6.38	7.37	7.33	7.28	6.86	6.4
$\sigma[\bar{r}]$	2.89	3.16	3.44	2.51	1.96	4.13	4.17	4.27	3.99	3.8	4.26	4.27	4.35	4.17	3.97
$\sigma[r_t]$	10.6	11.05	11.6	10.53	10.78	16.91	16.79	16.82	16.05	15.59	17.25	17.17	17.24	16.3	15.15
$\bar{\theta}$	45.5	46.56	47.08	37.6	33.03	71.36	69.88	68.43	62.34	57.2	71.8	70.59	69.38	61.52	49.85
θ^{tot}	50.95	62.59	73.54	37.6	33.03	74.22	78.92	84.22	62.34	57.2	74.62	79.41	84.69	61.52	49.85
P_{PL}	5.5	8.1	10.8	7.1	0	22.9	23.3	24.2	25.0	0	29.7	28.2	28.1	33.1	0

Table 6: Different risk measures for dynamic products with different multipliers (m) and different protection levels (PL) based on different low-risk investments (ICF, RB,TB) in the interest rate sensitivity scenario. All results are given in percent.

Preferences under EUT, CPT, MCPT and PMCPT

The results under the different preference theories are displayed in Figure 5 and Table 7. Firstly, we observe in Figure 5 that the structure of the results is very similar to the base case. In general, the CE values are higher than in the base case for all products. The products with the highest EUT values are very similar to the base case as the higher interest rates level shifts expected returns upwards but has only little influence on the volatility of the return. For MCPT and PMCT we find that the CE maximizing products have higher stock ratios than in the base case due to the lower probabilities for annual losses.



(a) EUT

(b) MCPT

(c) PMCPT

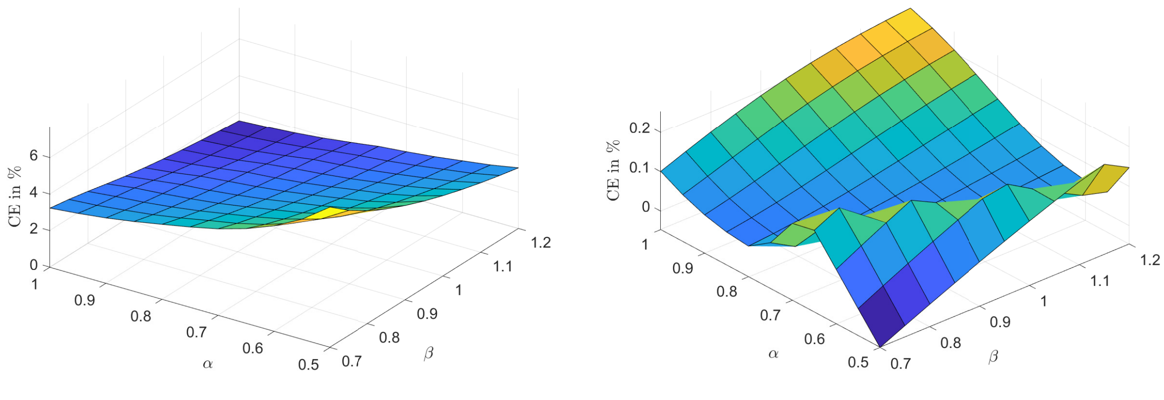
Figure 5: The maximum CE of different product types under different preference functions and for different levels of risk respectively loss aversion in the interest rate sensitivity scenario. The size of the circle corresponds to the (average) direct stock ratio of the product with the highest CE.

	$\gamma = 1.5$	$\gamma = 2.5$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 6$
EUT	static-ICF^{50} $\theta = 60\%$	static-ICF^{50} $\theta = 27.5\%$	static-ICF^{30} $\theta = 25\%$	static-ICF^{30} $\theta = 17.5\%$	static-TB $\theta = 20\%$
	$\lambda = 1.25$	$\lambda = 1.616$	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 3$
CPT	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$
MCPT	$\theta = 100\%$	$\theta = 100\%$	static-ICF^{50} $\theta = 47.5\%$	static-ICF^{50} $\theta = 30\%$	static-ICF^{50} $\theta = 22.5\%$
PMCPT	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	static-ICF^{50} 72.5% Stock	static-ICF^{50} 40% Stock

Table 7: The product with the highest CE value for different preference theories and for selected levels of risk and loss aversion in the interest rate sensitivity scenario, respectively.

4.2.2 Probability Weighting Parameters

Next, we perform a sensitivity analysis with respect to the probability weighting. We fix the loss aversion at $\lambda = 2$ and only vary the probability weighting parameters $\alpha^- = \alpha^+ \in [0.5, 1]$ and $\beta^- = \beta^+ \in [0.7, 1.2]$. While α governs the curvature of the function and hence the tendency to overweight low likelihood events and to underweight high likelihood events (likelihood-insensitivity), the choice of β primarily influences the point where the function (typically inverse S-shaped) crosses the 45-degree line (elevation).[20]



(a) MCPT (b) PMCPT

Figure 6: The difference between the maximum CE of an ICF-based investment strategy and the maximum of a market-based investment strategy for different levels of probability weighting.

