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Probabilistic prediction of solar power supply to distribution networks, using global radiation forecasts

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Outline

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Risk in feed-in of solar power

Motivation

- Increase in solar plants $\rightarrow$ voltage violations and overloading problems

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Timeframe: May, June and July of the years 2015-2017 (11-12 UTC)

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Goal

Predict the risk of solar power supply exceeding critical thresholds

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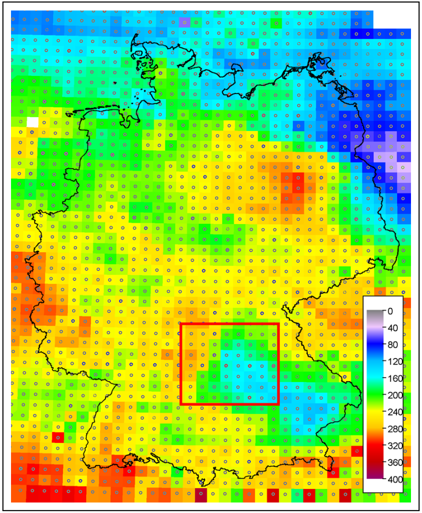
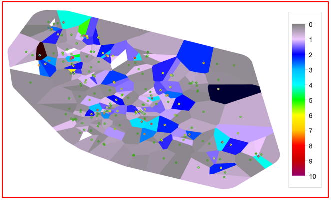
Visualization of data



Global radiation forecast (in J/cm^2) for July 07, 2017 11-12 UTC



Measured solar power supply (in *MW*) for July 07, 2017 11-12 UTC

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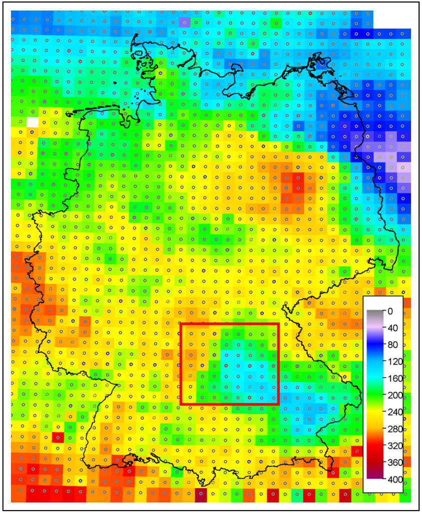

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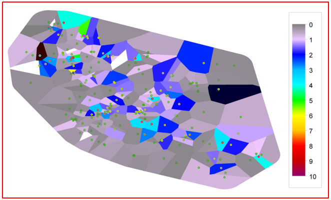

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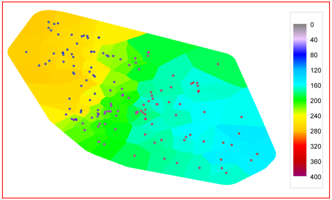

Interpolated global radiation forecast
for July 07, 2017 11-12 UTC

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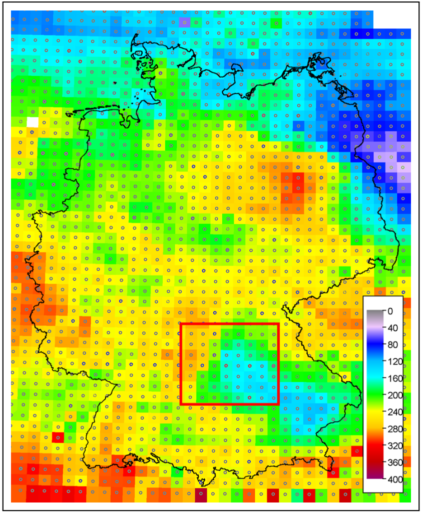

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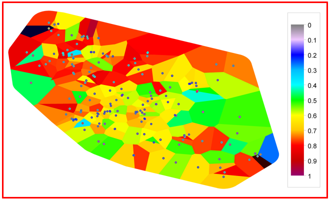
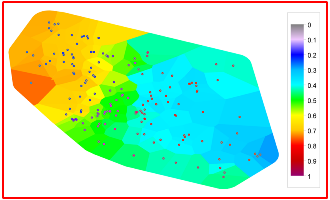


