Stochastic Finance 2010
Summer School Ulm
Lecture 1: Energy Derivatives

Professor Dr. Rüdiger Kiesel

21. September 2010

Agenda

History

Since the deregulation of electricity markets in the end of the 1990s, power can be traded at exchanges like the Nordpool, http://www.nordpoolspot.com/ or the European Energy Exchange (EEX), http://www.eex.com/en. All exchanges have established spot and futures markets.

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- Continuous day-ahead block trading for France 7:30 am to 11:30 am, 7 days a week, including holidays.
- Continuous Power intraday trading for Germany and France until 75 minutes before the beginning of delivery with delivery on the same or the following day in single hours or blocks.

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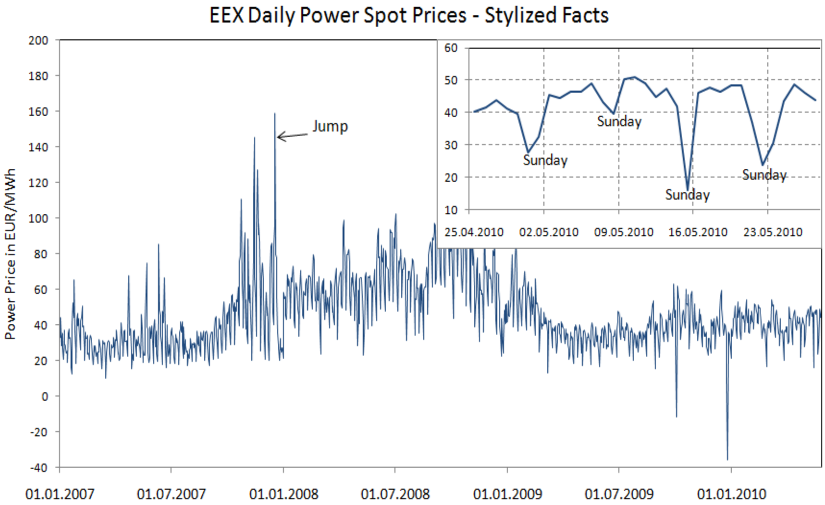
- Similar structures can be found on other power exchanges (Nord Pool, APX, etc.).

EEX Spot Market Price Processes



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- In addition, OTC transactions.

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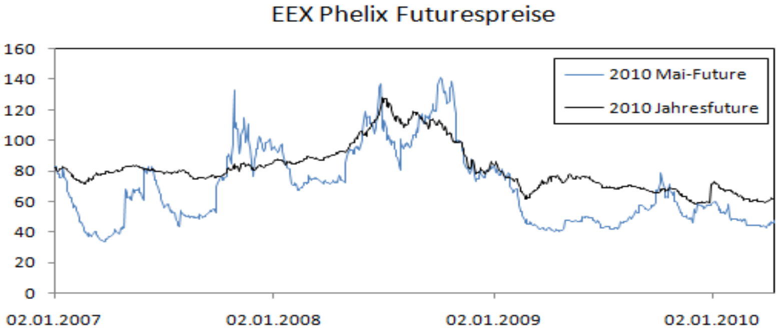
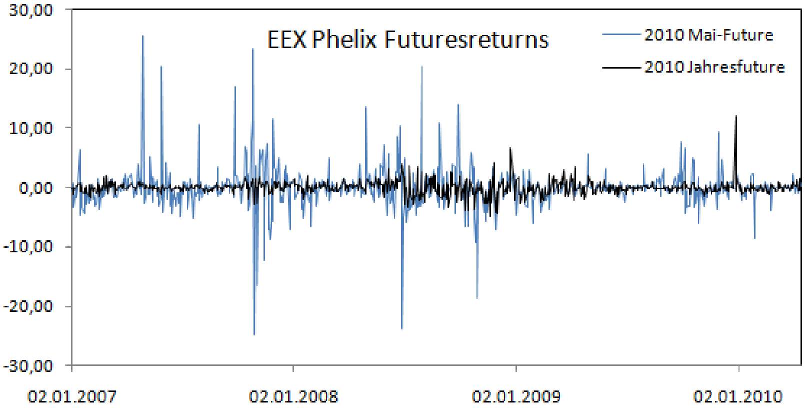


2010 Mai-Future: Actual 1-month future contract; the future prices are the quotations of the rolling contracts, i.e. the prices of the actual monthly contract (with delivery in the next month).