Note that under CPT with $\lambda = 2$ and for all considered specifications of the probability weight-

[20]Reducing α increases likelihood-insensitivity and reducing β shifts the crossing point right and upwards.

ing function the ICF-based investment strategy and market-based investment strategy with the highest CE are equal (always a pure stock investment). For MCPT and PMCPT, we display in Figure 6 the difference between the maximum CE of an ICF based investment product (static and dynamic) and the maximum CE of a market-based investment product (static and dynamic with RB and TB). The results are shown in Figure 6 (MCPT in panel (a) and PMCPT in panel (b)). In panel (a) of Figure 6, we observe that under MCPT the preferred ICF-based product always yields a higher CE than the preferred market-based product. The difference is particularly high for very low values of α and β (i.e., in case of strong overweighting of low-likelihood events). The optimal ICF-product is always a static-ICF^{50} with a direct stock ratio between 27.5% ($\theta^{tot} = 63.75\%$) and 37.5% ($\theta^{tot} = 68.75\%$), where the stock ratio is increasing for increasing α and β. Note that the CE of the preferred ICF-product is decreasing (from 4.9% to 2.3%) for increasing α and β.

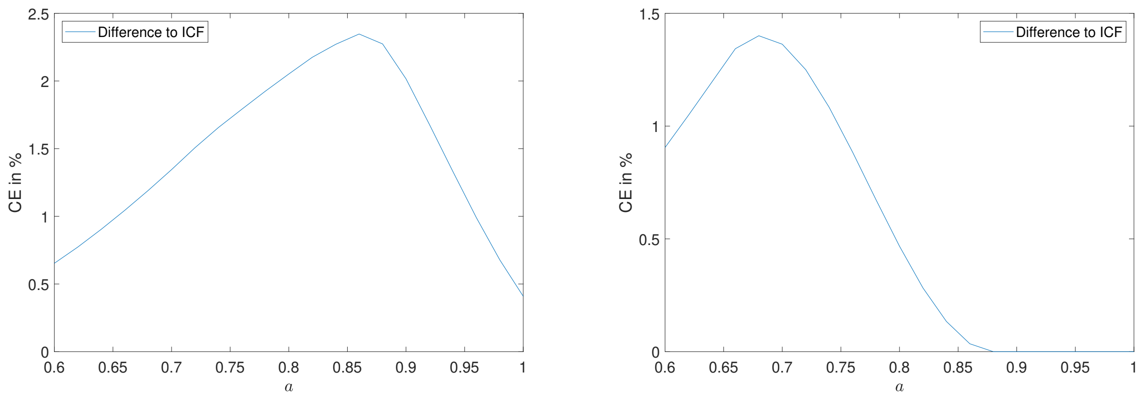
Under PMCPT, we observe that for most combinations of α and β, the preferred ICF-product yields a higher CE than the preferred market-based product. For medium and high α and β a static-ICF^{50} with a direct stock ratio between 65% and 95% yields the highest CE (where the stock ratio is decreasing for increasing α and β). Only for very low values of α and β, the market-based product has a higher CE by roughly 0.05%.[21] The shape of the surface is striking and caused by a number of superimposing effects. For example, the type and stock ratio of the optimal ICF-based vs. market-based product changes several times on the surface.

4.2.3 Risk Attitude

We also varied the risk attitude a between 0.6 and 1. We fix $\lambda = 2$ and the probability weighting parameters as stated for the base case. We again compute the difference between the maximum CE of an ICF-based investment product and the maximum of a market-based investment product. The results can be found in Figure 7.[22]

[21] Note that for low values of α and β, the preferred ICF product is a dynamic-ICF based on ICF^{10} with $PL = 80\%$ and $m = 5$, while the preferred market-based product is a dynamic-RB with $PL = 80\%$ and $m = 5$.

[22] Note that under CPT the optimal strategy is again a pure stock investment for all values of a.



(a) MCPT (b) PMCPT

Figure 7: The difference between the maximum CE of an ICF-based investment strategy and the maximum CE of a market-based investment strategy for different level of risk attitude (a) .

In Figure 7, we observe that for all levels of the risk attitude the maximum CE of the ICF-based investment products is higher or equal than the maximum CE of the market-based investment products. Under MCPT, we observe that an ICF-based investment product yields the highest CE for all values of the risk attitude as ICF products come with significantly smaller annual changes and thus lower probabilities for annual losses. Further, the direct stock ratio of the preferred ICF-products is between 20% and 60% (hence, θ^{tot} is between 60% and 80%) and is increasing for a decreasing risk attitude.

