The Multi-Year Non-Life Insurance Risk

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Abstract: The aim of this paper is to extend recent contributions in the field of risk modeling for non-life insurance companies by modeling insurance risk in a multi-year context. Academic literature on non-life insurance risk to date has only considered an ultimo perspective (using traditional methods) and, more recently, a one-year perspective (for solvency purposes). This paper is motivated by the fact that strategic management in an insurance company requires a multi-year time horizon for economic decision making, e.g., in the context of internal risk models. We extend the simulation-based method for quantifying the one-year non-life insurance risk presented in Ohlsson and Lauzeningks (2009) to a multi-year perspective. Moreover, we present a simulation approach for calculating the risk margin which can be consistently integrated in the model so that approximation approaches are no longer needed. The usefulness of the new multi-year horizon is illustrated in the context of internal risk models using an application to a claims development triangle based on Mack (1993) and England and Verrall (2006).

Keywords: Non-Life Insurance; Internal Risk Models; Claims Reserving; Risk Capital

1. Introduction

Typically, non-life insurance risk is divided into reserve risk and premium risk (see Ohlsson and Lauzeningks, 2009). For the modeling of reserve risk, the academic literature contains a variety of stochastic claims reserving methods that can be used for quantifying the risk on an ultimo view, including bootstrapping methods, regression approaches, and Bayesian techniques (see, e.g., England and Verrall 2002, 2006; Wüthrich and Merz, 2008).[1] Some of them are analytical (see, e.g., Buchwalder et al., 2006), while others are based on simulations (see, e.g., England and Verrall, 2006). Traditionally, all approaches are based on an ultimo view, which means reserve risk uncertainty is quantified up to final settlement. Recently,

[1] The literature on stochastic claims reserving methods based on an ultimo view is extensive. Schmidt (2011) provides a bibliography of more than 700 contributions in the field of loss reserving. Wüthrich and Merz (2008) summarize the most important methods.

however, regulators have required a one-year perspective of non-life insurance risk for solvency purposes, e.g., in Solvency II and the Swiss Solvency Test (see Eling et al., 2009), which means insurance risk uncertainty should be quantified for one calendar year only. This requirement has spurred a great deal of discussion by both academics and practitioners as to how one-year insurance risk should be quantified, and a new stream of literature has developed over a very short period (see Merz and Wüthrich, 2007, 2008; Bühlmann et al., 2009; Ohlsson and Lauzeningks, 2009; Wüthrich et al., 2009; Gault et al., 2010).

Merz and Wüthrich (2008) present an analytical approach for calculating the mean squared error of prediction (MSEP) of the claims development result (CDR) on a one-year perspective, i.e., for the next calendar year. However, especially due to the need for simulated cash flows of a future claims settlement process within internal risk models, analytical approaches need to be complemented by simulation-based approaches. Ohlsson and Lauzeningks (2009) describe such a simulation-based method – which we call stochastic re-reserving – for quantifying the one-year reserve risk. While Ohlsson and Lauzeningks (2009) give a general description for the process of simulating the one-year reserve risk, Kraus and Diers (2010) utilize this description and give a concrete mathematical formulation of the stochastic re-reserving process based on bootstrap methods and Bayesian techniques.

Both the one-year and the ultimo view are relevant and helpful in understanding the nature of reserve risk in non-life insurance. From a practitioner's point of view, however, a multi-year (m-year) time horizon (which means that reserve risk uncertainty is quantified up to calendar year m) is relevant for practical decision making and both these two approaches do not provide this view. The first considers only one year, while the second summarizes uncertainty over the whole projection horizon. To our knowledge, there is no model for analyzing multi-year non-life insurance risk. An internal risk model with a multi-year view might be useful, e.g., to calculate the necessary risk capital to cover those risks.

The aim of this paper is thus to develop a simulation-based model for the determination of multi-year risk capital based on the multi-year non-life insurance risk. Our model can be used to calculate both the reserve risk and the premium risk and we emphasize its use especially in the context of internal risk models. For example, by using these models, management will be able to answer the following important question: How many years of high aggregate losses or adverse claim developments is it possible to withstand at a certain confidence level without the need for external capital? We empirically illustrate the usefulness of our model using a claims development triangle that has been considered in academic literature several times (e.g., Mack, 1993; England and Verrall, 2006). We also address another recent related discussion, i.e., how to calculate the risk margin in a multi-year context and present an integrated simulation approach for the calculation of the risk margin. So far, mostly simplified methods for the approximation of the risk margin have been presented in academic literature (see, e.g., Ohlsson and Lauzeningks, 2009). In this paper we present a simulation approach so that approximations are no longer needed. We thus build upon and extend the work by Ohlsson and Lauzeningks (2009) in three ways: (1) next to a one-year view we allow for a multi-year time horizon; (2) we present a consistent and integrated approach for calculating the premium risk and the reserve risk; (3) we present a simulation-based approach for calculating the risk margin that can be integrated into internal risk models.

