Computation of Clinch and Elimination Numbers in League Sports Based on Integer Programming

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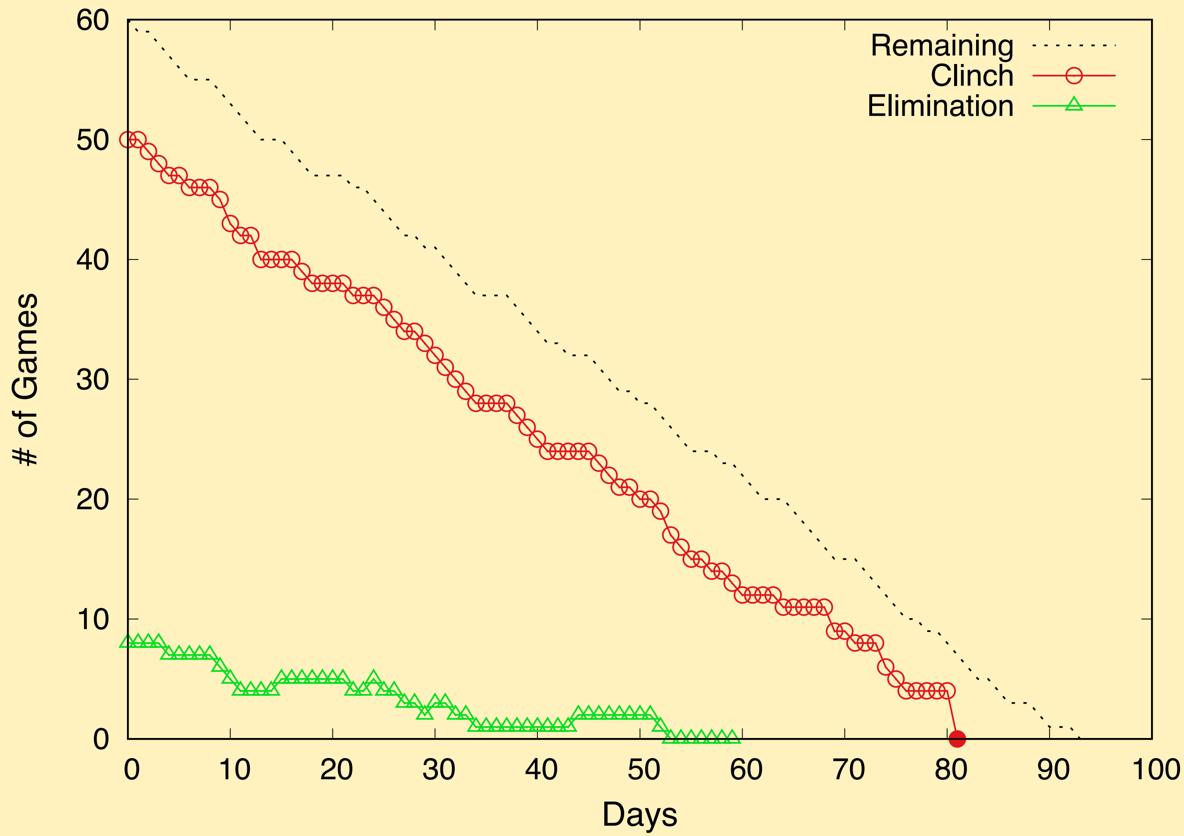
Introduction: clinch and elimination

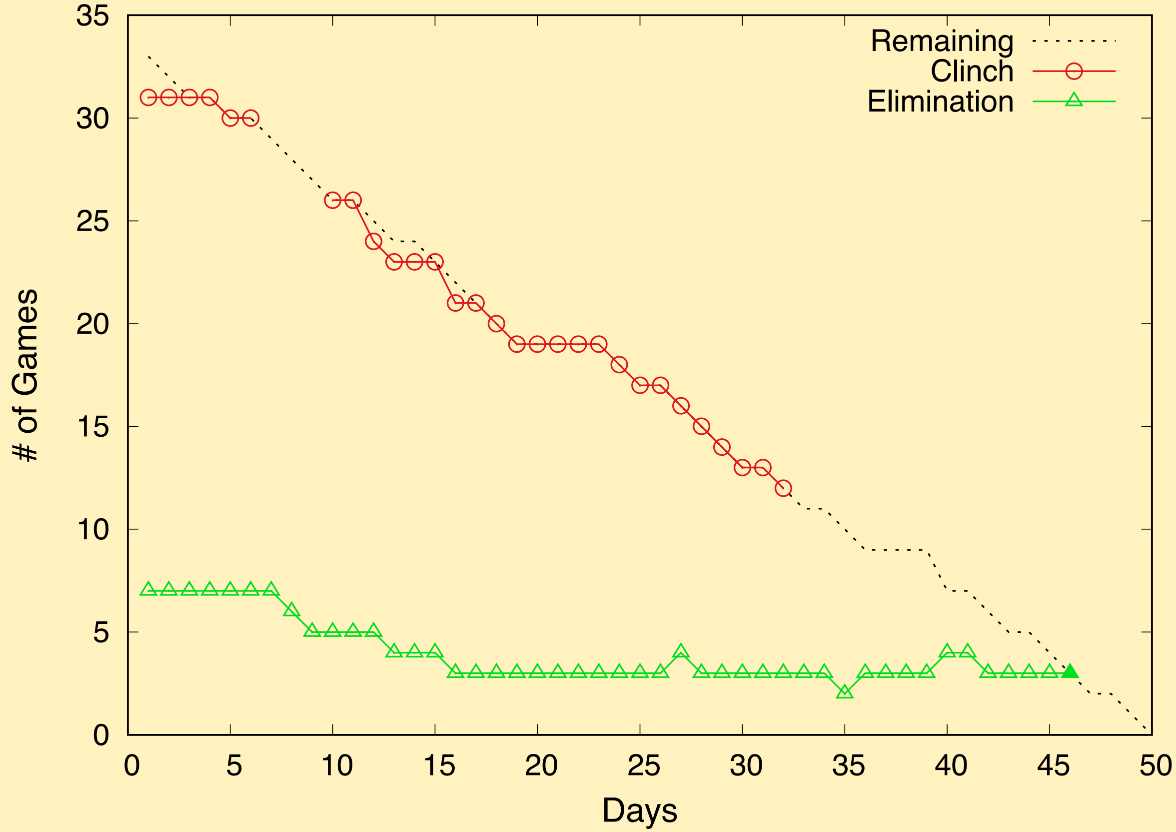


Uni-President Lions (統一獅)'s playoff C/E# during the 2nd half of the 2017 CPBL season.

Note: E# is expressed here as a maximal # of future wins, which is defined as

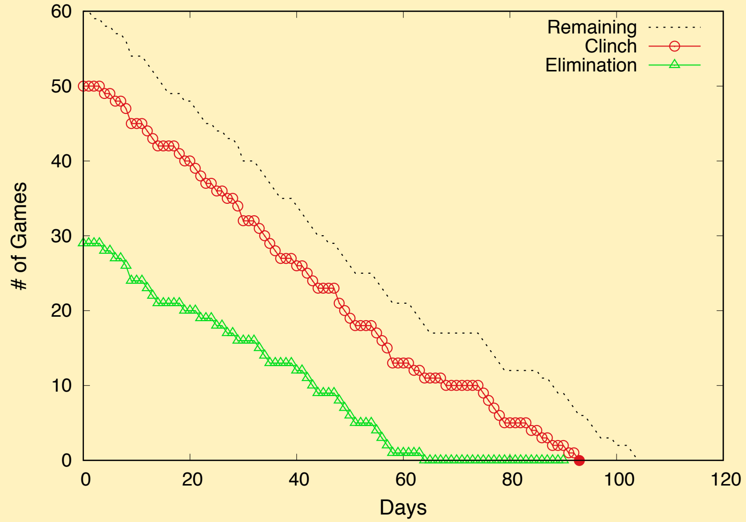
(the number of remaining games) − E#

in order to compare with C# within the same figure.

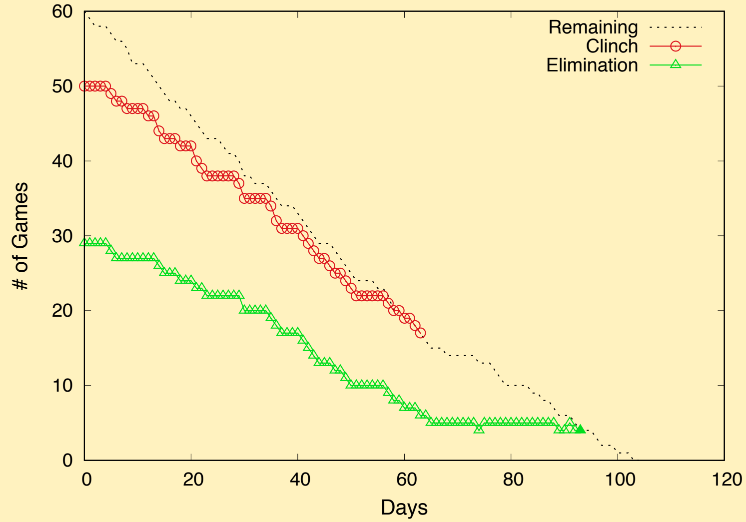


Kyoto Hannarys' C/E# for championship tournament qualification during the last 50 game days of the 2016–2017 B.LEAGUE season.

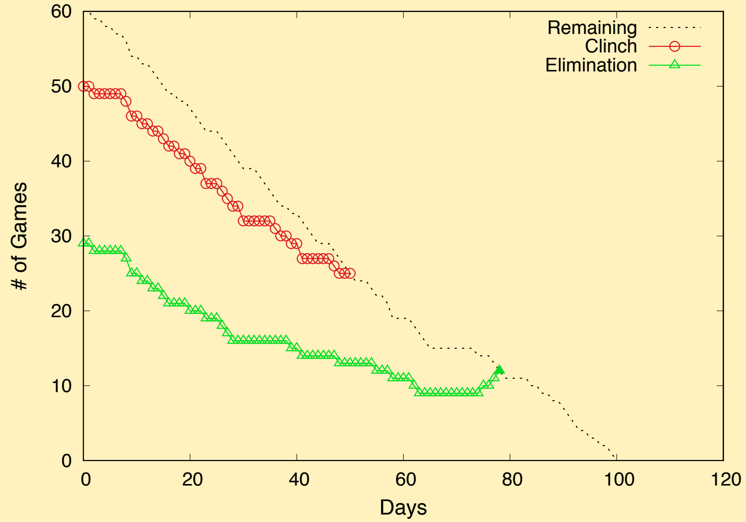
1st place C/E# during the 1st half



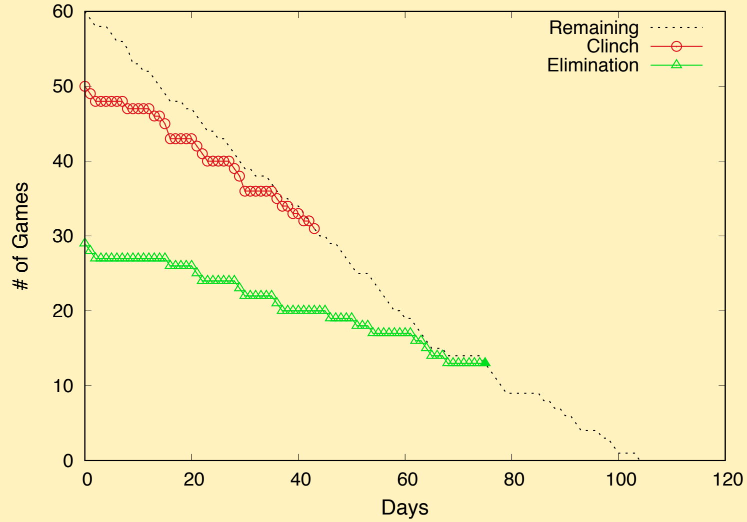
(a) Lamigo Monkeys



(b) Chinatrust Brothers



(c) Uni-President Lions



(d) Fubon Guardians

Clinch and elimination

At an early stage of the season, any team could be

- the *top* of the league if it *wins* all remaining games,
- the *bottom* of the league if it *loses* all remaining games.

At any stage of the season, every team has

- **a minimal number of future wins** sufficient to clinch a specified place (unless the team has a chance to be eliminated from the situation even when the team wins all remaining games),
- **a minimal number of future losses** sufficient to be eliminated from the specified place (unless the team has a chance to achieve the situation even when the team loses all remaining games).

These numbers are called the **clinch** and **elimination** numbers, respectively.

In this talk,

we will formulate several mathematical optimization models for finding these numbers and show a generic computational framework based on integer programming for solving these optimization models.

Computing cost

- Varies from league to league.
- Structural factors that significantly affects the cost:
  1. Treatment of ties (draws) for each game
     - Ties are not allowed (or as many overtimes as necessary).
     - A tie is converted to a fixed score (e.g., a loss, or a pair of a 1/2 win and a 1/2 loss).
     - Winning point system (e.g., 3/1/0 points for a win/tie/loss)
     - Winning persentage (WP) system (as in CPBL; a tie has a value of WP wins and $(1 - \mathrm{WP})$ losses)

$$\boxed{\mathrm{WP} = \frac{(\#\text{wins})}{(\#\text{wins}) + (\#\text{losses})}}$$

  2. Tiebreaking for season standings
     - Some leagues permit joint championships.
     - Extra games are played among the tied teams (as in CPBL).
     - Tiebreaking criteria are provided.
       (e.g., in B.LEAGUE) ① #wins → ② #wins among tied → ③ scoring differential among tied → ④ scoring average among tied → ⑤ scoring differential during the season → ⑥ total score → ⑦ drawing

Scenario-based modeling

Scenario set (1/2)

- Let L be the set of teams in a league, where $n = |L|$. (Sometimes, L consists of some disjoint districts; e.g., $L = \bigcup_{l=1}^{3} D_l$ in B.LEAGUE)。
- Suppose we are given the current win-loss records of all teams and the remaining schedule of games in L.
  - Let $w = (w_{ij})_{i,j \in L}$ denote the current number of wins of team $i \in L$ against team $j \in L$.
  - The current number of losses of team i against team j is then given by w_{ji}, and the current winning percentage of team i is then given by

$$\sum_{j \in L} w_{ij} \Bigg/ \sum_{j \in L} (w_{ij} + w_{ji}).$$

  - Let $g = (g_{ij})_{i,j \in L}$ denote the number of remaining games between teams i and j.
  - $w = (w_{ij})$ and $g = (g_{ij})$ can be respectively seen as nonsymmetric and symmetric square matrices of order n with zero diagonals.
- Assume each team in L plays M games in a season.

Scenario set (2/2)

- Let x_{ij} be the number of future wins of team i against team j. If there is no ties in each game, any matrix $x = (x_{ij}) \in \mathbb{Z}^{n\times n}$ satisfying

$$\begin{cases} x_{ij} + x_{ji} = g_{ij} & (\forall i, j \in L,\ i < j) \\ x_{ii} = 0 & (\forall i \in L) \\ x_{ij} \geq 0 & (\forall i, j \in L) \\ x_{ij} \in \mathbb{Z} & (\forall i, j \in L) \end{cases} \tag{S}$$

  represents a possible future scenario in the current season ($\mathbb{Z}$: integers).

- ▼ Given w and g, let X be the set of scenarios satisfying (S).
- ▼ If ties are allowed in each game, the first equality is replaced by

$$x_{ij} + x_{ji} \leq g_{ij} \quad (\forall i, j \in L,\ i < j).$$

- ▼ The final #wins and WP of team $i \in L$ under the scenario $x \in X$:

$$\sum_{j\in L}(w_{ij} + x_{ij}), \quad \frac{\displaystyle\sum_{j\in L}(w_{ij} + x_{ij})}{\displaystyle\sum_{j\in L}(w_{ij} + w_{ji} + x_{ij} + x_{ji})}$$

Optimization models to solve

C#: a min # of future wins sufficient to clinch some situation
E#: a min # of future losses sufficient to be eliminated from the situation

Clinching a situation means that there is no chance of missing the situation even if the team loses all of its remaining games.
Being eliminated from a situation means that there is no chance of clinching the situation even if the team wins all of its remaining games.

