Risk and Statistics

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大学共同利用機関法人 情報・システム研究機構
統計数理研究所
The Institute of Statistical Mathematics

2nd ISM-UUlm Joint Workshop

October 8 – October 10, 2019

Villa Eberhardt
Heidenheimer Straße 80
89075 Ulm

Tuesday, October 8

8:00 - 8:45	Registration
8:45 - 9:00	Vicepresident Dieter Rautenbach – Welcome address
9:00 - 9:45	Satoshi Ito (ISM) – Computation of clinch and elimination numbers in league sports based on integer programming
9:45 – 10:30	An Chen (UUlm) – Innovative retirement plans
10:30 – 11:00	Coffee break
11:00 – 11:45	Shunichi Nomura (ISM) – Cluster-based discrimination of foreshocks for earthquake prediction
11:45 – 12:30	Mitja Stadje (UUlm) – On dynamic deviation measures and continuous-time portfolio optimization
12:30 – 14:30	Lunch
14:30 – 15:15	Toshikazu Kitano (Nagoya Institute of Technology) - Applications of bivariate generalized Pareto distribution and the threshold choice
15:15-16:00	Yuma Uehara (ISM) – Bootstrap method for misspecified stochastic differential equation models
16:00 – 16:30	Coffee break
16:30 – 17:15	Motonobu Kanagawa (Eurecom) – Convergence guarantees for adaptive Bayesian quadrature methods
17:15 – 18:00	Rene Schilling (TU Dresden) – On the Liouville property for Lévy generators
18:00	Get together (Rittersaal)

Wednesday, October 9

9:00 – 9:45	Denis Belomestny (University of Duisburg-Essen) – Density deconvolution under general assumptions on the distribution of measurement errors
9:45 – 10:30	Teppei Ogihara (University of Tokyo) – Local asymptotic mixed normality for multi-dimensional integrated diffusion processes
10:30 – 11:00	Coffee break
11:00 – 11:45.	Satoshi Kuriki (ISM) – Perturbation of the expected Minkowski functional and its applications
11:45 – 12:30	Robert Stelzer, Bennet Ströh (UUlm) – Weak dependence of mixed moving average processes and random fields with applications
12:30 – 14:30	Lunch
14:30 – 16:00	Poster session incl. coffee break
16:00 – 19:00	Excursion
19:00	Conference dinner

Thursday, October 10

9:00 – 9:45	Jeannette Wörner (TU Dortmund) – Inference for periodic Ornstein-Uhlenbeck processes driven by fractional Brownian motion
9:45 – 10:30	Volker Schmidt (UUlm) – Probabilistic prediction of solar power supply to distribution networks, using forecasts of global radiation

10:30 – 11:00	Coffee break
11:00 – 11:45	Yasumasa Matsuda (Tohuku University) – Bivariate CARMA models
11:45 – 12:30	Alexander Lindner, David Berger (UUlm) – CARMA SPDEs and quasi-infinitely divisible distributions
12:30 – 14:30	Lunch
14:30 – 15:15	Takaaki Shimura (ISM) – Subexponential densities of infinitely divisible distributions
15:15 – 16:00	Evgeny Spodarev (UUlm) – Long range dependence for heavy-tailed random functions
16:0 – 16:30	Coffee break
16:30 – 16:45	Vitalii Makogin (UUlm) – Change-point methods for anomaly detection in fibrous media
16:45 – 17:00	Stefan Schelling (UUlm) – Return smoothing and risk sharing elements in life insurance from a client perspective
17:00 – 17:15	Imma Curato (UUlm) – Preservation of strong mixing and weak dependence under renewal sampling
17:15 – 18:00	Mark Podolskij (University of Aarhus) – Optimal estimation of certain random quantities
18:00 – 18:30	Coffee break
18:30	Panel discussion

Talks

Tuesday

- **Prof. Satoshi Ito (ISM), Computation of clinch and elimination numbers in league sports based on integer programming:**
  At any stage of the season, every team has a minimal number of future wins that is sufficient to clinch a specified place and/or a minimal number of future losses that is sufficient to be eliminated from the specified place. These numbers are respectively called the clinch and elimination numbers. In this talk, we will formulate some mathematical optimization models for finding these numbers and show a generic computational framework based on integer programming for solving these optimization models.

