Stochastic Finance 2010
Summer School Ulm
Lecture 2: Electricity Forwards

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22. September 2010

Agenda

Characteristics

- **Electricity Futures** – Obligation to buy/sell a specified amount of electricity during a **delivery period**, typically a month, quarter or year.

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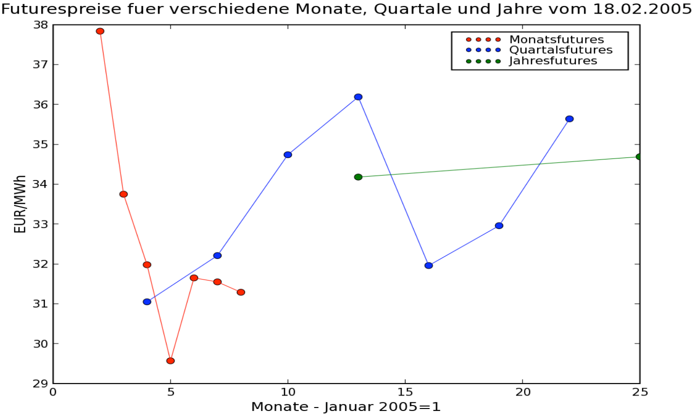
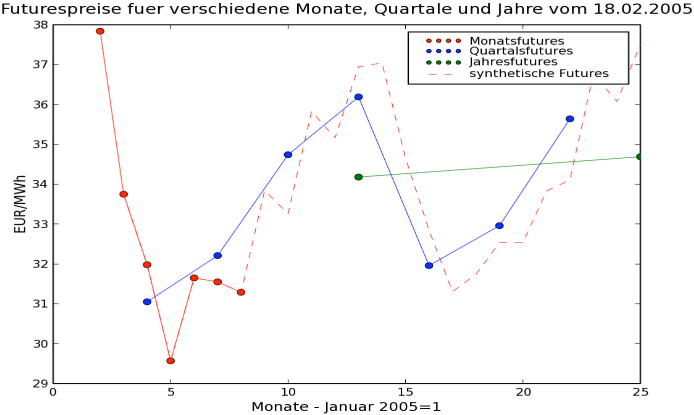
- Futures show a decreasing volatility term structure
- Level of volatility depends on length of delivery period

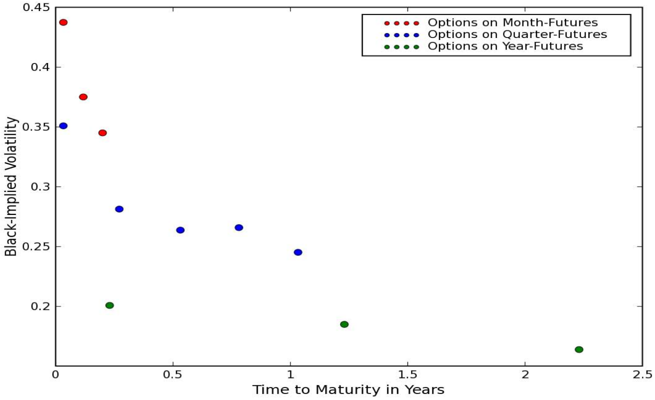
Prices of Futures



Implied Prices of Futures



Volatilities



Modelling Approach

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- We distinguish between forward contracts with a fixed time delivery and forward contracts with a delivery period, called *swaps*.
- A typical lognormal dynamics of the swap price is,

$$dF(t, T_1, T_2) = \Sigma(t, T_1, T_2)F(t, T_1, T_2)\, dW(t). \tag{1}$$

The only parameter in this model is the volatility function Σ which has to capture all movements of the swap price and especially the time to maturity effect.

Volatility Functions

We assume that the swap price dynamics for all swaps is given by (1) where $\Sigma(t, T_1, T_2)$ is a continuously differentiable and positive function.

Starting out with a given volatility function for a fixed time forward contract the volatility function Σ for the swap contract is given by

$$\Sigma(t, T_1, T_2) = \int_{T_1}^{T_2} \hat{w}(u, T_1, T_2)\sigma(t, u)\, du. \tag{2}$$

Schwartz Volatility

For the related volatility function of the forward we obtain

$$\sigma(t,u) = ae^{-b(u-t)} \tag{3}$$

where $a, b > 0$ are constant.

Schwartz Volatility

The time to maturity effect is modeled by a negative exponential function.

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The time to maturity effect is modeled by a negative exponential function.

- When the time to maturity tends to infinity the volatility function converges to zero.
- The exponential function causes that the volatility increases as the time to maturity decreases which leads to an increased volatility when the contract approaches the maturity.

Schwartz Volatility

Applying this forward volatility to (2) the swap volatility is:

$$\Sigma(t, T_1, T_2) = a\,\varphi(T_1, T_2) \tag{4}$$

where

$$\varphi(T_1, T_2) = \frac{e^{-b(T_1-t)} - e^{-b(T_2-t)}}{b(T_2 - T_1)} \tag{5}$$

The Black-76 specification of the forward volatility can be obtained if $\varphi(T_1, T_2) = 1$, that is $b = 0$ in (3).
The associated swap price volatility is then given by $\Sigma(t, T_1, T_2) = a$.

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  where $\sigma(t, T)$ is an adapted d-dimensional deterministic function and $W(t)$ a d-dimensional Brownian motion.
- Initial value of this SDE is the initial forward curve observed at the market.

Options on Building Blocks

A European call option on $F(t,T)$ with maturity T_0 and strike K can be easily evaluated by the Black-formula

$$V^{option}(0) = e^{-rT_0}\left(F(0,T)\mathcal{N}(d_1) - K\mathcal{N}(d_2)\right), \tag{6}$$

where $\mathcal{N}$ denotes the normal distribution, $\Sigma(T_0,T) = \int_0^{T_0} ||\sigma(s,T)||^2 ds$ and

$$\begin{aligned} d_1 &= \frac{\log \frac{F(0,T)}{K} + \frac{1}{2}\Sigma(T_0,T)}{\sqrt{\Sigma(T_0,T)}} \\ d_2 &= d_1 - \sqrt{\Sigma(T_0,T)} \end{aligned}$$

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  (Compare modelling of forward swap rates in terms of forward LIBOR rates)

- In case of 1-year-futures, the swap rate is the forward price of the 1-year-futures, which can be also observed in the market.

Options on n-period futures

- We need to compute

$$e^{-rT_0}\mathbb{E}\left[(Y(T_0)-K)^+\right],$$

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- An analysis of the goodness of the approximation can be found in Brigo-Mercurio (2003).

