ONLINE APPENDIX

to

The Impact of a Firm's Share of Exports on Revenue, Wages, and Measure of Workers Hired

—

Theory and Evidence

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This Appendix provides the basic derivations of the multi-country trade model of heterogeneous firms presented in Hesse (2014). The ensuing presentation borrows from Dixit & Stiglitz (1977), Melitz (2003) as well as Helpman et al. (2010*a*) and its technical appendix, Helpman et al. (2010*b*).

1. A FIRM'S REVENUE AND EXPORT DECISION

a. Domestic Demand

The preferences of a representative consumer are given by a C.E.S. utility function over a continuum of varieties indexed by ω:

$$U = \left[\int_{\omega \in \Omega} y(\omega)^{\rho} d\omega \right]^{\frac{1}{\rho}} ,$$

where $y(\omega)$ indexes the amount of variety ω and Ω represents the set of available varieties within the sector. These varieties are substitutes, implying $0 < \rho < 1$ and an elasticity of substitution between any two varieties of

$$\sigma = \frac{1}{1-\rho} > 1 \quad \Leftrightarrow \quad \rho = 1 - \frac{1}{\sigma} = \frac{\sigma - 1}{\sigma} . \tag{A.1}$$

The consumer's constrained maximization problem may be solved by the Lagrangian

$$\mathcal{L} = U^{\rho} - \lambda \left(\int_{\omega \in \Omega} p(\omega) y(\omega) d\omega - I \right) ,$$

where U^{ρ} is a strictly increasing transformation of U, $p(\omega)$ the price of variety ω, and I the consumer's income. The maximization problem yields the following first-order condition

$$\frac{\partial \mathcal{L}}{\partial y(\omega)} = \rho y(\omega)^{\rho - 1} - \lambda p(\omega) = 0 .$$

By dividing the first-order condition of one variety ω_1 by the first-order condition of another variety ω_2, we obtain the relative demand

$$\frac{y(\omega_1)}{y(\omega_2)} = \left(\frac{p(\omega_1)}{p(\omega_2)}\right)^{\frac{1}{\rho-1}} .$$

Multiplying both sides with $y(\omega_2)$ and using (A.1) yields

$$y(\omega_1) = y(\omega_2)\left(\frac{p(\omega_1)}{p(\omega_2)}\right)^{-\sigma} .$$

When multiplying both sides with $p(\omega_1)$ and taking the integral with respect to ω_1, we get

$$\int_{\omega\in\Omega} p(\omega_1)y(\omega_1)d\omega_1 = \int_{\omega\in\Omega} y(\omega_2)p(\omega_1)^{1-\sigma}p(\omega_2)^{\sigma}d\omega_1 .$$

On the left-hand side we now have the consumer's total expenditure on all varieties, R, which is assumed to be equal to his income I, i.e.,

$$R = I = y(\omega_2)p(\omega_2)^{\sigma}\int_{\omega\in\Omega} p(\omega_1)^{1-\sigma}d\omega_1 .$$

Solving for $y(\omega_2)$ yields the Marshallian demand for ω_2

$$y(\omega_2) = \frac{Ip(\omega_2)^{-\sigma}}{\int_{\omega\in\Omega} p(\omega_1)^{1-\sigma}d\omega_1} .$$

By defining an index of the overall price level

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}} ,$$

Marshallian demand for a variety ω simplifies to

$$y(\omega) = p(\omega)^{-\sigma}P^{\sigma-1}I = \left(\frac{p(\omega)}{P}\right)^{-\sigma}\frac{I}{P} .$$

Domestic demand, denoted by $y_d(\omega)$, can accordingly be written as

$$y_d(\omega) = p_d(\omega)^{-\sigma}P_d^{\sigma-1}I_d = \left(\frac{p_d(\omega)}{P_d}\right)^{-\sigma}\frac{I_d}{P_d} ,$$

where $p_d(\omega)$ denotes the price of the good in the domestic market while P_d and I_d indicate the domestic aggregate price and domestic income, respectively.