EEX Futures Market Price Processes

Returns seem to be stationary, no seasonality.



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- In addition, options on second period European Carbon Futures are traded.

Agenda

Problems

Modelling of electricity spot price dynamics is a delicate issue. Spot prices demonstrate the following typical features:

- *seasonality*: daily, weekly, monthly;
- *mean-reversion* or *stationarity*;
- *spikes*: may occur with some seasonal intensity;
- *high volatility*.

Basic model setup

Let $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ be a suitable filtered probability space:

- time horizon $t = 0, \ldots, T$ is fixed;
- electricity spot price at time $0 \leq t \leq T$ by $S(t)$ takes the form:

$$S(t) = e^{\mu(t)} X(t)\,, \tag{1}$$

- $\mu(t)$ is a deterministic function modelling the seasonal trend;
- $X(t)$ is some stochastic process modelling the random fluctuation.

Seasonality

Spot prices vary with seasons, so there is a need in some periodic function:

$$\mu(t) = \alpha + \beta t + \gamma \cos(\epsilon + 2\pi t) + \delta \cos(\zeta + 4\pi t)\,, \qquad (2)$$

where the parameters $\alpha, \beta, \gamma, \delta$, ϵ and ζ are all constants:

- α is fixed cost linked to the power production;
- β drives the long run linear trend in the total production cost;
- γ, δ, ϵ and ζ construct periodicity by adding two maxima per year with possibly different magnitude.

Definition of the model

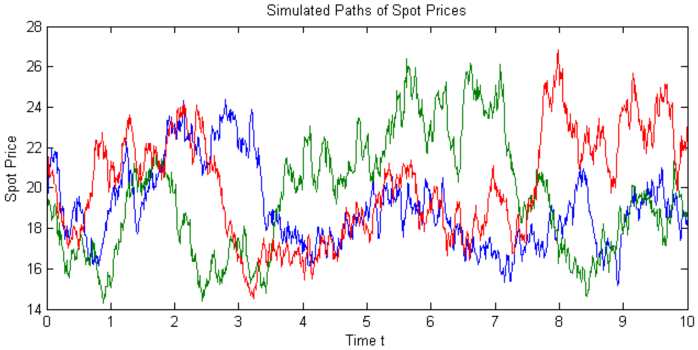
$$d \log X_t = \kappa(\ln \theta - \log X_t)dt + \sigma dW_t.$$

Thus, the logarithm of the prices follow a mean reverting diffusion process, the so-called Ornstein-Uhlenbeck-Process.

- κ - Speed of mean reversion
- θ - Level of mean reversion
- σ - Volatility of the process

Sample paths of the model

We simulate S_t (with $S_0 = 20, \theta = 20, \kappa = 1, \sigma = 0.2$) and get sample paths



Properties of the model

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- Mean reverting
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- Continuous paths
- Relative price changes are normally distributed
- Analytic results for the forward-curve and option prices exist
- Calibration easily possible

Model specification

Here the deseasonalized logarithmic spot prices are modelled by

$$d \ln X(t) = -\alpha \ln X(t)\, dt + \sigma(t)\, dW(t) + \ln J\, dq(t)\,, \tag{3}$$

where α is the speed of mean-reversion, W is a Brownian motion, $\sigma(t)$ is a time-dependent volatility, J is a proportional random jump size and dq_t is a Poisson process of intensity l with

$$dq_t = \begin{cases} 1 & \text{with probability} \quad l\, dt \\ 0 & \text{with probability} \quad 1 - l\, dt\,. \end{cases} \tag{4}$$

A typical assumption on the jump size distribution is $\ln J \sim \mathcal{N}(\mu_j, \sigma_j^2)$.

Basics of the models

The **the factor model**, was proposed by *Benth et al. 2007*. It is an additive linear model, where the price dynamics is a superposition of Ornstein-Uhlenbeck processes driven by subordinators to ensure positivity of the prices. It separates the modelling of spikes and base components.