Options on *n*-period futures

Using this approximation, it is possible to apply a Black-Option formula again to obtain the option value as

$$\begin{aligned} V^{option} &= e^{-rT_0}\mathbb{E}\left[(Y(T_0)-K)^+\right] \\ &\approx e^{-rT_0}\mathbb{E}\left[\left(\hat{Y}(T_0)-K\right)^+\right] \\ &= e^{-rT_0}\left(Y(0)\mathcal{N}(d_1)-K\mathcal{N}(d_2)\right) \end{aligned} \tag{7}$$

with

$$\begin{aligned} d_1 &= \frac{\log\frac{Y(0)}{K}+\frac{1}{2}s^2}{s} \\ d_2 &= d_1 - s \end{aligned}$$

A Two-Factor Model

- For a fixed delivery start T and delivery period 1 month, let the dynamics of a Future $F_{t,T}$ be given by the two factor model:

$$F(t,T) = F(0,T)\exp\left\{\mu(t,T) + \int_0^t \hat{\sigma}_1(s,T)dW_s^{(1)} + \sigma_2 W_t^{(2)}\right\}$$

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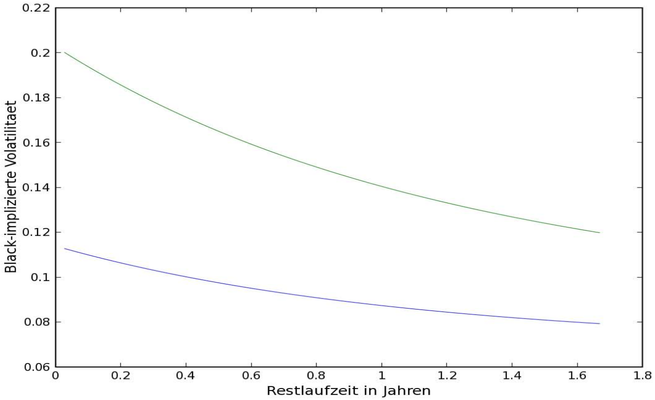
- $\sigma_1,\ \sigma_2,\ \kappa > 0$ constants
- $\mu(t,T)$ being the risk-neutral martingale drift

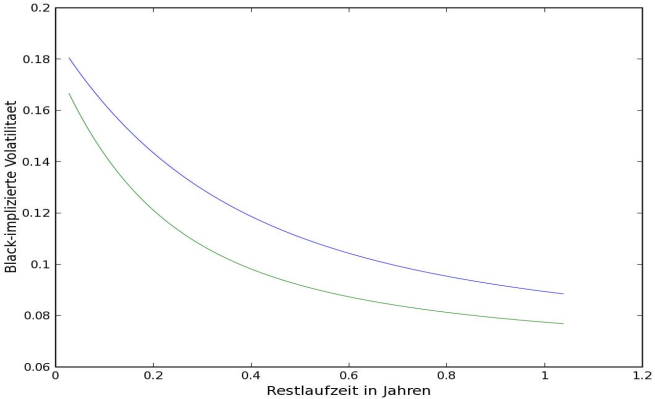
Model Parameters

σ_1 affects the level at the short end of the volatility curve



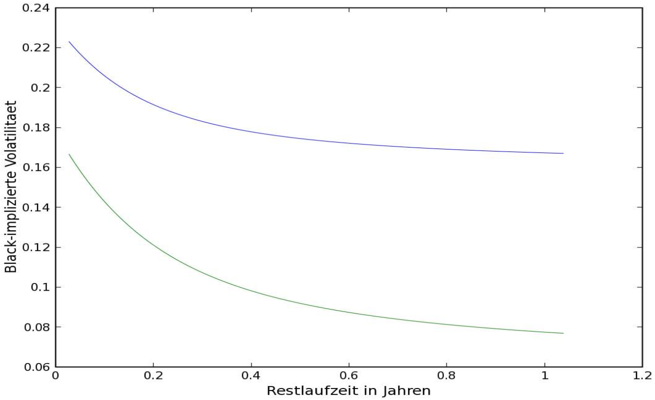
Model Parameters

κ affects the slope of the volatility curve at the short end



Model Parameters

σ_2 affects the level at the long end of the volatility curve



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- Prices of quarterly and yearly Futures are given as an average of the n corresponding monthly Futures.
- $Y_{t,T_1,\dots T_n} = Y = \frac{\sum e^{-rT_i} F_{t,T_i}}{\sum e^{-rT_i}}$ is the forward price of a n-month forward quoted in the market (cp. swap rate)

Pricing of Options on Month-Futures

- At time $t = 0$, the price of a Call-Option with strike K and maturity T_0 on a Month-Future $F_{t,T}$ is given by

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- Within the model, $F_{T_0,T}$ is log-normally distributed with known variance

$$\Sigma(T_0, T) = \frac{\sigma_1^2}{2\kappa}(e^{-2\kappa(T-T_0)} - e^{-2\kappa T}) + \sigma_2^2 T_0$$

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- Thus the option's value is given by the formula (Black 76):

$$e^{-rT_0}\mathbb{E}\left[(F_{T_0,T} - K)^+\right] \quad = \quad e^{-rT_0}\left(F_{0,T}\mathcal{N}(d_1) - K\mathcal{N}(d_2)\right)$$

with $d_{1,2}$ depending on the parameters $\sigma_1, \sigma_2, \kappa$.

Pricing of Options on quarterly and yearly Futures

- At time $t = 0$, the price of a Call-Option with strike K and maturity T_0 on a n-Month-Future Y is given by

$$e^{-rT_0}\mathbb{E}\left[(Y-K)^+\right] = e^{-rT_0}\mathbb{E}\left[\left(\frac{\sum e^{-rT_i}F_{t,T_i}}{\sum e^{-rT_i}} - K\right)^+\right]$$

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- Approximate the random variable Y by a log-normal random variable $\hat{Y}$ with same mean and variance (depending on the model parameters)

Matching the Variance

Using the moment-generating function of a normal random variable, we get

$$\exp(s^2) = \frac{\mathrm{Var}(Y)}{(\mathbb{E}(Y))^2} + 1 = \frac{\mathbb{E}(Y^2)}{\mathbb{E}(Y)^2}$$

From the martingale property $\mathbb{E}(F_{T_0,T_i}) = F_{0,T_i}$ and

$$\mathbb{E}(Y_{T_1,\dots T_n}(T_0)) = \frac{\sum e^{-r(T_i - T_0)} F_{0,T_i}}{\sum e^{-r(T_i - T_0)}}.$$

Matching the Variance

So

$$\mathbb{E}(Y_{T_1,\dots T_n}(T_0)^2) = \frac{\sum_{i,j} e^{-r(T_i+T_j-2T_0)} F_{0,T_i} F_{0,T_j} \cdot \exp Cov_{ij}}{\left(\sum e^{-r(T_i-T_0)}\right)^2}$$

with $Cov_{ij} = \mathrm{Cov}(\log F(T_0, T_i), \log F(T_0, T_j))$.
The covariance can be computed directly from the explicit solution of the SDE

$$\begin{aligned} & \mathrm{Cov}(\log F(T_0, T_i), \log F(T_0, T_j)) \\ = \quad & e^{-\kappa(T_i+T_j-2T_0)} \frac{\sigma_1^2}{2\kappa}(1 - e^{-2\kappa T_0}) + \sigma_2^2 T_0 \end{aligned}$$

Pricing of Options on quarterly and yearly Futures

The option value can be computed by Black's formula

$$\begin{aligned} e^{-rT_0}\mathbb{E}\left[(Y-K)^+\right] &\approx e^{-rT_0}\mathbb{E}\left[\left(\hat{Y}-K\right)^+\right] \\ &= e^{-rT_0}\left(Y(0)\mathcal{N}(d_1)-K\mathcal{N}(d_2)\right) \end{aligned}$$

with $d_{1,2}$ depending on the parameters $\sigma_1, \sigma_2, \kappa$.