Although our paper focuses on contributing to the academic discussion on risk modeling, this work is also highly relevant to practitioners and policymakers. Internal risk models are becoming increasingly important in the value-based management of non-life insurance companies and are an important tool for determining business decisions. Furthermore, regulators encourage insurers to develop internal risk models that might also be used to determine solvency capital requirements, e.g., under Solvency II and the Swiss Solvency Test. Our work, therefore, shall not only expand the academic discussion, but also provide a tool for modeling non-life insurance risk in insurance practice.

This paper is organized as follows. In Section 2 we describe a multi-year internal risk model framework and define the claims development result in a multi-year context. In Section 3, we present a mathematical formulation of the simulation-based stochastic re-reserving process and calculate multi-year risk capital for non-life insurance risk. Section 4 provides the associated simulation model for calculating the risk margin considering both a one-year and multi-year time horizon. The usefulness of the new multi-year model for practical applications is illustrated in Section 5. Finally, we conclude in Section 6.

2. Modeling the Multi-Year Non-Life Insurance Risk

Insurance risk is typically divided into reserve risk and premium risk. Reserve risk considers known and unknown claims that have already occurred in the past, it thus focuses on uncertainty about future payments due to a claims settlement process. In contrast, premium risk (also called pricing risk or underwriting risk) deals with the uncertainty that payments for future claims are higher than their expected value, so it deals with future accident years. Both risk categories constitute major risks for non-life insurers. Therefore, quantifying the reserve risk and the premium risk by means of stochastic claims reserving methods plays an essential role in risk modeling of non-life insurers.

In this paper, we analyze reserve and premium risk in the context of internal risk models, which have been developed since the 1990s and in the meantime play an essential role in analyzing the risk and return situation of non-life insurance companies. Internal risk models – also called dynamic financial analysis (DFA) models – project future cash flows of non-life insurance companies using stochastic simulation techniques (see, e.g., Kaufmann et al., 2001; Blum and Dacorogna, 2004; D'Arcy and Gorvett, 2004; Eling and Toplek, 2009). Internal risk models usually take into account management strategies in response to changing risk factors such as insurance risk and asset risk (see Blum and Dacorogna, 2004).

Within internal risk models many different scenarios are stochastically generated in order to derive the distribution of the economic earnings (EcE) for each future calendar year t up to final settlement ω ($t \in \{1, \ldots, \omega\}$). We assume management is interested in a multi-year planning horizon of m years, e.g., five years. Thus, based on the internal model description in Diers (2011), we define the m-year economic earnings ($\mathrm{EcE}_{[0,m]}$) in a multi-year context as the change in net asset value (NAV) over the period $t = 0$ and $t = m$. This can be calculated by adding the m-year investment result ($I_{[0,m]}$) and the m-year technical result ($T_{[0,m]}$):

$$\mathrm{EcE}_{[0,m]} = \mathrm{NAV}_m - \mathrm{NAV}_0 = I_{[0,m]} + T_{[0,m]}$$

As a simplifying assumption we do not consider taxes and dividends in this model. Moreover, we do not take into account inflation and discount effects. Thus the multi-year view of the economic earnings corresponds to a one-year view in such a way that m-year economic earnings equals the sum of the economic earnings of each calendar year t, i.e., $\mathrm{EcE}_{[0,m]} = \mathrm{EcE}_1 + \cdots + \mathrm{EcE}_m$. The technical result ($T_{[0,m]}$) is calculated using the m-year underwriting result ($U_{[0,m]}$) and the m-year claims development result ($\mathrm{CDR}_{[0,m]}$):[2]

$$T_{[0,m]} = U_{[0,m]} + \mathrm{CDR}_{[0,m]}$$

The reserve risk, i.e., risk with regard to past claims (which are settled in the future: IBNR, IBNER), and the premium risk, i.e., risk with regard to future claims (which will occur in the future), can be specified as follows.

