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Winter 2008/2009
16.12.2008
Sheet 9

Time Series

(Due: Tu., 13.1.2008, 1:15 pm, in the exercise classes)

1. If (X_t) is a casual AR(1) process with mean μ, show that
$$\sqrt{n}(\bar{X}_n - \mu) \to \mathcal{N}(0, \frac{\sigma^2}{(1-\alpha)^2}).$$
Moreover, assume you are given a sample of size 1000 from an AR(1) process with $\alpha = 0.6$ and $\varepsilon_t \sim \mathcal{N}(0, 2)$. We obtain $\bar{X}_n = 0.271$. Construct an approximate 95% confidence interval for the mean μ. Does the data suggest that $\mu = 0$?
Hint: $\bar{X}_n - \alpha \bar{X}_{n-1} = \bar{\epsilon}_n$

(5 Credits)

2. Assume we have a given stationary AR(p) process $(X_t, t \in \mathbb{Z})$, with coefficients $\alpha_1, \ldots, \alpha_p$ and white noise variance $\sigma^2 > 0$. Show that $\sigma^2 \Gamma_p^{-1} = (1 - \alpha_1 \rho_X(1) - \ldots, -\alpha_p \rho_X(p)) R_p^{-1}$, where $R_p^{-1} \in \mathbb{R}^{p \times p}$ contains the elements $\rho_X(|i - j|)$, $i, j \in \{1, \ldots, p\}$.

(2 Credits)

3. We observed $X_1, \ldots, X_{100}$ of a stationary AR(2) process with coefficients α_1, α_2 and the corresponding white noise process (ε_t) of iid random variables with variance $\sigma^2 > 0$ and finite fourth moment. Assume we have computed $\hat{\gamma}_X(0) = 1382.2$, $\hat{\gamma}_X(1) = 1114.4$ $\hat{\gamma}_X(2) = 591.72$ and $\hat{\gamma}_X(3) = 96.215$.
   (a) Find the Yule Walker estimates of α_1, α_2 and σ^2.
   (b) Compute the asymptotic correlation between $\hat{\alpha}_1$ and $\hat{\alpha}_2$ in general and interpret the result for the estimates obtained in (a).
   Hint: Problem 2 may help.
   (c) Assume ε_t to be Gaussian. Compute the quasi Maximum Likelihood Estimator for α_1, α_2 and σ^2 and compare it with the Yule Walker estimates.
   Hint: $\tilde{\gamma}(k) = \hat{\gamma}(k) + O_p\left(\frac{1}{n}\right)$

(2 + 3 + 3 Credits)

We wish you a merry Christmas and a happy new year!!!
http://www.uni-ulm.de/mawi/zawa/lehre/winter2008/ts20082009.html