Specification $X(t)$ for the factor model

$$S(t) = e^{\mu(t)} X(t) \,,$$

where $X(t)$ is a stochastic process represented as a weighted sum of n independent non-Gaussian Ornstein-Uhlenbeck processes $Y_i(t)$

$$X(t) = \sum_{i=1}^{n} w_i Y_i(t) \,, \tag{5}$$

where each $Y_i(t)$ is defined as

$$dY_i(t) = -\lambda_i Y_i(t) dt + dL_i(t) \,, \;\; Y_i(0) = y_i, i = 1, \ldots, n \,. \tag{6}$$

w_i are weighted functions; λ_i are mean-reversion coefficients; $L_i(t)$, $t = 1, \ldots n$ are independent *càdlàg* pure-jump additive processes with increasing paths.

Basics of the models

The **the threshold model**, was suggested by *Roncoroni 2002* and further developed by *Geman and Roncoroni 2006*. It represents an exponential Ornstein-Uhlenbeck process driven by a Brownian motion and a state-dependent compound Poisson process. It is designed to capture both statistical and pathwise properties of electricity spot prices.

Specification $X(t)$ for the threshold model

$$S(t) = e^{\mu(t)} X(t)\,,$$

$$d \ln X(t) = -\theta_1 \ln X(t)\, dt + \sigma\, dW(t) + h(\ln(X(t-)))\, dJ(t)\,, \tag{7}$$

θ_1 is one mean-reversion parameter, positive constant;
σ is Brownian volatility parameter, positive constant.

The Brownian component models the normal random variations of the electricity price around its mean, i.e., *the base signal*.

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where J is a time-inhomogeneous compound Poisson process:

$$J(t) = \sum_{i=1}^{N(t)} J_i\,,$$

and $N(t)$ counts the spikes up to time t and is a Poisson process with time-dependent jump intensity.
$J_1, J_2, \ldots$ model the magnitude of the spikes and are assumed to be i.i.d. random variables.
The function h attains two values, ± 1, indicating the direction of the jump.

Agenda

SME Group Energy Derivatives

CME Group Energy Futures and Options

PHYSICALLY SETTLED CONTRACTS

- Light Sweet Crude Oil
- Natural Gas
- Heating Oil
- RBOB Gasoline
- Singapore 380cst Fuel Oil
- Gulf Coast Gasoline
- Gulf Coast Ultra Low Sulfur Diesel (ULSD)
- New York Harbor Ultra Low Sulfur Diesel (ULSD)
- Russian Export Blend Crude Oil (REBCO)
- Ethanol

CASH SETTLED CONTRACTS

- Light Sweet Crude Oil
- Natural Gas Last-day
- Natural Gas Penultimate
- Heating Oil
- RBOB Gasoline
- Brent Crude Oil Penultimate
- Brent Crude Oil Last-day
- Propane
- Heating Oil and Gasoline Crack Spread
- Electricity
- Uranium
- E-mini Crude Oil
- E-mini Natural Gas
- E-mini RBOB Gasoline
- E-mini Heating Oil

CME Group Energy Derivatives

CME Group is built on heritage of CME, CBOT and NYMEX.

- World's largest and most diverse derivatives exchange
- Average daily volume of 1.25 million energy contracts
- Year-on-year volume growth – up 19 percent in 2008 alone

Size of Derivative Markets: NYMEX

Energy Futures on NYMEX:

NYMEX ENERGY FUTURES VOLUMES & OPEN INTEREST PAGE

DATED: 13-May POSTED: 14-May TIME: 14:00

MONTH	RIC	VOLUME	OP INT		CHANGE	EXPIRY
DEC13	<QLZ3>	0	5	=		26NOV13
TOTAL		1255	7940	↓	315	

NYMEX ENERGY FUTURES CONTRACTS VOLUME & OPEN INTEREST - TOTAL

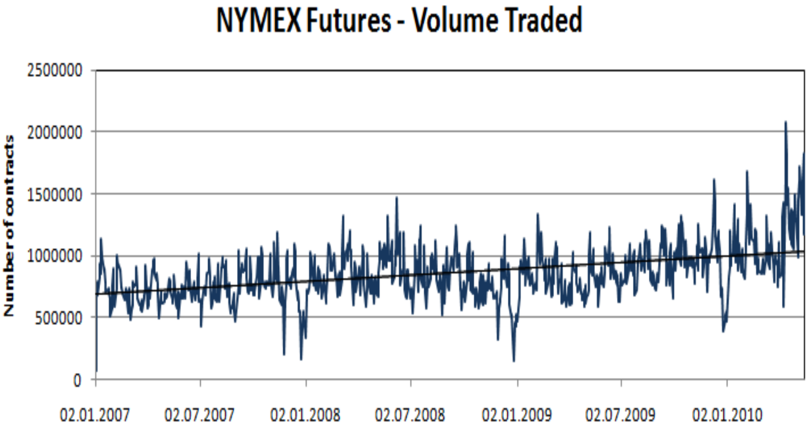
Name & Chain	RIC	VOLUME	OI		Net Change
LIGHT SWEET CRUDE <0#CL:>	<CL-TOT>	1169169	1482103	↑	12258
HEATING OIL <0#HO:>	<HO-TOT>	132425	320756	↑	452
RBOB GASOLINE <0#RB:>	<RB-TOT>	122025	278693	↓	2070
Natural Gas <0#NG:>	<NG-TOT>	402506	870095	↓	3662
BRENT LAST DAY <0#BZZ:>	<BZZ-TOT>	35	7504	↑	7
HENRY HUB <0#NN:>	<NN-TOT>	97764	2857819	↑	5539
BRENT FINANCIAL <0#B:>	<B-TOT>	2637	39752	↑	1121
CAPP COAL <0#QL:>	<QL-TOT>	1255	7940	↓	315

page above contains Volumes and Open Interest for contracts traded on NYMEX
Some of the Contracts may be expired, however they are still displayed by NYMEX
as they have Open Interest.

NYMEX GUIDE .....<NYM/FUTEX1> COMMODITIES MAIN GUIDE ..<COMMOD>

Source: Reuters

Size of Derivative Markets: NYMEX

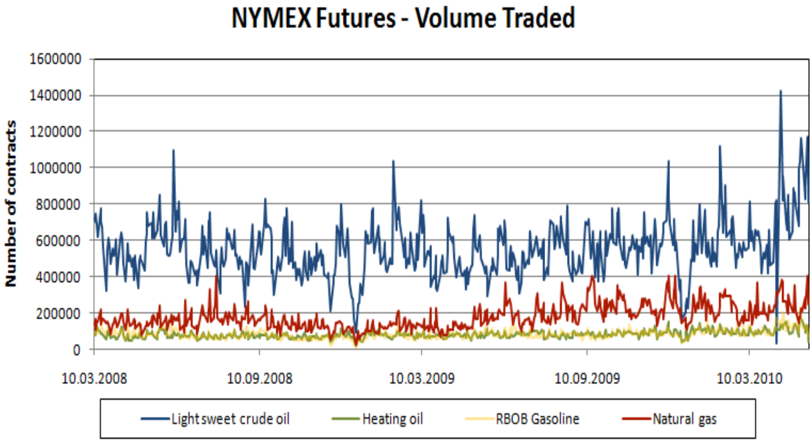


Size of Derivative Markets: NYMEX

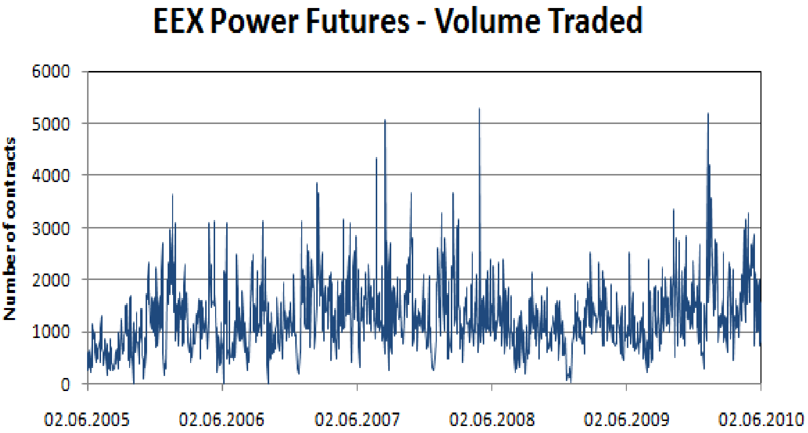
In practice, most futures contracts on NYMEX are liquidated via offset, so that physical delivery of the underlying commodity is relatively rare.