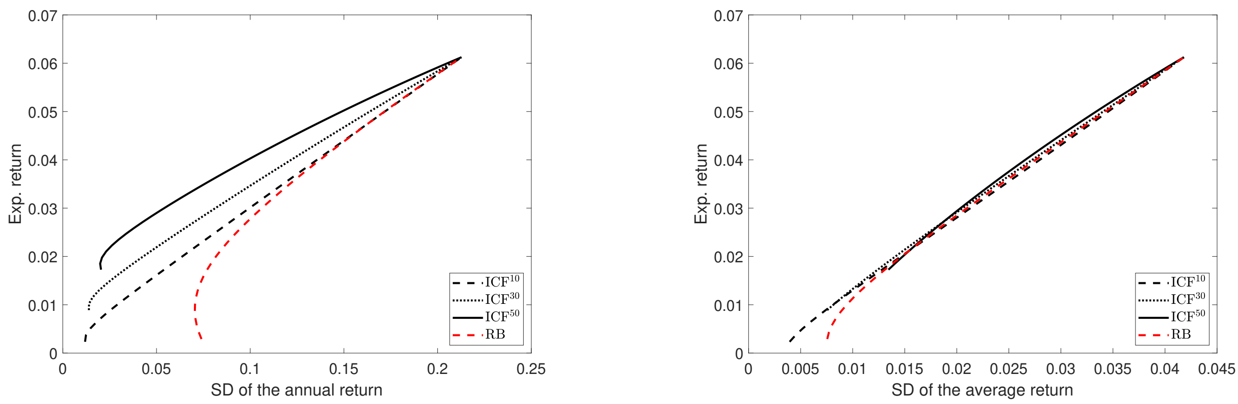
Under PMCPT and for a low risk attitude ($a \geq 0.9$) the market- and the ICF-based products yield the same maximum CE which in both cases is attained by a pure stock investment. Further, we observe that in these cases the CPT CE has a higher influence on the PMCPT CE than for higher risk attitudes (lower values of a). For a higher risk attitude a more stable annual return is more important and CPT has a lower impact on the PMCPT CE than in the case of low risk attitudes. Therefore, in these cases an ICF-based investment product is preferred. Also, the direct stock ratio of the strategy with the highest CE is increasing for a decreasing risk attitude (from 25% ($\theta^{tot} = 62.5\%$) to 100%).

4.2.4 The smoothing period

We have increased the smoothing period from 3 years to 10 years and we repeated the numerical analysis.[23] The fair participation rate is 0.3766 for a stock ratio of 10%, 0.4912 for a stock ratio of 30% and 0.6261 for $\phi = 50\%$. The values of α are significantly lower than for a smoothing period of 3 years because the historical returns are significantly higher.

Effects of Smoothing

As for the base case, we observe that smoothing in the insurer's cover fund reduces the volatility of the annual returns while not affecting the overall return, cf., Figure 8. In particular, for a low direct stock ratio the volatility of the annual return is reduced more strongly by the longer smoothing period. Note that the history has only a small influence on the volatility of the annual returns. We also studied other histories and smoothing periods and the results were qualitatively very similar. This indicates that the observed effects do qualitatively not depend on the smoothing period (for reasonable choices).



(a) Average SD of annual returns

(b) The SD of the average return

Figure 8: The efficient frontier for different static-ICF based on a smoothing period of 10 years and a static-RB investment based on the standard deviation of annual returns and based on the standard deviation of the average return.

[23]For this, Equation (1) has been adjusted accordingly. In particular, we use $sp = 10$ years (instead of 3) and the necessary historical stock and bond return are given in Table 11 in Appendix A.

The Dynamic Investment Products

Again, we have analyzed different figures for dynamic products based on the different low-risk assets and for different protection level and multiplier (similar to Table 2).[24] We observe similar results as in the base case, i.e., ICF products yield higher expected returns and higher average direct stock ratios, in particular for conservative dynamic products (high PL and low m). Also, the standard deviations of annual returns, the average annual returns, and the shortfall probabilities are very similar for the dynamic-ICFs compared to the base case. Consequently, this indicates that, independent of the smoothing period, dynamic-ICFs have a significant potential to earn a higher return than a comparable dynamic-RB.

