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Winter 2008/2009
20.1.2009
Sheet 11

Time Series

(Due: Tu., 27.1.2009, 1:15 pm, in the exercise classes)

1. Consider the following ARCH(2)-model:

$$\begin{aligned} X_t &= \sigma_t \cdot \varepsilon_t, \quad \varepsilon_t \text{ iid} \\ \sigma_t^2 &= \kappa + \alpha_1 X_{t-1}^2 + \alpha_2 X_{t-2}^2 \end{aligned}$$

   (a) Simulate a realization path of length 1000 of this ARCH-process with parameters $\kappa = 100$, $\alpha_1 = 0.4$ and $\alpha_2 = 0.3$. Let the process start with values $X_1 = 0$ and $X_2 = 1$. Assume ε_t distributed standardnormally. Repeat the simulation with ε_t with $t-$distributed with 2 and 10 degrees of freedom.

   (b) Compute in all three cases the ACF of the simulated process, the ACF of the absolute value and the process and the ACF of the squared process.

   (c) Given the plots from part b), would you decide for an ARCH-model in all three cases? What might be the reason for the unfamiliar behaviour of the ACF in the $t-$distribution case with 2 degrees of freedom?

   (d) Do part (a) and (b) for $\alpha_1 = 0.5$ and $\alpha_2 = 0.475$ and comment on the shapes of the different ACFs.

(5 Credits)

2. Show for a strongly stationary ARCH(1) process $(X_t)_{t\in\mathbb{Z}}$

   (a) (in case $\mathbb{E}\left(X^{2m}\right)$ exists) the recursion

$$\mathbb{E}\left(X^{2m}\right) = \frac{\mathbb{E}\left(\varepsilon^{2m}\right)}{1 - \alpha_1^m \mathbb{E}\left(\varepsilon^{2m}\right)} \sum_{k=0}^{m-1} \binom{m}{k} \alpha_1^k \alpha_0^{m-k} \mathbb{E}\left(X^{2k}\right),$$

   calculate $\mathbb{E}\left(X_0^4\right)$.

   (b) (in case $\mathbb{E}\left(X^4\right)$ exists) that

$$\mathrm{Cov}(X_t^2, X_0^2) = \alpha_1^t \mathrm{Var} X_0^2,$$

   (c*) that $X_0^2 \stackrel{\mathrm{d}}{=} \alpha_0 \sum_{j=1}^{\infty} \varepsilon_j^2 \prod_{\nu=1}^{j-1} \alpha_1 \varepsilon_\nu^2$ and $X_t \stackrel{\mathrm{d}}{=} |X_0| \, r_0$, where r_0 is an independent Bernoulli random variable taking values ± 1.

(2 + 3 + 3 Credits)

http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html