For calculating **C# and E# for a team a**, it is essential to solve

Clinch problem

$$\max_{x \in X} \ \# \text{ of future wins of team } a$$

$$\text{subject to team } a \text{ does not achieve the situation}$$

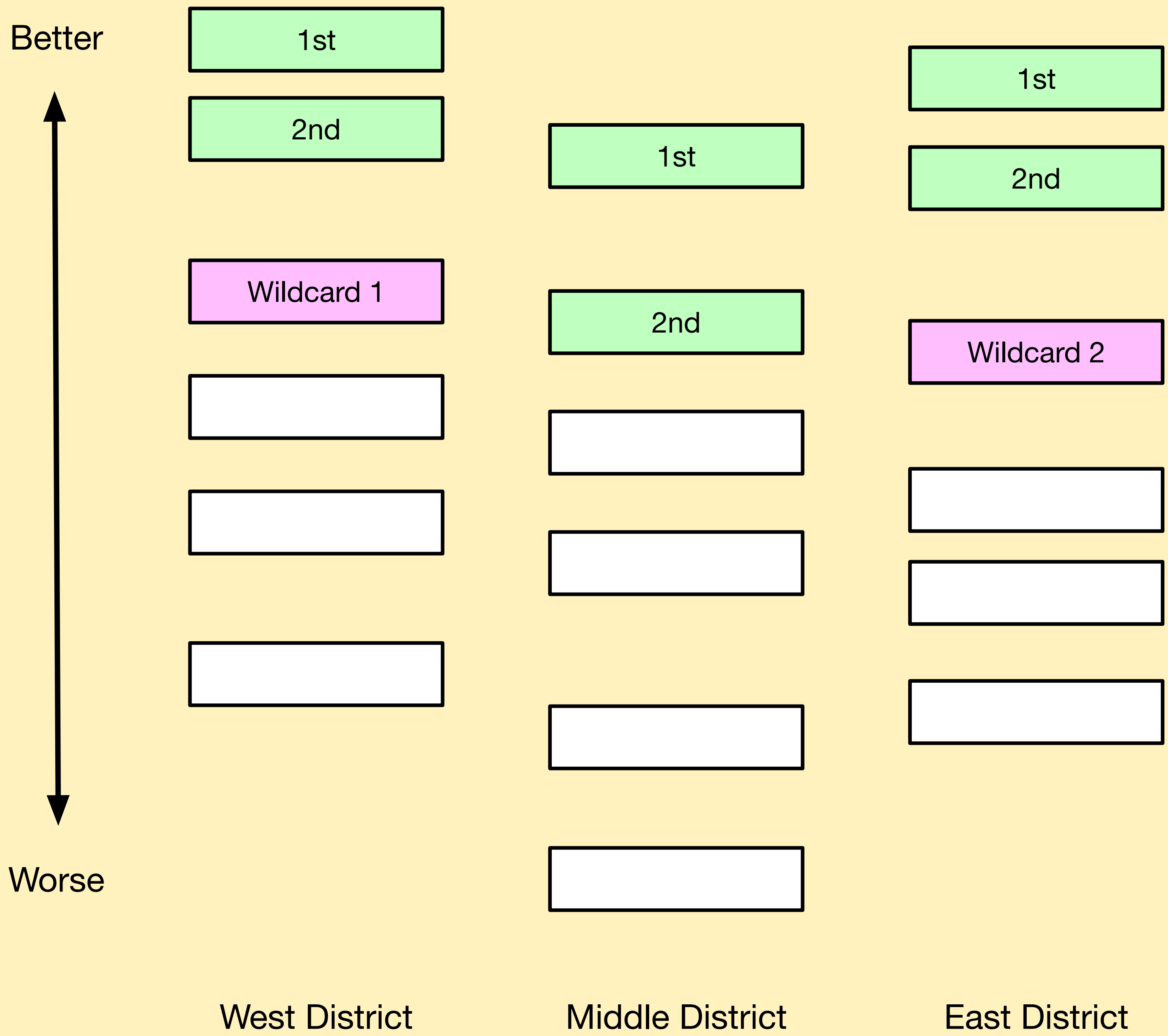
Elimination problem

$$\max_{x \in X} \ \# \text{ of future losses of team } a$$

$$\text{subject to team } a \text{ achieves the situation}$$

B.LEAGUE (JP Professional Basketball League)

Championship T. qualification



Scenario set for B.LEAGUE

- Let L be the set of teams in a division, and consists of 3 disjoint subsets D_l, $l = 1$, 2, 3, each of which respectively corresponds to the set of teams in a district; then $L = \bigcup_{l=1}^{3} D_l$.
- Let n_l be the number of teams in each district; namely $n_l := |D_l|$.
- $w = (w_{ij})$ and $g = (g_{ij})$ are now nonsymmetric and symmetric square matrices of order $n := \sum_{l=1}^{3} n_l$ with zero diagonals.
- Each team in L plays M games in a season.
- In current regulations, $n_l = 6$ (for all l) and $M = 60$.
- The set $X \subset \mathbb{Z}^{n \times n}$ of future scenarios:

$$\begin{cases} x_{ij} + x_{ji} = g_{ij} & (\forall i, j \in L,\ i < j) \\ x_{ii} = 0 & (\forall i \in L) \\ x_{ij} \geq 0 & (\forall i, j \in L) \\ x_{ij} \in \mathbb{Z} & (\forall i, j \in L) \end{cases}$$

Intra-district k-th place C# (1/3)

Intra-district k-th place clinch problem for team $a \in D_{l'}$ ($l' = 1$, 2 or 3):

$$\left[\begin{array}{ll} \displaystyle\max_{\substack{x \in X \\ \alpha,\beta,\lambda \in \{0,1\}^{n_{l'}}}} & \displaystyle\sum_{j \in L} x_{aj} \\ \text{subject to} & \boxed{\displaystyle\sum_{j \in L}(w_{aj} + x_{aj}) \leq \sum_{j \in L}(w_{ij} + x_{ij}) + (M+1)\,\alpha_i - 1 \quad (\forall i \in D_{l'})} \\ & \boxed{\begin{array}{l} \displaystyle\left|\sum_{j \in L}(w_{aj} + x_{aj}) - \sum_{j \in L}(w_{ij} + x_{ij})\right| \leq M\lambda_i \quad (\forall i \in D_{l'}) \\ \displaystyle\left|\sum_{j \in L}(w_{aj} + x_{aj}) - \sum_{j \in L}(w_{ij} + x_{ij})\right| \geq \lambda_i \quad (\forall i \in D_{l'}) \\ \displaystyle\sum_{j \in D_{l'}}(w_{aj} + x_{aj})(1 - \lambda_j) \leq \sum_{j \in D_{l'}}(w_{ij} + x_{ij})(1 - \lambda_j) + M\beta_i \quad (\forall i \in D_{l'}) \\ \lambda_i \leq \beta_i \quad (\forall i \in D_{l'}), \quad \beta_a = 1 \end{array}} \\ & \displaystyle\sum_{i \in D_{l'}} \alpha_i + \sum_{i \in D_{l'}} \beta_i = 2n_{l'} - k \end{array}\right.$$

where $\alpha_a = 1$, $\lambda_a = 0$, and $\alpha_i + \beta_i \geq 1$ $(\forall i \in D_{l'})$ hold for any feasible solution.

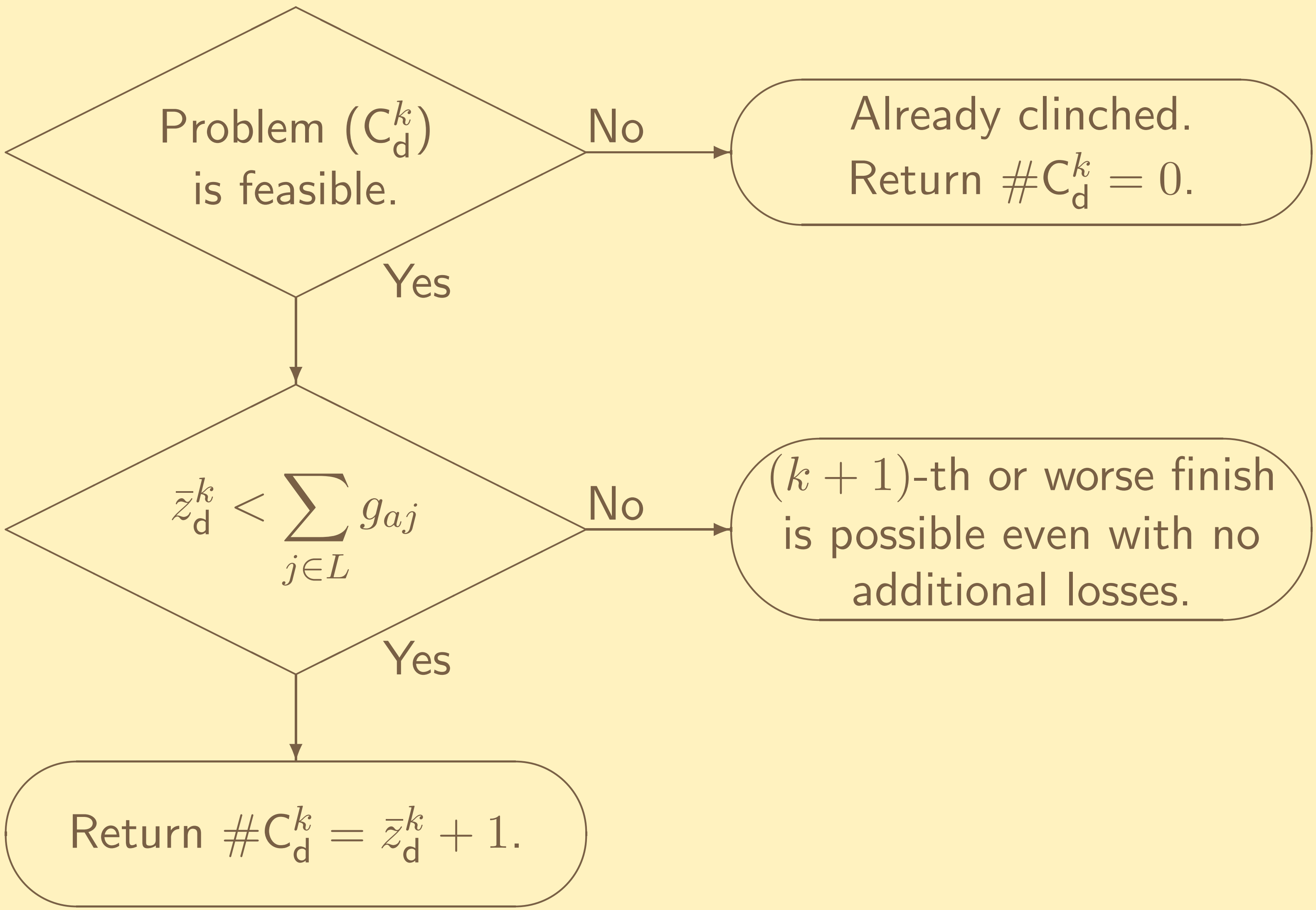
Intra-district k-th place C# (2/3)

With removing redundancy of the constraints, we have

$$
\left[
\begin{array}{lll}
\max\limits_{\substack{x \in X \\ \alpha,\beta,\lambda,\lambda_1,\lambda_2 \in \{0,1\}^{n_{l'}}}} & \sum\limits_{j \in L} x_{aj} & \\
\text{subject to} & \left| \sum\limits_{j \in L} (w_{aj} + x_{aj}) - \sum\limits_{j \in L} (w_{ij} + x_{ij}) \right| \le M\lambda_i \quad (\forall i \in D_{l'}) & \\
& \sum\limits_{j \in L} (w_{aj} + x_{aj}) - \sum\limits_{j \in L} (w_{ij} + x_{ij}) \le (M+1)\,\lambda_{1i} - 1 \quad (\forall i \in D_{l'}) & \\
& \sum\limits_{j \in L} (w_{aj} + x_{aj}) - \sum\limits_{j \in L} (w_{ij} + x_{ij}) \ge -(M+1)\,\lambda_{2i} + 1 \quad (\forall i \in D_{l'}) & (\mathsf{C}_\mathsf{d}^k) \\
& \lambda_i + \lambda_{1i} + \lambda_{2i} = 2 \quad (\forall i \in D_{l'}) & \\
& \boxed{\lambda_{1i} \le \alpha_i \quad (\forall i \in D_{l'})} & \\
& \boxed{\begin{array}{l} \sum\limits_{j \in D_{l'}} (w_{aj} + x_{aj})(1 - \lambda_j) \le \sum\limits_{j \in D_{l'}} (w_{ij} + x_{ij})(1 - \lambda_j) + M\beta_i \quad (\forall i \in D_{l'}) \\ \lambda_i \le \beta_i \quad (\forall i \in D_{l'}), \quad \beta_a = 1 \end{array}} & \\
& \sum\limits_{i \in D_{l'}} \alpha_i + \sum\limits_{i \in D_{l'}} \beta_i = 2n_{l'} - k &
\end{array}
\right.
$$

Intra-district k-th place C# (3/3)

After solving the clinch problem, we have the **intra-district k-th place clinch number $\#\mathsf{C}^k_{\mathsf{d}}$ of team a** as below, where $\bar{z}^k_{\mathsf{d}}$ denotes the optimal objective function value of problem ($\mathsf{C}^k_{\mathsf{d}}$).



Intra-district k-th place E# (1/3)

Intra-district k-th place elimination problem for team $a \in D_{l'}$ ($l' = 1$, 2 or 3):

$$
\left[\begin{array}{ll}
\displaystyle\max_{\substack{x \in X \\ \alpha,\beta,\lambda \in \{0,1\}^{n_{l'}}}} & \displaystyle\sum_{j \in L} x_{ja} \\
\text{subject to} & \boxed{\displaystyle\sum_{j \in L}(w_{aj} + x_{aj}) \geq \sum_{j \in L}(w_{ij} + x_{ij}) - (M+1)\,\alpha_i + 1 \quad (\forall i \in D_{l'})} \\
& \boxed{\begin{array}{l}
\displaystyle\left|\sum_{j \in L}(w_{aj} + x_{aj}) - \sum_{j \in L}(w_{ij} + x_{ij})\right| \leq M\lambda_i \quad (\forall i \in D_{l'}) \\
\displaystyle\left|\sum_{j \in L}(w_{aj} + x_{aj}) - \sum_{j \in L}(w_{ij} + x_{ij})\right| \geq \lambda_i \quad (\forall i \in D_{l'}) \\
\displaystyle\sum_{j \in D_{l'}}(w_{aj} + x_{aj})(1 - \lambda_j) \geq \sum_{j \in D_{l'}}(w_{ij} + x_{ij})(1 - \lambda_j) - M\beta_i \quad (\forall i \in D_{l'}) \\
\lambda_i \leq \beta_i \quad (\forall i \in D_{l'}), \quad \beta_a = 1
\end{array}} \\
& \displaystyle\sum_{i \in D_{l'}} \alpha_i + \sum_{i \in D_{l'}} \beta_i = n_{l'} + k
\end{array}\right.
$$

where $\alpha_a = 1$, $\lambda_a = 0$, and $\alpha_i + \beta_i \geq 1$ $(\forall i \in D_{l'})$ hold for any feasible solution.

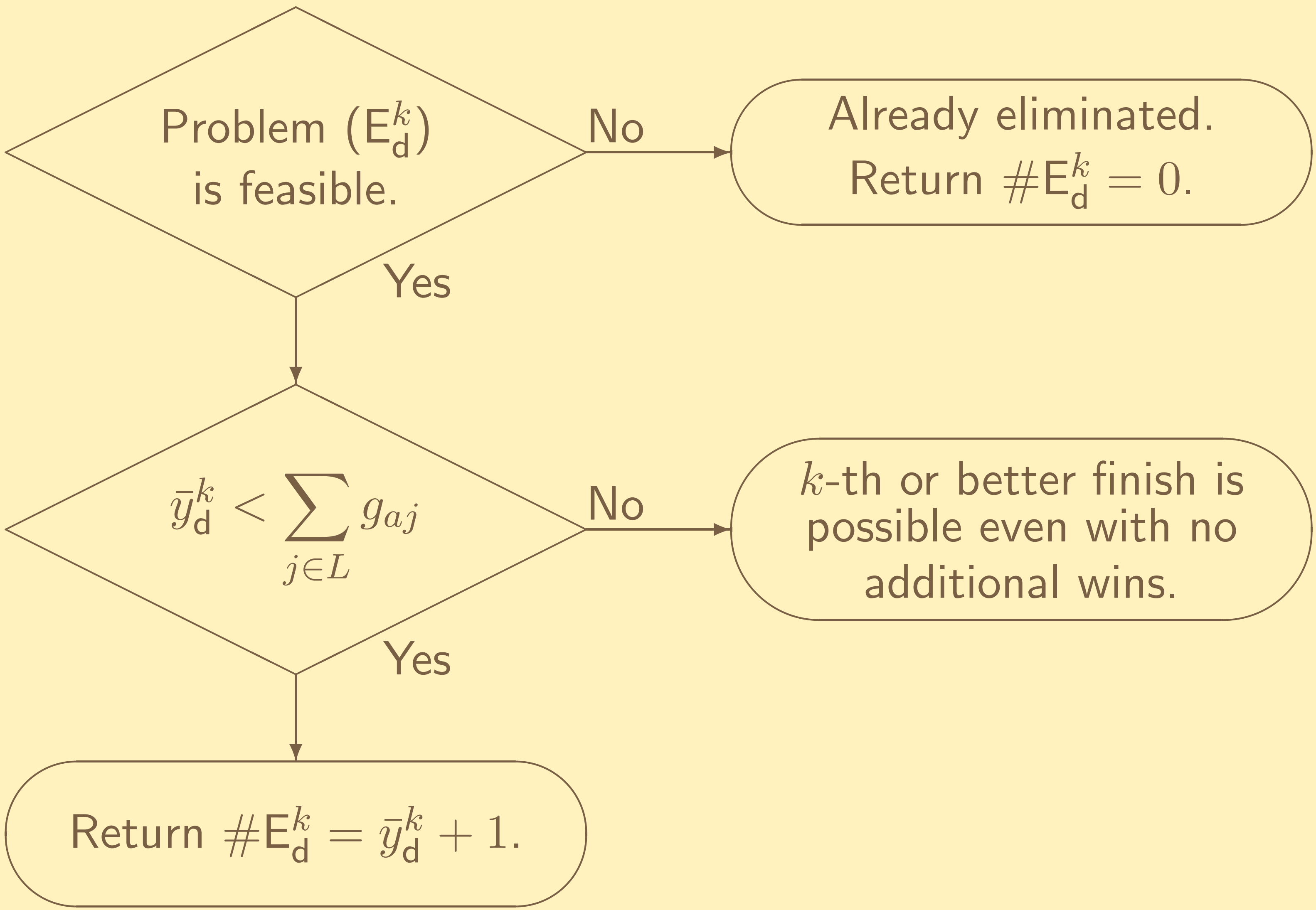
Intra-district k-th place E# (2/3)