Preferences under EUT, CPT, MCPT and PMCPT

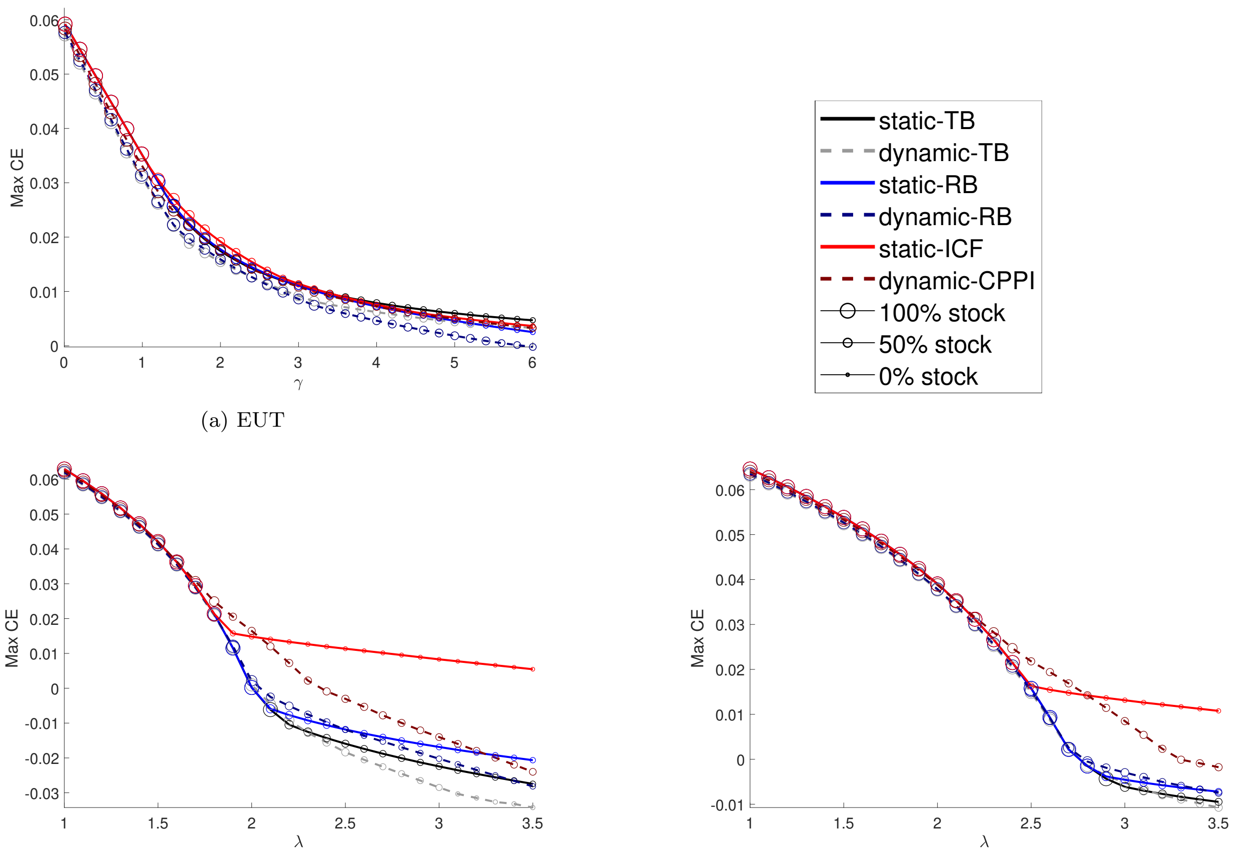
The results under the different preference theories are displayed in Figure 9 and Table 8. For EUT, we observe similar results as in the base case: static products have a higher CE than dynamic products. Further, up to a high risk aversion $\gamma = 4$ a static-ICF product and for $\gamma > 4$ a static-TB yield the highest CE, cf., Figure 9.

Under MCPT and PMCPT the products based on the insurer's cover fund yield the same or higher CE than the market-based products (as in the base case). However, for a medium level of loss aversion (MCPT $\lambda \approx 2$ and PMCPT $\lambda \approx 2.5$) a dynamic-ICF yields a higher CE than the static-ICFs, cf., Table 8. Note that we also analyzed other smoothing periods (longer than 3 years) and the results are qualitatively very similar. To sum up, also under longer smoothing periods, we observe that the ICF based products outperform the market-based products in many cases (or are at least comparable in terms of CE values).

[24]Note that the values for the dynamic-RB and dynamic-TB are identical to the base case setup as they are not influenced by a longer smoothing period.

	$\gamma = 1.5$	$\gamma = 2.5$	$\gamma = 3$	$\gamma = 3.5$	$\gamma = 6$
EUT	Static-ICF^{50} $\theta = 67.5\%$	Static-ICF^{50} $\theta = 35\%$	Static-ICF^{30} $\theta = 30\%$	Static-ICF^{30} $\theta = 22.5\%$	Static-TB $\theta = 20\%$
	$\lambda = 1.25$	$\lambda = 1.616$	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 3$
CPT	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$
MCPT	$\theta = 100\%$	dynamic-ICF^{10} $PL = 70\%$, $m = 4$	dynamic-ICF^{50} $PL = 90\%$, $m = 1$	Static-ICF^{50} $\theta = 10\%$	Static-ICF^{50} $\theta = 7.5\%$
PMCPT	$\theta = 100\%$	$\theta = 100\%$	$\theta = 100\%$	dynamic-ICF^{50} PL 90%, $m = 1$	Static-ICF^{50} $\theta = 15\%$

Table 8: The product with the highest CE value for different preference theories and for selected levels of risk and loss aversion in the smoothing period sensitivity scenario, respectively.



(a) EUT

(b) MCPT

(c) PMCPT

Figure 9: The maximum CE of different product types under different preference functions and for different levels of risk and loss aversion in the smoothing period sensitivity scenario, respectively. The size of the circle corresponds to the (average) direct stock ratio of the product with the highest CE.

5 Conclusion

In this paper, we investigated the effect of multiple, segregated cover funds equipped with different equity ratios on the short-term fluctuation risk as well as on the distribution of the terminal benefit of different retirement savings products. We found that smoothing mechanisms embedded in such cover funds can significantly reduce interim fluctuations while hardly impacting the long term risk-return profile. Hence, the existence of multiple, segregated cover funds that can be used as building blocks for more complex products enables insurers to offer different risk-return profiles of the terminal benefit in combination with a rather low pathwise volatility (when compared to investments without smoothing mechanisms). This appears highly relevant for the subjective attractiveness.

We analyzed various static and dynamic investment products based on cover funds with different stock ratios and compared them to purely market-based investment products. The results show that dynamic products based on the insurer's cover fund have a significantly higher return potential compared to dynamic market-based products with the same risk characteristics (in particular for conservative product settings, which is of particular interest for rather risk averse investors).