Parameter Estimation

- Use the approximating Black-formula

$$\begin{aligned}\text{Option value} &= e^{-rT_0}\left(Y(0)\mathcal{N}(d_1) - K\mathcal{N}(d_2)\right)\\ d_{1,2} &= d_{1,2}\left(Y(0), K, Var(\log \hat{Y}(T_0))\right)\end{aligned}$$

  Only the variance $Var(\log \hat{Y}(T_0))$ depends on the unknown parameters

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  Only the variance $Var(\log \hat{Y}(T_0))$ depends on the unknown parameters
- Compute the variances $Var(\log \hat{Y}(T_0))$ for products observable in the market
- Choose parameter σ_1, σ_2 and κ to minimize the distance of the model variances to the market variances in a given metric (in the least-square sense)

Data

Product	Delivery Start	Strike	Forward	Market Price	Implied Vola
Month	October 05	48	48.90	2.023	43.80%
Month	November 05	49	50.00	3.064	37.66%
Month	December 05	49	49.45	3.244	34.72%
Quarter	October 05	48	49.44	2.086	35.15%
Quarter	January 06	47	48.59	3.637	28.43%
Quarter	April 06	40	40.71	3.421	26.84%
Quarter	July 06	42	41.80	3.758	27.19%
Quarter	October 06	43	43.71	4.566	25.35%
Year	January 06	44	43.68	1.521	20.19%
Year	January 07	43	42.62	3.228	19.14%
Year	January 08	42	42.70	4.286	17.46%

Table: ATM calls and implied Black-volatility, Sep 14

Parameter Estimates

Method	Constraints	σ_1	σ_2	κ	Time
Function calls and numerical gradient	yes	0.37	0.15	1.40	<1min
Least Square Algorithm	no	0.37	0.15	1.41	<1min

Table: Parameter estimates with different optimizers, market data as of Sep 14

Options, which are far away from maturity, will have a volatility of about 15%, which can add up to more than 50%, when time to maturity decreases.
A κ value of 1.40 indicates, that disturbances in the futures market halve in $\frac{1}{\kappa} \cdot \log 2 \approx 0.69$ years.

Agenda

Spot-Forward Relationship: Classical theory

Under the no-arbitrage assumption we have the spot-forward relationship

$$F(t, T) = S(t)e^{(r-y)(T-t)} \tag{8}$$

where r is the interest rate at time t for maturity T and y is the convenience yield.

Spot-Forward Relationship: Classical theory

- In the stochastic model this means

$$F(t, T) = \mathbb{E}_{\mathbb{Q}}(S(T)|\mathcal{F}_t)$$

where $\mathcal{F}_t$ is the accumulated available market information (in most models the information generated by the spot price).

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  where $\mathcal{F}_t$ is the accumulated available market information (in most models the information generated by the spot price).
- $\mathbb{Q}$ is the risk-neutral probability
  - discounted spot price is a $\mathbb{Q}$-martingale
  - ..or, the expected return under $\mathbb{Q}$ is r

Spot-Forward Relationship: Classical theory

We observe normal backwardation: Futures prices are below spot price

- Producers accept paying a premium for securing future production

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- Producers accept paying a premium for securing future production
- This may be caused by hedging pressure for long term investments
- Convenience yield larger than risk-free rate

Spot-Forward Relationship: Classical theory

Most models give either normal backwardation or contango (Futures prices are above spot price)

- No stochastic change of sign (risk premium)

Spot-Forward Relationship: Classical theory

Most models give either normal backwardation or contango (Futures prices are above spot price)

- No stochastic change of sign (risk premium)
- True even for jump models

Case of Electricity

- Storage of spot is not possible (only indirectly in water reservoirs)

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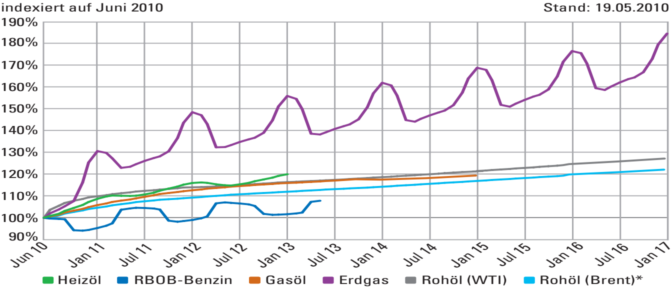
- Storage of spot is not possible (only indirectly in water reservoirs)
- Delivery periods for Futures
- Buy-and-Hold strategy fails
- No foundation for "classical" spot forward relations

Forward Curves

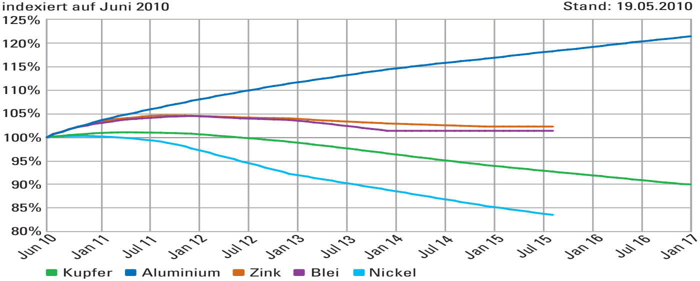
Abb. 1: Contango bei fast allen Energieträgern



Die Energieträger Heizöl, Gasöl, Erdgas sowie die Rohölsorten Brent und WTI zeigen am kurzen Ende Forwardkurvenverläufe im Contango. *Quelle: Bloomberg L.P.*
* indexiert auf Juli 2010