With removing redundancy of the constraints, we have

$$
\left[\begin{array}{ll}
\max\limits_{\substack{x \in X \\ \alpha,\beta,\lambda,\lambda_1,\lambda_2 \in \{0,1\}^{n_{l'}}}} & \sum\limits_{j \in L} x_{ja} \\
\text{subject to} & \boxed{\begin{array}{l}
\left| \sum\limits_{j \in L} (w_{aj} + x_{aj}) - \sum\limits_{j \in L} (w_{ij} + x_{ij}) \right| \le M\lambda_i \quad (\forall i \in D_{l'}) \\
\sum\limits_{j \in L} (w_{aj} + x_{aj}) - \sum\limits_{j \in L} (w_{ij} + x_{ij}) \le (M+1)\,\lambda_{1i} - 1 \quad (\forall i \in D_{l'}) \\
\sum\limits_{j \in L} (w_{aj} + x_{aj}) - \sum\limits_{j \in L} (w_{ij} + x_{ij}) \ge -(M+1)\,\lambda_{2i} + 1 \quad (\forall i \in D_{l'}) \\
\lambda_i + \lambda_{1i} + \lambda_{2i} = 2 \quad (\forall i \in D_{l'})
\end{array}} \\
& \boxed{\lambda_{2i} \le \alpha_i \quad (\forall i \in D_{l'})} \\
& \boxed{\begin{array}{l}
\sum\limits_{j \in D_{l'}} (w_{aj} + x_{aj})(1 - \lambda_j) \ge \sum\limits_{j \in D_{l'}} (w_{ij} + x_{ij})(1 - \lambda_j) - M\beta_i \quad (\forall i \in D_{l'}) \\
\lambda_i \le \beta_i \quad (\forall i \in D_{l'}), \quad \beta_a = 1
\end{array}} \\
& \sum\limits_{i \in D_{l'}} \alpha_i + \sum\limits_{i \in D_{l'}} \beta_i = n_{l'} + k
\end{array}\right. \qquad (\mathsf{E}_{\mathsf{d}}^{k})
$$

Intra-district k-th place E# (3/3)

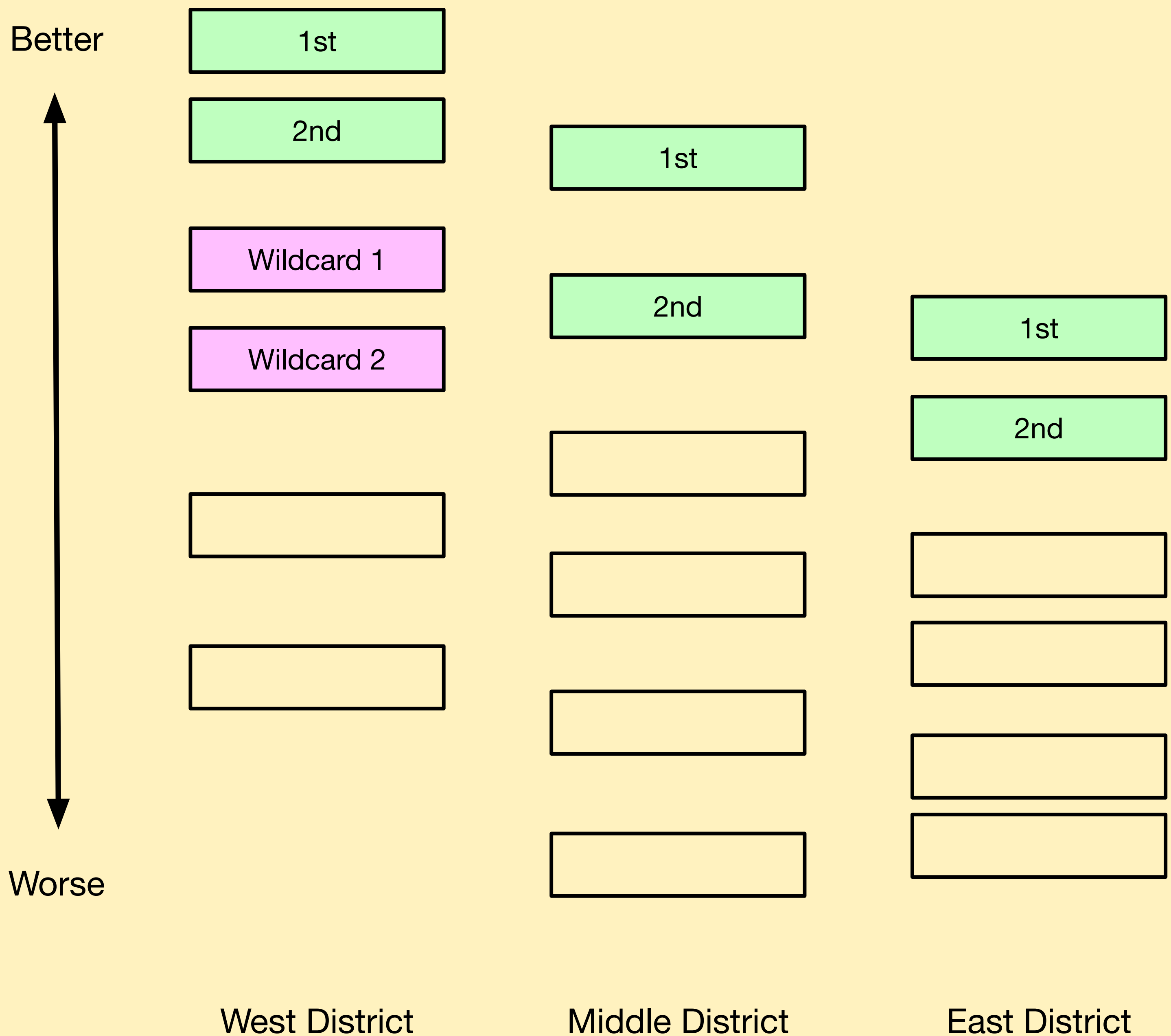
After solving the elimination problem, we have **the intra-district k-th place elimination number $\#\mathsf{E}_{\mathsf{d}}^{k}$ of team a** as below, where $\bar{y}_{\mathsf{d}}^{k}$ denotes the optimal objective function value of $(\mathsf{E}_{\mathsf{d}}^{k})$.

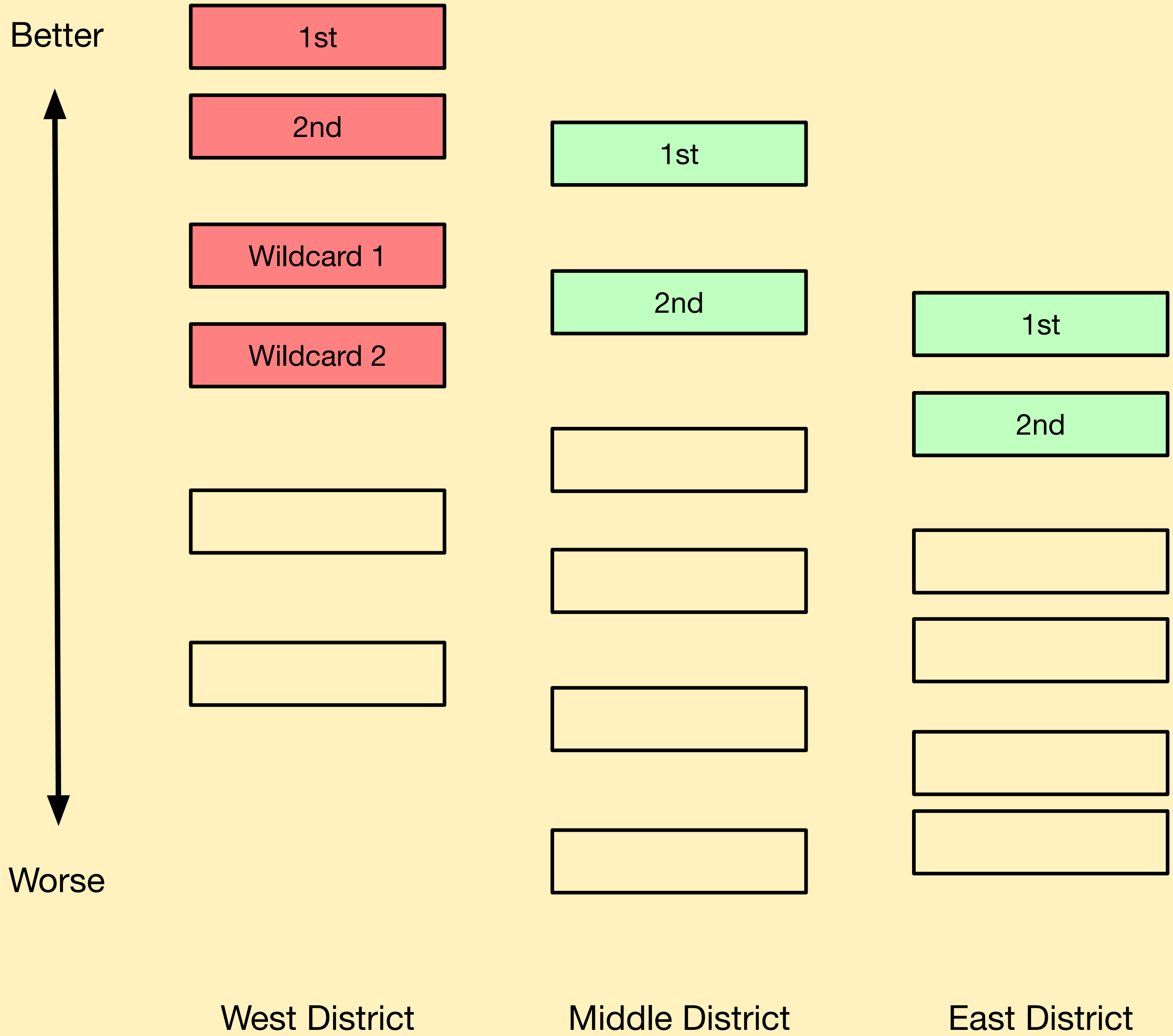


CT qualification C# (1/5)

- In division B1, a team will qualify for the championship tournament if the team **either** (1) finishes in the top 2 of its district **or** (2) finishes as a wildcard (within the best 2 records among all teams except the top 2 of each district).
- Therere, for calculating the C# for championship tournament qualification, we need to find a max # of future wins for each team under the conditions that $(\overline{1})$ the team does not finish within the top 2 of its district **and** $(\overline{2})$ does not finish as a wildcard.
- The former condition $(\overline{1})$ is obvious.
- The latter condition $(\overline{2})$ is equivalent to that **either** there exist 4 teams with better records in one district **or** there exist 3 teams with better records in each of two districts.

It's not so complicated if tiebreakers are additionally played among tied teams.


Better
Worse
1st
2nd
Wildcard 1
Wildcard 2
1st
2nd
1st
2nd
West District
Middle District
East District


Better
1st
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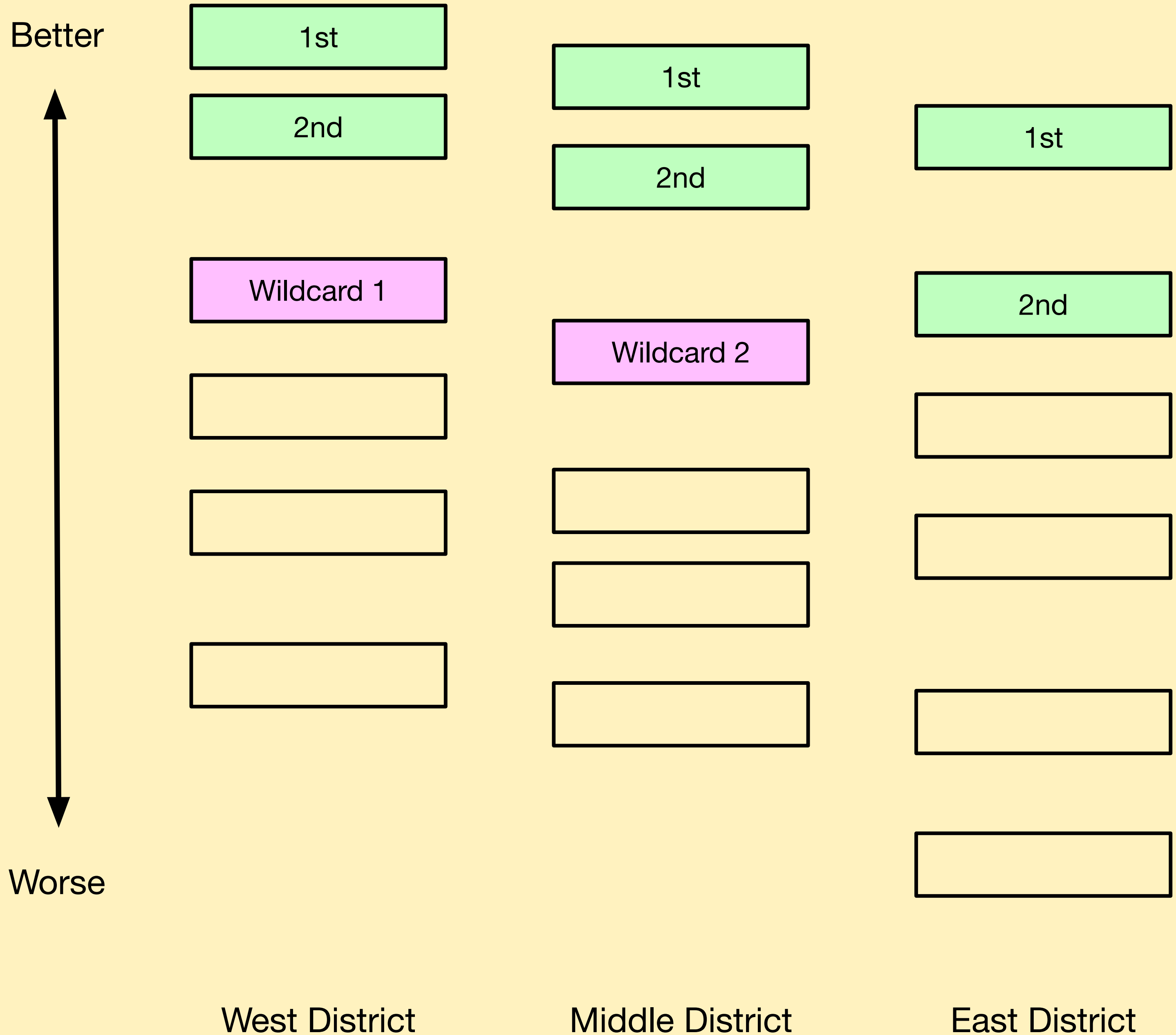
West District
Middle District
East District


Better
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1st
2nd
Wildcard 1
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2nd
Wildcard 2
1st
2nd
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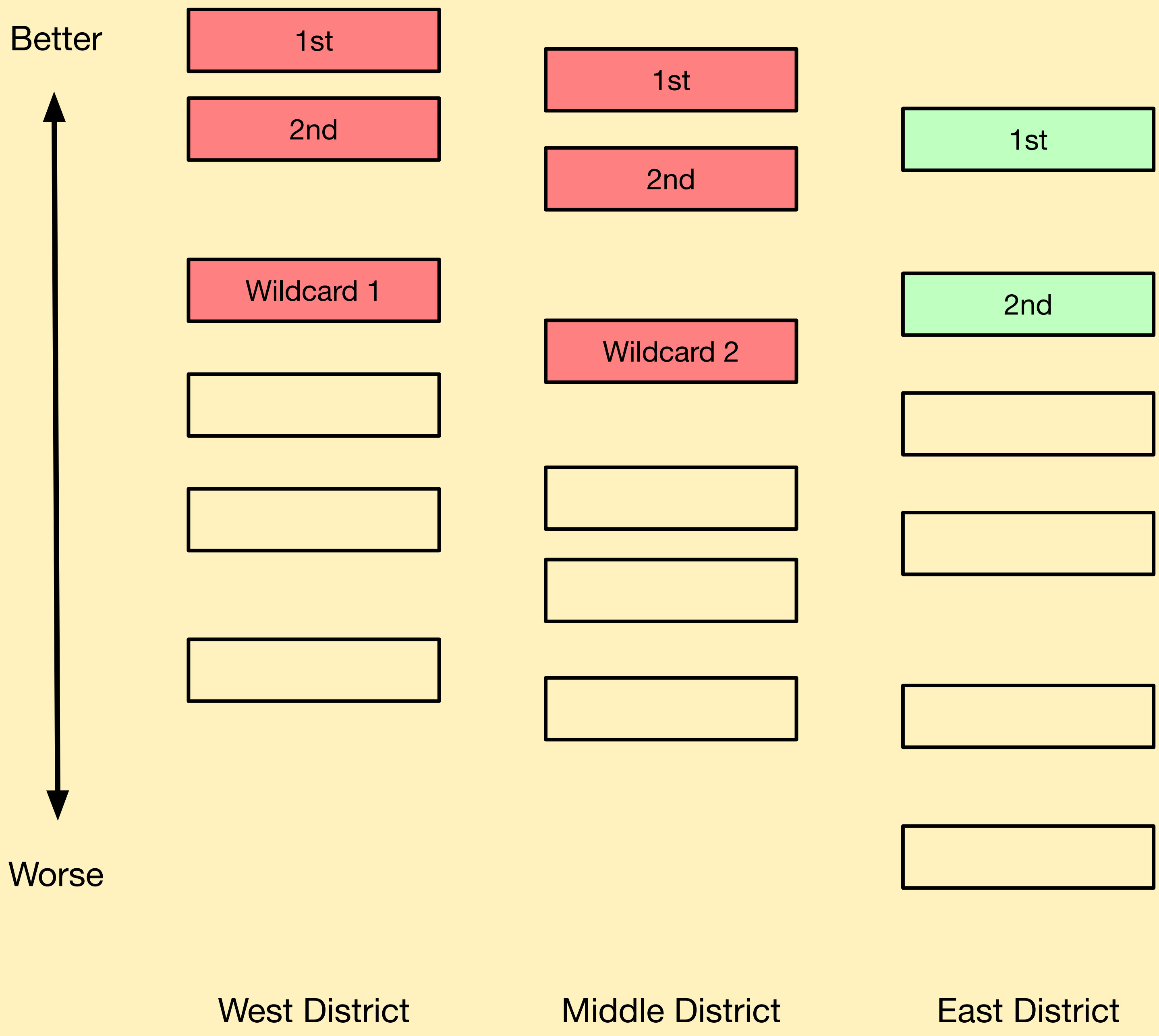
Middle District
East District