Moreover, we examined the effect of collective cover funds on the objective utility and subjective attractiveness. To this end, we compared the products under Expected Utility Theory as well as Cumulative Prospect Theory and extensions of it (MCPT and PMCPT). Overall, we found that in most cases static products based on collective cover funds (mostly equipped with high stock ratios) outperform all other products with respect to objective utility and subjective attractiveness. Although, unfortunately, not the same product design is at the same time objectively optimal and subjectively preferable. Also, the main results are qualitatively independent of the smoothing period and also hold for other (reasonable) interest rate environments. Consequently, our results clearly demonstrate the advantages of segregated cover funds with different equity ratios.

The current challenges caused mainly by consumers' demand for low-risk products and the difficulty to offer high guarantees in the low-interest environment, could potentially be overcome with products without guarantee that still come with a rather high degree of safety. Therefore, our findings should be of high topical interest for insurance companies and legislators/regulators. Suitably designed products based on insurer's cover funds with return smoothing elements can serve the consumers' desire for safety (in particular, avoiding high short-term losses) without limiting the long-term return potential. Nevertheless, there are several practical obstacles that need to be overcome. For instance, in some countries, e.g., Germany, regulation for surplus distribution in participating life insurance products was designed to distribute returns that exceed a guaranteed rate of return. It is not always clear, if and how this can be applied to products without a guarantee. Even if it might be formally straightforward to set the annual guaranteed rate to -100% (and hence let any and all return play the role of "surplus"), some laws and regulation needs to be rewritten to make sense also in such settings. Also, in particular when cover funds with a rather high stock ratio are being offered, there need to be rules that prevent individual consumers to speculate against the pool of policyholders, e.g. by surrendering their contract after a market crash, when the smoothed policy value significantly exceeds the market value of the corresponding assets. Hence, appropriate regulation for fair market value adjustments in case of surrender is needed. Moreover, there is a range of practical issues (operationally as well as economically) that need to be considered when a new, segregated fund is being set up, in particular in the current low interest rate environment.

There are numerous suggestions for further research: Mainly the impact of different kinds of smoothing mechanisms on the risk-return characteristics of the resulting products should be further analyzed. We have seen that the considered – purely formula based – smoothing algorithm is very effective to reduce interim fluctuation but does hardly impact the distribution of terminal wealth. While this may be desirable in some circumstances, one might also be interested in smoothing mechanisms that reduce the volatility of the probability distribution of terminal wealth (ideally without impacting expected return). It can be expected that path-dependent return smoothing (e.g., by building up certain "collective buffers" in good years which are used to increase returns in bad years or benefits at maturity that are particularly low) increases the advantages of smoothed products even more. Hence, a systematic analysis which type of smoothing mechanism is suitable to achieve which goal could stimulate the design of products that more effectively reduce those risks that are relevant to an individual consumer without harming the return potential too much. Since some smoothing mechanisms applied in practice have characteristics of "collective buffers" described above, such analyses might reveal that purely formula based smoothing rules systematically underestimate the effect of actually used smoothing mechanisms on the probability distribution of terminal wealth. This might impact the question how smoothing should be approximated, e.g., in models that calculate risk-return profiles and risk-return classes of retirement savings products.

A The Hull-White Model and the Stock Market Model

The two-factor Hull-White model (cf., Brigo & Mercurio (2007)) is given by the following dynamics for $t \in [0, T]$ with $T \in \mathbb{N}$:

$$dx(t) = -ax(t)dt + \sigma dW_1(t), \quad x(0) = 0, \tag{8}$$

$$dy(t) = -by(t)dt + \eta\Big(\rho dW_1(t) + \sqrt{1-\rho^2}dW_2(t)\Big), \quad y(0) = 0, \tag{9}$$

$$r^*(t) = x(t) + y(t) + \zeta(t), \tag{10}$$

where $r^*(t)$ denotes the short rate at time t under the risk-neutral measure Q, $\{W_i(t)\}_{t\geq 0}$ are independent one-dimensional Brownian motions for $i = 1, 2$, and $\zeta(t)$ is a deterministic function given by

$$\zeta(t) = f^M(0,t) + \frac{\sigma^2}{2a^2}(1 - e^{-at})^2 + \frac{\eta^2}{2b^2}(1 - e^{-bt})^2 + \rho\frac{\sigma\eta}{ab}(1 - e^{-at})(1 - e^{-bt}), \tag{11}$$

where $f^M(0,t)$ is the initial market forward rate curve and can be obtained by the Nelson-Siegel-Svensson curve (NSS). The interest rate model is exponentially affine. Hence, based on $r(t)$ we can determine the value of a zero coupon bond $P(t,T)$ at time t with maturity T.

Under the real-world measure P, the short rate r is given by,

$$r(t) = x(t) + d_x(1 - e^{-at}) + y(t) + d_y(1 - e^{-bt}) + \zeta(t), \tag{12}$$

where $\{x(t)\}_{t\geq 0}$ and $\{y(t)\}_{t\geq 0}$ are the processes under Q and d_x and d_y are constants where $d_x + d_y$ can be interpreted as the long-run risk premium of the short-rate, cf., Berninger & Pfeiffer (2020).