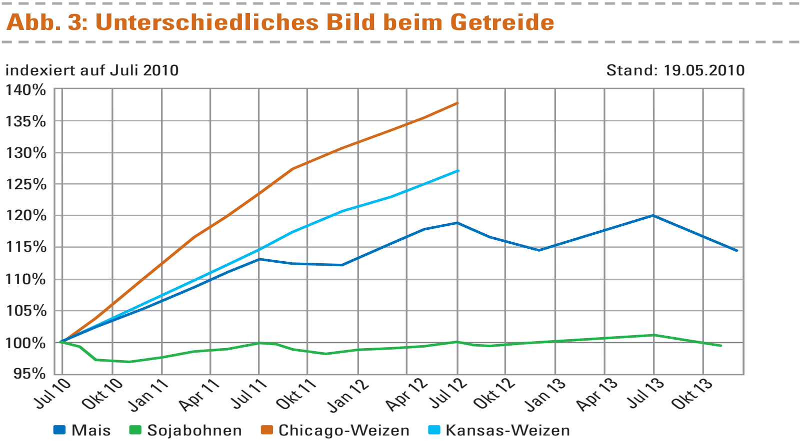
Forward Curves

Abb. 2: Contango und Backwardation bei Industriemetallen



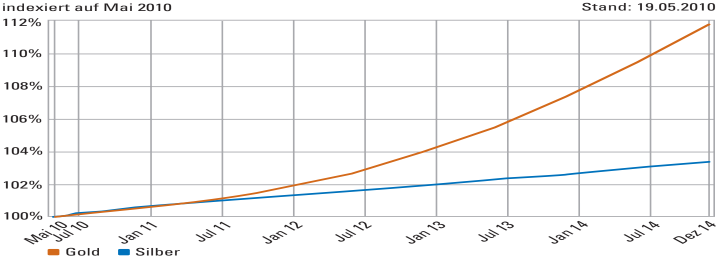
Am langen Ende zeigt nur die Aluminium-Forwardkurve den für Industriemetalle eher untypischen steigenden Verlauf, auch Contango genannt. *Quelle: Bloomberg L.P.*

Forward Curves



Forward Curves

Abb. 6: Contango bei Edelmetallen



Bei den beiden im S&P Goldman Sachs Commodity Index (S&P GSCI®) enthaltenen Edelmetallen Gold und Silber stellt der Contango den normalen Verlauf der Forwardkurve dar. *Quelle: Bloomberg L.P.*

KnowHow **06/2010**

Forward Curves

The shape of commodities forward curves for different delivery periods indicates the market players' (producers, retailers and speculators) 'attitudes' towards risk bearing in these markets.

Contrast to e.g. equity: Here, if interest rates and dividends are assumed deterministic, simple no-arbitrage arguments are employed to show that the arbitrage-free forward price will be the cost of borrowing net of collected dividends yielded by the equity.

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- for 'long' dated forward contracts, markets are in backwardation (forward below spot)
- for 'shorter' maturities the markets are in contango (forward above spot).
- See e.g. Longstaff & Wang (2004, JF); Diko, Lawford & Limpens (2006, Studies in Nonlinear Dynamics & Econometrics)

Market Risk Premium

The *market risk premium* or *forward bias* $\pi(t,T)$ relates forward and expected spot prices
It is defined as the difference, calculated at time t, between the forward $F(t,T)$ at time t with delivery at T and expected spot price:

$$\pi(t,T) = F(t,T) - \mathbb{E}^{\mathbb{P}}[S(T)|\mathcal{F}_t]. \tag{9}$$

Here $\mathbb{E}^{\mathbb{P}}$ is the expectation operator, under the historical measure $\mathbb{P}$, with information up until time t and $S(T)$ is the spot price at time T.

Agenda

Market Risk Premium – Players

The main motivation for players to engage in forward contracts is that of risk diversification.
Producers have made large investments with the aim of recouping them over a long period of time as well as making a return on them.
Retailers (which might be intermediaries and/or use the commodity in their production process) also have an incentive to hedge their positions in the market by contracting forwards that help diversify their risks.

Market Risk Premium – Qualitative

Exposure to the market will differ both between producers and retailers as well as within their own group.
For example, a large producer will generally be exposed to market uncertainty for a longer period of time, perhaps determined by the remaining life of the assets, whilst retailers will tend to make decisions based on a shorter time scale.
So the need for risk-diversification has a temporal dimension.

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- Retailers are less incentivized to contract commodity forwards the further out we look into the market.
- In contrast, on the producers' side the need to hedge in the long-term does not fade away as quickly.

Market Risk Premium

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- The mirror image is therefore one where the producers' desire to hedge their positions outweighs that of the retailers resulting in a negative market risk premium.

Representative Agents

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- A key question for the producer is how much of his future production, which cannot be predicted with total certainty, will he wish to sell on the forward market or, when the time comes, sell it on the spot market.
- Similarly, the retailer must decide how much of her future needs, which cannot be predicted with full certainty either, will be acquired via the forward markets and how much on the spot.

Representative Agents

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- Second, we discuss how the relative willingness of producers and retailers to hedge their exposures determines market clearing prices.

Representative Agents

We assume that the risk preferences of the representative agents are expressed in terms of an exponential utility function parameterized by the risk aversion constant $\gamma > 0$;

$$U(x) = 1 - \exp(-\gamma x) .$$

We let $\gamma := \gamma_p$ for the producer and $\gamma := \gamma_c$ for the retailer.

The Model

Following Lucia and Schwartz (Rev. Derivatives Research, 2002) and Benth, Kallsen and Meyer-Brandis (Appl. Math. Finance, 2007), we assume that the electricity spot price follows a mean-reverting multi-factor additive process

$$S_t = \Lambda(t) + \sum_{i=1}^{m} X_i(t) + \sum_{j=1}^{n} Y_j(t) \tag{10}$$

where $\Lambda(t)$ is the deterministic seasonal spot price level, while $X_i(t)$ and $Y_j(t)$ are the solutions to the stochastic differential equations

$$dX_i(t) = -\alpha_i X_i(t)\,dt + \sigma_i(t)\,dB_i(t) \tag{11}$$

and

$$dY_j(t) = -\beta_j Y_j(t)\,dt + dL_j(t). \tag{12}$$

$B_i(t)$, $i = 1, \dots, m$, are standard independent Brownian motions,

The Model

The processes $Y_j(t)$ are zero-mean reverting processes responsible for the spikes or large deviations which revert at a fast rate $\beta_j > 0$.

$X_i(t)$ are zero-mean reverting processes that account for the normal variations in the spot price evolution with lower degree of mean-reversion $\alpha_i > 0$.

Indifference Prices

Assume that the producer will deliver the spot over the time interval $[T_1, T_2]$.

He has the choice to deliver the production in the spot market, where he faces uncertainty in the prices over the delivery period, or to sell a forward contract with delivery over the same period.

The producer takes this decision at time $t \leq T_1$.