- **Prof. An Chen (UUlm), Innovative retirement plans:**
  Tontines and pooled annuities, which are retirement products in which the longevity risk is shared in a pool of policyholders, have recently gained vast attention from researchers and practitioners.
  These products are cheaper than annuities, but on the other side do not provide stable payments to policyholders. This raises the question whether the advantages of annuities and tontines can be combined to form a retirement plan which is cheaper than an annuity and carries less risk than a tontine. In this talk, we analyze and compare three approaches of combining annuities and tontines in an expected utility framework: The tonuity introduced in Chen, Hieber and Klein (2019) (this product will be introduced in detail), a product very similar to the tonuity which we call "antine" and a portfolio consisting of conventional annuities and tontines. Our results show that the novel retirement products antine and tonuity are outperformed by the portfolio. The reason for this is that a policyholder holding a portfolio of annuities and tontines can replicate the payoff of any tonuity and antine.

- **Prof. Shunichi Nomura (ISM), Cluster-based discrimination of foreshocks for earthquake prediction:**
  We propose a short-term forecasting model for real-time earthquake forecasting. The model focuses on large mainshocks by discriminating the foreshocks. Using the single-link clustering method, the model updates the expanding seismic clusters and determines in real time the probabilities that larger subsequent events will occur. The foreshock clusters and other cluster types show different trends of certain feature statistics with respect to their magnitudes and spatio–temporal distances. Based on those features and the epicentral location, a non-linear logistic regression model is used to evaluate the probabilities that growing seismic clusters will be foreshocks triggering the mainshock within 30 days. The log of the odds ratio is estimated between the foreshock clusters and other clusters for respective feature values as nonlinear spline functions from a Japanese hypocenter catalogue of 1926 to 1999. It is determined that foreshock clusters tend to have smaller differences in their two largest magnitudes and shorter time durations; however, they have a weak tendency of being more distant from their epicenters. When a foreshock cluster is detected, its mainshock magnitude can be predicted by the Gutenberg–Richter law over the largest foreshock magnitude. We validate the predictive performance of our model using a Japanese hypocenter catalog of 2000 to date.

- **Prof. Mitja Stadje (UUlm), On dynamic deviation measures and continuous-time portfolio optimisation:**
  In this paper we propose the notion of dynamic deviation measure, as a dynamic time-consistent extension of the (static) notion of deviation measure. To achieve time-consistency we require that a dynamic deviation measures satisfies a generalised conditional variance formula. We show that, under a domination condition, dynamic deviation measures are characterised as the solutions to a certain class of backward SDEs. We establish for any dynamic deviation measure an integral representation, and derive a dual characterisation result in terms of additively m-stable dual sets. Using this notion of dynamic deviation measure we formulate a dynamic mean-deviation portfolio optimisation problem in a jump-diffusion setting and identify a subgame-perfect Nash equilibrium strategy that is linear as function of wealth by deriving and solving an associated extended HJB equation.

- **Prof. Toshikazu Kitano (Nagoya Institute of Technology), Applications of bivariate generalized Pareto distribution and the threshold choice:**
  Climate change will impact on the occurrence of heavy rain and it concerns us for the near future of our activities. Large size datasets, like as d4PDF, "database for Policy Decision-Making for Future Climate Change", will want to be checked in the statistical point of view for practical applications. Then the establishments and the diagnostics of the theoretical consistency should be suitable rather than the estimation of parameters of the distribution to be fit. We utilize the unique feature of d4PDF that the numerous ensemble members are available, to examine the properties of bivariate extremes exceeding over the thresholds, display the mathematical meanings as the graphical images, and demonstrate procedures for the dependency.