Futures trading volume data display strong seasonality due to the 'rolling over' of positions close to the expiry date of the near contract.

Size of Derivative Markets: NYMEX



Size of Derivative Markets: EEX

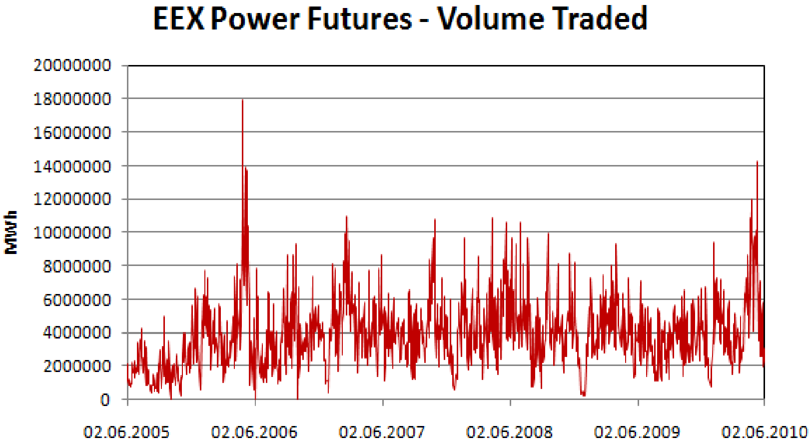


Size of Derivative Markets: EEX

Number of contracts reflects the total number of all power futures contracts traded on a particular day on EEX.

EEX power futures are available as base load and peak load contracts each with month, quarter and year futures. The contract volumes range from 240MWh for the smallest peak load month contract to up to 8 784MWh for the biggest base load year contract. The delivery rate amounts to 1MWh pro contract.

Size of Derivative Markets: EEX



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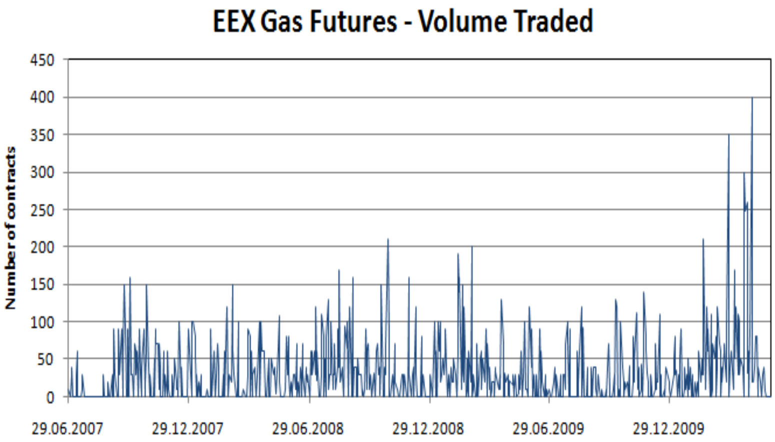
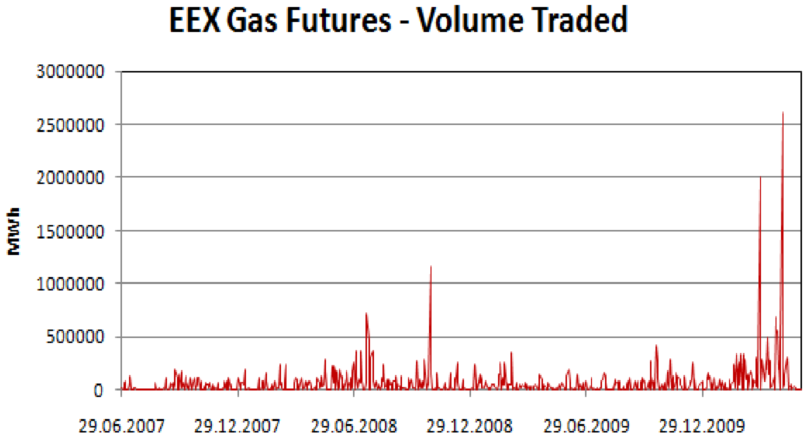


Size of Derivative Markets: EEX

Number of contracts reflects the total number of all natural gas futures contracts traded on a particular day on EEX.