The dynamics of the stock market is modeled by a generalized Black-Scholes model, particularly the price of a stock at time t is given by

$$S^*(t) = s_0 \exp\left(\int_0^t r^*(s)ds - 0.5\sigma_S^2 t + \sigma_S W(t)\right) \quad \text{under } Q, \tag{13}$$

$$S(t) = s_0 \exp\left(\int_0^t r(s)ds + (\lambda_S - 0.5\sigma_S^2)t + \sigma_S W(t)\right) \quad \text{under } P, \tag{14}$$

where s_0 is the initial price of the stock (at time 0), λ_S is the risk premium, σ_S is the volatility of the stock market, and $\{W(t)\}_{t\geq 0}$ is a one-dimensional Brownian motion independent of the short rate processes. The parameters used in our numerical analyses are given in Tables 9-11.

HW-Model	a	b	σ	η	ρ	d_x	d_y
	0.3912	0.0785	0.0201	0.0135	-0.6450	-0.0033	0.0255
NSS Curve	β_1	β_2	β_3	β_4	τ_1	τ_2	
	0.27173	-0.37865	-2.5003	-1.43785	2.95077	0.21103	

Table 9: The base case parameters of the short rate model (in line with Graf *et al.* (2021)).

NSS Curve	β_1	β_2	β_3	β_4	τ_1	τ_2	
	2.14449	-2.37645	26.11241	-29.99782	1.8297	1.99969	
History	2012	2013	2014	History	2012	2013	2014
Stock	17.65%	21.61%	3.93%	Rolling Bond	9.44%	-3.34%	18.19%

Table 10: The NSS-parameters and the historical returns (stock and rolling bond) as at the end of 2014 (used for the sensitivity analysis, cf., Section 4.2.1).

Year	2010/1	2011/2	2012/3	2013/4	2014/5	2015/6	2016/7	2017/8	2018/9	2019/20
Stock	14.36%	-17.29%	19.2%	26.68%	8.83%	-13.89%	23.27%	1.28%	5.12%	-5.05%
R-Bond	1.10%	19.39%	2.61%	8.15%	6.83%	11.11%	-4.49%	2.33%	7.40%	1.81%

Table 11: The historical stock and rolling bond returns for a smoothing period of 10 years, cf., Section 4.2.4.

B Cumulative Prospect Theory

In Cumulative Prospect Theory (CPT), cf., Kahneman & Tversky (1979) and Tversky & Kahneman (1992), an investment A with (random) final outcomes E is valued with an S-shaped value function v and relative to a given reference point χ. The gains and losses are described by the random variable $X := E - \chi$. Then the CPT utility is defined as

$$CPT(X) = \int_{-\infty}^{0} v(x)d\left(w\left(F(x)\right)\right) + \int_{0}^{\infty} v(x)d\left(-w\left(1 - F(x)\right)\right), \tag{15}$$

where $F(x) = \mathbb{P}(X \leq x)$ and v is the investor's value-function which is defined as $v(x) := x^a \mathbb{1}\left\{x \geq 0\right\} - \lambda |x|^a \mathbb{1}\left\{x < 0\right\}$ where $\lambda > 0$ is the loss aversion parameter and $a \in \mathbb{R}_+$ controls the risk appetite. The probability distortion function is given by $w^s(p) = \exp(-\beta^s(-\ln(p))^{\alpha^s})$, where $\alpha > 0$ governs the curvature, $\beta > 0$ the elevation, and $s \in \{+, -\}$ indicates gains or losses since probabilities for losses (α^-, β^-) and gains (α^+, β^+) are distorted differently.

In Multi Cumulative Prospect Theory (MCPT), cf., Ruß & Schelling (2018), the annual gains and losses (X_t) of an investment A are taken into account, i.e., $X_t := A_t - \chi_t$, where $t \in \{1, \ldots, T\}$, T is the maturity of the investment, A_t is the account value at time t, and χ_t is the reference point at time t. The MCPT value of investment A is then defined by

$$MCPT(A) := \sum_{t=1}^{T} CPT(X_t) \tag{16}$$

with $CPT(X)$ as defined in (15) and we assume no subjective discounting.

In Partial Multi Cumulative Prospect Theory (PMCPT) the terminal value (X) of an investment A as well as potential interim changes (X_t) are considered and thus it combines features of CPT and MCPT. The combination is defined as

$$PMCPT(A) := \omega CPT(X) + (1 - \omega) MCPT(A), \tag{17}$$

where $\omega \in [0, 1]$ denotes the weight, $CPT(X)$ as defined in (15) and $MCPT(A)$ as defined in (16).

C Computation of the Certainty Equivalents

For all preference formulations it is possible to compute certainty equivalent returns which describe the fixed annual return that an investor would regard equally desirable as the considered contract. We denote the value of choice c under a preference formulation G by $G(c)$, where $G \in \{EUT, CPT, MCPT, PMCPT\}$. Further, we compute the corresponding fixed annual returns r^G by the following formulas:

- Under EUT:

$$EUT(c)^{\frac{1}{\gamma}} = e^{T \cdot rEUT}.$$

- Under CPT:

$$\begin{aligned}
CPT(c) &= \left(e^{T \cdot r^{CPT}} - 1\right)^a \quad \text{if } CPT(c) \geq 0 \\
CPT(c) &= -\lambda \left|e^{T \cdot r^{CPT}} - 1\right|^a \quad \text{if } CPT(c) < 0.
\end{aligned}$$