Normalized solar power supply for July 07, 2017 11-12 UTC



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Copula models

Random variables

R: (Normalized) global radiation forecast
S: (Normalized) solar power supply

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Goals

For a predefined threshold v and feed-in points $p_1, \ldots, p_n$ compute the conditional probabilities

- $P(S_1 \geq v \mid R_1 = r(p_1, t))$
- $P(S_1 + \ldots + S_n \geq v \mid R_1 = r(p_1, t), \ldots, R_n = r(p_n, t))$

given global radiation forecasts $r(p_1, t), \ldots, r(p_n, t)$ and forecast time t

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Modeling approach

- Fit univariate marginal distributions
- Fit bivariate and multivariate distributions using bivariate copulas and D-vine copulas

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Copula theory

Bivariate copulas

A bivariate copula is the joint distribution function $C : [0,1] \times [0,1] \to [0,1]$ of a 2-dimensional random vector (U, V) with components U and V uniformly distributed on $[0,1]$

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Theorem of Sklar

Let (R, S) be a 2-dimensional random vector with joint distribution function $F_{(R,S)} : \mathbb{R}^2 \to [0,1]$ and marginal distribution functions F_R and F_S. Then, a bivariate copula function $C : [0,1] \times [0,1] \to [0,1]$ exists such that

$$F_{(R,S)}(r,s) = C(F_R(r), F_S(s)) \qquad \text{for all } r, s \in \mathbb{R}$$

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Differential form of Sklar's theorem

For the density functions $f_{(R,S)}$, f_R, f_S and c it holds that

$$f_{(R,S)}(r,s) = f_R(r) \cdot f_S(s) \cdot c(F_R(r), F_S(s)) \qquad \text{for all } r, s \in \mathbb{R}$$

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Definition of Archimedean copulas

Archimedean generator

A function $g : [0, 1] \to [0, \infty]$ is called Archimedean generator if g is continuous, strictly decreasing and solves $g(1) = 0$.

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Pseudo-inverse

The pseudo-inverse $g^{[-1]}$ of an Archimedean generator g is an extension of the inverse function $g^{(-1)}$ defined as

$$g^{[-1]}(t) = \begin{cases} g^{(-1)}(t), & \text{if } 0 \leq t \leq g(0) \\ 0, & \text{if } g(0) \leq t \leq \infty. \end{cases}$$

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Arichmedian copula

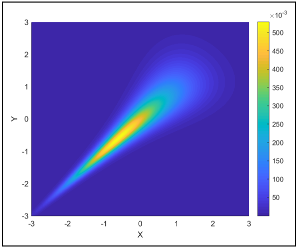
The Arichmedian copula generated by g is given by

$$C(u,v) = g^{[-1]}(g(u) + g(v)) \qquad u, v \in [0,1].$$

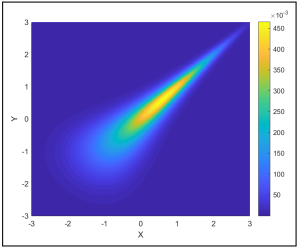
Examples of Archimedean copulas

copula family	Archimedean generator	parameter
Joe	$g(t) = -\log(1-(1-t)^{\theta})$	$\theta \in [1,\infty]$
Frank	$g(t) = (-\log(\frac{\exp(-\theta t-1)}{\exp(-\theta)-1}))^{\theta}$	$\theta \in \mathbb{R}\setminus\{0\}$
Clayton	$g(t) = \frac{1}{\theta}(t^{-\theta}-1)$	$\theta \in [-1,\infty)\setminus\{0\}$
Gumbel	$g(t) = (-\log(t))^{\theta}$	$\theta \in [1,\infty)$

Visualization of the copula types



Clayton with parameter $\theta = 5$



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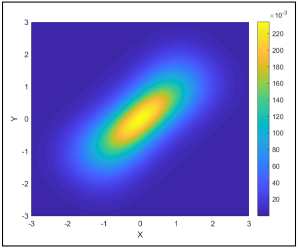
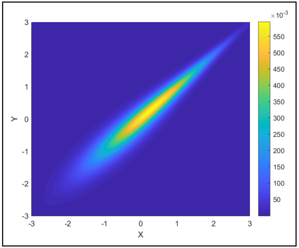