Better
Worse
1st
2nd
Wildcard 1
1st
2nd
Wildcard 2
1st
2nd
West District
Middle District
East District

CT qualification C# (2/5)

$$\max_{\substack{x\in X\\ \alpha\in\{0,1\}^{n_{l'}}\\ \gamma^l\in\{0,1\}^{n_l},\ l\in\{1,2,3\}\\ \sigma\in\{0,1\}^3,\ \theta\in\{0,1\}}} \sum_{j\in L} x_{aj}$$

subject to

$$\boxed{\begin{aligned}&\sum_{j\in L}(w_{aj}+x_{aj}) \le \sum_{j\in L}(w_{ij}+x_{ij}) + (M+1)\,\alpha_i - 1 \quad (\forall i\in D_{l'})\\ &\sum_{i\in D_{l'}} \alpha_i = n_{l'} - 2\end{aligned}}$$

$$\boxed{\begin{aligned}&\sum_{j\in L}(w_{aj}+x_{aj}) \le \sum_{j\in L}(w_{ij}+x_{ij}) + (M+1)\,\gamma_i^l - 1 \quad (\forall i\in D_l,\ \forall l\in\{1,2,3\})\\ &(1-\theta)\left(\sum_{i\in D_l}\gamma_i^l - n_l + 4\sigma_l\right) = 0 \quad (\forall l\in\{1,2,3\})\\ &(1-\theta)\left(\sum_{l=1}^{3}\sigma_l - 1\right) = 0\\ &\theta\left(\sum_{i\in D_l}\gamma_i^l - n_l + 3\sigma_l\right) = 0 \quad (\forall l\in\{1,2,3\})\\ &\theta\left(\sum_{l=1}^{3}\sigma_l - 2\right) = 0\end{aligned}}$$

$$\alpha_a = \gamma_a^{l'} = 1$$

CT qualification C# (3/5)

$$\max_{\substack{x\in X\\ \alpha,\beta\in\{0,1\}^{n_{l'}}\\ \gamma^l,\delta^l,\lambda^l,\lambda^l_1,\lambda^l_2\in\{0,1\}^{n_l},\ l\in\{1,2,3\}\\ \sigma\in\{0,1\}^3,\ \theta\in\{0,1\}}} \quad \sum_{j\in L} x_{aj}$$

subject to

$$\left|\sum_{j\in L}(w_{aj}+x_{aj})-\sum_{j\in L}(w_{ij}+x_{ij})\right| \le M\lambda^l_i \quad (\forall i\in D_l,\ \forall l\in\{1,2,3\})$$

$$\sum_{j\in L}(w_{aj}+x_{aj})-\sum_{j\in L}(w_{ij}+x_{ij}) \le (M+1)\,\lambda^l_{1i}-1 \quad (\forall i\in D_l,\ \forall l\in\{1,2,3\})$$

$$\sum_{j\in L}(w_{aj}+x_{aj})-\sum_{j\in L}(w_{ij}+x_{ij}) \ge -(M+1)\,\lambda^l_{2i}+1 \quad (\forall i\in D_l,\ \forall l\in\{1,2,3\})$$

$$\lambda^l_i+\lambda^l_{1i}+\lambda^l_{2i}=2 \quad (\forall i\in D_l,\ \forall l\in\{1,2,3\})$$

$$\boxed{\lambda^{l'}_{1i}\le\alpha_i \quad (\forall i\in D_{l'})}$$

$$\sum_{j\in D_{l'}}(w_{aj}+x_{aj})(1-\lambda^{l'}_j) \le \sum_{j\in D_{l'}}(w_{ij}+x_{ij})(1-\lambda^{l'}_j)+M\beta_i \quad (\forall i\in D_{l'})$$

$$\lambda^{l'}_i\le\beta_i \quad (\forall i\in D_{l'}),\quad \beta_a=1$$

$$\sum_{i\in D_{l'}}\alpha_i+\sum_{i\in D_{l'}}\beta_i=2n_{l'}-2$$

(C_c)

CT qualification C# (4/5)

$$\boxed{\lambda_{1i}^l \le \gamma_i^l \quad (\forall i \in D_l,\ \forall l \in \{1,2,3\})}$$

$$\frac{\sum_{l=1}^{3}\sum_{j\in D_l}(w_{aj}+x_{aj})(1-\lambda_j^l)}{\sum_{l=1}^{3}\sum_{j\in D_l}(w_{aj}+w_{ja}+g_{aj})(1-\lambda_j^l)} \le \frac{\sum_{l=1}^{3}\sum_{j\in D_l}(w_{ij}+x_{ij})(1-\lambda_j^l)}{\sum_{l=1}^{3}\sum_{j\in D_l}(w_{ij}+w_{ji}+g_{ij})(1-\lambda_j^l)} + M\delta_i^l \quad (\forall i \in D_l,\ \forall l \in \{1,2,3\})$$

$$\lambda_i^l \le \delta_i^l \quad (\forall i \in D_l,\ \forall l \in \{1,2,3\}), \quad \delta_a^{l'} = 1$$

$$(1-\theta)\left(\sum_{i\in D_l}\gamma_i^l + \sum_{i\in D_l}\delta_i^l - 2n_l + 4\sigma_l\right) = 0 \quad (\forall l \in \{1,2,3\})$$

$$(1-\theta)\left(\sum_{l=1}^{3}\sigma_l - 1\right) = 0$$

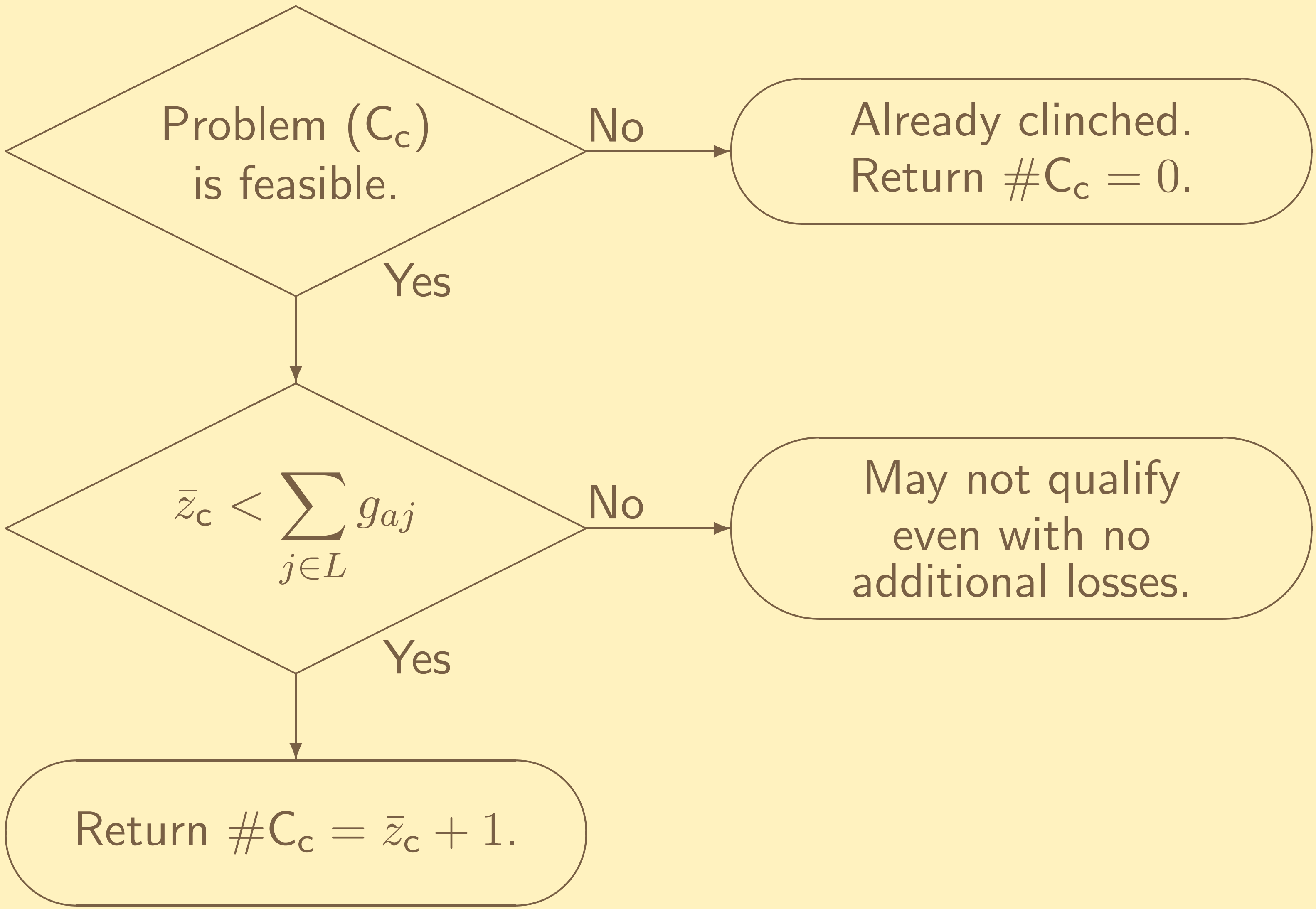
$$\theta\left(\sum_{i\in D_l}\gamma_i^l + \sum_{i\in D_l}\delta_i^l - 2n_l + 3\sigma_l\right) = 0 \quad (\forall l \in \{1,2,3\})$$

$$\theta\left(\sum_{l=1}^{3}\sigma_l - 2\right) = 0$$

where $\alpha_a = \gamma_a^{l'} = 1,\ \lambda_a^{l'} = 0,\ \alpha_i + \beta_i \ge 1\ (\forall i \in D_{l'})$, and $\gamma_i^l + \delta_i^l \ge 1\ (\forall i \in D_l,\ \forall l \in \{1,2,3\})$ hold for any feasible solution.

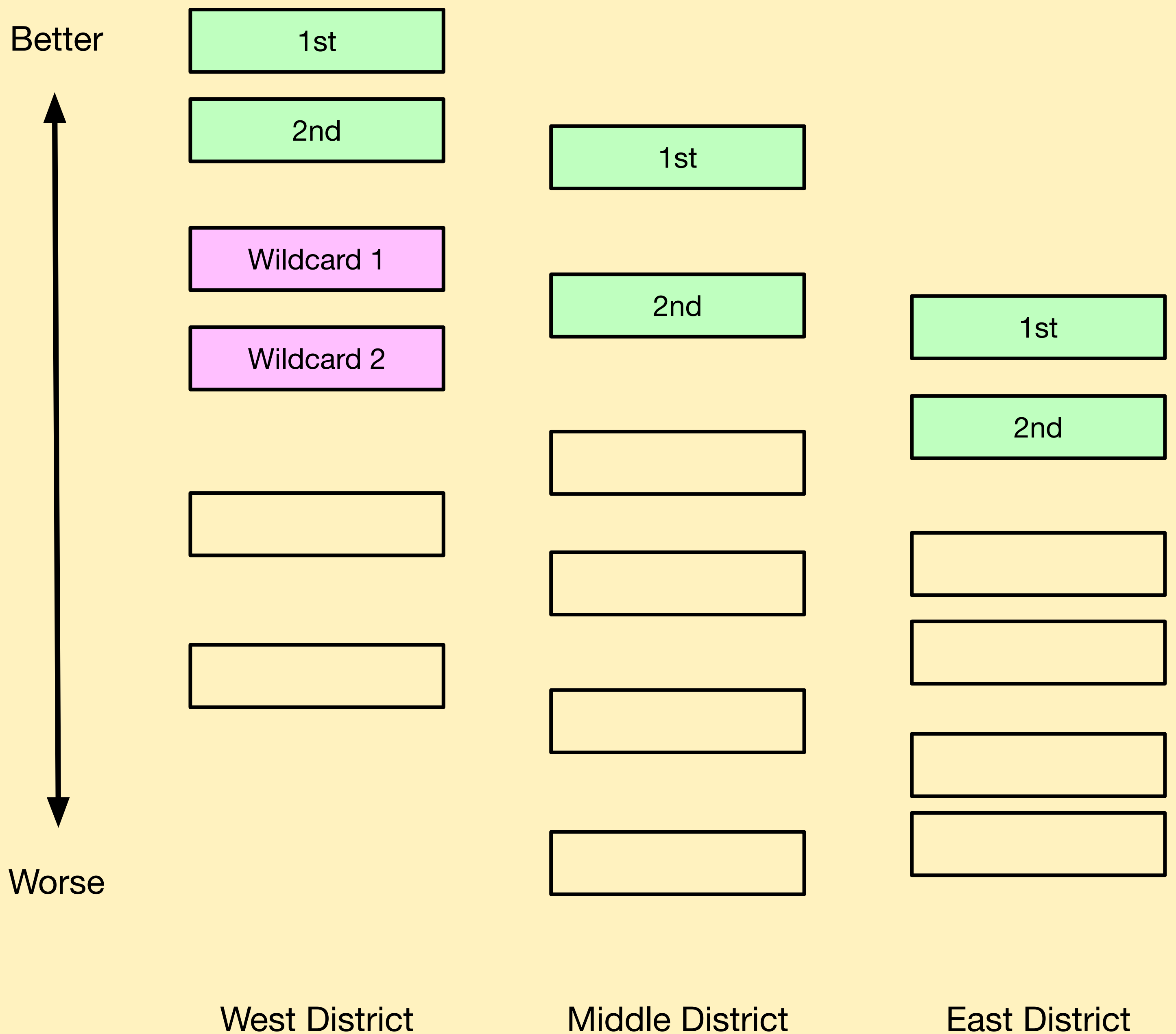
CT qualification C# (5/5)

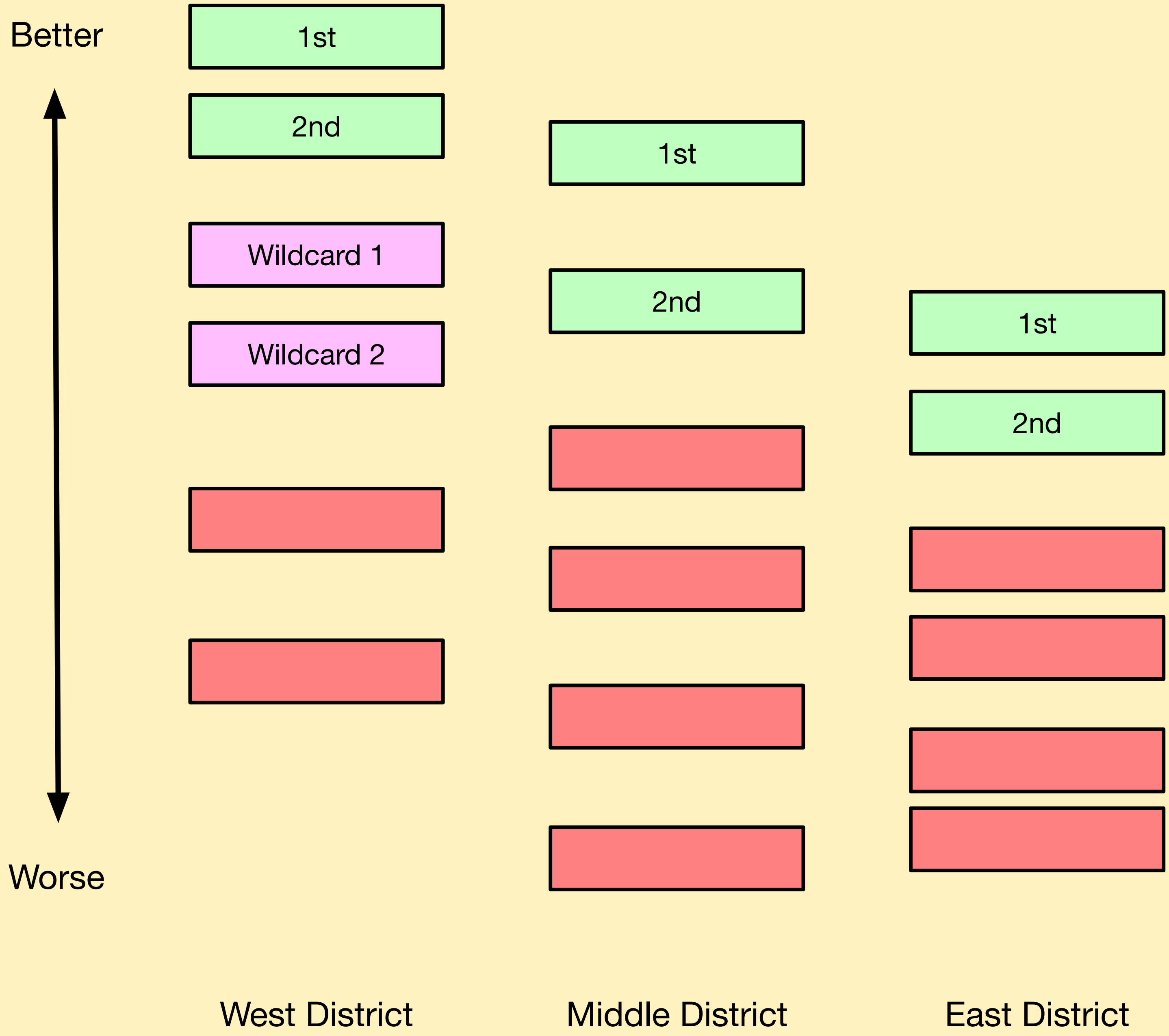
The clinch number #C$_\mathsf{c}$ of team a can be obtained **for the championship tournament qualification** as below, where $\bar{z}_\mathsf{c}$ denotes the optimal objective function value of problem (C$_\mathsf{c}$).