b. Domestic Revenue

With a firm's domestic output being equal to domestic demand, domestic firm revenue can be written as

$$r_d(\omega) = y_d(\omega)p_d(\omega) = I_d\left(\frac{p_d(\omega)}{P_d}\right)^{1-\sigma} .$$

Note that with $p_d(\omega) = y_d(\omega)^{\frac{1}{-\sigma}}P_d^{\frac{\sigma-1}{\sigma}}I_d^{\frac{1}{\sigma}}$ and (A.1) domestic revenue can also be written as in HIR, i.e.,

$$r_d(\omega) = y_d(\omega)^{1-\frac{1}{\sigma}}P_d^{\frac{\sigma-1}{\sigma}}I_d^{\frac{1}{\sigma}} = y_d(\omega)^{\rho}P_d^{\rho}I_d^{1-\rho} = y_d(\omega)^{\rho}A_d ,$$

where A_d is called the domestic demand shifter, with $A_d = P_d^{\rho} I_d^{1-\rho}$. As with increasing productivity a firm's output and thereby its domestic revenue will increase continuously, we can write domestic revenue — and revenue in general — as

$$r_d(\varphi) = y_d(\varphi)^{\rho} A_d \,.$$

c. Revenue from Exporting

By assuming country specific iceberg trading costs, τ_c, such that $\tau_c > 1$ units of a variety must be exported for a unit to arrive in country c, we can write the revenue from exporting to country c as

$$r_{x,c}(\varphi) = \frac{y_{x,c}(\varphi)}{\tau_c} p_{x,c}(\varphi) = \left(\frac{y_{x,c}(\varphi)}{\tau_c}\right)^{\rho} P_{x,c}^{\rho} I_{x,c}^{1-\rho} = \left(\frac{y_{x,c}(\varphi)}{\tau_c}\right)^{\rho} A_{x,c} \,,$$

where $A_{x,c} = I_{x,c}^{1-\rho} P_{x,c}^{\rho}$ is the demand shifter of country c.

d. Υ and a Derivation of $y_d(\varphi) = y(\varphi)/\Upsilon$

Using the first-order conditions (5), we can write a firm's total output,

$$y(\varphi) = y_d(\varphi) + \sum_{c=1}^{c'} y_{x,c}(\varphi) \,,$$

as

$$y(\varphi) = y_d(\varphi) + \sum_{c=1}^{c'} \mathbb{I}_c \tau_c^{\frac{\rho}{\rho-1}} y_d(\varphi) \left(\frac{A_{x,c}}{A_d}\right)^{\frac{1}{1-\rho}} = y_d(\varphi)\left(1 + \sum_{c=1}^{c'} \mathbb{I}_c \tau_c^{\frac{\rho}{\rho-1}} \left(\frac{A_{x,c}}{A_d}\right)^{\frac{1}{1-\rho}}\right) ,$$

where $\mathbb{I}_c$ equals 1 if the firm exports to country c and 0 otherwise. By defining $\Upsilon \equiv 1 + \sum_{c=1}^{c'} \mathbb{I}_c \tau_c^{\frac{\rho}{\rho-1}} \left(\frac{A_{x,c}}{A_d}\right)^{\frac{1}{1-\rho}}$, we obtain

$$y_d(\varphi) = y(\varphi)/\Upsilon \,.$$

e. Total Revenue

A firm's total revenue is given by

$$r(\varphi) \equiv r_d(\varphi) + \sum_{c=1}^{c'} r_{x,c}(\varphi) = y_d(\varphi)^{\rho} A_d + \sum_{c=1}^{c'} \tau_c^{-\rho} y_{x,c}(\varphi)^{\rho} A_{x,c} \,.$$

Using again the first-order conditions (5) , this can be written as

$$r(\varphi) = y_d(\varphi)^{\rho} A_d + \sum_{c=1}^{c'} \tau_c^{\frac{\rho}{\rho-1}} y_d(\varphi)^{\rho} A_{x,c} \left(\frac{A_{x,c}}{A_d}\right)^{\frac{\rho}{1-\rho}}$$

$$= y_d(\varphi)^{\rho} A_d \left(1 + \sum_{c=1}^{c'} \mathbb{I}_c \tau_c^{\frac{\rho}{\rho-1}} \left(\frac{A_{x,c}}{A_d}\right)^{\frac{1}{1-\rho}}\right) = y_d(\varphi)^{\rho} A_d \Upsilon \,.$$