Indifference Prices

We determine the forward price that makes the producer indifferent between the two alternatives: denote by $F_{\text{pr}}(t, T_1, T_2)$ the forward price derived from the equation

$$
\begin{aligned}
& 1 - \mathbb{E}^P \left[\exp\left(-\gamma_p \int_{T_1}^{T_2} S(u)\, du \right) \,\middle|\, \mathcal{F}_t \right] \\
= \quad & 1 - \mathbb{E}^P \left[\exp\left(-\gamma_p (T_2 - T_1) F_{\text{pr}}(t, T_1, T_2) \right) \,\middle|\, \mathcal{F}_t \right]
\end{aligned}
$$

Indifference Prices

Equivalently,

$$F_{\mathrm{pr}}(t, T_1, T_2) = -\frac{1}{\gamma_p}\frac{1}{T_2 - T_1}\ln \mathbb{E}^P\left[\exp\left(-\gamma_p\int_{T_1}^{T_2} S(u)\,du\right) \mid \mathcal{F}_t\right] ; \tag{13}$$

where for simplicity we have assumed that the risk-free interest rate is zero.

$\int_{T_1}^{T_2} S(u)\,du$ is what the producer collects from selling the commodity on the spot market over the delivery period $[T_1, T_2]$, while he receives $(T_2 - T_1)F_{\mathrm{pr}}(t, T_1, T_2)$ from selling it on the forward market.

Notation

For $i = 1, \dots, m$ and $j = 1, \dots, n$,

$$\bar{\alpha}_i(s, T_1, T_2) = \begin{cases} \frac{1}{\alpha_i}\left(e^{-\alpha_i(T_1 - s)} - e^{-\alpha_i(T_2 - s)}\right) & , \quad s \le T_1 \,, \\ \frac{1}{\alpha_i}\left(1 - e^{-\alpha_i(T_2 - s)}\right) & , \quad s \ge T_1 \,. \end{cases} \tag{14}$$

and

$$\bar{\beta}_j(s, T_1, T_2) = \begin{cases} \frac{1}{\beta_j}\left(e^{-\beta_j(T_1 - s)} - e^{-\beta_j(T_2 - s)}\right) & , \quad s \le T_1 \,, \\ \frac{1}{\beta_j}\left(1 - e^{-\beta_j(T_2 - s)}\right) & , \quad s \ge T_1 \,. \end{cases} \tag{15}$$

Indifference Prices

The price for which the producer is indifferent between the forward and spot market is given by

$$\begin{aligned} F_{\mathrm{pr}}(t, T_1, T_2) &= \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u)\, du \\ &+ \sum_{i=1}^{m} \frac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) + \sum_{j=1}^{n} \frac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\ &- \frac{\gamma_p}{2(T_2 - T_1)} \int_t^{T_2} \sum_{i=1}^{m} \sigma_i^2(s) \bar{\alpha}_i^2(s, T_1, T_2)\, ds \\ &- \frac{1}{\gamma_p} \frac{1}{T_2 - T_1} \int_t^{T_2} \sum_{j=1}^{n} \phi_j \left(-\gamma_p \bar{\beta}_j(s, T_1, T_2) \right) ds\,, \end{aligned}$$

where $\bar{\alpha}_i$ and $\bar{\beta}_j$ are given by (14) and (15) respectively.

Indifference Price – Consumer

The retailer will derive the indifference price from the incurred expenses in the spot or forward market, which entails

$$
\begin{aligned}
& 1-\mathbb{E}^P\left[\exp\left(-\gamma_c\left(-\int_{T_1}^{T_2} S(u)\,du\right)\right) \mid \mathcal{F}_t\right] \\
= \quad & 1-\mathbb{E}^P\left[\exp\left(-\gamma_c(-(T_2-T_1)F_c(t,T_1,T_2))\right) \mid \mathcal{F}_t\right],
\end{aligned}
$$

or,

$$
F_c(t,T_1,T_2)=\frac{1}{\gamma_c}\frac{1}{T_2-T_1}\ln\mathbb{E}^P\left[\exp\left(\gamma_c\int_{T_1}^{T_2} S(u)\,du\right) \mid \mathcal{F}_t\right]. \tag{16}
$$

Indifference Price – Consumer

The price that makes the retailer indifferent between the forward and the spot market is given by

$$
\begin{aligned}
F_c(t, T_1, T_2) = {} & \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u)\, du + \sum_{i=1}^{m} \frac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) \\
& + \sum_{j=1}^{n} \frac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\
& + \frac{\gamma_c}{2(T_2 - T_1)} \int_t^{T_2} \sum_{i=1}^{m} \sigma_i^2(s) \bar{\alpha}_i^2(s, T_1, T_2)\, ds \\
& + \frac{1}{\gamma_c} \frac{1}{T_2 - T_1} \int_t^{T_2} \sum_{j=1}^{n} \phi_j \left(\gamma_c \bar{\beta}_j(s, T_1, T_2) \right) ds\,.
\end{aligned}
$$

Indifference Price – Bounds

Note that the producer prefers to sell his production in the forward market as long as the market forward price $F(t, T_1, T_2)$ is higher than $F_{pr}(t, T_1, T_2)$. On the other hand, the retailer prefers the spot market if the market forward price is more expensive than his indifference price $F_c(t, T_1, T_2)$. Thus, we have the bounds

$$F_{pr}(t, T_1, T_2) \leq F(t, T_1, T_2) \leq F_c(t, T_1, T_2) . \tag{17}$$

Market Power

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- For $p(t, T_1, T_2) = 1$ the producer has full market power and can charge the maximum price possible in the forward market (short-term positions), namely $F_c(t, T_1, T_2)$.
- If the retailer has full power, ie $p(t, T_1, T_2) = 0$ (long-term positions), she will drive the forward price as far down as possible which corresponds to $F_{pr}(t, T_1, T_2)$.

Market Power

For any market power $0 < p(t, T_1, T_2) < 1$,
the forward price $F^p(t, T_1, T_2)$ is defined to be

$$\begin{aligned} F^p(t, T_1, T_2) &= p(t, T_1, T_2) F_c(t, T_1, T_2) \\ &+ (1 - p(t, T_1, T_2)) F_{pr}(t, T_1, T_2). \end{aligned} \tag{18}$$

Market Power

For $0 \leq t \leq T_1 < T_2$ the forward prices are

$$
\begin{aligned}
&F^p(t, T_1, T_2) \\
&= \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u)\, du + \sum_{i=1}^{m} \frac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) + \sum_{j=1}^{n} \frac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\
&\quad + \frac{p(t, T_1, T_2)(\gamma_{\mathrm{pr}} + \gamma_{\mathrm{c}}) - \gamma_{\mathrm{pr}}}{2(T_2 - T_1)} \int_t^{T_2} \sum_{i=1}^{m} \sigma_i^2(s) \bar{\alpha}_i^2(s, T_1, T_2)\, ds \\
&\quad + \frac{p(t, T_1, T_2)}{\gamma_{\mathrm{c}}(T_2 - T_1)} \int_t^{T_2} \sum_{j=1}^{n} \phi_j(\gamma_{\mathrm{c}} \bar{\beta}_j(s, T_1, T_2))\, ds \\
&\quad - \frac{1 - p(t, T_1, T_2)}{\gamma_{\mathrm{pr}}(T_2 - T_1)} \int_t^{T_2} \sum_{j=1}^{n} \phi_j(-\gamma_{\mathrm{pr}} \bar{\beta}_j(s, T_1, T_2))\, ds\,,
\end{aligned}
$$

Risk-Neutral Probabilities

Suppose that we want to price a forward contract with delivery over the period $[T_1, T_2]$. The forward price is defined as

$$F^Q(t, T_1, T_2) = \mathbb{E}^Q\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(u)\,du \,\middle|\, \mathcal{F}_t\right],$$

where we use F^Q to indicate the dependency on the chosen risk-neutral probability Q.