Frank with parameter $\theta = 5$



Gumbel with parameter $\theta = 5$

Outline

Fitting of univariate marginal distributions

- Fit mixed beta densities $f(x) = qf_1(x) + (1-q)f_2(x)$ with mixture parameter $q \in [0,1]$ and beta densities $f_i : (0,1) \to [0,\infty)$ with $f_i(x) = \frac{\Gamma(a_i+b_i)}{\Gamma(a_i)\Gamma(b_i)} x^{a_i-1}(1-x)^{b_i-1}$ and two parameters $a_i, b_i > 0$

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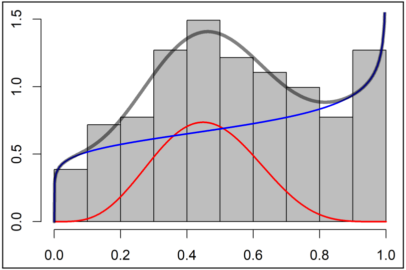
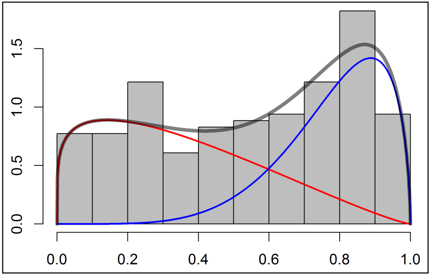


Global radiation forecast data



Solar power supply data

Fitting bivariate copulas

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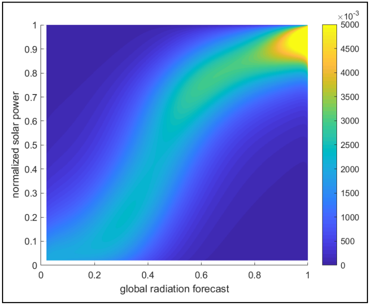

Fitted bivariate joint density

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Computation of conditional probabilities

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- Based on Sklar's theorem compute the conditional density
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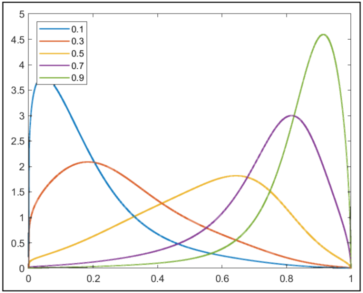

Conditional densities given the global radiation forecasts r for $r = 0.1, 0.3, 0.5, 0.7$ and 0.9

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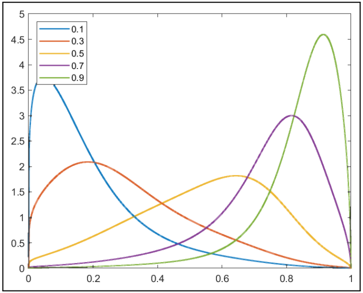
  $P(v, r) = P(S \geq v \mid R = r) = \int_v^1 f_S(s \mid R = r)\mathrm{d}s$



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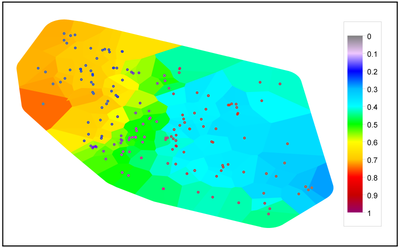
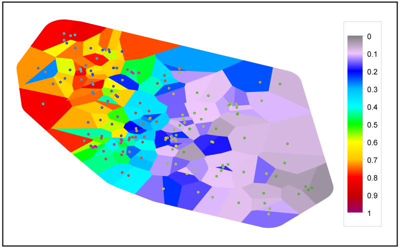
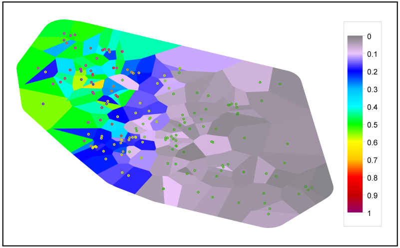
Conditional probabilities for a critical event



Normalized global radiation forecast



Conditional probabilities for threshold $v = 0.7$



Conditional probabilities for threshold $v = 0.8$

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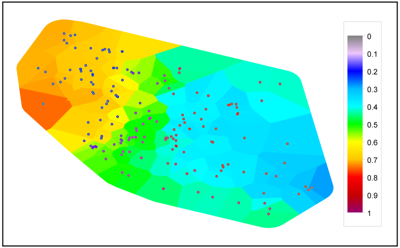