With $y_d(\varphi) = y(\varphi)/\Upsilon$ we obtain

$$r(\varphi) = y(\varphi)^{\rho} A_d \Upsilon^{1-\rho}. \tag{A.2}$$

f. Revenue as a Function of a Firm's Productivity

Using the earlier definition of $r(\varphi)$ in (A.2), the production function (2), and the first-order conditions (8) and (9), we are now able to express revenue as

$$r(\varphi) = \left(\frac{\zeta_d}{\zeta_d - 1} a_{\min,d}^{\gamma\zeta_d} \varphi \left(\frac{\rho\gamma}{(1+\rho\gamma)b} \right)^{\gamma} \left(\frac{\rho(1-\gamma\zeta_d)}{\varepsilon(1+\rho\gamma)} \right)^{\frac{1-\gamma\zeta_d}{\delta}} \right)^{\frac{\rho}{\Gamma}} A_d^{\frac{1}{\Gamma}} \Upsilon^{\frac{1-\rho}{\Gamma}} , \tag{A.3}$$

where $\Gamma \equiv 1 - \rho\gamma - \rho(1-\gamma\zeta_d)/\delta$. In a next step, we compute the firm's profits by making once more use of the first-order conditions

$$\pi(\varphi) = \frac{\Gamma}{1+\rho\gamma} r(\varphi) - f_d - \sum_{c=1}^{c'} \mathbb{I}_c f_{x,c} .$$

Furthermore, we know that the firm with the lowest productivity, φ_d, makes zero profit and is not exporting, hence no productivity gains from exporting are possible, i.e., $\varphi_d \equiv \varphi_d'$. It follows

$$\frac{\Gamma}{1+\rho\gamma} r(\varphi_d) = f_d \quad \Rightarrow \quad r(\varphi_d) \equiv r_d' = \frac{1+\rho\gamma}{\Gamma} f_d . \tag{A.4}$$

In the following, we use the expression for $r(\varphi)$ from (A.3) and determine the relative revenue of a firm in comparison to the firm with the lowest productivity. We obtain

$$\frac{r(\varphi)}{r_d'} = \Upsilon^{\frac{1-\rho}{\Gamma}} \left(\frac{\varphi}{\varphi_d} \right)^{\frac{\rho}{\Gamma}} \quad \Rightarrow \quad r(\varphi) = r_d' \left(\frac{\varphi}{\varphi_d} \right)^{\frac{\rho}{\Gamma}} \Upsilon^{\frac{1-\rho}{\Gamma}} . \tag{A.5}$$

Since we can decompose a firm's productivity into its initial productivity, φ', and the possible productivity gain from exporting, $e^{\mathbb{I}\iota(\varphi)}$, we can write revenue as

$$r(\varphi') = r_d' \left(\frac{\varphi'}{\varphi_d} \right)^{\frac{\rho}{\Gamma}} \Upsilon^{\frac{1-\rho}{\Gamma}} e^{\frac{\rho \mathbb{I}\iota(\varphi')}{\Gamma}} .$$

2. A FIRM'S AVERAGE WAGE

By the same token, we are able to compute $a_\varepsilon(\varphi)$. We employ the first-order condition (9) and get

$$\frac{a_\varepsilon(\varphi)^\delta}{a_\varepsilon(\varphi_d)^\delta} = \Upsilon^{\frac{1-\rho}{\Gamma}} \left(\frac{\varphi}{\varphi_d} \right)^{\frac{\rho}{\Gamma}} \quad \Rightarrow \quad a_\varepsilon(\varphi) = a_\varepsilon(\varphi_d) \left(\frac{\varphi}{\varphi_d} \right)^{\frac{\rho}{\delta\Gamma}} \Upsilon^{\frac{1-\rho}{\delta\Gamma}} . \tag{A.6}$$

