

A41

ZZ: X, Y mit $E X < \infty$ und $E Y^2 < \infty \Rightarrow \text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2 \text{Cov}(X, Y)$,

Ergibt: $\text{Var}(X+Y) = E((X+Y)^2) - (E(X+Y))^2$

lim. d. Erwart.

$$= (E(X^2) + 2E(XY) + E(Y^2)) - (E(X)^2 + 2E(X)E(Y) + E(Y)^2)$$

$$= (E(X^2) - (E(X))^2) + (E(Y^2) - (E(Y))^2) + 2(E(XY) - E(X)E(Y))$$

$$= \text{Var}(X) + \text{Var}(Y) + 2 \text{Cov}(X, Y).$$

□

$$Z = 60.000(1+X) + 40.000(1+Y)$$

Dann gilt: $E Z = 60.000(1+E X) + 40.000(1+E(Y))$

$$= 60.000(1+0,08) + 40.000(1+0,06) = \underline{107.200 \text{ €}}.$$

$$\text{Var } Z = \text{Var}(60.000(1+X) + 40.000(1+Y))$$

$$= \text{Var}(60.000(1+X)) + \text{Var}(40.000(1+Y)) + 2 \text{Cov}(60.000(1+X), 40.000(1+Y))$$

$$\stackrel{\text{Prop. 3.4}}{=} 60.000^2 \text{Var}(X) + 40.000^2 \text{Var}(Y) + 2 \cdot 60.000 \cdot 40.000 \cdot \text{Cov}(X, Y)$$

a) $\text{Cov}(X, Y) = 0$

$$\Rightarrow \text{Var } Z = 60.000^2 \cdot 0,0004 + 40.000^2 \cdot 0,0001 = 1.600.000$$

b) $\text{Cov}(X, Y) = \sqrt{\text{Var } X} \sqrt{\text{Var } Y} \cdot \rho(X, Y) = -0,3 \sqrt{0,0004} \sqrt{0,0001}$

$$\Rightarrow \text{Var } Z = 1.600.000 + 2 \cdot 60.000 \cdot 40.000 \cdot (-0,3) \sqrt{0,0004} \sqrt{0,0001} = 1.312.000$$