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Winter 2008/2009
11.11.2008
Sheet 4

Time Series

(Due: Tu., 18.11.2008, 1:15 pm, in the exercise classes)

1. (a) Show that in order for an AR(2) process with $\Psi(z) = 1 - \alpha_1 z - \alpha_2 z^2$ to have a casual solution the parameters α_1, α_2 have to lie in a triangular array determined by

$$\begin{aligned} \alpha_2 + \alpha_1 &< 1 \\ \alpha_2 - \alpha_1 &< 1 \\ |\alpha_2| &< 1. \end{aligned}$$

   (b) Let $X_t = 0.7X_{t-1} - 0.1X_{t-2} + \varepsilon_t, t \in \mathbb{Z}$, be a stochastic process, where (ε_t) is white noise. Compute the coefficients $(c_k)_{k=0}^{\infty}$ so that $X_t = \sum\limits_{k=0}^{\infty} c_k \varepsilon_{t-k}$.

(3 + 2 Credits)

2. Let $(X_t, t \in \mathbb{Z})$ be an MA(1) process with parameter $\theta \in \mathbb{R}$ (assume $\sigma^2 = 1$ for the corresponding $((\varepsilon_t), t \in \mathbb{Z})$). Consider the linear filter $\alpha = (\alpha_t)_{t\in\mathbb{Z}}$

$$\alpha_t = \begin{cases} \frac{1}{2} & t = 0 \\ \frac{1}{4} & |t| = 1 \\ 0 & \text{else} \end{cases}$$

   (a) Compute the transfer function A_α and the power transfer function T_α of the linear filter $\boldsymbol{\alpha}$. Sketch the power transfer function for $\lambda \in [-\pi, \pi]$.
   (b) Compute the spectral density f_X of (X_t) and the spectral density f_Y of the linear filtered process (Y_t) with $Y_t = \sum_{k\in\mathbb{Z}} \alpha_k X_{t-k}$.
   (c) Sketch the spectral densities f_X and f_Y in the same diagram. Use $\theta = \pm 1$ as parameters.

(2 + 2 + 1 Credits)

3. Simulate and plot the following processes and compute pacf and acf (choose n=500).

$$\begin{aligned} X_t^{(1)} - 0.3X_{t-1}^{(1)} - 0.4X_{t-2}^{(1)} &= \varepsilon_t \\ X_t^{(2)} &= \varepsilon_t - 0.3\varepsilon_{t-1} - 0.4\varepsilon_{t-2} \\ X_t^{(3)} - 0.3X_{t-1}^{(3)} - 0.4X_{t-2}^{(3)} &= \varepsilon_t - 0.3\varepsilon_{t-1} - 0.4\varepsilon_{t-2} \end{aligned}$$

   Hint: A useful R command is arima.sim(.).

(5 Credits)

http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html