Reserve Risk

The academic literature on stochastic claims reserving methods concentrates mostly on an ultimo view, which means reserve risk uncertainty would be quantified up to final settlement $t = \omega$ (see, e.g., England and Verrall, 2002; Wüthrich and Merz, 2008). Calculations are based on an ultimate claims development result ($\mathrm{CDR}_{[0,\omega]}$), which can be determined by the

[2] The technical result typically represents catastrophe risks and non-catastrophe risks, which due to the different statistical behavior should be modeled separately (see Kaufmann et al., 2001). Non-catastrophe losses are further divided in losses caused by large claims and attritional claims, which again should be modeled separately (see Kaufmann et al., 2001). As a simplification, we concentrate on the attritional claims and model their reserve risk (represented by the claims development result of attritional claims) and their premium risk (represented by the underwriting result of attritional claims). For the stochastic modeling of catastrophe risks in internal risk models, we refer to Diers (2009).

difference of the opening best estimate claims reserve ($\widehat{R}^{\mathfrak{D}}$) based on all past observations $\mathfrak{D}$ and future cash flows (the sum of all future claim payments) based on previous accident years ($C_{[0,\omega]}$) simulated up to final settlement $t = \omega$:

$$CDR_{[0,\omega]} = \widehat{R}^{\mathfrak{D}} - C_{[0,\omega]}$$

Merz and Wüthrich (2008) calculate a one-year claims development result ($CDR_{[0,1]}$) as the difference of the opening best estimate claims reserve ($\widehat{R}^{\mathfrak{D}}$), the claim payments based on previous accident years for the next calendar year $t = 1$ ($C_{[0,1]}$), and the closing best estimate claims reserve ($\widehat{R}^{\mathfrak{D}^1}$) at the end of period $t = 1$, based on the updated information $\mathfrak{D}^1$ (see, e.g., Merz and Wüthrich, 2007, 2008; Wüthrich et al., 2009; Bühlmann et al., 2009; Ohlsson and Lauzeningks, 2009):

$$CDR_{[0,1]} = \widehat{R}^{\mathfrak{D}} - \left(C_{[0,1]} + \widehat{R}^{\mathfrak{D}^1}\right)$$

The innovative element of this paper is to consider a multi-year time horizon, which means we have to define the m-year claims development result ($CDR_{[0,m]}$). Reserve risk uncertainty should thus be quantified up to calendar year $t = m$. This is then defined as the difference between the opening best estimate claims reserve ($\widehat{R}^{\mathfrak{D}}$), the sum of claim payments based on previous accident years up to calendar year $t = m$ ($C_{[0,m]}$), and the closing best estimate claims reserve ($\widehat{R}^{\mathfrak{D}^m}$) at the end of period $t = m$, based on the updated information $\mathfrak{D}^m$:

$$CDR_{[0,m]} = \widehat{R}^{\mathfrak{D}} - \left(C_{[0,m]} + \widehat{R}^{\mathfrak{D}^m}\right) \quad (1)$$

Premium Risk

In the context of Solvency II, premium risk is defined as the risk that results from fluctuations in the timing of frequency and severity of insured events (see CEIOPS, 2010). Ohlsson and Lauzeningks (2009) suggest how to calculate one-year premium risk from an economic perspective. They take into account the earned premium for the next calendar year (${}^{NY}P_{[0,1]}$), corresponding operating expenses (${}^{NY}E_{[0,1]}$), claim payments based on future accident years

($^{NY}C_{[0,1]}$), and the (closing) best estimate claims reserve ($^{NY}\widehat{R}^{\mathfrak{D}^1}$) at the end of period $t = 1$, based on the information of simulated first-year payments $\mathfrak{D}^1$:

$$U_{[0,1]} = {}^{NY}P_{[0,1]} - {}^{NY}E_{[0,1]} - \left({}^{NY}C_{[0,1]} + {}^{NY}\widehat{R}^{\mathfrak{D}^1}\right)$$

We follow this approach and integrate the premium risk into our multi-year internal risk model by defining the m-year underwriting result as the difference between the sum of earned premiums ($^{NY}P_{[0,m]}$), the sum of operating expenses ($^{NY}E_{[0,m]}$) and the sum of ultimate future claim payments over the next m calendar years ($^{NY}S_{[0,m]}$):

$$U_{[0,m]} = {}^{NY}P_{[0,m]} - {}^{NY}E_{[0,m]} - {}^{NY}S_{[0,m]}$$

Initially (in $t = 0$) the m-year underwriting result is forecasted by:

$$\widehat{U}^{\mathfrak{D}}_{[0,m]} = {}^{NY}\widehat{P}^{\mathfrak{D}}_{[0,m]} - {}^{NY}\widehat{E}^{\mathfrak{D}}_{[0,m]} - {}^{NY}\widehat{R}^{\mathfrak{D}}_{[0,m]}$$