The tradable delivery periods are the balance of month, the following six month, seven quarters, four seasons and six calender years. All prices are quoted in €/MWh. The contract volumes range from 720MWh for the month contract to up to 8 760MWh for the year contract. The delivery rate amounts to 1MWh pro contract.

Size of Derivative Markets: EEX



Spread Options

Some market participants are exposed to the difference of commodity prices. Examples are

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- the spark spread between power and gas (model for a gas-fired power plant)

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- the dark spread between power and coal (model for a coal-fired power plant)
- the spark spread between power and gas (model for a gas-fired power plant)
- the crack spread between different refinements of oil (model for a refinement plant)

Spark Spread

$$\text{Spark\_Spread} = \text{Power\_Price} - \text{Heat\_Rate} \times \text{Fuel\_Price}.$$

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- Heat rate is the number of Btus needed to make 1kWh of electricity.
- In the absence of any inefficiency it takes 3412Btu to produce 1kWh of electricity.

Clean Spreads

In countries covered by the European Union Emissions Trading Scheme, utilities have to consider also the cost of carbon dioxide emission allowances. Emission trading has started in the EU in January 2005.

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- Clean dark spread represents the net revenue a coal-fired power plant makes from selling power, having bought coal and the required number of carbon allowances.
- The difference between the clean dark spread and the clean spark spread is known as the climate spread.

Clean Spark Spread

$$\begin{aligned} & \text{Clean\_Spark\_Spread} \\ = \; & \text{Power\_Price} - \text{Heat\_Rate} \times \text{Gas\_Price} \\ & - \text{Gas\_Emission\_Intensity\_Factor} \times \text{Carbon\_Price} \end{aligned}$$

- Clean spark spread reflects the cost of generating power from gas after taking into account gas and carbon allowance costs.

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- Clean spark spread reflects the cost of generating power from gas after taking into account gas and carbon allowance costs.
- A positive spread effectively means that it is profitable to generate electricity, while a negative spread means that generation would be a loss-making activity.
- However, it is important to note that the clean spark spreads do not take into account additional generating charges beyond gas and carbon, such as operational costs.

Valuation of Spread Options

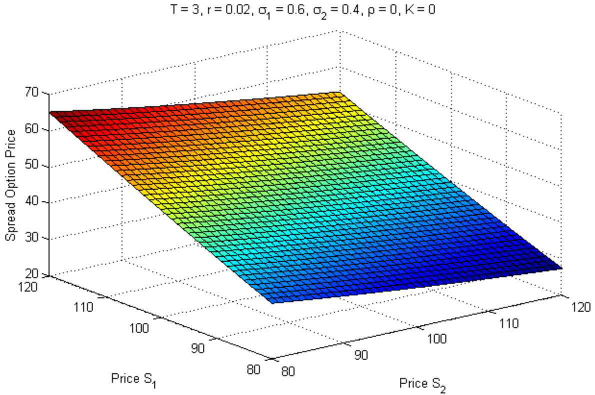
For Black-Scholes-type models and $K = 0$ (exchange option) there is an analytic formula due to Margrabe (1978).

$$
\begin{aligned}
C_{\text{spread}}(t) &= e^{-r(T-t)}(S_1(t)\Phi(d_1) - S_2(t)\Phi(d_2)) \\
P_{\text{spread}}(t) &= e^{-r(T-t)}(S_2(t)\Phi(-d_2) - S_1(t)\Phi(-d_1)) \\
\text{where} \quad d_1 &= \frac{\log(S_1(t)/S_2(t))+\sigma^2(T-t)/2}{\sqrt{\sigma^2(T-t)}}, \quad d_2 = d_1 - \sqrt{\sigma^2(T-t)} \\
\text{and} \quad \sigma &= \sqrt{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}
\end{aligned}
$$

where ρ is the correlation between the two underlyings.
For $K \neq 0$ no easy analytic formula is available.

Valuation of Spread Options - Price

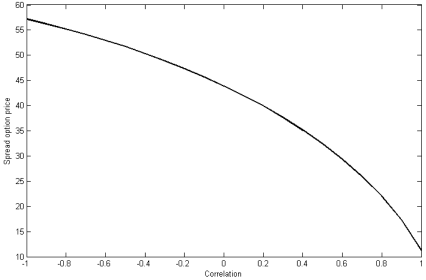
In this case, the price of the option depending on the underlying prices has the following structure:



Spread Option Value and Correlation

The value of a spread option depends strongly on the correlation between the two underlyings.