Risk-Neutral Probabilities

We parameterize the market price of risk by introducing a probability measure $Q^{\theta} := Q_B \times Q_L$, where Q_B is a Girsanov transform of the Brownian motions $B_i(t)$, Q_L is an Esscher transform of the jump processes $L_j(t)$, and θ is an $\mathbb{R}^{n+m}$-valued function describing the market price of risk.

Risk-Neutral Probabilities -Brownian Motions

For $t \leq T$, with $T \geq T_2$ being a finite time horizon encapsulating all the delivery periods in the market, let the probability Q_B have the density process

$$Z_B(t) = \exp\left(- \int_0^t \sum_{i=1}^m \frac{\theta_{B,i}(t)}{\sigma_i(s)}\, dB_i(s) - \frac{1}{2} \int_0^t \sum_{i=1}^m \frac{\theta_{B,i}^2(s)}{\sigma_i^2(s)}\, ds \right),$$

where we have supposed that the functions $\theta_{B,i}/\sigma_i$, $i = 1, \ldots, m$, are square integrable over $[0, T]$.

Risk-Neutral Probabilities -Brownian Motions

This measure change in the Wiener coordinates is given by the Girsanov transform,

$$dW_i(t) = -\frac{\theta_{B,i}(t)}{\sigma_i(t)}\,dt + dB_i(t)\,,$$

where $W_i(t)$ become Brownian motions on $[0, T]$, $i = 1, \ldots, m$. The functions $\theta_{B,i}$ represent the compensation market players obtain for bearing the risk introduced by the non-extreme variations in the market, i.e. the diffusion component. We let it be time dependent to allow for variations across different seasons throughout the year.

Risk-Neutral Probabilities -Brownian Motions

This Girsanov change gives the dynamics (for $1 \leq i \leq m$)

$$dX_i(t) = (\theta_{B,i}(t) - \alpha_i X_i(t))\, dt + \sigma_i(t)\, dW_i(t)\,,$$

and thus we have added a time-dependent level of mean-reversion to the processes $X_i(t)$.

Risk-Neutral Probabilities

We let $\theta := (\theta_B, \theta_L)$, where $\theta_B := (\theta_{B,i})_{i=1}^m$ and $\theta_L := (\theta_{L,j})_{j=1}^n$.

The density process of the probability Q^θ becomes
$Z(t) := Z_B(t) Z_L(t)$.

We denote by $\mathbb{E}^{Q^\theta}$ the expectation with respect to the probability measure Q^θ.

Forward Price

The forward price $F^{\theta}(t, T_1, T_2)$ is given by

$$
\begin{aligned}
& F^{\theta}(t, T_1, T_2) \\
= \quad & \tfrac{1}{T_2 - T_1} \int_{T_1}^{T_2} \Lambda(u)\, du + \sum_{i=1}^{m} \tfrac{\bar{\alpha}_i(t, T_1, T_2)}{T_2 - T_1} X_i(t) + \sum_{j=1}^{n} \tfrac{\bar{\beta}_j(t, T_1, T_2)}{T_2 - T_1} Y_j(t) \\
& \quad + \int_t^{T_2} \sum_{i=1}^{m} \theta_{B,i}(s) \tfrac{\bar{\alpha}_i(s, T_1, T_2)}{T_2 - T_1}\, ds \\
& \quad + \int_t^{T_2} \sum_{j=1}^{n} \phi_j'(\theta_{L,j}(s)) \tfrac{\bar{\beta}_j(s, T_1, T_2)}{T_2 - T_1}\, ds \,.
\end{aligned}
$$

for $0 \leq t \leq T_1 < T_2$.

Risk Premium without Jump Risk

Suppose that the market price of jump risk is zero, i.e. $\theta_{L,j}(s) = 0$ for $j = 1, \dots, n$. Then

$$\begin{aligned} F^{\theta}(t, T_1, T_2) &= \mathbb{E}^P\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(u)\,du \,\middle|\, \mathcal{F}_t\right] \\ &+ \int_t^{T_2}\sum_{i=1}^{m}\theta_{B,i}(s)\frac{\bar{\alpha}_i(s, T_1, T_2)}{T_2 - T_1}\,ds\,. \end{aligned}$$

Risk Premium without Jump Risk

Thus, we see that when market players are not compensated for bearing jump risk, the market risk premium is positive as long as

$$\pi(t, T_1, T_2) = \int_t^{T_2} \sum_{i=1}^{m} \theta_{B,i}(s) \frac{\bar{\alpha}_i(s, T_1, T_2)}{T_2 - T_1}\, ds$$

is positive.

Risk Premium without Jump Risk

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- In general, one can obtain a change in the sign of the market risk premium over time t by appropriate specification of the functions $\theta_{B,i}(t)$.

Example: Model Specification

We consider a forward market consisting of 52 contracts with weekly delivery. The market power is supposed to be constant $p(t, T_1, T_2) = p \in [0, 1]$. Assume that the spot model has $m = 52$ diffusion components $X_i(t)$, and one $(n = 1)$ jump component $Y(t)$. Suppose that the seasonal function is

$$\Lambda(t) = 150 + 20\cos(2\pi t/365)\,,$$

and the mean-reversion parameters for the diffusion components are $\alpha_i = 0.067/i$, with volatility $\sigma_i = 0.3/\sqrt{i}$, for $i = 1, \ldots, 52$.

Model Specification

- We mimic here a sequence of mean-reverting processes with decreasing speeds of mean reversion and with decreasing volatility.

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- The speed of mean reversion equal to 0.067 means that a shock will be halved over 10 days.
- The jump process is driven by $L(t) = \eta N(t)$, where $N(t)$ is a Poisson process with intensity λ and the jump size is constant, equal to η.

Model Specification

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Model Specification

- The mean-reversion for the jump component is $\beta = 0.5$, meaning that a jump will, on average, revert back in two days.
- We have a combination of slow mean reverting normal variations and fast mean reverting spikes in the spot market.
- The frequency of spikes is set to $\lambda = 2/365$, i.e. two spikes, on average, per year.

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- The risk aversion coefficients of the producer and retailer are set equal to $\gamma_c = \gamma_{pr} = 0.5$.
- We derive forward curves for weakly settled forward contracts over a year.