Hereby ${}^{NY}\widehat{P}^{\mathfrak{D}}_{[0,m]}$ and ${}^{NY}\widehat{E}^{\mathfrak{D}}_{[0,m]}$ denote forecasts for premium income ${}^{NY}P_{[0,m]}$ and operating expenses ${}^{NY}E_{[0,m]}$ over the next m calendar years. ${}^{NY}\widehat{R}^{\mathfrak{D}}_{[0,m]}$ represents the best estimate of future claim payments ${}^{NY}S_{[0,m]}$ (cash flows) for future accident years given the current information $\mathfrak{D}$. After m further years the m-year underwriting result is calculated from the effectively earned premiums ${}^{NY}\widehat{P}^{\mathfrak{D}^m}_{[0,m]}$, incurred expenses ${}^{NY}\widehat{E}^{\mathfrak{D}^m}_{[0,m]}$, and the sum of observed claim payments (${}^{NY}C_{[0,m]}$) up to calendar year $t = m$ and the closing best estimate claims reserve (${}^{NY}\widehat{R}^{\mathfrak{D}^m}_{[0,m]}$) for future accident years at the end of period $t = m$, based on the updated information $\mathfrak{D}^m$:

$$\widehat{U}^{\mathfrak{D}^m}_{[0,m]} = {}^{NY}\widehat{P}^{\mathfrak{D}^m}_{[0,m]} - {}^{NY}\widehat{E}^{\mathfrak{D}^m}_{[0,m]} - \left({}^{NY}C_{[0,m]} + {}^{NY}\widehat{R}^{\mathfrak{D}^m}_{[0,m]}\right)$$

We now define the m-year premium risk for new accident years as the deviation of the estimated underwriting result $\widehat{U}^{\mathfrak{D}^m}_{[0,m]}$ after m years from the initial forecast $\widehat{U}^{\mathfrak{D}}_{[0,m]}$. For the reason of simplification we assume, that the initially forecasted technical underwriting result

$\widehat{U}^{\mathfrak{D}}_{[0,m]}$ directly leads to an increase / decrease of own funds in t=0. The deviation can be expressed in the following decomposition:

$$\Delta = \widehat{U}^{\mathfrak{D}^m}_{[0,m]} - \widehat{U}^{\mathfrak{D}}_{[0,m]}$$

$$= \left({}^{NY}\widehat{P}^{\mathfrak{D}^m}_{[0,m]} - {}^{NY}\widehat{P}^{\mathfrak{D}}_{[0,m]}\right) - \left({}^{NY}\widehat{E}^{\mathfrak{D}^m}_{[0,m]} - {}^{NY}\widehat{E}^{\mathfrak{D}}_{[0,m]}\right) - \left({}^{NY}\widehat{R}^{\mathfrak{D}^m}_{[0,m]} + {}^{NY}C_{[0,m]} - {}^{NY}\widehat{R}^{\mathfrak{D}}_{[0,m]}\right)$$

If premiums and expenses are regarded as deterministic and known, the premium risk can be directly calculated from the claims development result for new accident years

$$CDR^{NY}_{[0,m]} = {}^{NY}\widehat{R}^{\mathfrak{D}}_{[0,m]} - \left({}^{NY}C_{[0,m]} + {}^{NY}\widehat{R}^{\mathfrak{D}^m}_{[0,m]}\right), \tag{2}$$

which is consistent and directly comparable with the claims development result for previous accident years (reserve risk, see equation (1)). Note, that it is a simplifying assumption that premiums and expenses are deterministic. Usually in internal risk models premiums and expenses are modeled stochastically and premium cycles have to be taken into account (see, e.g., Kaufmann et al., 2001).

Insurance Risk

To derive an integrated approach of modeling the non-life insurance risk we combine the claims development result for previous accident years (see equation (1)), i.e., reserve risk and for future accident years (see equation (2)), i.e., premium risk, and thus define the m-year claims development result for the non-life insurance risk as:

$$CDR^{PY+NY}_{[0,m]} := CDR_{[0,m]} + CDR^{NY}_{[0,m]} \tag{3}$$

Note that in general the m-year premium risk and m-year reserve risk have an implicit dependency due to the joint estimation and re-reserving process.

Based upon this definition we can now use stochastic re-reserving techniques to derive the empirical frequency distribution of $CDR^{PY+NY}_{[0,m]}$ (see Section 3.1.) and then compute any risk measure of interest to derive the insurance risk as well as the reserve risk and premium risk (see Section 3.2.). Furthermore, we might determine the risk capital (RC), i.e. the amount the insurance company needs to hold to cover non-life insurance risk.

