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Winter 2008/2009
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Sheet 10

Time Series

(Due: Tu., 20.1.2009, 1:15 pm, in the exercise classes)

1. Consider the data sets aic1.dat, aic2.dat and aic3.dat on the lecture's homepage. All of them come from some ARMA(p,q) process. Use Akaike's Information Criterion (AIC) and Bayes Information Criterion (BIC) to determine the orders p and q.

(5 Credits)

2. Show for the periodogram

$$I_n(\lambda_j) = \begin{cases} n\hat{\mu}^2, & j = 0 \\ \sum\limits_{|k|<n} \hat{\gamma}_X(k) e^{-ik\lambda_j}, & j = -\left[\frac{n-1}{2}\right], \ldots, -1, 1, \ldots, \left[\frac{n}{2}\right] \end{cases}$$

(a) that for an arbitrary stationary process X with $\gamma_X(.) \in l_1$

$$\mathbb{E}\left(I_n(0)\right) - n\hat{\mu}^2 \to 2\pi f(0) \quad (n \to \infty) \quad \text{and} \quad \mathbb{E}\left(I_n(\lambda_j)\right) \to 2\pi f(\lambda_j) \quad (n \to \infty)$$

(b) that in the case where X is a white noise process with existing fourth moments $\mathbb{E}\left(X^4\right) = \mu_4$, $\mathbb{E}\left(X^3\right) = 0$ and $\mathbb{E}\left(X^2\right) = \sigma^2$ we have

$$\mathrm{Var}(I_n(\lambda_j)) = \begin{cases} 2\sigma^4 + \frac{1}{n}(\mu_4 - 3\sigma^4), & \lambda_j = 0, \lambda_j = \pm\pi \\ \sigma^4 + \frac{1}{n}(\mu_4 - 3\sigma^4), & \text{otherwise} \end{cases}$$

(5 Credits)

3.* Given observations from an ARMA(p,q) process $(X_t, t \in \mathbb{Z})$, one way to determine p and q is to use the *Extended Autocorrelation Function (EACF)*. The idea is to first compute the ACF of $(X_t, t \in \mathbb{Z})$ and then successively fit AR processes of increasing order i ($i = 1, 2, 3, \ldots$) to the data, estimate the corresponding coefficients $\alpha_j^{(i)}$, $j \in \{1, \ldots, i\}$ and determine if the sample ACF of the fitted process $Y_t^{(i)} = X_t - \sum_{j=1}^{i} \alpha_j^{(i)} B^j X_t$ significantly differs from zero. To simplify this procedure, one may look at a simplified table, where $\forall\, i, j \in \mathbb{N}_0$, the entry in the $(i+1)$th row and $(j+1)$th column is "X" if the sample ACF of lag $j+1$ of the fitted AR(i) process significantly differs from zero, and "O" otherwise.
For example, if $(X_t, t \in \mathbb{Z})$ follows a MA(1) process, i.e. $X_t = (1 - \beta B)\varepsilon_t$, we have $\gamma_X(h) \neq 0$ if $|h| \leq 1$ and zero else. This determines the first row of Table 1. For the second row, fit an AR(1) process to the data, i.e. $(1 - \alpha_1^{(1)} B)X_t = (1 - \alpha_1^{(1)} B)(1 - \beta B)\epsilon_t$, and compute the ACF of the fitted process. On the right hand side, we are left with an MA(2) process, so $\gamma_X(h) \neq 0$ if $|h| \leq 2$ and zero else. This determines the two "X" in the second line. Continuing this way, we obtain a triangular with vertex at row 1 and column 2 which corresponds (the table labels are chosen accordingly) to a MA(1) process.

	MA				
AR	0	1	2	3	4
0	X	O	O	O	O
1	X	X	O	O	O
2	X	X	X	O	O
3	X	X	X	X	O

Tabelle 1: Simplified EACF table for a MA(1) process

	MA				
AR	0	1	2	3	4
0	X	X	X	X	X
1	X	O	O	O	O
2	X	X	O	O	O
3	X	X	X	O	O

Tabelle 2: Simplified EACF table for an ARMA(?,?) process

(a) How does the table look like for an AR(1) process?

(b) Given Table 2, try to identify p and q for the corresponding ARMA process.

(3 + 2 Credits)

http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html