- **Dr. Yuma Uehara (ISM), Bootstrap method for misspecified stochastic differential equation models:**
  In this talk, we focus on the statistical theory of misspecified stochastic differential equation models driven by Lévy processes.
  Due to the model misspecification, it is known that the asymptotic behavior of the tractable Gaussian quasi-likelihood estimator of drift and scale parameters is different from the correctly specified case.
  More specifically, its asymptotic variance contains the solution of the extended Poisson equation which makes it difficult to construct a consistent estimator of the asymptotic variance, and thus hypothesis testing and confidence interval.
  To solve this problem, we propose a block bootstrap type estimator, and theoretically show that the approximation based on the (normalized) estimator works well.

- **Prof. Motonobu Kanagawa (Eurecom), Convergence guarantees for adaptive Bayesian quadrature methods:**
  Adaptive Bayesian quadrature (ABQ) is a powerful approach to numerical integration that empirically compares favorably with Monte Carlo integration on problems of medium dimensionality (where non-adaptive quadrature is not competitive). Its key ingredient is an acquisition function that changes as a function of previously collected values of the integrand. While this adaptivity appears to be empirically powerful, it complicates analysis. Consequently, there are no theoretical guarantees so far for this class of methods. In this work, for a broad class of adaptive Bayesian quadrature methods, we prove consistency, deriving non-tight but informative convergence rates. To do so we introduce a new concept we call weak adaptivity. In guaranteeing consistency of ABQ, weak adaptivity is notionally similar to the ideas of detailed balance and ergodicity in Markov Chain Monte Carlo methods, which allow sufficient conditions for consistency of MCMC.

Likewise, our results identify a large and flexible class of adaptive Bayesian quadrature rules as consistent, within which practitioners can develop empirically efficient methods.

- **Prof. Rene Schilling (TU Dresden), On the Liouville property for Lévy generators:**
  The classical Liouville property says that a bounded harmonic function is constant. We formulate the analogue for harmonic functions for a Lévy process and we give a necessary and sufficient condition for the Liouville property to hold.

Wednesday

- **Prof. Denis Belomestny (University of Duisburg-Essen), Density deconvolution under general assumptions on the distribution of measurement errors:**
  In this talk we study the problem of density deconvolution under general assumptions on the measurement error distribution. Typically deconvolution estimators are constructed using Fourier transform techniques where it is assumed that the characteristic function of the measurement errors does not have zeros on the real line. This assumption is rather strong and is not fulfilled in many cases of interest. In this paper we develop a methodology for constructing optimal density deconvolution estimators in the general setting that covers vanishing and non-vanishing characteristic functions of the measurement errors. We derive upper bounds on the risk of the proposed estimators and provide sufficient conditions under which zeros of the corresponding characteristic function have no effect on estimation accuracy.

- **Prof. Teppei Ogihara (University of Tokyo), Local asymptotic mixed normality for multi-dimensional integrated diffusion processes:**
  We study statistical inference for integrated diffusion processes and consider asymptotic properties of this model in a high-frequency limit. This model arises when we observe a process after passage through an electric filter, and is also related to modeling of the stochastic volatility in finance. Gloter and Gobet (2008) studied this model and showed the local asymptotic mixed normality (LAMN) when the latent diffusion process is one-dimensional. The LAMN property is important in asymptotic statistical theory and enables us to discuss the asymptotic efficiency of estimators. We extend their results of the LAMN property to multi-dimensional diffusion processes which may have feedback from the integral process. Then we can apply these results to the Langevin equation which is a model for molecular motion. We also consider the construction of an efficient estimator. This talk is based on joint work with Masaaki Fukasawa in Osaka University.