3. Stochastic Re-Reserving and Calculation of Multi-Year Non-Life Insurance Risk

3.1. Stochastic Re-Reserving

Based on the model description in Ohlsson and Lauzeningks (2009), we now present a simulation-based modeling approach for quantifying the m-year claims development result of the non-life insurance risk (see equation (3)), which is called stochastic re-reserving. As the underlying stochastic reserving method we use bootstrapping and Bayesian techniques, implemented using Markov Chain Monte Carlo (MCMC) methods (see England and Verrall, 2006). Stochastic re-reserving allows us to quantify the empirical probability distribution of the one-year and multi-year claims development result, which is then the basis for risk capital calculations. Since stochastic re-reserving is a simulation-based approach, it can be easily integrated into internal risk models.

Besides the modeling of reserve risk, which has been the focus of much academic debate, we also incorporate premium risk in our analysis. Ohlsson and Lauzeningks (2009) describe the quantification of premium risk in a one-year perspective. Gault et al. (2010) represent a stochastic simulation model for measuring premium risk over a one-year and an ultimo risk horizon. We, however, integrate the premium risk in our stochastic re-reserving model and thus present an integrated approach for modeling non-life insurance risk in a one-year, multi-year, and ultimo perspective.

Following Mack (2002) we denote incremental payments for accident year $i \in \{1, \dots, n\}$ and $k \in \{1, \dots, K\}$ by $S_{i,k}$. Cumulative payments are given by $C_{i,j}$ (where $C_{i,k} = \sum_{j=1}^{k} S_{i,j}$). Hereby $C_{i,K}$ is called the ultimate claim amount for accident year i. At time $t = 0$, having $1 \leq n \leq K$ years of claims development observed, a set of all past observations $\mathfrak{D}$ is given by

$$\mathfrak{D} = \left\{S_{i,k} : i + k - 1 \leq n, 1 \leq i \leq n, 1 \leq k \leq K\right\}.$$

If we now go ahead $m \in \{1, 2, \dots, \omega\}$ years in time, from $t = 0$ to $t = m$, a new set of observations $\mathfrak{D}^m$ (including future accident years) is given by

$$\mathfrak{D}^m = \left\{S_{i,k}: i + k - 1 \leq n + m, 1 \leq i \leq n + m, 1 \leq k \leq K\right\}.$$

For simplicity, we assume $K = n$, so that for each accident year i we have a complete settlement of our claims in development year n; we thus do not take into account any tail factors. We choose the distribution-free Mack model (1993) as the underlying reserving model for the re-reserving process and make a slight extension to it by adding some additional assumptions about the claim payments in the first development period.

Definition 1 (Extended Mack Model) There exist parameters $f_k, \sigma_k > 0,\ 1 \le k \le n$ such that for all $1 \le i \le n + m\ , 1 \le k \le n$ we have:

- $E\left[C_{i,k} \mid C_{i,k-1}\right] = f_k \cdot C_{i,k-1}$
- $Var\left[C_{i,k} \mid C_{i,k-1}\right] = \sigma_k^2 \cdot C_{i,k-1}$
- Different accident years i are independent

Hereby $C_{i,0}$ represents an appropriate volume measure such as premiums or number of insurance contracts for the accident year i (which have to be forecasted for n), and f_1 represents its respective incremental loss ratio or average loss (see Merz and Wüthrich, 2010) in the first development period. Then, according to Mack (2002), unbiased and uncorrelated estimators $\hat{f}_k$ for f_k (at time $t = 0$) are given by

$$\hat{f}_k = \frac{\sum_{i=1}^{n-k+1} C_{i,k}}{\sum_{i=1}^{n-k+1} C_{i,k-1}},$$

and unbiased estimators $\hat{\sigma}_k^2$ for σ_k^2 are given by

$$\hat{\sigma}_k^2 = \frac{1}{n-k} \cdot \sum_{i=1}^{n+1-k} C_{i,k-1} \cdot \left(\frac{C_{i,k}}{C_{i,k-1}} - \hat{f}_k\right)^2, 1 \le k < n.$$

For simplification we set $\hat{\sigma}_n^2 \coloneqq \min\{\hat{\sigma}_{n-1}^2, \hat{\sigma}_{n-2}^2, \hat{\sigma}_{n-3}^2\}$. For an extensive description of different extrapolation rules we refer to Mack (2002).