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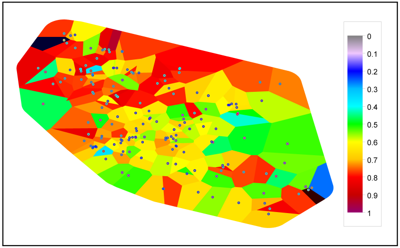
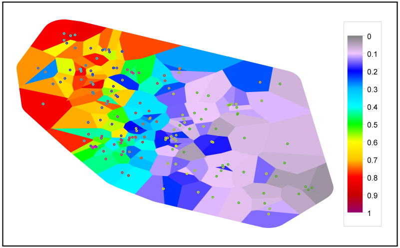


Normalized solar power supply



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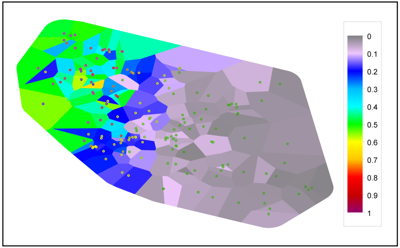

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Vine copulas

Multivariate copulas

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Vine copulas as families of trees

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Vine copulas as families of trees

We can interpret a vine copula as a family of trees, where

- each edge is a conditional bivariate copula
- each node is a conditional cumulative distribution function

R-vine

A regular vine (short: R-vine) V on n elements is a family of trees $\{\mathcal{T}_1, \ldots, \mathcal{T}_{n-1}\}$ with edges $E(V) = E_1 \cup \ldots \cup E_{n-1}$, such that

1. $\mathcal{T}_1 = (N_1, E_1)$ is a connected tree with nodes $N_1 = \{1, \ldots, n\}$ and edges E_1
2. $\mathcal{T}_k$ is a tree with nods $N_k = E_{k-1}$ for all $k \in \{2, \ldots, n-1\}$
3. $\#(e_1 \Delta e_2) = 2$ for all $\{e_1, e_2\} \in E_k$ with $k \in \{2, \ldots, n-1\}$

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R-vine decomposition

The decomposition of a n-dim. density $f_{1,\ldots,n}$ corresponding to an R-vine V with edges $E(V)$ is given by

$$f_{1,\ldots,n} = \prod_{e \in E(V)} c_{t_1,t_2|S(e)}(F_{t_1|S(e)}, F_{t_2|S(e)}) \cdot \prod_{j=1}^{n} f_j,$$

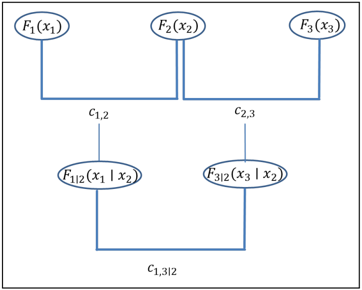
where $S(e)$ is the so-called conditioning set, $T(e) = \{t_1, t_2\}$ is the conditioned set of the edge e and f_j are the one-dim. marginal densities

Outline

D-vine copulas

D-vines

- D-vines are a special type of R-vine



D-vine structure for 3 random variables

D-vine copulas

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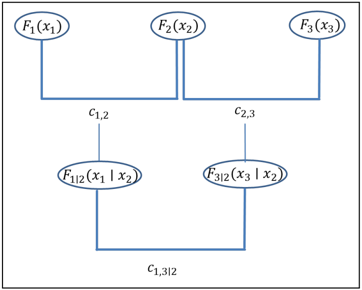
- Each node is connected to not more than 2 edges



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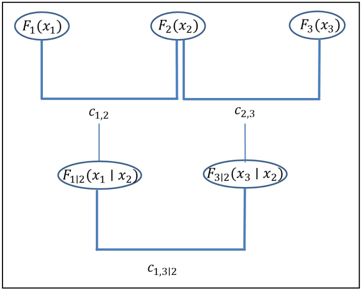
- For each edge in $E_k = \{e_1, \ldots, e_{n-k}\}$ the conditioned set is $T(e_i) = \{i, i+k\}$ and the conditioning set is $S(e_i) = \{i+1, \ldots, i+k-1\}$