- **Prof. Satoshi Kuriki (ISM), Perturbation of the expected Minkowski functional and its applications:**
  The Minkowski functional is a series of geometric quantities including the volume, the surface area, and the Euler characteristic. In this talk, we consider the Minkowski functional of the excursion set (sup-level set) of an isotropic smooth random field on arbitrary dimensional Euclidean space. Under the setting that the random field has weak non-Gaussianity, we provide the perturbation formula of the expected Minkowski functional. This result is a generalization of Matsubara (2003) who treated the 2- and 3-dimensional cases. The Minkowski functional is used in astronomy and cosmology as a test statistic for testing Gaussianity of the cosmic microwave background (CMB), and to characterize the large-scale structures

of the universe. Besides, the expected Minkowski functional of the highest degree is the expected Euler-characteristic of the excursion set, which approximates the upper tail probability of the maximum of the random field. This methodology is referred to as the Euler-characteristic method (the expected Euler-characteristic heuristic), and is used in multiple testing problems. We explain some applications of the perturbation formulas in these contexts. This talk is based on joint work with Takahiko Matsubara.

- **Prof. Robert Stelzer, Bennet Ströh (UUlm), Weak dependence of mixed moving average processes and random fields with applications:**
  The first part presented by Robert Stelzer considers a mixed moving average (MMA) process X driven by a Lévy basis and proves that it is weakly dependent with rates computable in terms of the moving average kernel and the characteristic quadruple of the Lévy basis. Using this property, we show conditions ensuring that sample mean and autocovariances of X have a limiting normal distribution. We extend these results to stochastic volatility models and then investigate a Generalized Method of Moments estimator for the supOU process and the supOU stochastic volatility model proving conditions for its asymptotic normality.
  The second part of the talk by Bennet Ströh extends the theory to random fields. A particular focus is on how to generalize the concept of "causality" from the realm of stochastic processes so that one covers many cases of interest and obtains central limit theorems under mild moment assumptions. To this end a new notion of weak dependence for random fields will be introduced and it will be shown that this leads to a suitable central limit theorem. As examples, mixed moving average random fields and ambit processes will be briefly discussed.

Thursday

- **Prof. Jeannette Wörner (TU Dortmund), Inference for periodic Ornstein-Uhlenbeck processes driven by fractional Brownian motion:**
  Fractional Ornstein Uhlenbeck processes are popular models for applications in science and finance, combining the mean reverting structure of the ergodic version or the explosive structure of a non-ergodic version with long range dependence. We now consider an extension of these processes incorporating a periodic mean function as it is for example desirable for modelling electricity prices or other commodities. For these models we construct a least squares estimator for the drift parameters and prove consistency and an asymptotic theory, both in the ergodic and non-ergodic setting. The technique of the proofs uses methods of stochastic analysis.

- **Prof. Volker Schmidt (UUlm), Probabilistic prediction of solar power supply to distribution networks, using forecasts of global radiation:**
  Renewable energy sources are continuously gaining in importance as reserves of fossil energy decline and concerns about global warming increase. Consequently, the number of installed solar plants is steadily raising. The resulting high reverse power flow in distribution networks leads to challenges for network operators, since overloading problems and voltage violations can occur causing great economic damages and endangering secure network operation. In response to these problems new computer-based tools are developed, which aim to analyze the dependency between solar power supply and related weather phenomena, predict overloading problems and generate automatic warnings. This talk presents a

mathematical model for the prediction of the probabilities of reverse power flow exceeding predefined critical thresholds at feed-in points of a distribution network. The parametric prediction model is based on hourly forecasts of global radiation and uses copulas, a probabilistic tool for modeling the joint probability distribution of two or more strongly correlated random variables with non-Gaussian (marginal) distributions. The model is used for determining the joint distribution of forecasts of global radiation and measured solar power supply at given feed-in points, where respective sample datasets were provided by Deutscher Wetterdienst and the Main-Donau Netzgesellschaft. It is shown that the fitted model replicates important characteristics of the data such as the corresponding marginal densities. Moreover, the validation results highlight strong performances of the model, i.e., very low bias and high Brier skill scores for most of the considered feed-in points are obtained. The proposed copula-based model enables us to predict the solar power supply conditioned on the forecasts of global radiation, thus anticipating great fluctuations in the distribution network.
This talk is based on joint research with F. von Loeper, P. Schaumann, M. de Langlard, R. Hess, and R. Bäsmann.