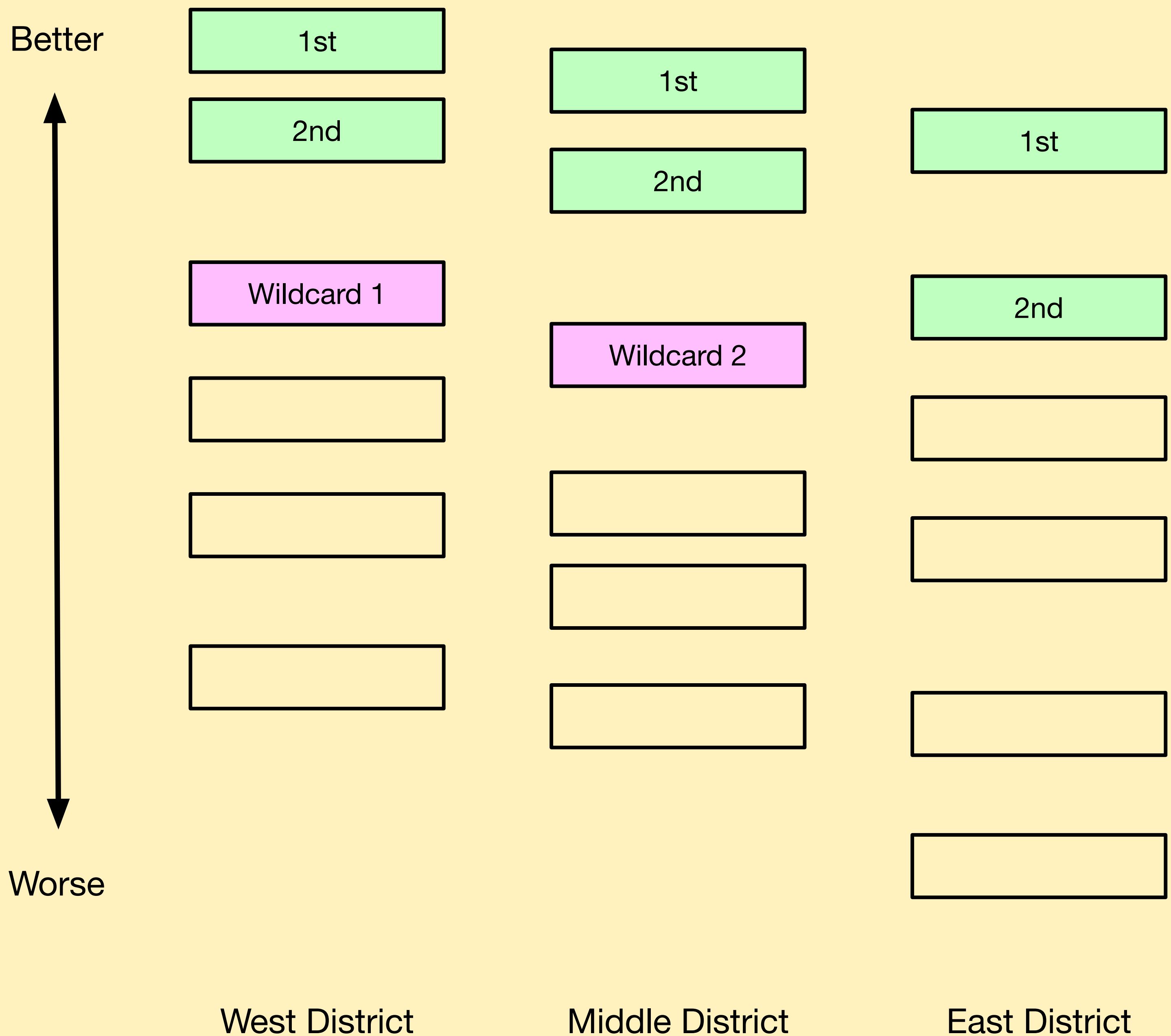


CT qualification E# (1/4)

- For elimination from the championship tournament, we will find a max # of future losses for each team under the condition that the team **either** (1) finishes within the top two of its district **or** (2) earns one of the two wildcard berths.
- One can separately calculate and combine **the intra-district 2nd place elimination number** and **the wildcard elimination number** under the respective condition.
- For the former, We already have problem ($\mathsf{E}_{\mathsf{d}}^{2}$).
- For the latter, it is not difficult to see that a team can earn a wildcard berth if and only if **either** there exist $(n_l - 4)$ teams with lower records in one district and $(n_l - 2)$ teams with lower records in each of the other two districts **or** there exist $(n_l - 3)$ teams with lower records in each of two districts and $(n_l - 2)$ teams with lower records in the other district.


Better
Worse
1st
2nd
Wildcard 1
Wildcard 2
1st
2nd
1st
2nd
West District
Middle District
East District


Better
Worse
1st
2nd
Wildcard 1
Wildcard 2
1st
2nd
1st
2nd
West District
Middle District
East District


Better
1st
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Wildcard 1
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Wildcard 2
1st
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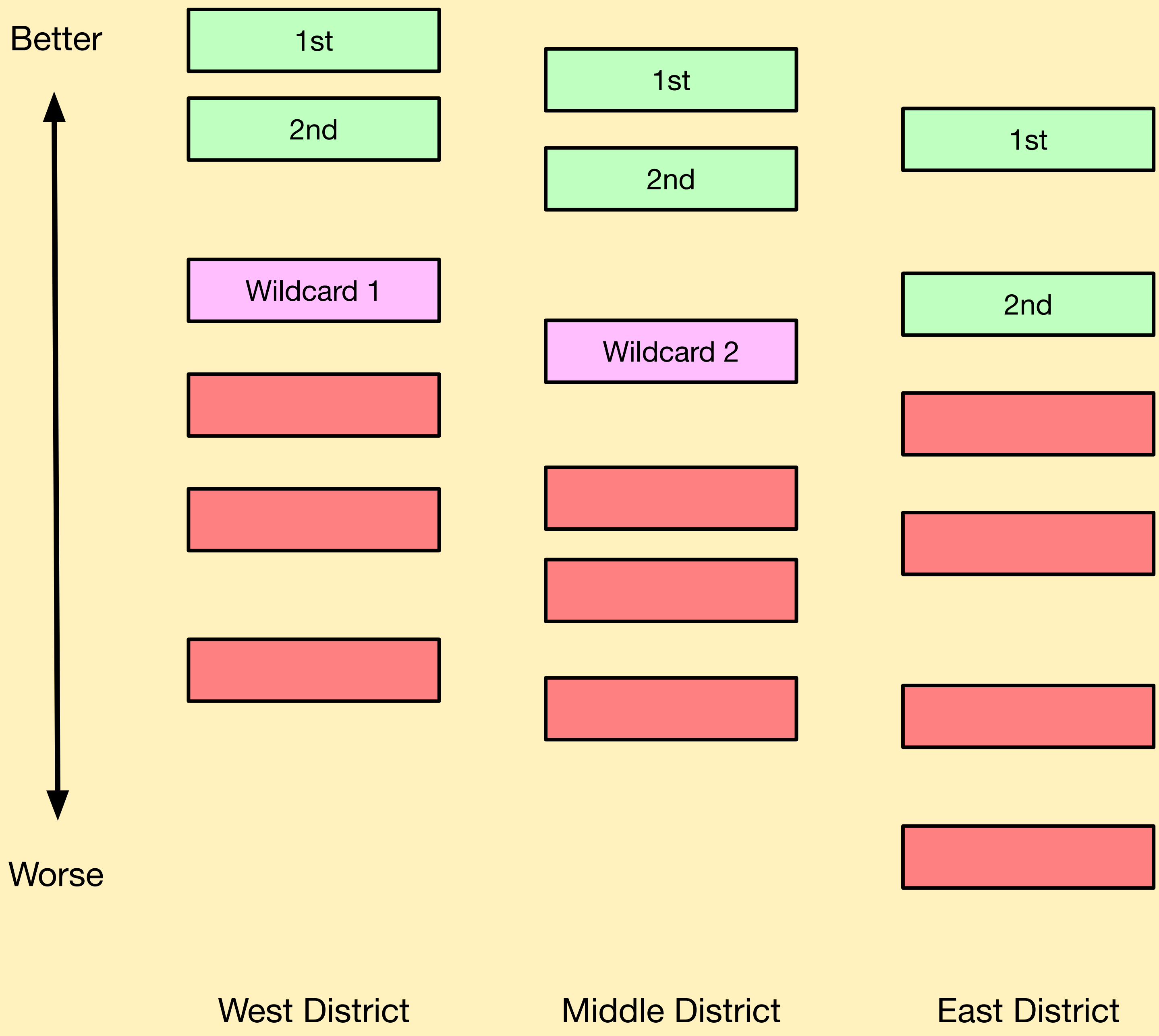
Worse
West District
Middle District
East District


Better
1st
2nd
Wildcard 1
1st
2nd
Wildcard 2
1st
2nd
Worse
West District
Middle District
East District

CT qualification E# (2/4)

$$
\left[\begin{array}{ll}
\displaystyle\max_{\substack{x\in X\\ \gamma^l,\delta^l,\lambda^l,\lambda_1^l,\lambda_2^l\in\{0,1\}^{n_l},\ l\in\{1,2,3\}\\ \sigma\in\{0,1\}^3\\ \theta\in\{0,1\}}} & \displaystyle\sum_{j\in L} x_{ja} \\
\text{subject to} & \boxed{\begin{array}{l}
\displaystyle\left|\sum_{j\in L}(w_{aj}+x_{aj})-\sum_{j\in L}(w_{ij}+x_{ij})\right| \le M\lambda_i^l \quad (\forall i\in D_l,\ \forall l\in\{1,2,3\}) \\
\displaystyle\sum_{j\in L}(w_{aj}+x_{aj})-\sum_{j\in L}(w_{ij}+x_{ij}) \le (M+1)\,\lambda_{1i}^l-1 \quad (\forall i\in D_l,\ \forall l\in\{1,2,3\}) \\
\displaystyle\sum_{j\in L}(w_{aj}+x_{aj})-\sum_{j\in L}(w_{ij}+x_{ij}) \ge -(M+1)\,\lambda_{2i}^l+1 \quad (\forall i\in D_l,\ \forall l\in\{1,2,3\}) \\
\lambda_i^l+\lambda_{1i}^l+\lambda_{2i}^l=2 \quad (\forall i\in D_l,\ \forall l\in\{1,2,3\})
\end{array}}
\end{array}\right.
$$

CT qualification E# (3/4)

$$\boxed{\lambda_{2i}^l \le \gamma_i^l \quad (\forall i \in D_l,\ \forall l \in \{1,2,3\})}$$

$$\boxed{\begin{array}{l}\dfrac{\sum_{l=1}^{3}\sum_{j\in D_l}(w_{aj}+x_{aj})(1-\lambda_j^l)}{\sum_{l=1}^{3}\sum_{j\in D_l}(w_{aj}+w_{ja}+g_{aj})(1-\lambda_j^l)} \ge \dfrac{\sum_{l=1}^{3}\sum_{j\in D_l}(w_{ij}+x_{ij})(1-\lambda_j^l)}{\sum_{l=1}^{3}\sum_{j\in D_l}(w_{ij}+w_{ji}+g_{ij})(1-\lambda_j^l)} - M\delta_i^l \\ \hfill (\forall i \in D_l,\ \forall l \in \{1,2,3\}) \\ \lambda_i^l \le \delta_i^l \quad (\forall i \in D_l,\ \forall l \in \{1,2,3\}), \quad \delta_a^{l'} = 1\end{array}}$$

$$(1-\theta)\left(\sum_{i\in D_l}\gamma_i^l + \sum_{i\in D_l}\delta_i^l - n_l - 2 - 2\sigma_l\right) = 0 \quad (\forall l \in \{1,2,3\})$$

$$(1-\theta)\left(\sum_{l=1}^{3}\sigma_l - 1\right) = 0$$

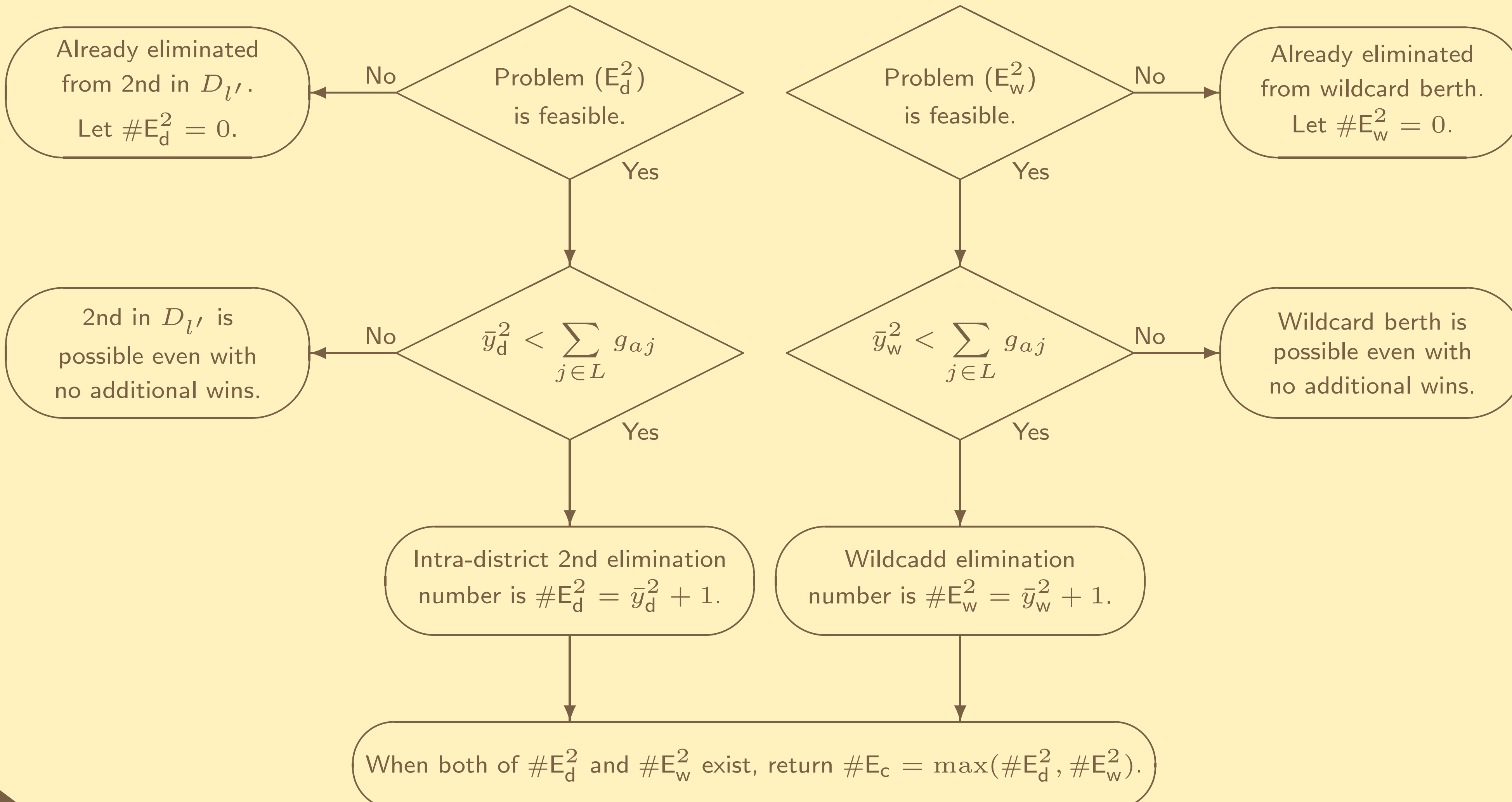
$$\theta\left(\sum_{i\in D_l}\gamma_i^l + \sum_{i\in D_l}\delta_i^l - n_l - 2 - \sigma_l\right) = 0 \quad (\forall l \in \{1,2,3\})$$

$$\theta\left(\sum_{l=1}^{3}\sigma_l - 2\right) = 0 \qquad (\mathsf{E}_\mathsf{w}^2)$$

where $\gamma_a^{l'} = 1$, $\lambda_a^{l'} = 0$, and $\gamma_i^l + \delta_i^l \ge 1$ $(\forall i \in D_l,\ \forall l \in \{1,2,3\})$ hold for any feasible solution.

