Prof. Dr. U. Stadtmüller
C. Hering

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Sheet 1

Time Series

(Due: Tu., 28.10.2008, 13:15 Uhr, in the exercise classes)

1. Assume the following process

$$X_t = \xi + \sum_{i=1}^{p} \alpha_i X_{t-i} + \epsilon_t - \sum_{j=1}^{q} \beta_j \epsilon_{t-j}$$

to be stationary, where $(\epsilon_t)_{t\in\mathbb{Z}}$ is “white noise” and ξ a constant. Calculate the mean of X_t.

(2 Credits)

2. Let $(Z_t,\, t \in \mathbb{Z})$ be independent random variables each with mean 0 and variance σ^2. Moreover let $c \in \mathbb{R}$ be an arbitrary constant. Which, if any, of the following processes are stationary? Specify the mean and the autocovariance function for the stationary processes.

   (a) $X_t = Z_1 \cos(ct) + Z_2 \sin(ct)$
   (b) $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$
   (c) $X_t = Z_t - X_{t-1}, X_0 = 0$

(6 Credits)

3. Let $(X_t,\, t \in \mathbb{Z})$ and $(Y_t,\, t \in \mathbb{Z})$ be stationary time series with $\mathrm{Cov}(X_s, Y_t) = 0$ for every $s, t \in \mathbb{Z}$. Define $Z_t := X_t + Y_t,\, t \in \mathbb{Z}$. Show that $(Z_t,\, t \in \mathbb{Z})$ is also stationary.

(2 Credits)

4. The data set fuel.dat contains monthly fuel prices in (US) cents per gallon.

   (a) Read the data set and compute mean, median and variance. Plot the time series.
   (b) Discuss whether the time series is stationary.
   (c) Try to fit a suitable linear model of the form $X_t = \alpha + \beta t + \gamma t^2 + \delta \sin(.) + \lambda \cos(.)$ to the time series. Does the graph look like a stationary time series?

(5 Credits)

http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html