Since the Mack (1993) model produces the same reserve estimates as the deterministic chain-ladder algorithm (see, e.g., Mack, 1993, 1994; England and Verrall, 1999; Verrall, 2000), we can use the chain-ladder algorithm to give best estimates for the opening and closing reserve estimates. The modeling steps for quantifying the multi-year reserve risk and the multi-year premium risk using the re-reserving process are shown in Figure 1. The derivation of the modeling steps comes from the definition of the m-year claims development result for the non-life insurance risk (see equation (3)):

$$\mathrm{CDR}_{[0,m]}^{\mathrm{PY+NY}} = \underbrace{\widehat{\mathrm{R}}^{\mathfrak{D}} + {}^{\mathrm{NY}}\widehat{\mathrm{R}}^{\mathfrak{D}}}_{\text{Step 1}} - \underbrace{\left(\underbrace{\mathrm{C}_{[0,m]} + {}^{\mathrm{NY}}\mathrm{C}_{[0,m]}}_{\text{Step 2}} + \underbrace{\widehat{\mathrm{R}}^{\mathfrak{D}^{\mathrm{m}}} + {}^{\mathrm{NY}}\widehat{\mathrm{R}}^{\mathfrak{D}^{\mathrm{m}}}}_{\text{Step 3}} \right)}_{\text{Z simulations}}$$

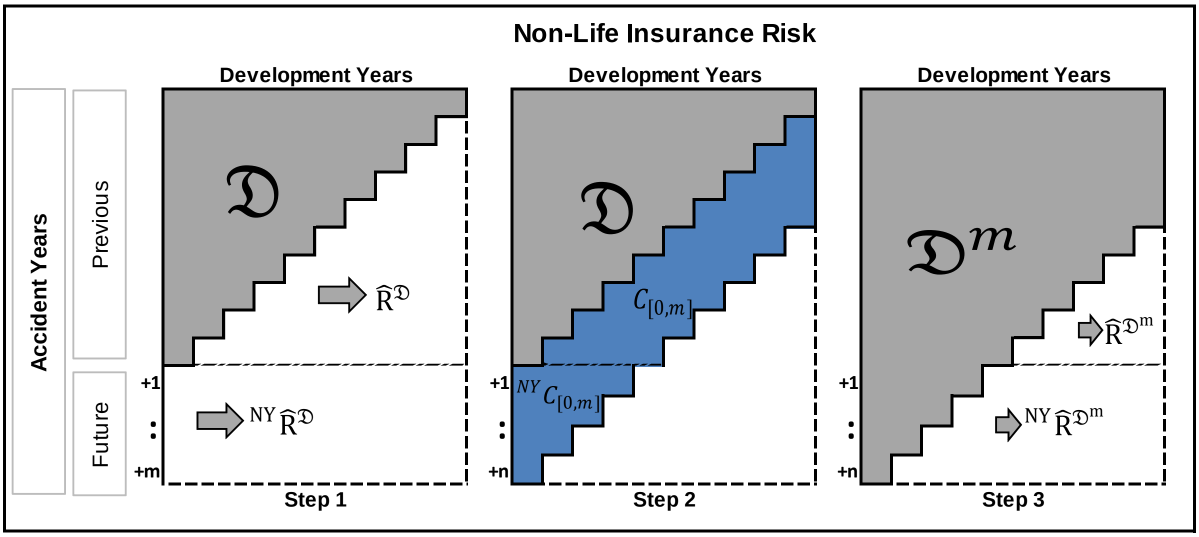

Figure 1: Modeling Steps for the Re-Reserving Process

In the following we present an integrated and detailed mathematical framework for the use of the multi-year stochastic claims reserving process for modeling the multi-year reserve risk (previous accident years) and the multi-year premium risk (future accident years) and thus the multi-year non-life insurance risk.

<u>Step 1: Calculating the opening reserve estimate</u>

In **Step 1** we calculate an estimator for the opening reserve of previous accident years as well as a forecast for the ultimate claims of new accident years (based on all past observations $\mathfrak{D}$). The opening reserve estimate / initial ultimate claim forecast can be calculated from any underlying reserving model and should agree with the actuary's best estimate for outstanding claims in time $\mathrm{t} = 0$ (see Ohlsson and Lauzeningks, 2009) and the underwriting assumptions for new business (future accident years). We thus calculate a predictor for the (ultimate) best estimate claims reserve $R^{\mathfrak{D}}$ at the beginning of period $t = 0$. For this purpose we chose the deterministic chain-ladder algorithm as described in Wüthrich and Merz (2008):

$\hat{R}^{\mathfrak{D}} = \sum_{i=1}^{n} \hat{R}_i^{\mathfrak{D}}$ and $^{NY}\hat{R}^{\mathfrak{D}} = \sum_{i=n+1}^{n+m} \hat{R}_i^{\mathfrak{D}}$

