Prof. Dr. U. Stadtmüller
C. Hering

Winter 2008/2009
3.2.2009
Sheet 13

Time Series

(Due: Tu., 10.2.2009, 1:15 pm, in the exercise classes)

1. Let $(\varepsilon_t)_{t\in\mathbb{Z}}$,iid be a real white noise process. For $\mu \in \mathbb{R}$ let the time series

$$\begin{aligned} X_{t,n} &= \varepsilon_t - \mu, \quad t = 1, \dots, n \\ X_{t,n} &= \varepsilon_t + \mu, \quad t = n+1, \dots, 2n. \end{aligned}$$

   Show that $\mathbb{E}\left(\frac{1}{2n}\sum_{t=1}^{2n} X_{t,n}\right) = 0$ and that for $\hat{\gamma_n}(h) = \frac{1}{2n}\sum_{t=1}^{2n-h} X_{t,n}X_{t+h,n}$ we have $\hat{\gamma_n}(h) \xrightarrow{P} \mu^2$ $(n \to \infty)$ for every $h \geq 1$, although $(X_t)_{t\in\mathbb{Z}}$ are independent.

   (5 Credits)

2. Show that in the strongly stationary GARCH(1,1)-model the conditions

$$\mathbb{E}\left(X_1^4\right) < \infty \quad \text{and} \quad \mathbb{E}\left(a\varepsilon_1^2 + b\right)^2 < 1,$$

   are equivalent. Moreover show that in this case

$$\mathbb{E}\left(X_1^4\right) = \eta\frac{1}{1-\theta^2}\frac{1+a+b}{1-a-b},$$

   where $\eta = \mathbb{E}\left(X_1^4\right)$ and $\theta = \mathbb{E}\left(\left(a\varepsilon_1^2 + b\right)^2\right) = a^2\eta + 2ab + b^2$

   (5 Credits)

http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html