Prof. Dr. U. Stadtmüller
C. Hering

Winter 2008/2009
2.12.2008
Sheet 7

Time Series

(Due: Tu., 9.12.2008, 1:15 pm, in the exercise classes)

1. Show that for an AR(p) process (white noise variance 1) the Kalman filter algorithm ($C_t := (1, 0, \ldots, 0)^T$) transfers the starting value $\hat{Z}_t := (X_t, \ldots, X_{t+1-p})$ into Z_{t+1}, i.e. show that $\hat{Z}_{t+1} = Z_{t+1}$.

(6 Credits)

2. Let X_t be an ARMA(2,1) process with coefficients $\alpha_1 = 0.5$, $\alpha_2 = 0.25$ and $\beta_1 = 0.75$. Moreover, assume that the white noise variance is $\sigma^2 = 1$ and assume we can only observe a disturbed process $(Y_t, t \in \mathbb{Z})$ with

$$Y_t = X_t + \tilde{\varepsilon}_t \quad \forall t \in \mathbb{Z},$$

where $(\tilde{\varepsilon}_t, t \in \mathbb{Z})$ is a white noise with variance $\tilde{\sigma}^2 = 2$, which is pairwise uncorrelated to $(\varepsilon_t, t \in \mathbb{Z})$.

(a) Set up a "state space model" for this situation, i.e. specify how the system and observation equation look like.

(b) Explain in words, how the Kalman filter algorithm works.

(c) Use the Kalman filter algorithm to determine the best linear (one step ahead) forecast for the system assuming we observe the value 2.

(9 Credits)

3.* Consider a state space model with the "system equation"

$$Z_{t+1} = AZ_t + D\xi_{t+1} \quad (*),$$

where $Z_t \in \mathbb{R}^p$, $\xi_t \in \mathbb{R}^m$ are random vectors and $A \in \mathbb{R}^{p\times p}$, $B \in \mathbb{R}^{p\times m}$ are matrices which do not depend on t. Moreover $\mathbb{E}\xi_t = 0$ and $Cov(\xi_t) = Q \in \mathbb{R}^{m\times m}$. Show that if the eigenvalues $(\lambda_i)_{i=1}^p$ of A satisfy $|\lambda_i| < 1$ then the unique stationary solution to $(*)$ is given by

$$Z_t = \sum_{k=0}^{\infty} A^k D\xi_{t-k}$$

(5 Credits)

http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html