Indifference price with forward curves for positive jumps

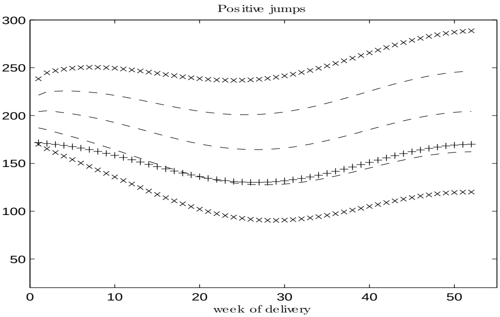

Figure 1: The indifference price curves together with the forward curves for market powers equal to $p = 0.25, 0.5$ and $p = 0.75$, in increasing order. The forecasted curve is depicted '+'. The jumps are positive of size 10.

Market risk premium – positive jumps

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- For a low market power of 0.25, we observe that the forecasted price curve is below the forward curve in the shorter end, while in the medium to long end we see the opposite.
- This corresponds to a positive market risk premium in the shorter end, whereas it becomes negative in the medium and longer end.

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- For a low market power of 0.25, we observe that the forecasted price curve is below the forward curve in the shorter end, while in the medium to long end we see the opposite.
- This corresponds to a positive market risk premium in the shorter end, whereas it becomes negative in the medium and longer end.
- The retailer wishes to avoid upward jumps in the price and is, even for a weak producer, willing to accept a positive market risk premium in the short end. In the long end, the effect of jumps vanish as a consequence of mean reversion, so the retailer will have more power.

Market risk premium – positive jumps

To illustrate this particular example we have plotted the difference of the forward curve with market power 0.25 and the forecasted curve. For the contracts with delivery up to approximately week 20, the market premium is positive. The premium decreases with time to delivery, and becomes negative in the medium and long end.

Market risk premium – positive jumps

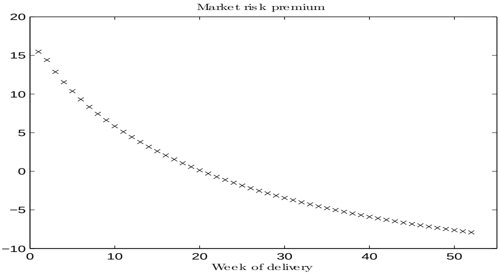

Figure 2: The market risk premium given by the difference of the forward curve with market power 0.25 and the forecasted curve.

Indifference price curves forward curves for negative jumps

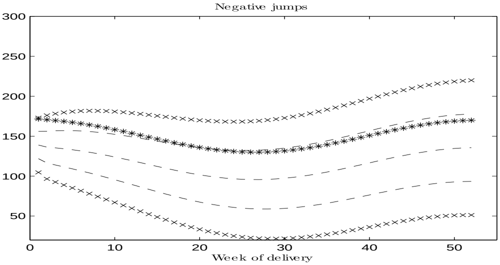

Figure 3: The indifference price curves together with the forward curves for market powers equal to $p = 0.25, 0.5$ and $p = 0.75$, in increasing order. The forecasted curve is depicted with '*'. The jumps are negative of size 10.

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- We observe that all curves are shifted downwards, indicating that the producer is willing to accept lower forward prices to hedge the possibility of sudden drops in prices.

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Market risk premium – negative jumps

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- In the short-term we observe, for all cases of market power, that the forecasted spot price is above forward prices, i.e. negative market risk premium.
- In the long-term, only when producer's market power is high, that is 0.75, we have the situation where the forecasted curve is below the forward curve signaling that the retailer bears a positive risk premium.

Market risk premium – negative jumps

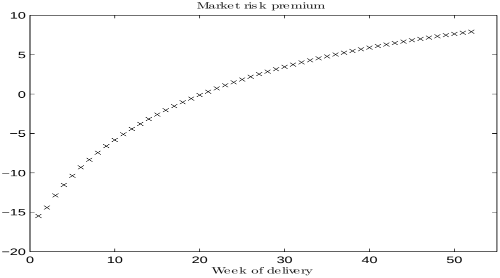


Figure 4: The market risk premium given by the difference of the forward curve with market power 0.75 and the forecasted curve.

Estimation problems

- We need to estimate the physical parameters of our two-factor model.
- From forward market data, denoted by $F(t, T_1, T_2)$, we estimate the risk-aversion coefficients for both producers and retailers and estimate the producer's market power.

Data used

- Spot prices: Phelix base load traded at the EEX.
- Forward contract prices with delivery periods: monthly, quarterly and yearly.
- Period covered: January 2 2002 to January 1 2006 with 1461 spot price observations.
- Forward data: 108 contracts with monthly delivery, 35 contracts with quarterly delivery and 12 contracts with yearly delivery.

Spot model specification

We apply the model to

$$S(t) = \Lambda(t) + X(t) + Y(t)$$

where, $\Lambda(t)$ is the seasonal component,

$$dX(t) = -\alpha X(t)dt + \sigma dB(t) \tag{19}$$

where $\alpha \geq 0$, $\sigma \geq 0$ and $B(t)$ is a standard Brownian motion,

Spot model specification

$$dY(t) = -\beta Y(t)dt + dL(t) \tag{20}$$

with $\beta \geq 0$ and

$$L(t) = \sum_{i}^{N(t)} J_i \tag{21}$$

is a compound Poisson process.
$N(t)$ is a homogeneous Poisson process with intensity λ and J_i's are i.i.d. with exponential density function

$$f(j) = p\lambda_1 e^{-\lambda_1 j}\mathbf{1}_{j>0} + (1-p)\lambda_2 e^{-\lambda_2 |j|}\mathbf{1}_{j<0},$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$ are responsible for the decay of the tails for the distribution.
We assume that $N(t)$, J and $B(t)$ are independent.

Spot model specification

For the seasonal component we assume

$$\begin{aligned}\Lambda(t) &= a_0 + a_1 \mathbf{1}_{\{t=Su\}} + a_2 \mathbf{1}_{\{t=Mo\}} + a_3 \mathbf{1}_{\{t=Tu,We,Th\}} + a_4 \mathbf{1}_{\{t=Sa\}} \\ &\quad + a_5 \cos\left[\frac{6\pi}{365}(t+a_6)\right] + a_7 t,\end{aligned}$$

where the indicator function is acting on the different days of the week.

Risk aversion coefficients

Recall that $F_c(t, T_1, T_2)$ (upper bound) and $F_{pr}(t, T_1, T_2)$ (lower bound) depend on the choice of γ_c and γ_{pr}, we estimate γ_{pr} and γ_c by minimizing the distance between $F_c(t, T_1, T_2)$, $F_{pr}(t, T_1, T_2)$ and the market prices of forwards $F(t, T_1, T_2)$, respectively, in the following way.