CT qualification E# (4/4)

With the optimal objective function values of problem (E^2_d) and (E^2_w), **three elimination numbers of team a** can be obtained **for intra-district 2nd-place, wildcard, and championship tournament qualification** as below, where $\bar{y}^2_\mathsf{d}$ and $\bar{y}^2_\mathsf{w}$ respectively denote the optimal objective function values of problems (E^2_d) and (E^2_w).



Upper and lower bounds (1/4)

- We utilize **upper and lower bounds of each optimization problem** since some of them are computationally expensive to solve.
- All optimization models we have seen so far have **two constraint blocks**, which respectively correspond to the first and second criteria.
- These criteria are expressed as **inequalities** in the constraint blocks of each problem.
- **The first constraint blocks do not logically include equal signs but the second constraint blocks do**, which means that ties are not allowed in the first criteria but are allowed in the second criteria.
- The second criteria are only used in case of ties in the first criteria, which is why **if equal signs are logically added to the first blocks, then** the second blocks essentially disappear and **the resulting problems respectively provide upper bounds to the original problems**.
- Conversely, **if we add an additional constraint that the binary variables for team selection in the second criteria must be one,** then this makes the second constraint blocks invisible and **the resulting problems respectively provide lower bounds to the original problems**.

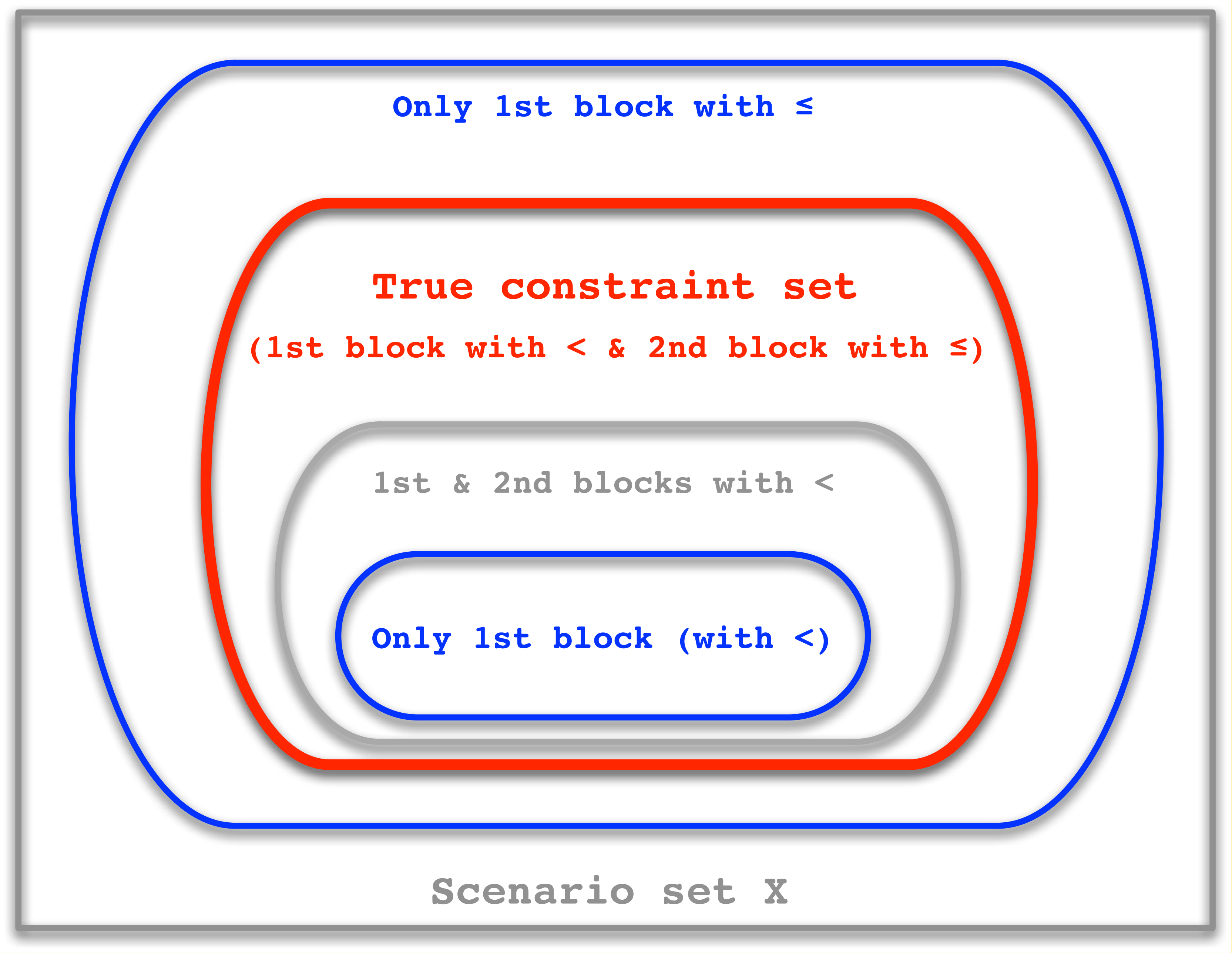
Upper and lower bounds (2/4)

Intra-district k-th place clinch problem for team $a \in D_{l'}$ ($l' = 1$, 2 or 3):

$$
\left[\begin{array}{ll}
\displaystyle\max_{\substack{x \in X \\ \alpha,\beta,\lambda \in \{0,1\}^{n_{l'}}}} & \displaystyle\sum_{j \in L} x_{aj} \\
\text{subject to} & \boxed{\displaystyle\sum_{j \in L}(w_{aj} + x_{aj}) \le \sum_{j \in L}(w_{ij} + x_{ij}) + (M+1)\,\alpha_i - 1 \quad (\forall i \in D_{l'})} \\
& \boxed{\begin{array}{l}
\displaystyle\left|\sum_{j \in L}(w_{aj} + x_{aj}) - \sum_{j \in L}(w_{ij} + x_{ij})\right| \le M\lambda_i \quad (\forall i \in D_{l'}) \\
\displaystyle\left|\sum_{j \in L}(w_{aj} + x_{aj}) - \sum_{j \in L}(w_{ij} + x_{ij})\right| \ge \lambda_i \quad (\forall i \in D_{l'}) \\
\displaystyle\sum_{j \in D_{l'}}(w_{aj} + x_{aj})(1 - \lambda_j) \le \sum_{j \in D_{l'}}(w_{ij} + x_{ij})(1 - \lambda_j) + M\beta_i \quad (\forall i \in D_{l'}) \\
\lambda_i \le \beta_i \quad (\forall i \in D_{l'}), \quad \beta_a = 1
\end{array}} \\
& \displaystyle\sum_{i \in D_{l'}} \alpha_i + \sum_{i \in D_{l'}} \beta_i = 2n_{l'} - k \qquad (\mathsf{C}_{\mathsf{d}}^{k})
\end{array}\right.
$$

The first constraint block does not *logically* include equal signs but the second constraint block does, which means that ties are not allowed in the first criterion but are allowed in the second criterion.

Upper and lower bounds (3/4)



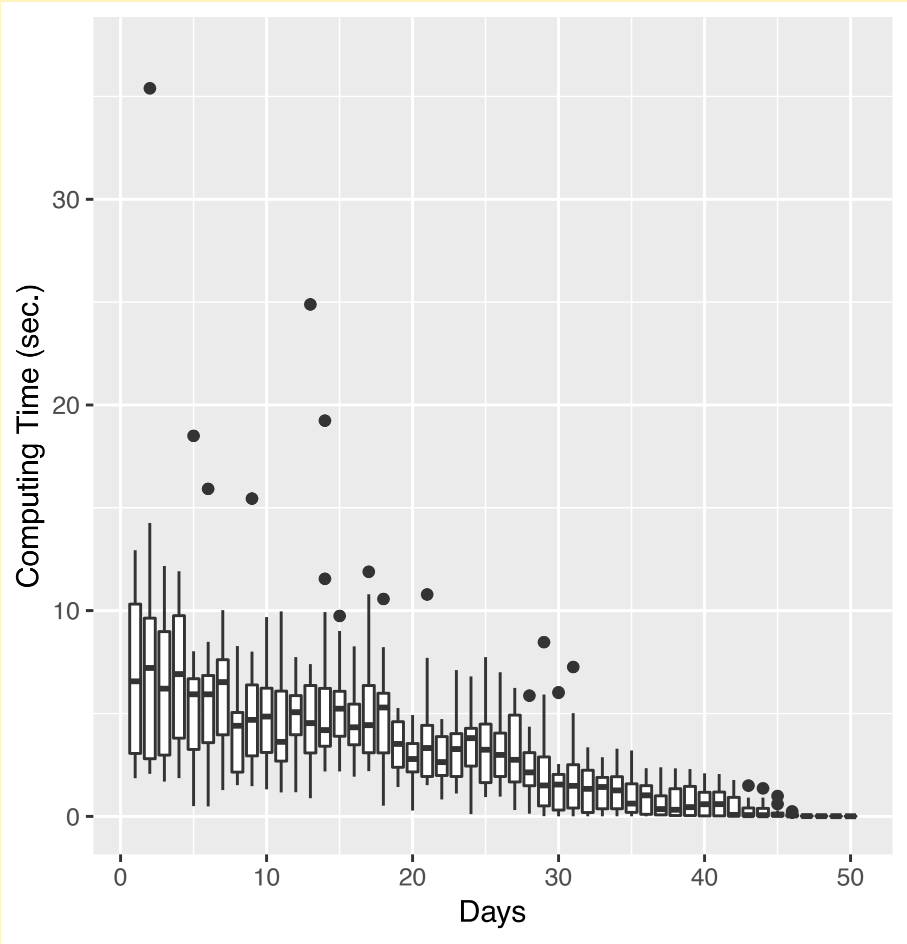
Upper and lower bounds (4/4)

$$\left[\begin{array}{lll} \max\limits_{\substack{x\in X\\ \alpha\in\{0,1\}^{n_{l'}}}} & \sum\limits_{j\in L} x_{aj} & \\ \text{subject to} & \boxed{\sum\limits_{j\in L}(w_{aj}+x_{aj}) \leq \sum\limits_{j\in L}(w_{ij}+x_{ij}) + M\alpha_i \quad (\forall i\in D_{l'})} & \\ & \sum\limits_{i\in D_{l'}} \alpha_i = n_{l'} - k & \end{array}\right. \qquad (\hat{\mathsf{C}}_{\mathsf{d}}^{k})$$

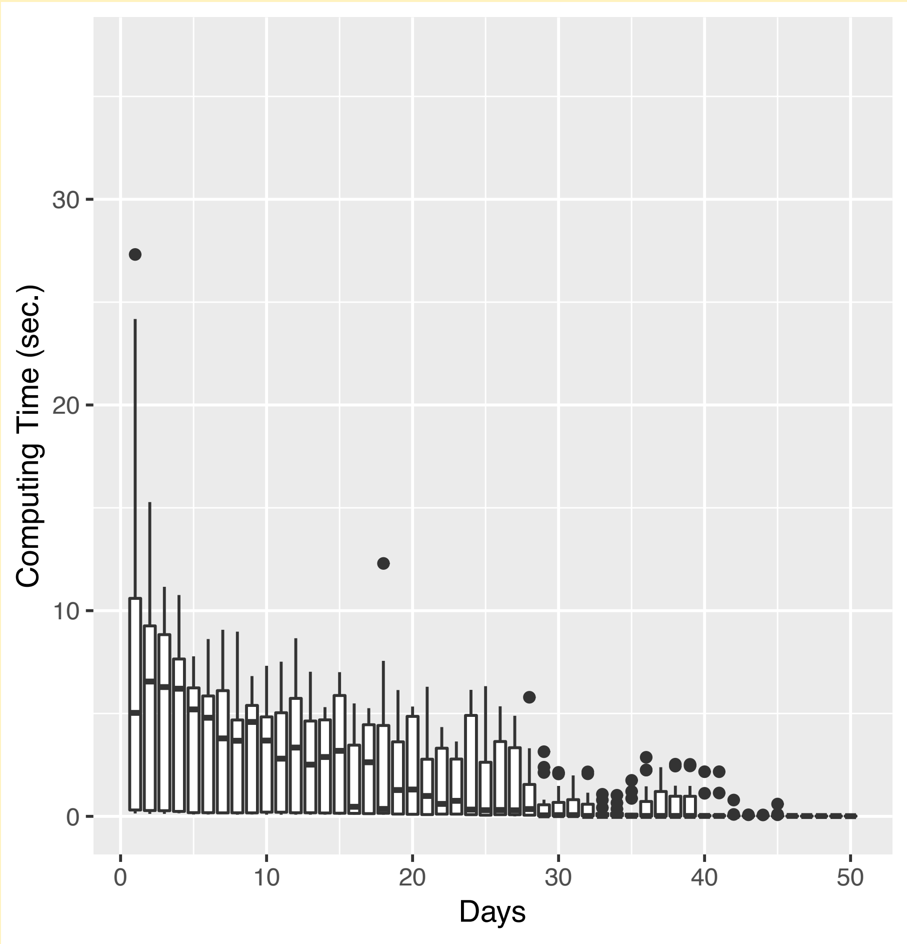
$$\left[\begin{array}{lll} \max\limits_{\substack{x\in X\\ \alpha\in\{0,1\}^{n_{l'}}}} & \sum\limits_{j\in L} x_{aj} & \\ \text{subject to} & \boxed{\sum\limits_{j\in L}(w_{aj}+x_{aj}) \leq \sum\limits_{j\in L}(w_{ij}+x_{ij}) + (M+1)\,\alpha_i - 1 \quad (\forall i\in D_{l'})} & \\ & \sum\limits_{i\in D_{l'}} \alpha_i = n_{l'} - k & \end{array}\right. \qquad (\check{\mathsf{C}}_{\mathsf{d}}^{k})$$

Computational experiments

- B.LEAGUE's **regular season starts in late September or early October and ends in early May.**
- Using **the win-loss records of division B1 during the last 50 game days (starting January 1st) of the 2016–2017 season**, we did several computational experiments.
- All experiments were performed with **SCIP version 6.0.1 and SoPlex 4.0.1 on a 2.8 GHz Intel Core i7 processor.**
- **A Python script** was written to read win-loss records, to generate intermediate ZIMPL files for the optimization models, and to perform algorithms for finding the clinch and elimination numbers based on the results of SCIP calculations.
- **The computation times for championship tournament qualification clinch and elimination numbers are respectively shown as box plots of 18 instances on each day.** (The ends of the whiskers represent the shortest time within 1.5 IQR (interquartile range) of the lower quartile and the longest time within 1.5 IQR of the upper quartile, and any data outside the whisker is plotted as an outlier with a dot.)