- **Prof. Yasumasa Matsuda (Tohoku University), Bivariate CARMA models:**
  Brockwell and Matsuda (2017) extended CARMA models for time series to those for random fields, which we call as "CARMA random fields". In this talk, we consider a bivariate extension of CARMA random fields to analyze spatially scattered bivariate observations. After defining bivariate CARMA random fields, we introduce Whittle likelihoods to estimate the parameters with applications to imputation for missing components of bivariate observations. There is a gap between discrete observations and continuous models that needs to be accounted to conduct the imputation. We employ Bayesian ways to fill the gap. We demonstrate them by applying bivariate CARMA random fields to precipitation and temperature data observed at around 7000 irregularly scattered points in US continent.

- **Prof. Alexander Lindner, David Berger (UUlm), CARMA SPDEs and quasi-infinitely divisible distributions:**
  We give a new definition of a Lévy driven CARMA random field, defining it as a generalized solution of a stochastic partial differential equation (SPDE). Furthermore, we give sufficient conditions for the existence of a mild solution of our SPDE. Our model finds a connection between all known definitions of CARMA random fields, and especially for dimension 1 we obtain the classical CARMA process. We also consider quasi-infinitely divisible distributions.
  A quasi-infinitely divisible distribution on the line is a probability distribution whose characteristic function allows a Lévy-Khintchine type representation with a "signed Lévy measure", rather than a Lévy measure.
  Quasi-infinitely divisible distributions appear naturally in the factorization of infinitely divisible distributions. Namely, a distribution of a random variable X is quasi-infinitely divisible if and only if there are two independent random vectors Y and Z with infinitely divisible distributions such that X=Y+Z.
  In this talk, we study certain properties of quasi-infinitely divisible distributions and give some examples.
  In particular, it is shown that the set of quasi-infinitely divisible distributions is dense in the set of all probability distributions with respect to weak convergence. Further, it is proved that a distribution concentrated on the integers is quasi-infinitely divisible if and only if its characteristic function does not have zeroes, with the use of the Wiener-Lévy theorem on absolutely convergent Fourier series. This talk is based on joint work with Ken-iti Sato and Lei Pan.

- **Prof. Takaaki Shimura (ISM), Subexponential densities of infinitely divisible distributions:**
  Embrechts et al. (1979) showed that the subexponentiality of an infinitely divisible distribution on $[0, \infty)$

and that of its normalized Lévy measure are equivalent.
The relationship between the behavior of an infinitely divisible distribution and one of its Lévy measure has been investigated for various properties related to subexponentiality. In this talk, we consider this relationship for subexponential densities. This is a joint work with Toshiro Watanabe (The University of Aizu).

- **Prof. Evgeny Spodarev (UUlm), Long range dependence for heavy-tailed random functions:**
Long range dependence is usually related to the asymptotic behavior of (co)variance, spectral density, etc. of a stationary square integrable random function in infinity or at zero. For that, the second moment of the random function should be finite. We give a new definition of long range dependence of stationary random functions with an infinite second moment which is based on the asymptotic variance of the volume of their level sets. It follows that all mixing random functions are short range dependent. We also show how this definition can be applied to different heavy-tailed processes and fields such as subordinated Gaussian, random volatility, α- and max-stable. A connection to limit theorems for random volatility fields on $\mathbb{Z}^d$ is proven. This talk is based on joint work with R. Kulik, V. Makogin, M. Oesting, A. Rapp.

- **Dr. Vitalii Makogin (UUlm), Change-point methods for anomaly detection in fibrous media:**
We consider the problem of detecting anomalies in the directional distribution of fibre materials observed in 3D images. We divide the image into a set of scanning windows and classify them into two clusters: homogeneous material and anomaly. Based on a sample of estimated local fibre directions, for each scanning window we compute several classification attributes, namely the coordinate-wise means of local fibre directions, the entropy of the directional distribution, and a combination of them. We consider testing the significance of anomalies. To this end, we apply a change point technique for random fields and derive the exact inequalities for tail probabilities of a test statistics. The proposed methodology is first validated on simulated images. Finally, it is applied to a 3D image of a fibre reinforced polymer.

