Preservation of strong mixing and weak dependence under renewal sampling

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Sampling Schemes

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- Equidistant sampling $\rightarrow$ General asymptotic theory for sample moment statistics, i.e. when X is strong mixing (Bradley, 2007).
- Random Sampling $\rightarrow$?

Definition

Let $\tau = (\tau_i)_{i\in\mathbb{Z}\setminus\{0\}}$ be a non-negative sequence of i.i.d. random variable with distribution function μ such that $\mu(\{0\}) < 1$. For $i \in \mathbb{Z}$, we define $(T_i)_{i\in\mathbb{Z}}$ as

$$T_0 := 0 \quad \text{and} \quad T_i := \begin{cases} \sum_{j=1}^{i} \tau_j \,, & i \in \mathbb{N}^*, \\ -\sum_{j=i}^{-1} \tau_j \,, & -i \in \mathbb{N}^*. \end{cases} \qquad (1)$$

The sequence $(T_i)_{i\in\mathbb{Z}}$ is called a renewal sampling sequence.

Let $X = (X_t)_{t\in\mathbb{R}}$ a stationary process with values in $\mathbb{R}^d$-valued and let $(T_i)_{i\in\mathbb{Z}}$ be a sequence of random times as defined in (1) and independent of X , we define the sequence $Y = (Y_i)_{i\in\mathbb{Z}}$ as a stochastic process with values in $\mathbb{R}^{d+1}$ given by

$$Y_i = \begin{pmatrix} X_{T_i} \\ \tau_i \end{pmatrix}.$$

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- X is strictly-stationary and satisfies a weak dependent property
- X admits exponential or power decaying weak dependent coefficients

Then, we can apply to Y the existing asymptotic theory for equidistant sampling.

Definition

Let T a non empty index set equipped with a distance d and $X = (X_t)_{t\in T}$ a process with values in $\mathbb{R}^d$. The process is called a Ψ-weak dependent process if there exists a function Ψ and a sequence of coefficients $\iota = (\iota(r))_{r\in\mathbb{R}^+}$ converging to zero satisfying

$$|Cov(F(X_{i_1},\ldots,X_{i_u}),G(X_{j_1},\ldots,X_{j_v}))| \leq c\,\Psi(F,G,u,v)\,\iota(r) \qquad (2)$$

for all

$$\begin{cases} (u,v)\in\mathbb{N}^*\times\mathbb{N}^*; \\ r\in\mathbb{R}^+; \\ (i_1,\ldots,i_u)\in T^u \text{ and } (j_1,\ldots,j_v)\in T^v, \\ \text{such that } r=\min\{d(i_l,j_m): 1\leq l\leq u, 1\leq m\leq v\} \\ \text{for functions } F\colon (\mathbb{R}^d)^u\to\mathbb{R} \text{ and } G\colon (\mathbb{R}^d)^v\to\mathbb{R} \end{cases}$$

and where c is a constant independent of r. ι is called the sequence of the weak dependent coefficients.

η-weak dependence

Let $\mathcal{F}_u = \mathcal{G}_u$ be classes of bounded and Lipschitz functions with

$$\Psi(F, G, u, v) = uLip(F)\|G\|_\infty + vLip(G)\|F\|_\infty,$$

then ι corresponds to the η**-coefficients** defined in Doukhan and Louhichi, (1999).

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Also λ**-weak dependence** and κ**-weak dependence**, as defined in Doukhan and Wintenberger (2007), are encompassed by (2).

BL-dependence

If, instead,

$$\Psi(F, G, u, v) = min(u, v)Lip(F)Lip(G),$$

then ι corresponds to the **BL-weak dependent coefficients** defined in Bulinski and Sashkin (2005).

θ-weak dependence

Let $\mathcal{F}_u$ be the class of bounded functions and $\mathcal{G}_v$ the class of bounded and Lipschitz functions with

$$\Psi(F, G, u, v) = v\|F\|_\infty Lip(G),$$

then ι corresponds to the θ**-coefficients** defined in Dedecker and Doukhan, (2003).

Proposition (Brandes, C., Stelzer)

Let $X = (X_t)_{t \in T}$ be a process with values in $\mathbb{R}^d$ and $\mathcal{F}_u = \mathcal{G}_u$ are classes of bounded functions. X is α**-mixing** (Rosenblatt, 1956) if and only if there exists a sequence $(\iota(r))_{r \in \mathbb{R}^+}$ converging to zero such that (2) is satisfied for

$$\Psi(F, G, u, v) = \|F\|_\infty \|G\|_\infty$$

.

Theorem (Brandes, C., Stelzer)

Let $Y = (Y_i)_{i\in\mathbb{Z}}$ be a $\mathbb{R}^{d+1}$-valued process with $X = (X_t)_{t\in\mathbb{R}}$ being strictly-stationary and Ψ-weak dependent with coefficients $\iota = (\iota(r))_{r\in\mathbb{R}^+}$. Then, it exists a sequence $(I(n))_{n\in\mathbb{N}^*}$ satisfying

$$|Cov(F(Y_{i_1},\dots,Y_{i_u}),G(Y_{j_1},\dots,Y_{j_v}))| \leq C\Psi(F,G,u,v)\,I(n)$$

where C is a constant independent of n and Ψ satisfies the same weak dependence conditions of the data generating process X. Moreover,

$$I(n) = \int_{\mathbb{R}^+} \iota(r)\,\mu^{*n}(dr),$$

with μ^{*n} the n-fold convolution of μ.

Exponential decay

If X is a Ψ-weak dependent process with coefficients $\iota(r) = c\mathrm{e}^{-\gamma r}$ with $\gamma > 0$ and μ a distribution function in $\mathbb{R}^+$, then Y is Ψ-weak dependent with coefficients

$$I(n) = C\Big(\Big(\frac{1}{\mathcal{L}_\mu(\gamma)}\Big)^{-n}\Big),$$

where $\mathcal{L}_\mu(\gamma) = \int_{\mathbb{R}^+} \mathrm{e}^{-\gamma r}\mu(dr)$ is the **Laplace transform** of the distribution function μ.

Power decay

If X is a Ψ-weak dependent process with power decaying coefficients such that $\iota(r) = cr^{-\gamma}$ for $\gamma > 0$. Then, the process Y is Ψ-weak dependent with coefficients $I(n) \leq Cn^{-\gamma}$ for large n.

Thank you

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