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Winter 2008/2009
25.11.2008
Sheet 6

Time Series

(Due: Tu., 2.12.2008, 1:15 pm, in the exercise classes)

1. Suppose that X_t is a stationary process with mean zero and spectral density

$$f_X(\lambda) = \frac{\pi - |\lambda|}{\pi^2}.$$

Find the coefficients $\{\phi_{ij}, j = 1, \ldots, i, i = 1, 2, 3\}$ and the mean squared errors $\{v_i, i = 1, 2, 3\}$.

(4 Credits)

2. Let X_t be a MA(1) process with white noise variance σ^2. Compute the 1-step linear forecast and the forecast variance given 2 past values by using

   (a) the Innovation algorithm.

   (b)* the Durbin-Levinson algorithm

   Now Assume the parameter to be $\theta = 0.5$ and assume that $X_1 = 0.5$ and $X_2 = 0.75$, which value is predicted by each algorithm for the next step? What is the variance of the forecast?

(6 Credits)

3. Let A and B be two uncorrelated random variables with mean 0 and variance 1. Moreover, let

$$X_t = A\cos(ct) + B\sin(ct),$$

where $c \in \mathbb{R}$ is some constant. The goal of this exercise is to find the best estimate for X_{t+1} given $\{X_s : s \leq t\}$. Proceed by doing the following steps:

   - Find the best linear predictor for X_2 given X_1.
   - Find the best linear predictor for X_3 given X_1, X_2.
   - Use a stationarity argument to get the claim.

(5 Credits)

http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html