Prof. Dr. U. Stadtmüller
C. Hering

Winter 2008/2009
4.11.2008
Sheet 3

Time Series

(Due: Tu., 11.11.2008, 1:15 pm, in the exercise classes)

1. Let Let $(X_t, t \in \mathbb{Z})$ be a stochastic process with

$$X_t = \varepsilon_t - \theta\varepsilon_{t-1},$$

where $(\varepsilon_t)_{t\in\mathbb{Z}}$ is white noise.

(a) Show, that the coefficients $(\phi_{n,j})$for the best linear prediction of X_{n+1} given $X_1, \ldots, X_n$, i.e. $\widehat{X}_{n+1} = \sum_{j=1}^{n} \phi_{n,j} X_{n+1-j}$, fulfill:

$$\begin{aligned} -\theta\phi_{n,j-1} + (1+\theta^2)\phi_{n,j} - \theta\phi_{n,j+1} &= 0, \quad \forall\, j = 2, \ldots, n-1 \\ (1+\theta^2)\phi_{n,n} - \theta\phi_{n,n-1} &= 0 \\ (1+\theta^2)\phi_{n,1} - \theta\phi_{n,2} &= -\theta \end{aligned}$$

(b) Show that

$$\phi_{n,n} = -\frac{\theta^n(1-\theta^2)}{1-\theta^{2(n+1)}},\ n \geq 2.$$

(5 Credits)

2. The files S1.dat, S2.dat, S3.dat and S4.dat each contain a realization of one of the following stochastic processes (X_t):

$$\begin{aligned} \text{P1: } X_t &= 2t\sin t + \varepsilon_t \\ \text{P2: } X_t &= 10 + 20t + \varepsilon_t \\ \text{P3: } X_t &= 0.3X_{t-1} + 0.4X_{t-2} + \varepsilon_t \\ \text{P4: } X_t &= \varepsilon_t - 0.7\varepsilon_{t-1}, \end{aligned}$$

where ε_t is white noise. Compute the theoretical PACF in the P3 case. Use R (or any other convenient software package) to compute the (partial) autocorrelation function of the realizations. Decide which path belongs to which model and give reason for your decision.

(5 Credits)

http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html