- Under MCPT:

$$\begin{aligned}
MCPT(c) &= \sum_{t=1}^{T} \left(e^{t \cdot r^{MCPT}} - e^{(t-1) \cdot r^{MCPT}}\right)^a \quad \text{if } MCPT(c) \geq 0 \\
MCPT(c) &= -\lambda \sum_{t=1}^{T} \left|e^{t \cdot r^{MCPT}} - e^{(t-1) \cdot r^{MCPT}}\right|^a \quad \text{if } MCPT(c) < 0.
\end{aligned}$$

- Under PMCPT:

$$\begin{aligned}
PMCPT(c) &= \omega \left(e^{T \cdot r^{PMCPT}} - 1\right)^a + (1-\omega) \sum_{t=1}^{T} \left(e^{t \cdot r^{PMCPT}} - e^{(t-1) \cdot r^{PMCPT}}\right)^a \\
&\qquad \text{if } PMCPT(c) \geq 0 \\
PMCPT(c) &= -\lambda \left[\omega \left|e^{T \cdot r^{PMCPT}} - 1\right|^a + (1-\omega) \sum_{t=1}^{T} \left|e^{t \cdot r^{PMCPT}} - e^{(t-1) \cdot r^{PMCPT}}\right|^a\right] \\
&\qquad \text{if } PMCPT(c) < 0.
\end{aligned}$$

Note that for EUT and CPT we can also derive certainty equivalent values which describe the fixed terminal payoff that an investor would regard as equally desirable as the considered contract. For these preference formulations, the certainty equivalent returns are equal to the annualized returns which correspond to the certainty equivalent values. However, under MCPT and PMCPT the preferences are based on interim value changes (in our case annual), i.e., from the paths which result in the terminal payoff. Therefore, the consideration of a single certainty equivalent value is not possible and we rather rely on fixed annual returns.

D Additional Numerical Results

Table 12 shows the shortfall probability, the expected shortfall and the mean for different dynamic strategies and protection levels.

		Shortfall probability in %				Exp. shortfall				Mean			
	PL	50%	70%	80%	90%	50%	70%	80%	90%	50%	70%	80%	90%
$m = 1$	ICF^{10}	0.94	3.37	6.12	11.04	0.4606	0.6324	0.7158	0.7958	4.1351	3.4098	3.0474	2.6875
	ICF^{30}	2.43	5.94	8.68	12.06	0.4353	0.599	0.6796	0.7557	4.4258	3.821	3.5228	3.2306
	ICF^{50}	4.46	8.87	11.53	14.51	0.4028	0.5533	0.6263	0.6971	4.6716	4.1767	3.9389	3.7125
	RB	0.75	4.12	9.06	18.71	0.4587	0.6276	0.7074	0.7805	3.6734	2.765	2.3171	1.8944
	TB	0	0	0	0	-	-	-	-	3.3235	2.2734	1.7484	1.2234
$m = 2$	ICF^{10}	6.16	11.05	14.31	18.69	0.4477	0.6216	0.7062	0.7878	5.7042	5.2553	4.9143	4.4812
	ICF^{30}	8.39	13.57	16.5	19.8	0.4279	0.5939	0.6763	0.7566	5.6321	5.1585	4.8511	4.5011
	ICF^{50}	10.62	16.33	19.11	21.81	0.3978	0.5513	0.627	0.6998	5.5546	5.0494	4.7653	4.4784
	RB	6.15	12.53	17.76	26.29	0.4479	0.6203	0.7035	0.7817	5.5208	4.5851	3.8995	3.0821
	TB	0	0	0	0	-	-	-	-	5.441	4.257	3.3268	2.0273
$m = 3$	ICF^{10}	10.7	17.92	21.6	26.04	0.4391	0.6092	0.6935	0.7767	5.7765	5.4521	5.2025	4.8669
	ICF^{30}	11.92	18.82	22.09	25.46	0.4231	0.5872	0.6693	0.7507	5.7221	5.3459	5.0937	4.7962
	ICF^{50}	13.26	20.42	23.57	26.48	0.3943	0.5466	0.6229	0.698	5.6669	5.235	4.9727	4.6981
	RB	11.25	20.85	26.93	34.74	0.4423	0.6137	0.6984	0.779	5.6551	4.9366	4.3042	3.4653
	TB	0	0	0	0	-	-	-	-	5.5984	4.6636	3.7922	2.4251
$m = 4$	ICF^{10}	13.08	21.76	26.16	30.69	0.4336	0.6009	0.6847	0.7675	5.7937	5.4948	5.2708	4.9761
	ICF^{30}	13.59	21.58	25.22	28.67	0.4201	0.5829	0.6647	0.746	5.7484	5.3991	5.1667	4.8935
	ICF^{50}	14.38	22.38	25.88	29.03	0.3923	0.5438	0.6204	0.6968	5.7032	5.3003	5.0519	4.7863
	RB	14.03	25.57	32.11	39.97	0.4388	0.6089	0.6927	0.7747	5.6785	5.0271	4.4356	3.6075
	TB	0	0	0	0	-	-	-	-	5.6124	4.7149	3.854	2.4983
$m = 5$	ICF^{10}	14.37	24.05	28.75	33.42	0.4295	0.5951	0.6779	0.7601	5.8014	5.5099	5.2976	5.0187
	ICF^{30}	14.43	23.12	27.07	30.65	0.4176	0.5798	0.6613	0.7425	5.7612	5.4214	5.2007	4.9382
	ICF^{50}	14.97	23.54	27.27	30.58	0.3911	0.5426	0.6193	0.6957	5.7212	5.3304	5.0942	4.8328
	RB	15.7	28.35	35.37	43.31	0.4365	0.6059	0.6898	0.7721	5.6868	5.0545	4.4907	3.6674
	TB	0	0	0	0	-	-	-	-	5.6058	4.684	3.8117	2.4562

Table 12: Risk measures for different dynamic strategies and protection levels (PL).