Hereby the predictors for single accident years i are given by

$$\hat{R}_i^{\mathfrak{D}} = \begin{cases} \hat{C}_{i,n}^{\mathfrak{D}} - C_{i,n+1-i} & \text{for } 1 \le i \le n \\ \hat{C}_{i,n}^{\mathfrak{D}} & \text{for } n+1 \le i \le n+m \end{cases}$$

whereas

$$\hat{C}_{i,n}^{\mathfrak{D}} = C_{i,\max(n-i+1,0)} \cdot \hat{f}_{\max(n-i+2,1)} \cdot \ldots \cdot \hat{f}_{n-1} \cdot \hat{f}_n \text{ , with } \hat{f}_k = \frac{\sum_{i=1}^{n-k+1} C_{i,k}}{\sum_{i=1}^{n-k+1} C_{i,k-1}}.$$

<u>Step 2: Calculating the cumulative payments</u>

Then, in **Step 2**, payments for the next m calendar years of previous accident years $C_{[0,m]}$ and of future (upcoming) accident years $^{NY}C_{[0,m]}$ are simulated. For this purpose, we use simulation-based reserving methods such as bootstrapping and Bayesian techniques (see, e.g., England and Verrall, 2006; Bjoerkwall et al., 2009) for previous accident years and direct parameterization of the first-year payment for future accident years.[3] We thus obtain a new level of knowledge at the end of calendar year $t = m$, and new payments for the next m diagonals can be derived by

$C_{[0,m]} = \sum_{i=1}^{n} S_i^m$ and $^{NY}C_{[0,m]} = \sum_{i=n+1}^{n+m} S_i^m$

whereas

$$S_i^m = \begin{cases} C_{i,n} - C_{i,n-i+1} & \text{for } 1 \le i \le m+1 \\ C_{i,n-i+m+1} - C_{i,n-i+1} & \text{for } m+2 \le i \le n \\ C_{i,m} & \text{for } n+1 \le i \le n+m \end{cases}.$$

[3] There are many different possibilities of how to simulate the next m diagonals. Instead of using bootstrapping and Bayesian techniques, Ohlsson and Lauzeningks (2009) propose to simulate from any distribution that fits the data (e.g., normal or lognormal) with mean given by the best estimate and variance given by σ_j^2 according to Mack (1993). For the simulation process of future accident years, as already mentioned by Ohlsson and Lauzeningks (2009) aggregate loss models might be used, where frequency and severity are simulated separately (see Klugman et al., 2004). Kaufmann et al. (2001) first simulate the ultimate claim of each future accident year and then model the incremental payments of those ultimate loss amounts over the development periods by using a beta probability distribution.

To simulate all cumulative payments for the next m diagonals $C_{i,k}$, for $1 \leq k \leq m$, we use bootstrap methods and Bayesian methods implemented using MCMC techniques based on the classic Mack (1993) model as presented in England and Verrall (2006). Hereby, the procedure to obtain predictive distributions for outstanding claims can be divided into three steps. As a starting point, a well-specified underlying statistical model needs to be declared. For this purpose the classic Mack (1993) model can be embedded within the framework of generalized linear models (GLM) and then leads to Mack's bootstrapping model (see England and Verrall, 2006). In a second step the estimation error needs to be incorporated. This can be done either by the use of bootstrapping or by the use of MCMC techniques. As a last step the process error needs to be described. This can be done by choosing appropriate assumptions for the underlying process variance. An appropriate assumption can be, e.g., a normal distribution, an over-dispersed Poisson distribution, a gamma distribution or a lognormal distribution (see, e.g., Bjoerkwall et al., 2009; England and Verrall, 2002, 2006).

In this paper, we use the normal distribution as a process distribution and simulate with the mean and variance given by the "pseudo" chain-ladder factors $\hat{f}_k^*$ and the estimated variance parameters $\hat{\sigma}_k^2$ based on the underlying Mack (1993) model:

$$C_{i,k}|C_{i,k-1} \sim \mathrm{Normal}\left(\hat{f}_k^* \cdot C_{i,k-1}\, ,\ \hat{\sigma}_k^2 \cdot C_{i,k-1}\right)$$

Hereby, in order to incorporate the estimation error, $\hat{f}_k^*$ is derived by a new set of "pseudo data" created using the data in the original claims development triangle based on all past observations $\mathfrak{D}$ with the help of bootstrapping techniques as described in England and Verrall (2006).