$S_1 = S_2 = 100, \ T = 3, \ r = 0.02, \ \sigma_1 = 0.6, \ \sigma_2 = 0.4.$



- The higher the correlation between the two underlyings the lower is the volatility of the spread and hence the value of the spread option.

Caps

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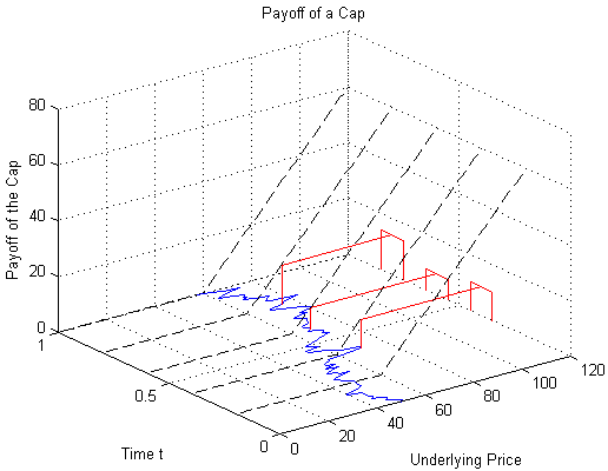
- It can be viewed as a strip of independent call options, for each time t_i the holder of the cap holds call options with maturity t_i and strike K.
- The static factors describing the cap are:
  - times $t_1, \ldots, t_N$ (how often? when?)
  - strike K (price?)
  - amount of the underlying (how much?)

Cap - Payoff


Payoff of a Cap
Payoff of the Cap
80
60
40
20
0
1
0.5
0
Time t
0
20
40
60
80
100
120
Underlying Price

Caps - Pricing

- Whenever the price of the underlying exceeds the strike K at one of the dates $t_1, \ldots, t_N$, the seller of the cap pays the holder of the cap the difference between the price of the underlying and the strike K or - in case one agreed on physical delivery - the underlying is delivered for the price K.

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- Typically, the price of a cap is quoted as price per delivery hours to make different delivery periods comparable. In this case we get a price per MWh.
- The formula is

$$U_c(t) = \frac{1}{N} \sum_{i=1}^{N} e^{-r(t_i - t)} \mathbb{E}[\max(S(t_i) - K, 0)].$$

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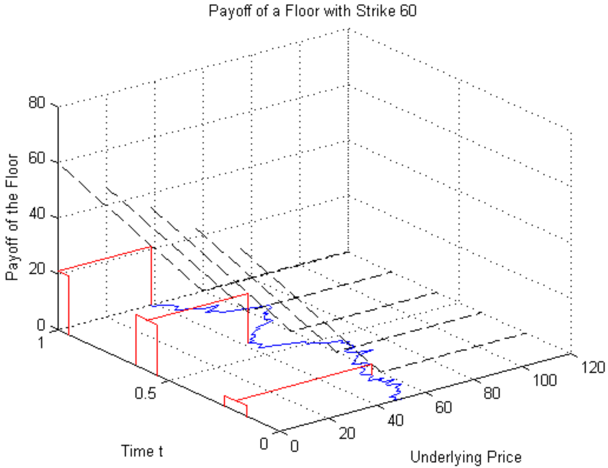
- Similar to the case of a cap, the pricing formula is

$$U_f(t) = \frac{1}{N} \sum_{i=1}^{N} e^{-r(t_i - t)} \mathbb{E}[\max(K - S(t_i), 0)].$$

As with the cap, the price is quoted in Euro/MWh.

Floor - Payoff



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- The holder of a short position might write a floor to produce liquidity upfront. The maximum gain from the short position is then limited to the strike K.

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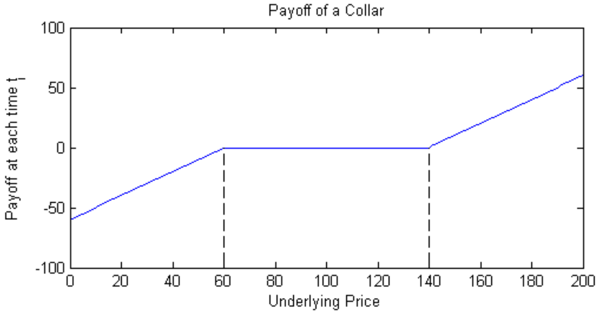
- As long as the price of the underlying is between K_1 and K_2 at one of the dates t_i, no cash flows are exchanged.
- If the underlying is above K_2, the holder of the long collar position receives the difference of the actual price and K_2. If the underlying is below K_1, the short collar position receives the difference between K_1 and the actual price.