References

Bauer, Daniel, Kiesel, Rüdiger, Kling, Alexander, & Ruß, Jochen. 2006. Risk-neutral valuation of participating life insurance contracts. *Insurance: Mathematics and Economics*, **39**(2), 171–183.

Berninger, Christoph, & Pfeiffer, Julian. 2020. The Gauss2++ Model–A Consistent Risk Neutral and Real World Calibration.

Bohnert, Alexander, Born, Patricia, & Gatzert, Nadine. 2014. Dynamic hybrid products in life insurance: Assessing the policyholders' viewpoint. *Insurance: Mathematics and Economics*, **59**, 87–99.

Brigo, Damiano, & Mercurio, Fabio. 2007. *Interest rate models-theory and practice: with smile, inflation and credit*. Springer Science & Business Media.

Chiappori, Pierre-André, & Paiella, Monica. 2011. Relative risk aversion is constant: Evidence from panel data. *Journal of the European Economic Association*, **9**(6), 1021–1052.

Deutsche Börse AG. 2021. *EURO STOXX 50 (Performance)*. `https://www.boerse-frankfurt.de/indices/euro-stoxx-50-performance/price-history/historical-prices-and-volumes`. accessed 25-February-2021.

Deutsche Bundesbank. 2021. *Tägliche Zinsstruktur für börsennotierte Bundeswertpapiere*. `https://www.bundesbank.de/de/statistiken/geld-und-kapitalmaerkte/zinssaetze-und-renditen/taegliche-zinsstruktur-fuer-boersennotierte-bundeswertpapiere-650724`. accessed 25-February-2021.

Gatzert, Nadine, & Kling, Alexander. 2007. Analysis of participating life insurance contracts: a unification approach. *Journal of Risk and Insurance*, **74**(3), 547–570.

Goecke, Oskar. 2013. Pension saving schemes with return smoothing mechanism. *Insurance: Mathematics and Economics*, **53**(3), 678–689.

Graf, Stefan, Ruß, Jochen, & Schelling, Stefan. 2019. As you like it: Explaining the popularity of life-cycle funds with multi cumulative prospect theory. *Risk Management and Insurance Review*, **22**(2), 221–238.

Graf, Stefan, Kling, Alexander, & Ruß, Jochen. 2021. *Auswirkungen unterschiedlich hoher garantierter Rentenfaktoren bei aufgeschobenen Rentenversicherungen*. `https://www.ifa-ulm.de/fileadmin/user_upload/download/sonstiges/2021-03-03_Inflationsbereinigte_Chancen_und_Risiken_langfristiger_Sparprozesse.pdf`. accessed 25-July-2021.

Guillén, Montserrat, Jørgensen, Peter Løchte, & Nielsen, Jens Perch. 2006. Return smoothing mechanisms in life and pension insurance: Path-dependent contingent claims. *Insurance: Mathematics and Economics*, **38**(2), 229–252.

Hansen, Mette, & Miltersen, Kristian R. 2002. Minimum rate of return guarantees: the Danish case. *Scandinavian Actuarial Journal*, **2002**(4), 280–318.

Kahneman, Daniel, & Tversky, Amos. 1979. Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society*, 263–291.

Korn, Ralf, Wagner, Andreas, *et al.* 2018. Chance-risk classification of pension products: scientific concepts and challenges. *Innovations in insurance, risk-and asset management*, 381–398.

l'Haridon, Olivier, & Vieider, Ferdinand M. 2019. All over the map: A worldwide comparison of risk preferences. *Quantitative Economics*, **10**(1), 185–215.

Lichtenstern, Andreas, & Zagst, Rudi. 2021. Optimal investment strategies for pension funds with regulation-conform dynamic pension payment management in the absence of guarantees. *European Actuarial Journal*, 1–54.

Løchte Jørgensen, Peter. 2007. Lognormal approximation of complex path-dependent pension scheme payoffs. *The European Journal of Finance*, **13**(7), 595–619.

Maurer, Raimond, Mitchell, Olivia S, Rogalla, Ralph, & Siegelin, Ivonne. 2016. Accounting and actuarial smoothing of retirement payouts in participating life annuities. *Insurance: Mathematics and Economics*, **71**, 268–283.

Prelec, Drazen. 1998. The probability weighting function. *Econometrica: Journal of the Econometric Society*, 497–527.

Ruß, Jochen, & Schelling, Stefan. 2018. Multi cumulative prospect theory and the demand for cliquet-style guarantees. *Journal of Risk and Insurance*, **85**(4), 1103–1125.

Ruß, Jochen, & Schelling, Stefan. 2021. Return smoothing in life insurance from a client perspective. *Insurance: Mathematics and Economics*, **101**, 91–106.

Tversky, Amos, & Kahneman, Daniel. 1992. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, **5**(4), 297–323.