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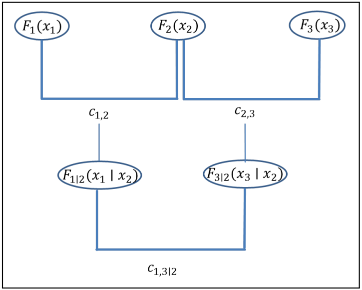
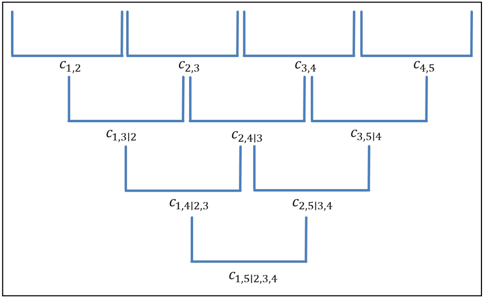


D-vine structure for 3 random variables

For 3-dim. densities a D-vine corresponds to following decomposition:

$$\begin{aligned} f_{1,2,3}(x_1, x_2, x_3) &= f_{3|1,2}(x_3 \mid x_1, x_2) f_{2|1}(x_2 \mid x_1) f_1(x_1) \\ &= c_{1,3|2}(F_{1|2}(x_1 \mid x_2), F_{3|2}(x_3 \mid x_2)) f_{3|2}(x_3 \mid x_2) f_{2|1}(x_2 \mid x_1) f_1(x_1) \\ &= c_{1,3|2}(F_{1|2}(x_1 \mid x_2), F_{3|2}(x_3 \mid x_2)) c_{2,3}(F_2(x_2), F_3(x_3)) f_3(x_3) \\ &\quad c_{1,2}(F_1(x_1), F_2(x_2)) f_2(x_2) f_1(x_1) \end{aligned}$$

D-vine copulas



D-vine structure for 5 random variables

The decomposition of an *n*-dim. density corresponding to D-vines is:

$$f_{1,\dots,n}(x_1,\dots,x_n) = \prod_{k=1}^{n-1}\prod_{i=1}^{n-k} c_{i,i+k|i+1,\dots,i+k-1}\big(F_{i|i+1,\dots,i+k-1}(x_i \mid x_{i+1},\dots,x_{i+k-1}),$$

$$F_{i+k|i+1,\dots,i+k-1}(x_{i+k} \mid x_{i+1},\dots,x_{i+k-1})\big) \cdot \prod_{j=1}^{n} f_j(x_j)$$

Outline

Fitting of vine copulas

Sequential estimation

The following steps are applied, starting with the first row:

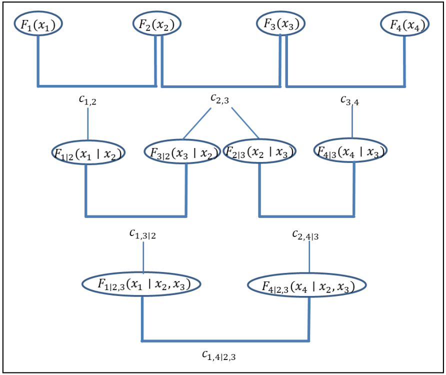
Step 0: Fit the marginal cdfs

Step 1: Transform the data based on the computed cdfs

Step 2: Fit bivariate copulas to the transformed data

Step 3: Compute conditional cdfs using the bivariate copulas

Step 4: Repeat Step 1-3 till the end



Fitting a D-vine with 4 random variables

Fitting of vine copulas

Sequential estimation

The following steps are applied, starting with the first row:

Step 0: Fit the marginal cdfs

Step 1: Transform the data based on the computed cdfs

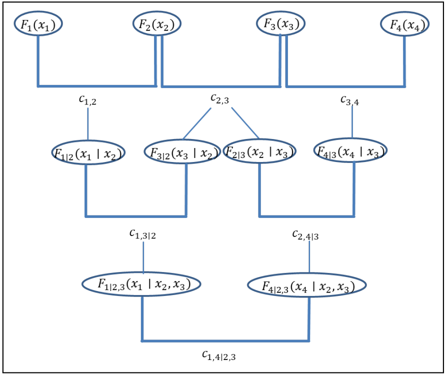
Step 2: Fit bivariate copulas to the transformed data