Collar - Payoff

As a long collar position is a strip of call options minus a strip of put options, the payoff of a collar at each time point t_i is the following:



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$$U^{K_1,K_2}_{collar}(t) = U^{K_2}_{cap}(t) - U^{K_1}_{floor}(t)$$
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- The price of a collar might be positive or negative - or even zero. In case the price is zero, the collar is called zero-cost collar.

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- A short collar protects against falling prices. At the same time, the ability to participate on rising prices is limited to K_2.
- Having a long position in the underlying, a short collar ensures that the position can be closed for prices in the range of $[K_1, K_2]$.

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- A possible extension is to include a short position in one cap with strike $K_3 >> K_2$ in order to reduce the cost of the collar. This extension is called 3-way-collar.
- The price of a 3-way-collar is thus:

$$\begin{aligned} U^{K_1,K_2,K_3}_{3-way}(t) &= U^{K_2}_{cap}(t) - U^{K_3}_{cap}(t) - U^{K_1}_{floor}(t) \\ &= \frac{1}{N}\sum_{i=1}^{N} e^{-r(t_i-t)}\mathbb{E}[(S(t_i) - K_2)^+ \\ &- (S(t_i) - K_3)^+ - (K_1 - S(t_i))^+] \end{aligned}$$

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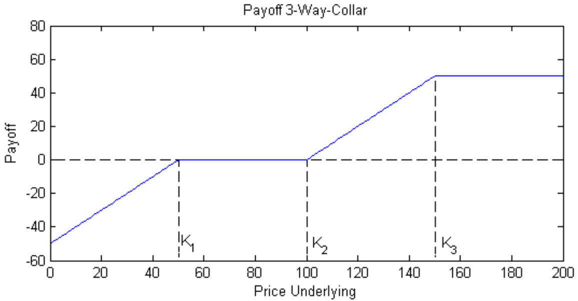
- The holder of the 3-way-collar is protected against increases in the underlying price above K_2, but only till K_3. Afterwards, no protection exists anymore.
- This strategy might be a good choice if one wants to protect its buying costs but is able to stop its business if prices rally unexpectedly high (above K_3).

3-Way-Collar - Payoff

The payoff is



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- We may also require that the number of exercises is between $E_{\min}$ and $E_{\max}$.

Swing Options

To determine the swing option value, we have to find an optimal exercise strategy $\Phi = (\phi_1, \ldots, \phi_N)$ maximising the expected payoff

$$\sum_{i=1}^{N} e^{-r(t_i - t)} \mathbb{E}[\phi_i (S(t_i) - K)] \quad \to \max$$

subject to

$$E_{\min} \leq \sum_{i=1}^{N} \phi_i \leq E_{\max}.$$

Bounds for Swing Options

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For deterministic spot prices, we

- Calculate the discounted payoffs $P(t_i) = e^{-r(t_i - t)}(S(t_i) - K)$.

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- Calculate the discounted payoffs $P(t_i) = e^{-r(t_i - t)}(S(t_i) - K)$.
- Sort the discounted payoffs $P(t_i)$ in descending order.
- Take the first $E_{\min}$ payoffs regardless of their value and subsequent payoffs up to $E_{\max}$ until their sign become negative.

Bounds for Swing Options

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- For stochastic spot prices the MC-approach gives an upper bound, since information on the whole path is used, but in reality only information up to time t is available when deciding at time t.
- A lower bound is given by the intrinsic value

$$\sum_{i=1}^{N} e^{-r(t_i-t)}\phi_i^F(F(t,t_i)-K) \quad \rightarrow \max$$

subject to

$$E_{\min} \leq \sum_{i=1}^{N} \phi_i^F \leq E_{\max}$$

where $\phi_i^F = \mathbf{1}_{\{F(t,t_i)>K\}}$, unless the restriction on $E_{\min}$ is in force.