- **Dr. Stefan Schelling (UUlm), Return smoothing and risk sharing elements in life insurance from a client perspective:**
In many countries, traditional participating life insurance (TPLI) contracts are typically equipped with a cliquet-style (year-to-year) guarantee. Life insurers pool the assets and liabilities of a heterogeneous portfolio of TPLI contracts. This allows for intergenerational risk sharing. Together with certain smoothing elements in the collective investment, it also results in rather stable returns for the policyholders. Despite the current low interest rate environment, TPLI contracts are still popular in the segment of retirement savings. Standard approaches which focus solely on the cash-flow at maturity cannot explain their popularity. In a recent paper, Ruß and Schelling (2018) have introduced a descriptive model of decision making which takes into account that potential future changes in the account value impact the decision of long-term investors at outset. Based on this, we illustrate how smoothing and risk sharing elements provided by a life insurer can significantly increase the subjective utility for such investors. Furthermore, we show that for these investors TPLI contracts are more attractive than common unit-linked (guaranteed) products. Hence, our findings explain the popularity of TPLI contracts and provide helpful insights into decision making in the context of retirement savings.

- **Dr. Imma Curato (UUlm), Preservation of strong mixing and weak dependence under renewal sampling:**
For a stationary process X being strongly mixing or weakly dependent, we investigate how its asymptotic dependence structure behaves under renewal sampling. We explicitly compute the strong mixing or weak dependence coefficients of

the renewal sampled process and show that the sampling preserves (asymptotically) the exponential or power decay of the coefficients of X.

- **Prof. Marc Podolskij (Aarhus University), Optimal estimation of certain random quantities:**
  In this talk we present some optimal estimation methods for random quantities, such as supremum/infimum, local time or occupation time measure of stochastic processes, in the setting of high frequency data. The theory is mostly presented in the case of Brownian motion, but we highlight how certain theoretical results can be extended to Levy processes and/or continuous diffusion models. As optimality criteria we focus on L^2 and L^1 optimality, and compare our theory to existing methods. This is a joint work with J. Ivanovs.

Posters

- **Valerii Dashuk (LMU Munich), Joint confidence regions for bivariate normal distributions. Theory and applications:**
  The usage of the random variables in economics and econometrics is widespread. Inflation, GDP, individual preferences may be treated as random variables. In financial econometrics stock returns is the classic example of a random variable. To be able to investigate stochasticity of the process more thoroughly, joint tests of both mean and variance are applied.
  A new approach for testing such hypotheses and constructing confidence sets is introduced in my work. This test allows setting an assessment of whether the data, from which mean and variance were derived, can be taken as normally distributed with the given parameters. After the test joint confidence sets are provided.
  This method, called CCR (Cumulative distribution function confidence region), is based on the difference of probability density functions. The starting point is two sets of Normal distribution parameters that should be compared (whether they may be considered as identical with given significance level). Then the absolute difference in probabilities at each "point" of the domain of these distributions is calculated. This measure is transformed to a function of cumulative distribution functions and compared to the critical values. Critical values table was designed from the simulations. Confidence regions appeared to be pseudo-ellipses in mean-variance space (2-dimensional figure for univariate distribution and 5-dimensional figure for bivariate case of the Normal distribution).
  Then CCR technique was also investigated for the effectiveness, robustness and area of the confidence region in comparison with the other techniques: an exact method, derived by Mood (1950), based on the standard pivotal quantities of the parameters of Normal distribution; Maximum-likelihood based methods, presented by Barry C. Arnold & Robert M. Shavelle (1998); method derived from Rao-distances by Jensen (1995), etc. It differs qualitatively and quantitatively in easiness of implementation, computation speed, accuracy of the critical region (theoretical vs. real significance level) from the other approaches. Stable results when working with outliers and non-Normal distributions as well as scaling possibilities are also strong sides of CCR method.
  The derived CCR method is equivalent to classic tests in standard situations, but has a bunch of advantages: CCR test is equivalent to classic tests in standard situations (i.e. in univariate case), this approach gives unique alternatives in more complex situations (increase in dimensions), additionally this test is efficient on big amounts of data and it can be expanded to more general distributions without loss of quality.