Step 3: Compute conditional cdfs using the bivariate copulas

Step 4: Repeat Step 1-3 till the end

Fitting of bivariate copulas

Apply ML estimation to fit one-parametric Archimedean copulas



Fitting a D-vine with 4 random variables

Outline

Compute conditional probabilities

Goal

For a predefined threshold v and feed-in points $p_1, \ldots, p_n$ compute the conditional probabilities $P(S_1 + \ldots + S_n \geq v \mid R_1 = r(p_1, t), \ldots, R_n = r(p_n, t))$ given global radiation forecasts $r(p_1, t), \ldots, r(p_n, t)$ and forecast time t

Application of D-vine copulas

Fit an $n+1$-dim. D-vine copula to the random vector $(R_1, \ldots, R_n, S_1 + \ldots + S_n)$

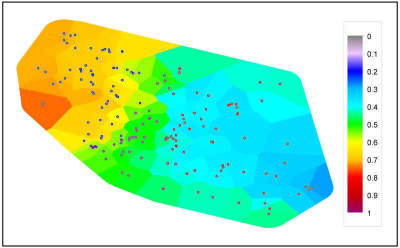
Computation of conditional probabilities

Based on the fitted D-vine copula we compute

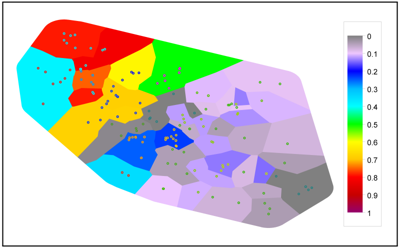
$$P(S_1 + \ldots + S_n \geq v \mid R_1 = r(p_1, t), \ldots, R_n = r(p_n, t)) =$$

$$\int_v^1 c_{1,n+1|2,\ldots,n}(F_{1|2,\ldots,n}(r(p_1,t) \mid r(p_2,t), \ldots, r(p_n,t)), F_{n+1|2,\ldots,n}(s \mid r(p_1,t), \ldots, r(p_{n-1},t)))\mathrm{d}s$$

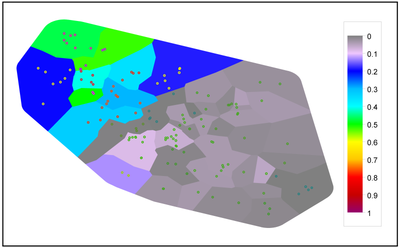
Conditional probabilities calculated by multivariate D-vines



Normalized global radiation forecast

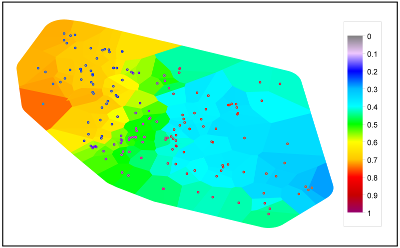


Conditional probabilities for threshold $v = 0.7$

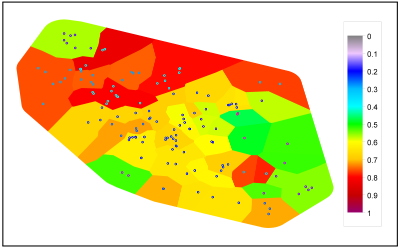


Conditional probabilities for threshold $v = 0.8$

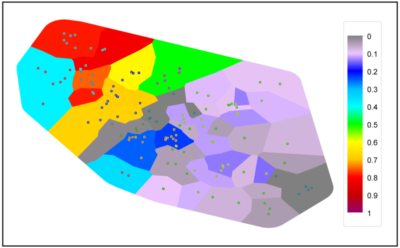
Conditional probabilities calculated by multivariate D-vines



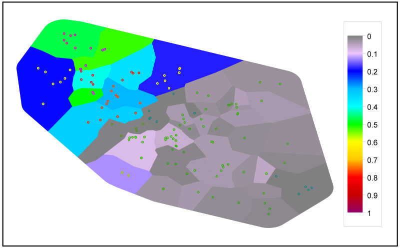
Normalized global radiation forecast



Normalized aggregated solar power supply



Conditional probabilities for threshold $v = 0.7$



Conditional probabilities for threshold $v = 0.8$

Literature

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