Risk aversion coefficients

- For all trading days $t \in [1, 1461]$, we determine all values of γ_{pr} and γ_c such that

$$F_{pr}(t, T_1, T_2) \leq F(t, T_1, T_2) \leq F_c(t, T_1, T_2). \tag{22}$$

- We define the intervals I_{pr}^t and I_c^t containing values for γ_{pr} and γ_c by guaranteeing that (22) holds.
- For the intersection of all these interval no forward prices $F(t, T_1, T_2)$ will lay outside the bounds $F_{pr}(t, T_1, T_2)$ and $F_c(t, T_1, T_2)$.
- We find that $\gamma_{pr} \in [0.421, \infty)$ and $\gamma_c \in [0.701, \infty)$.
- Thus we choose $\gamma_{pr} = 0.421$ and $\gamma_c = 0.701$.

Market power and market risk

Recall

$$p(t, T_1, T_2) = \frac{F(t, T_1, T_2) - F_{pr}(t, T_1, T_2)}{F_c(t, T_1, T_2) - F_{pr}(t, T_1, T_2)}$$

and

$$\pi(t, T_1, T_2) = F(t, T_1, T_2) - \mathbb{E}^P\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(u)du|\mathcal{F}_t\right].$$

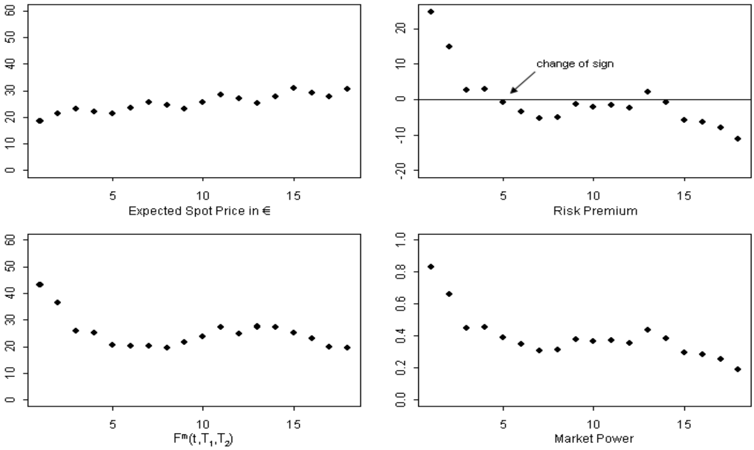
Market power and market risk

We consider three periods

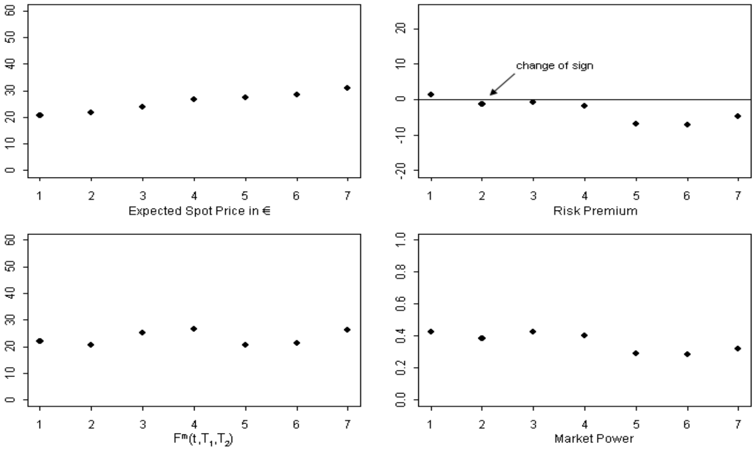
t	Type	# Contracts	Delivery Periods
01/Jan/2002	monthly	18	Jan 2002 - May 2003
01/Jan/2002	quarterly	7	2nd qtr 2002 - 4th qtr 2003
01/Jan/2002	yearly	3	2003 - 2005
03/Mar/2003	monthly	7	Feb 2003 - Aug 2003
03/Mar/2003	quarterly	7	2nd qtr 2003 - 4th qtr 2004
03/Mar/2003	yearly	3	2004 - 2006
04/Oct/2005	monthly	7	Oct 2005 - Apr 2006
04/Oct/2005	quarterly	7	1st qtr 2006 - 3rd qtr 2007
04/Oct/2005	yearly	6	2006 - 2011

Table: Forward contracts

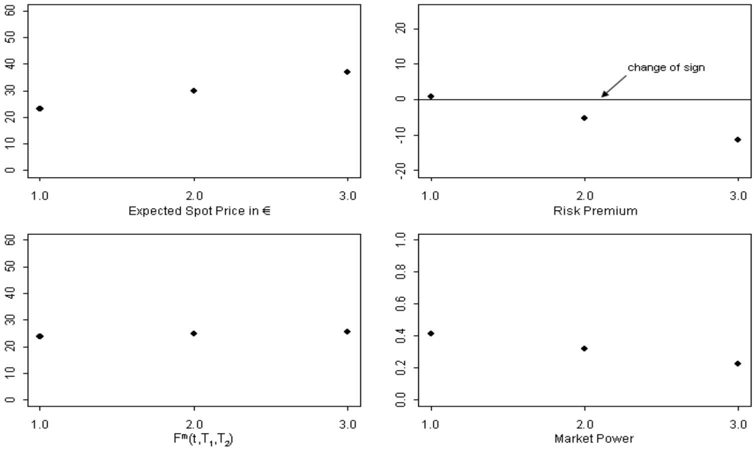
Producer's market power and market risk premium, 18 monthly contracts with $t =$ January 2 2002



Producer's market power and market risk premium, 7 quarterly contracts with t = second quarter 2002



Producer's market power and market risk premium, 3 yearly contracts with $t = 2002$



Agenda

Market Risk Premium – Information Approach

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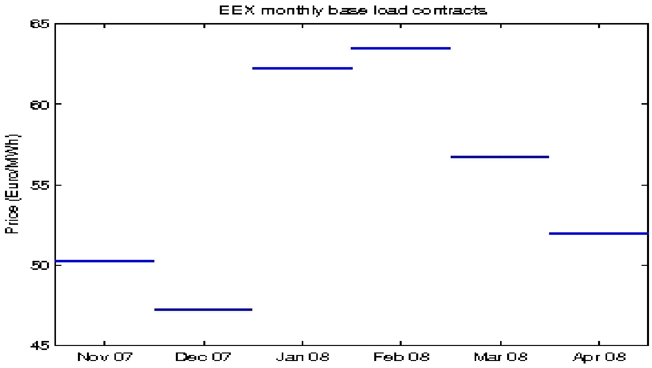
- Stylized example: planned outage of a power plant in one month
- Market example: in 2007 the market knew that in 2008 CO_2 emission costs will be introduced; this had a clearly observable effect on the forward prices!

Information Approach – Market Example



Information Approach – Definition

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