Using (A.4) together with (9), we can compute

$$a_\varepsilon(\varphi_d) = \left(\frac{\rho(1-\gamma k)}{(1+\rho\gamma)\varepsilon} \frac{1+\rho\gamma}{\Gamma} f_d \right)^{\frac{1}{\delta}} = \left(\frac{\rho(1-\gamma\zeta_d)}{\varepsilon\Gamma} f_d \right)^{\frac{1}{\delta}} .$$

With the wage condition from (10), the lowest wage paid by a domestic firm is then

$$w(\varphi_d) \equiv w_d' = b \left(\frac{a_\varepsilon(\varphi_d)}{a_{\min}} \right)^{\zeta_d} = \left(\frac{\rho(1-\gamma\zeta_d)}{\varepsilon\Gamma a_{\min}^\delta} f_d \right)^{\frac{\zeta_d}{\delta}} .$$

This yields a wage relation that is solely dependent on φ, $\Upsilon(\varphi)$, φ_d, and parameters, namely

$$\frac{w(\varphi)}{w_d'} = \left(\frac{a_\varepsilon(\varphi)}{a_\varepsilon(\varphi_d)} \right)^{\zeta_d} = \left(\frac{\varphi}{\varphi_d} \right)^{\frac{\rho\zeta_d}{\delta\Gamma}} \Upsilon^{\frac{\zeta_d(1-\rho)}{\delta\Gamma}} \quad \Rightarrow \quad w(\varphi) = w_d' \left(\frac{\varphi}{\varphi_d} \right)^{\frac{\rho\zeta_d}{\delta\Gamma}} \Upsilon^{\frac{\zeta_d(1-\rho)}{\delta\Gamma}} .$$

As can be seen from this last equation, wages increase with firm productivity and are always higher for exporting firms than for non-exporting firms. Ultimately, we decompose productivity into its components and obtain

$$w(\varphi') = w'_d \left(\frac{\varphi'}{\varphi_d}\right)^{\frac{\rho\zeta_d}{\delta\Gamma}} \Upsilon^{\frac{\zeta_d(1-\rho)}{\delta\Gamma}} e^{\frac{\rho\zeta_d \mathbb{I}\iota(\varphi')}{\delta\Gamma}} .$$

3. A FIRM'S MEASURE OF WORKERS HIRED

In a similar manner, we can derive the lowest measure of workers hired

$$h(\varphi_d) \equiv h'_d = m(\varphi_d)\left(\frac{a_{\min,d}}{a_\varepsilon(\varphi_d)}\right)^{\zeta_d} = \frac{\rho\gamma}{1+\rho\gamma}\frac{r'_d}{b}\left(\frac{a_{\min,d}}{a_\varepsilon(\varphi_d)}\right)^{\zeta_d} .$$

Using (A.5) and (A.6), the relation to $h(\varphi)$ is then given by

$$\frac{h(\varphi)}{h'_d} = \frac{r(\varphi)}{r'_d}\left(\frac{a_\varepsilon(\varphi_d)}{a_\varepsilon(\varphi)}\right)^{\zeta_d} = \Upsilon^{\frac{1-\rho}{\Gamma}}\left(\frac{\varphi}{\varphi_d}\right)^{\frac{\rho}{\Gamma}} \Upsilon^{\frac{\zeta_d(\rho-1)}{\delta\Gamma}}\left(\frac{\varphi}{\varphi_d}\right)^{\frac{-\zeta_d\rho}{\delta\Gamma}}$$

$$= \Upsilon^{\frac{(1-\rho)(1-\zeta_d/\delta)}{\Gamma}}\left(\frac{\varphi}{\varphi_d}\right)^{\rho\left(1-\frac{\zeta_d}{\delta}\right)} ,$$

which ultimately leads with (1) to

$$h(\varphi') = h'_d\left(\frac{\varphi'}{\varphi_d}\right)^{\rho\left(1-\frac{\zeta_d}{\delta}\right)} \Upsilon^{\frac{(1-\rho)(1-\zeta_d/\delta)}{\Gamma}} e^{\rho\left(1-\frac{\zeta_d}{\delta}\right)\mathbb{I}\iota(\varphi')} .$$

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