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Sheet 5

# Time Series

(Due: Tu., 25.11.2008, 1:15 pm, in the exercise classes)

1. So called feedback processes are sometimes used in econometrics. Assume $(\varepsilon_t)_{t\in\mathbb{Z}}$ and $(\widetilde{\varepsilon}_t)_{t\in\mathbb{Z}}$ to be two independent white noise processes with variance $\sigma^2$ and $\widetilde{\sigma}^2$, respectively. Moreover, let the two stochastic processes $(X_t)_{t\in\mathbb{Z}}$ and $(Y_t)_{t\in\mathbb{Z}}$ be defined via

$$\begin{aligned} X_t &= \alpha_1 Y_{t-1} + \varepsilon_t \\ Y_t &= \alpha_2 X_{t-1} + \widetilde{\varepsilon}_t \end{aligned}$$

where $\alpha_1$ and $\alpha_2$ are some constants. Formulate conditions for $\alpha_1$ and $\alpha_2$ so that $(X_t)_{t\in\mathbb{Z}}$ is a causal ARMA-process.

(4 Credits)

2. Consider the data sets sarima1.dat, sarima2.dat and sarima3.dat on the lecture's homepage.
   (a) Plot the corresponding time series and comment on its shape. Also plot the corresponding acf and pacf and comment on their shapes.
   (b) Assuming the data comes from an SARIMA process $(X_t)_{t\in\mathbb{Z}}$. Try to get stationary processes using differencing methods.
   *Hint:* The following steps might be useful.
   - Consider the differences $Y_t = (1-B)X_t$ for $t \in \mathbb{Z}$. Plot $(Y_t)_{t\in\mathbb{Z}}$, its acf and its pacf.
   - Also consider the differences $\widetilde{Y}_t = (1-B^{12})Y_t$ for $t \in \mathbb{Z}$. Plot $(\widetilde{Y}_t)_{t\in\mathbb{Z}}$, its acf and its pacf.

(6 Credits)

3.* Let $(X_t)_{t\in\mathbb{Z}}$ be a stationary process with mean 0, $\gamma_X(h) = 0$, if $|h| > q$ and $\gamma(q) \neq 0$, where $q \in \mathbb{N}$. Show that $(X_t)_{t\in\mathbb{Z}}$ is an MA(q) process.

(5 Credits)

**http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html**