- **Viet Hoang (UUlm), Spectral density estimation of stationary real harmonizable symmetric alpha-stable processes:**
  We consider stationary real harmonizable symmetric α-stable random processes which are uniquely characterized by their control measure. Assuming this measure possesses a density, we aim to retrieve this density from the codifference function of the random process which involves the inversion of a sine integral transform. To ensure injectivity of the inverse operator we restrict ourselves to symmetric density functions. We show that the aforementioned sine integral transform belongs to a broader class of integral operators whose inversion involves the Fourier transform on the multiplicative group on the positive reals. To illustrate our results, MATLAB simulation examples are presented.

- **Albert Rapp (UUlm), Long range dependence for stable random processes:**
  We investigate long and short memory in α -stable moving averages and max-stable processes with α -Fréchet marginal distributions.

As these processes are heavy-tailed, we rely on the notion of long range dependence suggested by Kulik and Spodarev (2019) based on the covariance of excursions. Sufficient conditions for the long and short range dependence of α-stable moving averages are proven in terms of integrability of the corresponding kernel functions. For max-stable processes, the extremal coefficient function is used to state a necessary and sufficient condition for long range dependence.
This poster is based on joint work with Vitalii Makogin, Marco Oesting and Evgeny Spodarev.

- **Jana Reker (UUlm), A functional equation for the law of the killed exponential functional:**
  We consider distributional properties of exponential functionals of the form $V_{q,\xi,\eta} = \int_0^\tau e^{-\xi_{s-}} d\eta_s$, where ξ and η are two independent Lévy processes and τ denotes an exponentially distributed random variable that is independent of (ξ, η). Observing that the law of $V_{q,\xi,\eta}$ arises as the invariant probability measure of certain Markov processes, we calculate the infinitesimal generator of the process and apply Schwartz theory of distributions to derive a functional equation for the law of the killed exponential functional. We then use this tool to study absolute continuity and investigate properties of the density. This poster is based on joint work with Anita Behme, Alexander Lindner and Victor Rivero.

- **Xing Yuan (Tohoku University), Whittle estimation for bivariate CARMA random fields:**
  We consider the missing value problems using a model-based technique on a bivariate CARMA random field. We are dealing with point observations in a random field which is two-dimensional rather than discrete observations. The problem occurs when the amount of observations is too large and there are a large number of missing values which is impossible to be removed. So, we create the bivariate continuous ARMA model, which is an analogue of ARMA model in continuous context, to give an estimation of the missing data. The bivariate CARMA model defines the CARMA random field via an integral of a kernel function g(x) over Levy noises, which is a hierarchical Bayes model. We apply empirical Bayes method for parameter estimation. Especially for setting hyperparameters, we choose the whittle likelihood where the model is in Fourier domain and the formulation is based on the discrete Fourier transform and its power spectral density.

- **Fangyuan Zhang (UUlm), On the equivalence between Value-at-Risk and expected shortfall:**
  This paper studies the optimal asset allocation problem of a non-concave utility- maximizing investor who has to simultaneously satisfy a given risk control con- straint, Value-at-Risk (VaR) or Expected Shortfall (ES). The non-concavity of the problem stems from the non-linear payoff structure to some investor in reality, e.g., the equity holder. We obtain the closed-form optimal wealth under VaR constraint as well as ES constraint respectively and explicitly calculate the optimal trading strategy considering a CRRA utility function. We find that the distribution of the optimal wealth under VaR constraint is similar to the one under ES constraint in non-concave optimization problem, which differs from the conclusion in concave optimization case studied by Basak and Shapiro [2001]. Further we show that it is always possible to transfer the ES constraint to an equivalent VaR constraint in the non-concave optimization problem.