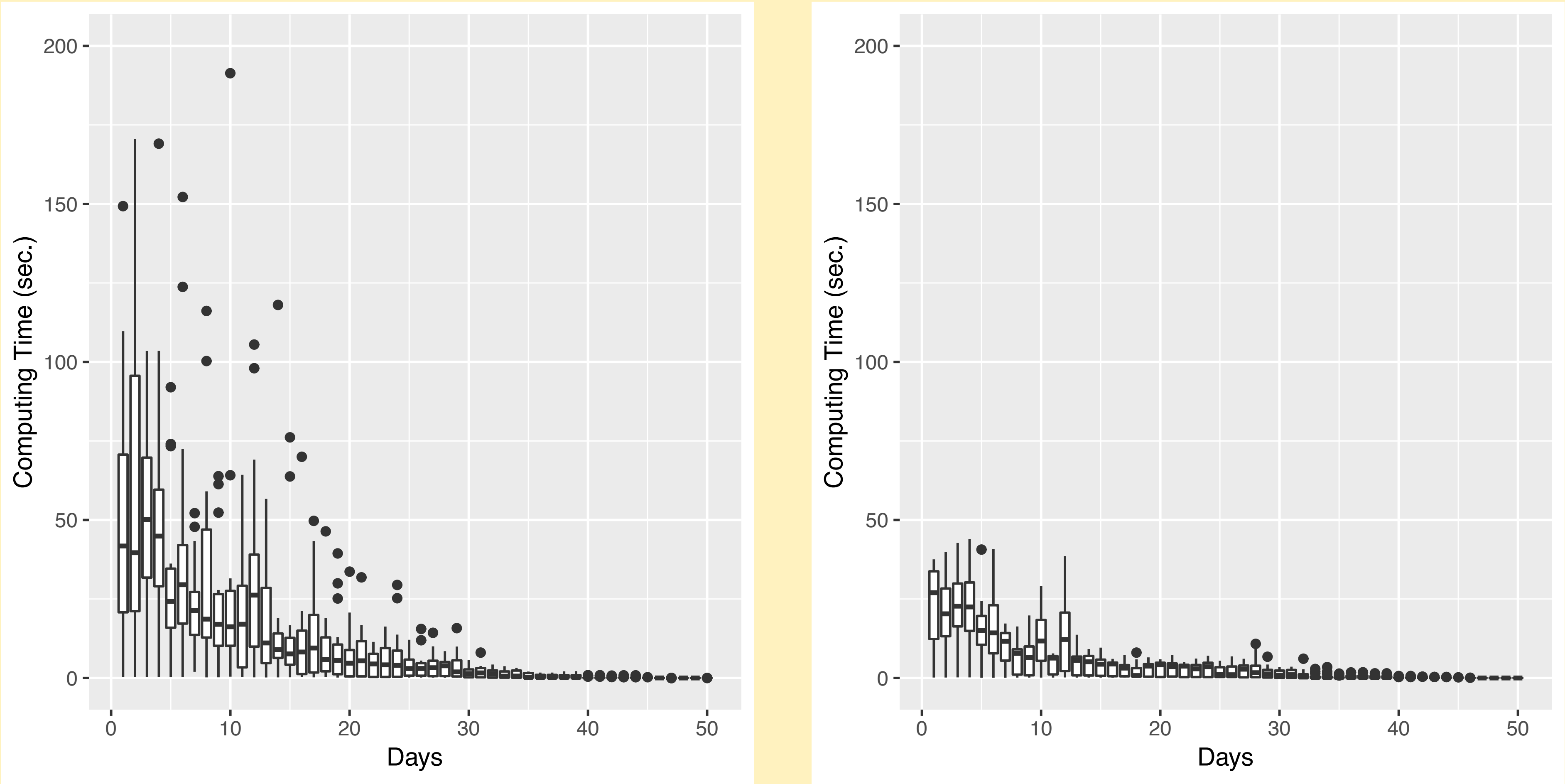


(a) Results from scratch



(b) Results with using upper/lower bounds

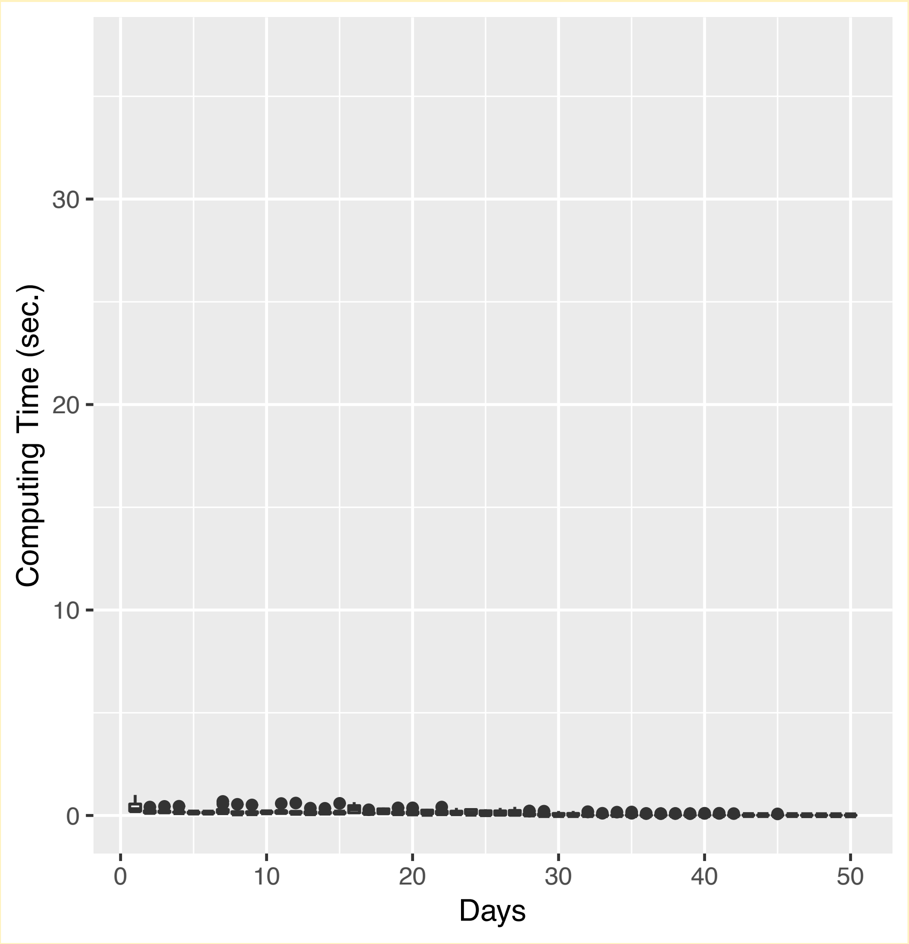
Figure 1: Computation time for championship tournament qualification clinch numbers.



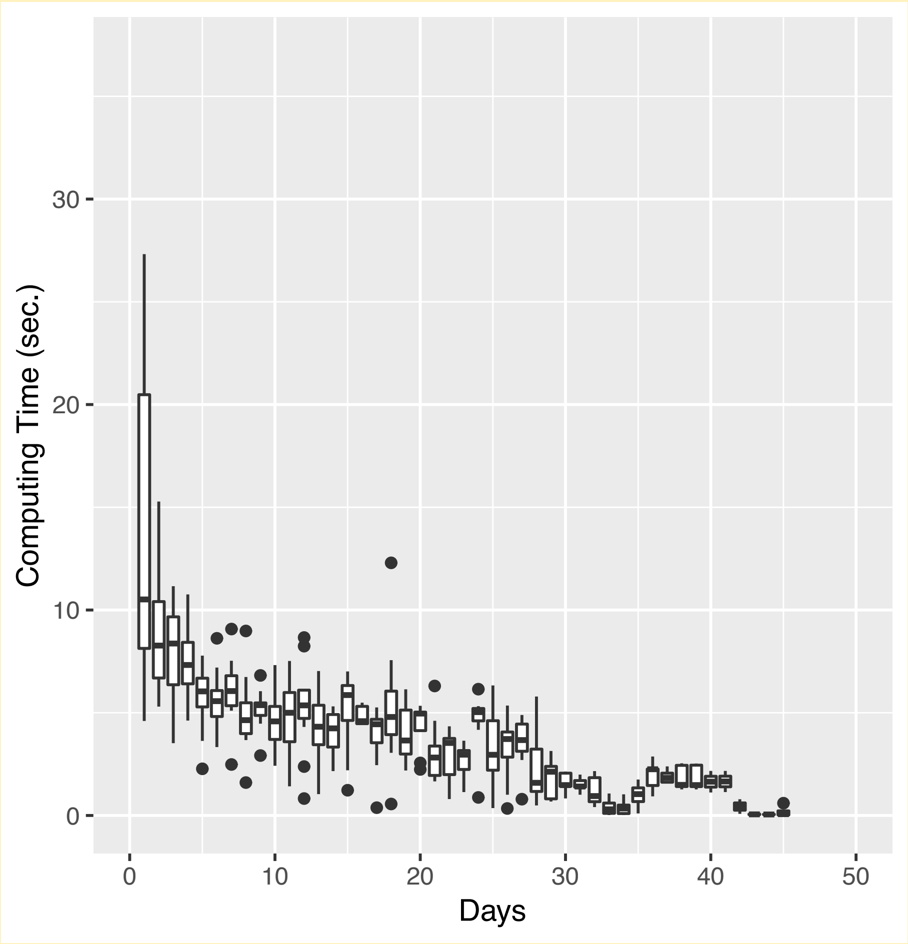
(a) Results from scratch

(b) Results with using upper/lower bounds

Figure 2: Computation time for championship tournament qualification elimination numbers.

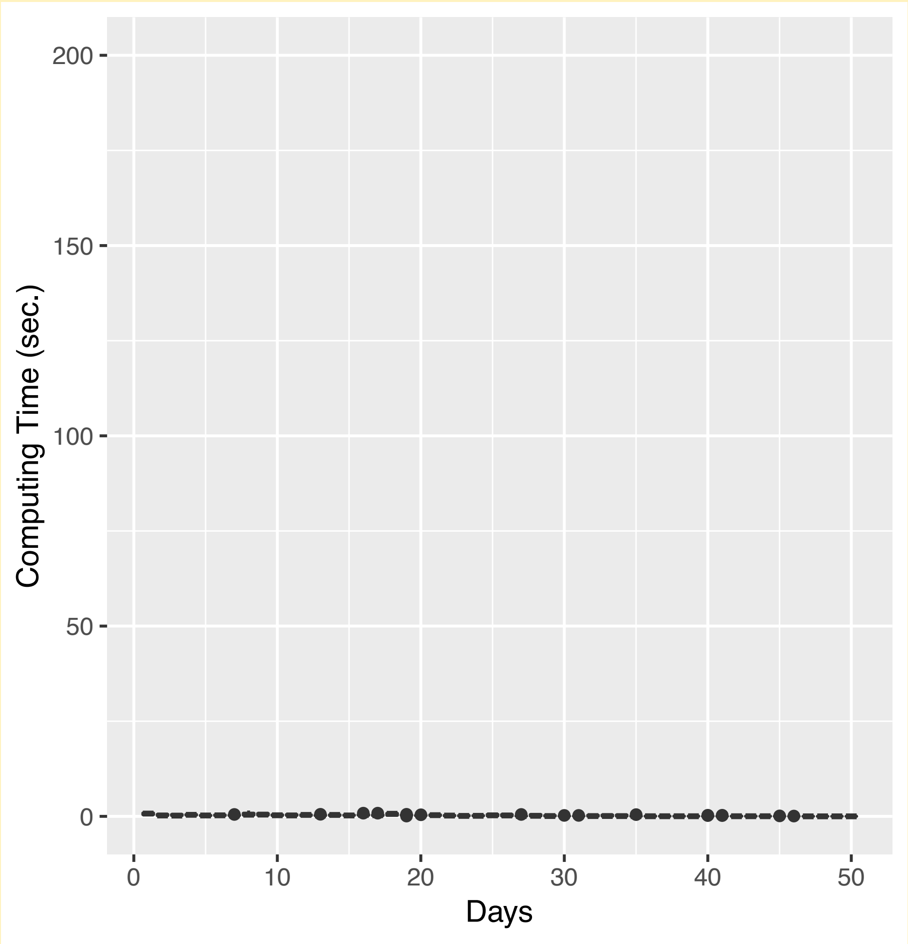


(a) Results for the case without solving (C_c)

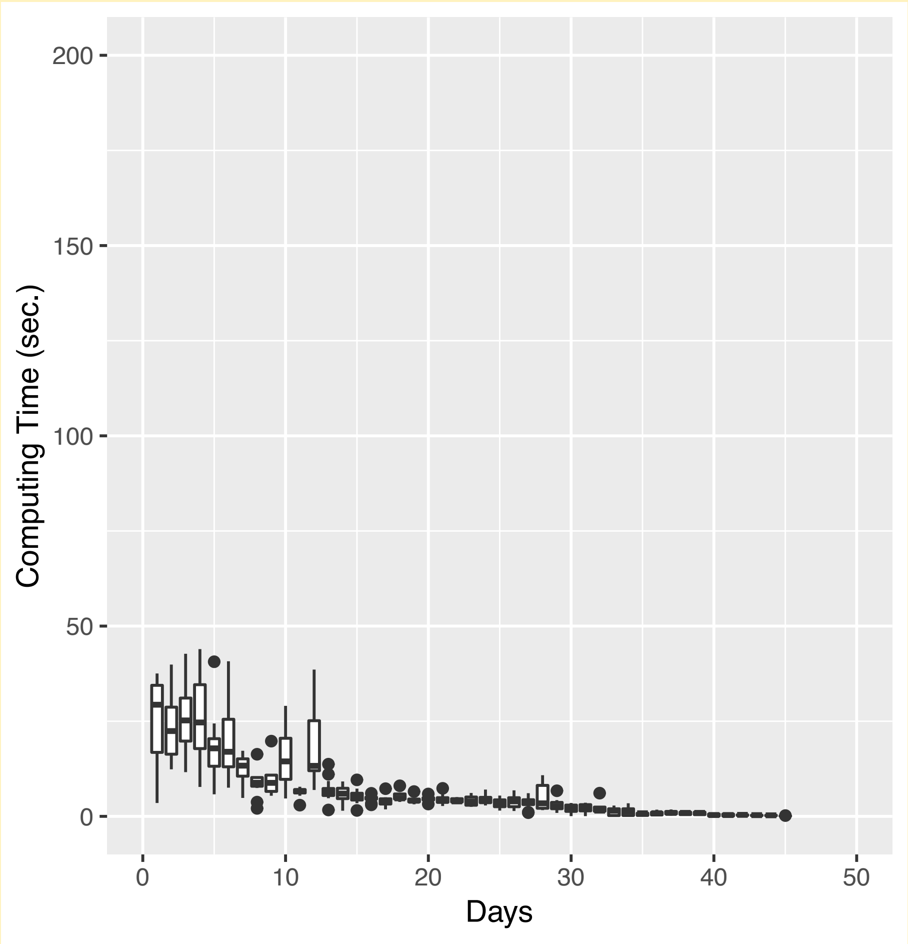


(b) Results for the case with solving (C_c)

Figure 3: Two cases for championship tournament qualification clinch number computations; (a) 549 instances, (b) 351 instances.



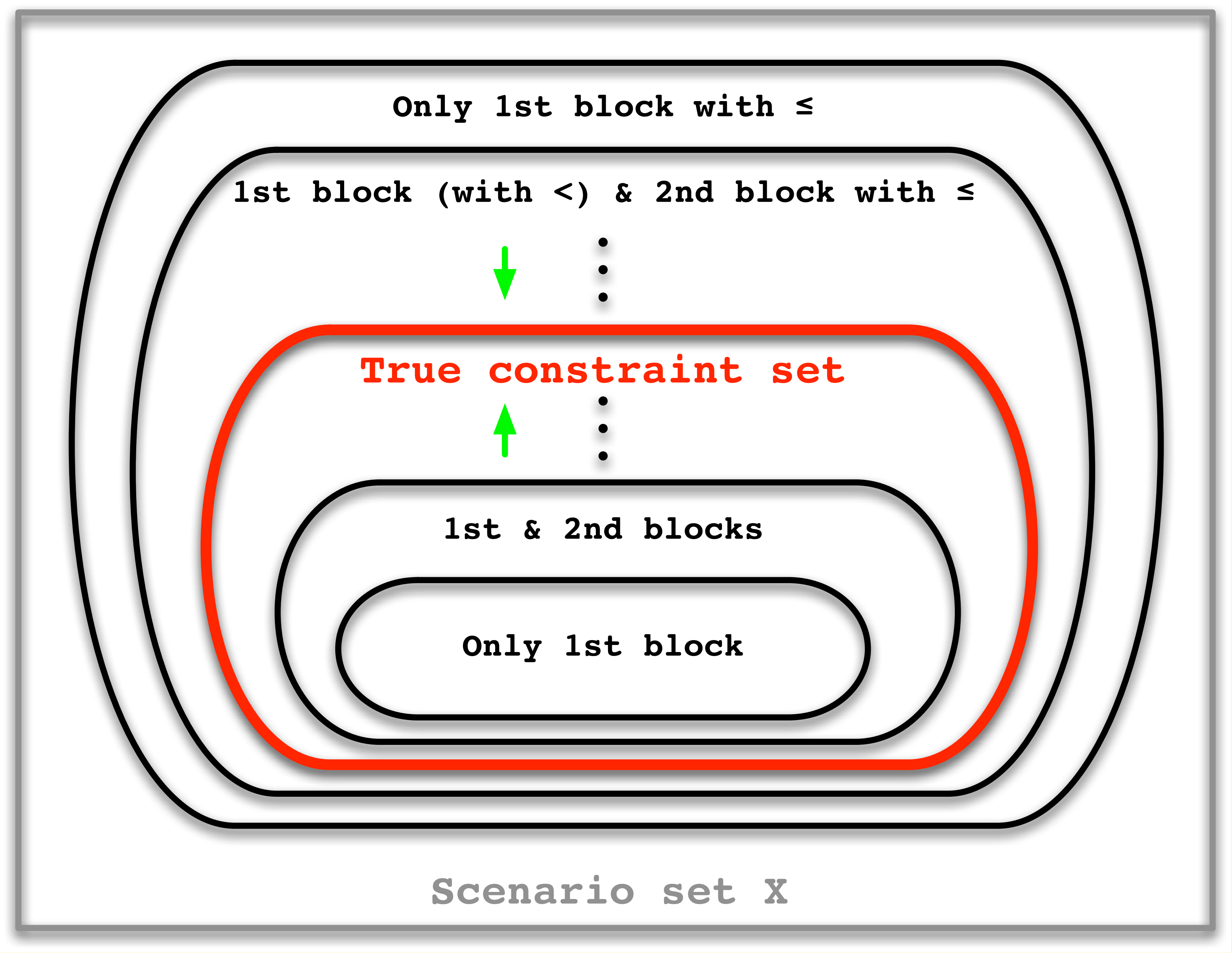
(a) Results for the case without solving (E_w^2)



(b) Results for the case with solving (E_w^2)

Figure 4: Two cases for championship tournament qualification elimination number computations; (a) 450 instances, (b) 450 instances.

Multilayered structure



Ex: k-place C# in NPB (1/2)

$$\left[\begin{array}{ll}
\max\limits_{\substack{x \in X \\ \alpha,\beta,\gamma,\delta \in \{0,1\}^n}} & \sum\limits_{j \in \bar{L}} x_{aj} \\
\text{subject to} & x_{aj} + x_{ja} = g_{aj} \quad \forall j \in L \\
& \boxed{n - \sum\limits_{i \in L} \alpha_i \text{ teams exist st } (\mathrm{C1})_a < (\mathrm{C1})_i} \\
& \boxed{n - \sum\limits_{i \in L} \beta_i \text{ teams exist st } (\mathrm{C1})_a = (\mathrm{C1})_i \text{ and } (\mathrm{C2})_a < (\mathrm{C2})_i} \\
& \boxed{\begin{array}{r} n - \sum\limits_{i \in L} \gamma_i \text{ teams exist st } (\mathrm{C1})_a = (\mathrm{C1})_i,\ (\mathrm{C2})_a = (\mathrm{C2})_i \\ \text{and } (\mathrm{C3})_a < (\mathrm{C3})_i \end{array}} \\
& \boxed{\begin{array}{r} n - \sum\limits_{i \in L} \delta_i \text{ teams exist st } (\mathrm{C1})_a = (\mathrm{C1})_i,\ (\mathrm{C2})_a = (\mathrm{C2})_i \\ (\mathrm{C3})_a = (\mathrm{C3})_i \text{ and } (\mathrm{C4})_a < (\mathrm{C4})_i \end{array}} \\
& \alpha_a = \beta_a = \gamma_a = \delta_a = 1, \quad \sum\limits_{i \in L} (\alpha_i + \beta_i + \gamma_i + \delta_i) = 4n - k
\end{array}\right.$$

Ex: k-place C# in NPB (2/2)

- NPB: Nippon (JP) Professional Baseball, $n = 6$.
- Computed the playoff C$\#$ for $k = 3$ with a full-season win-loss records.
- Without using bounds, about 5% instances takes more than 2 hours.
- When using the outermost (or secondmost) outer problem and the innermost inner problem for upper and lower bounds, we have the results:

  304 secs for 1,134 instances
  Average computing time $=$ 0.268 secs

CPBL（中華職業棒球大聯盟）

CPBL（中華職業棒球大聯盟）

- CPBL (Chinese Professional Baseball League) has currently 4 teams.
- A regular season is divided into 2 half seasons, and each team plays 60 games in each half season (120 games in total for the whole season).
- Teams are ranked after each half season and after the whole season.

- ▼ If the half-season winners are different, then those two winners and the whole-season winner can advance to the playoff round.
- ▼ If the half-season winners are the same, then the half-season winner and those teams which are not ranked the last plae in the whole season can advance to the playoff round.

- Therefore, in order to advance to the playoff round, a team must achieve one of the following:

  1) being ranked the 1st place in any half season,
  2) being ranked the 1st place in the whole season,
  3) not being raked the last place in the whole season when the half-season winners are the same.

Playoff C/E#

- Equal to the 1st place C/E# **during the 1st half season**
- 3 conditions for advancement to the playoff round:

  1) being ranked the 1st place in any half season (P)
  2) being ranked the 1st place in the whole season (Q)
  3) not being raked the last place in the whole season (R) when the half-season winners are the same (S)

- Simple logical relation

$$\overline{P \vee Q \vee (R \wedge S)} = \overline{P} \wedge \overline{Q} \wedge \overline{R \wedge S} = \overline{P} \wedge \overline{Q} \wedge (\overline{R} \vee \overline{S})$$

- 3 conditions for not advancing to the playoff round (**during the 2nd half**):

  1) not being ranked the 1st place in the 2nd half season ($\overline{P}$)
  2) not being ranked the 1st place in the whole season ($\overline{Q}$)
  3) being raked the last place in the whole season ($\overline{R}$) or the 1st-season winner is not the 1st place in the 2nd half season ($\overline{S}$)

1st-place C# during the 1st half

$$
\left[\begin{array}{ll}
\displaystyle\max_{\substack{x' \in X' \\ \alpha \in \{0,1\}^4}} & \displaystyle\sum_{j \in L} x'_{aj} \\
\text{subj. to} & x'_{aj} + x'_{ja} = g'_{aj} \quad (\forall j \in L) \\
& \dfrac{\displaystyle\sum_{j \in L} w'_{aj} + x'_{aj}}{\displaystyle\sum_{j \in L} w'_{aj} + w'_{ja} + g'_{aj}} \leq \dfrac{\displaystyle\sum_{j \in L} w'_{ij} + x'_{ij}}{\displaystyle\sum_{j \in L} w'_{ij} + w'_{ji} + x'_{ij} + x'_{ji}} + \mathcal{M}' \alpha_i - \epsilon' \quad (\forall i \in L) \\
& \displaystyle\sum_{i=1}^{4} \alpha_i = 3, \quad \alpha_a = 1
\end{array}\right.
$$

where ϵ' is a small positive number less than $\big(M'(M'+1)\big)^{-1}$, typically $\epsilon' = \big(2\,M'(M'+1)\big)^{-1}$, and $\mathcal{M}'$ is a big positive number greater than $1 + \epsilon'$.

The inequality above realizes the condition that team i has a higher winning percentage than team a when $\alpha_i = 0$.

Playoff C# during the 2nd half

$$\max_{\substack{x'' \in X'' \\ \alpha,\beta,\gamma,\delta \in \{0,1\}^4 \\ \zeta \in \{0,1\}}} \sum_{j \in L} x''_{aj}$$

subj. to $x''_{aj} + x''_{ja} = g''_{aj} \quad (\forall j \in L)$

$$\frac{\sum_{j\in L} w''_{aj} + x''_{aj}}{\sum_{j\in L} w''_{aj} + w''_{ja} + g''_{aj}} \leq \frac{\sum_{j\in L} w''_{ij} + x''_{ij}}{\sum_{j\in L} w''_{ij} + w''_{ji} + x''_{ij} + x''_{ji}} + \mathcal{M}''\alpha_i - \epsilon'' \quad (\forall i \in L)$$

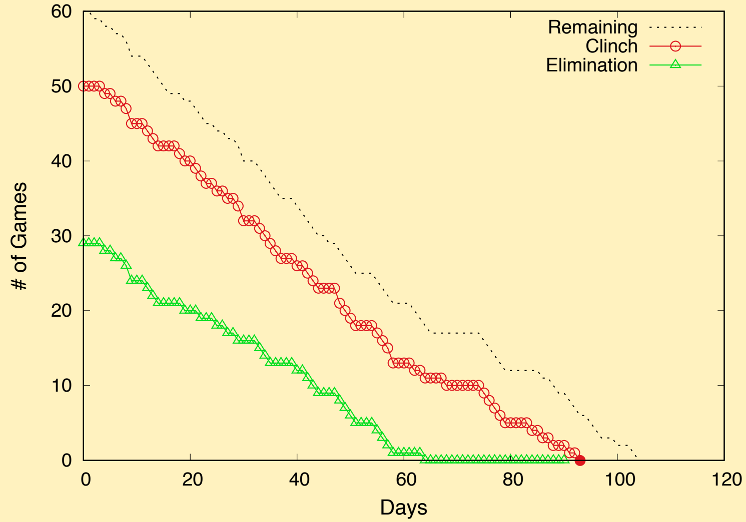
$$\frac{\sum_{j\in L} w_{aj} + x''_{aj}}{\sum_{j\in L} w_{aj} + w_{ja} + g''_{aj}} \leq \frac{\sum_{j\in L} w_{ij} + x''_{ij}}{\sum_{j\in L} w_{ij} + w_{ji} + x''_{ij} + x''_{ji}} + \mathcal{M}\beta_i - \epsilon \quad (\forall i \in L)$$

$$\zeta \frac{\sum_{j\in L} w''_{bj} + x''_{bj}}{\sum_{j\in L} w''_{bj} + w''_{jb} + x''_{bj} + x''_{jb}} \leq \zeta \left(\frac{\sum_{j\in L} w''_{ij} + x''_{ij}}{\sum_{j\in L} w''_{ij} + w''_{ji} + x''_{ij} + x''_{ji}} + \mathcal{M}''\gamma_i - \epsilon'' \right) \quad (\forall i \in L)$$

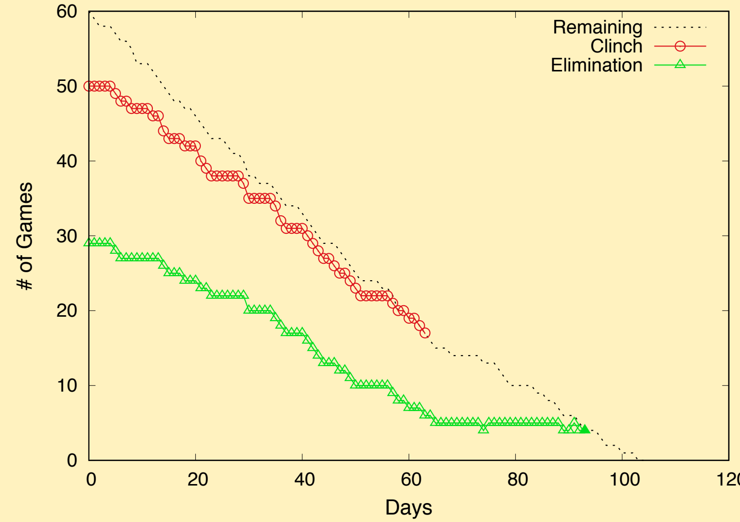
$$(1-\zeta) \frac{\sum_{j\in L} w_{aj} + x''_{aj}}{\sum_{j\in L} w_{aj} + w_{ja} + g''_{aj}} \leq (1-\zeta) \left(\frac{\sum_{j\in L} w_{ij} + x''_{ij}}{\sum_{j\in L} w_{ij} + w_{ji} + x''_{ij} + x''_{ji}} + \mathcal{M}\delta_i - \epsilon \right) \quad (\forall i \in L)$$

$$\sum_{i=1}^{4} \alpha_i = 3, \quad \sum_{i=1}^{4} \beta_i = 3, \quad \sum_{i=1}^{4} \gamma_i = 3, \quad \sum_{i=1}^{4} \delta_i = 1, \quad \alpha_a = \beta_a = \gamma_b = \delta_a = 1$$

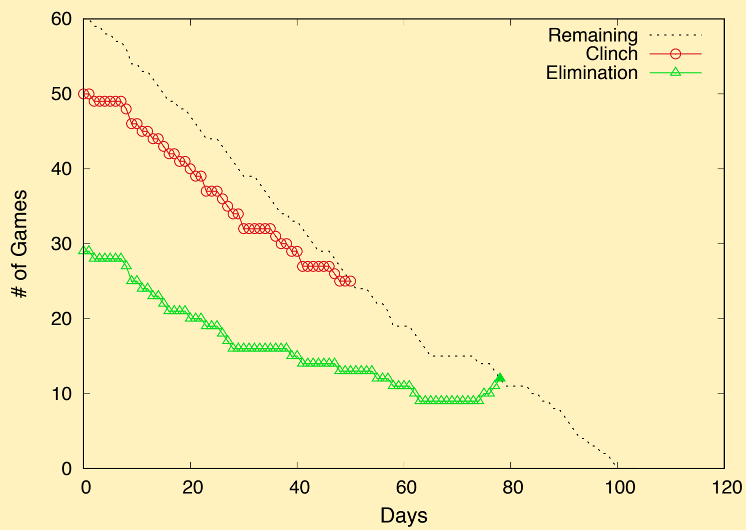
1st place C/E# during the 1st half



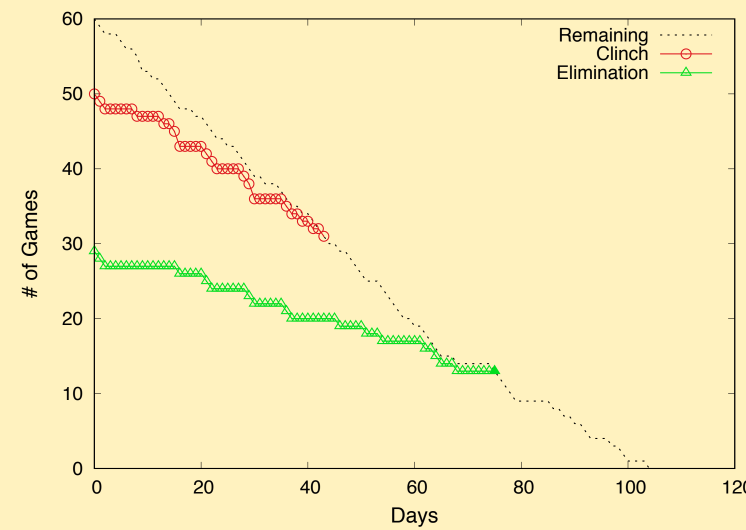
(a) Lamigo Monkeys



(b) Chinatrust Brothers



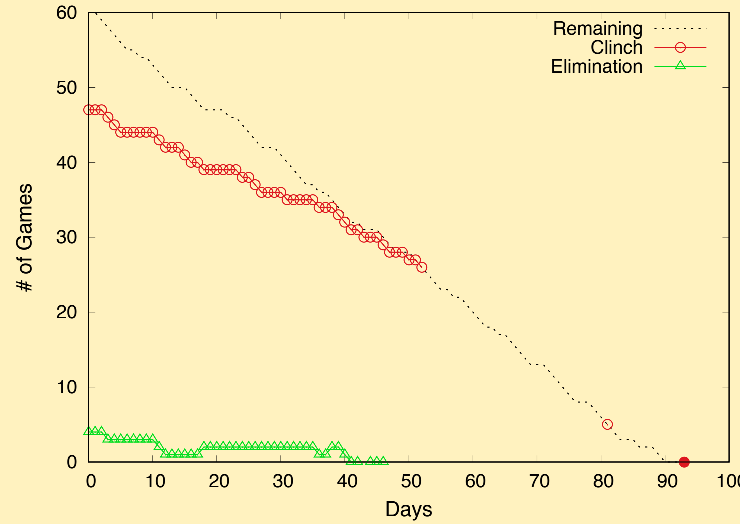
(c) Uni-President Lions



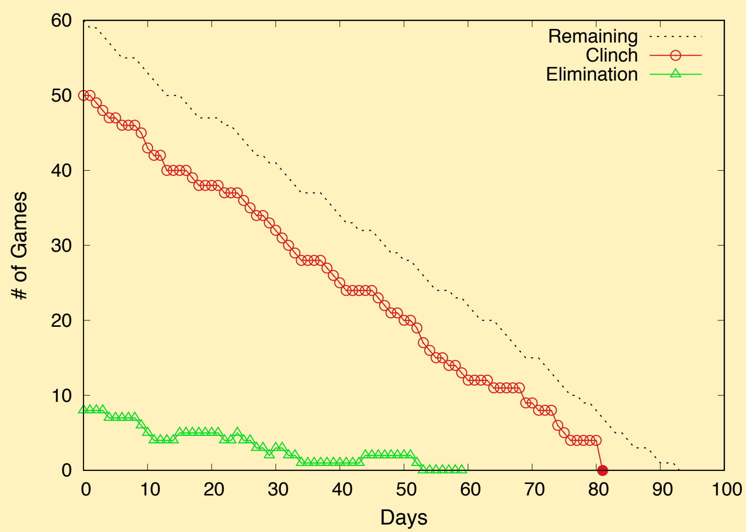
(d) Fubon Guardians

Playoff C/E# during the 2nd half

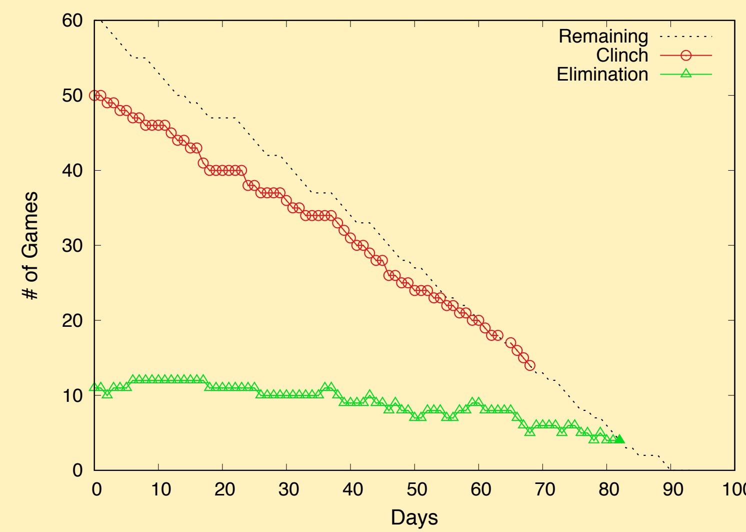
(a) Lamigo Monkeys



(b) Chinatrust Brothers



(c) Uni-President Lions



(d) Fubon Guardians

Concluding remarks

Concluding remarks

- We have formulated optimization models for calculating C/E numbers in the presence of predefined tiebreaking rules and winning percentage.
- The former affetcs the computing cost more significantly.
- Exploiting (multi-layered) upper and lower bounds seems to be effective in shortening the computation time of extremely time-consuming cases.
- Many sports leagues have tiebreking criteria that are not based on simple win-loss records.
- ▼ It is natural not to take such criteria into consideration in general except at the very last stage of the season.
- ▼ With another lower bound, one can check if there is a chance of clinching (or being eliminated) by such criteria that are not based on win-loss records.