<u>Step 3: Calculating the closing reserve estimates</u>

Finally, in **Step 3**, an estimator for the closing reserve estimate of previous and future accident years (based on all the updated observations $\mathfrak{D}^m$) needs to be calculated. This process is called re-reserving. Hereby, $\mathfrak{D}^m$ is composed by all past observations $\mathfrak{D}$ and the increase in information about the claims development process for the new simulated m diagonals from Step 2 (see Merz and Wüthrich, 2008). The closing reserve estimate should then be derived by the same reserving model as chosen within Step 1 (see Ohlsson and Lauzeningks, 2009).

Thus, we calculate a predictor $\widehat{R}^{\mathfrak{D}^m}$ for the (ultimate) best estimate claims reserve at the end of period $t = m$. For this purpose we need to use the same deterministic algorithm as chosen within Step 1, i.e., the chain-ladder algorithm:

$$\widehat{R}^{\mathfrak{D}^m} := \sum_{i=1}^{n} \widehat{R}_i^{\mathfrak{D}^m} \quad \text{and} \quad {}^{NY}\widehat{R}^{\mathfrak{D}^m} := \sum_{i=n+1}^{n+m} \widehat{R}_i^{\mathfrak{D}^m}$$

Hereby the predictors for the single accident years $1 \leq i \leq n + m$ are given by

$$\widehat{R}_i^{\mathfrak{D}^m} = \begin{cases} 0 & \text{for } 1 \leq i \leq m+1 \\ \widehat{C}_{i,n}^{\mathfrak{D}^m} - C_{i,n-i+m+1} & \text{for } m+2 \leq i \leq n+m \end{cases}$$

whereas

$$\widehat{C}_{i,n}^{\mathfrak{D}^m} = C_{i,n-i+m+1} \cdot \hat{f}_{n-i+m+2}^{(m)} \cdot \ldots \cdot \hat{f}_n^{(m)} \quad \text{with } \hat{f}_k^{(m)} = \frac{\sum_{i=1}^{n-k+m+1} C_{i,k}}{\sum_{i=1}^{n-k+m+1} C_{i,k-1}} .$$

Step 2 and Step 3 refer to the simulation process and are carried out many times (Z simulation steps) to derive the empirical frequency distribution of $CDR_{[0,m]}^{PY+NY}$. With a growing number of simulation steps, the empirical frequency distribution of the multi-year CDR converges against the underlying theoretical frequency distribution.

3.2. Calculation of Multi-Year Non-Life Insurance Risk

The information from step 1 to 3 delivers the empirical frequency distribution of the multi-year claims development result for the non-life insurance risk ($CDR_{[0,m]}^{PY+NY}$). A selected risk measure ρ can now be applied to derive the multi-year reserve risk and premium risk as well as the resulting multi-year risk capital. The multi-year risk capital corresponds to the amount of equity capital necessary to withstand years of worst-case scenarios at a predefined confidence level over a predefined time horizon; it is also often referred to as risk-based capital or economic capital (see Porteous and Tapadar, 2008). The risk capital can be calculated by using appropriate risk measures ρ such as value at risk (VaR) or tail value at risk (TVaR) (for a discussion of these risk measures we refer to Artzner et al., 1999; Tasche, 2002; Acerbi and Tasche, 2002; Heyde et al., 2007; Cont et al., 2010). To obtain the risk capital for non-life insurance risk $RC_{[0,m]}$, the risk measure ρ is applied to the random variable of $CDR_{[0,m]}^{PY+NY}$:

$$RC_{[0,m]} = \rho\left(-CDR_{[0,m]}^{PY+NY}\right) \tag{4}$$

However, since we consider reserve risk in a multi-year context, management also faces the risk of running out of capital before the end of period $t = m$. To address this issue, we follow Diers (2011) and define the multi-year risk capital by considering the following definition of a loss random variable L ($MaxLoss_{[0,m]}$) for the multi-year reserve risk, taking into account m future accident years:

$$MaxLoss_{[0,m]} = \max_{1 \le t \le m}\left(-CDR_{[0,t]}^{PY+NY}\right) \tag{5}$$

The risk measure ρ can now be applied to the probability function of the random variable $MaxLoss_{[0,m]}$ to calculate the amount of the m-year risk capital needed to cover the multi-year non-life insurance risk. To withstand years of adverse claim developments at a certain confidence level without the need for external capital, the insurance company needs to hold the following risk capital:

$$RC_{[0,m]}^{MAX} = \rho\left(MaxLoss_{[0,m]}\right) \tag{6}$$

For illustration purposes of the different effects between reserve risk and premium risk we restrict the application in Section 5 of this paper to a separate calculation of the multi-year risk capital for previous accident years (see equation (1)), i.e., reserve risk, and for future accident years (see equation (2)), i.e., premium risk. Note that the within the modeling approach described in Section 3.1 the overall (combined) non-life insurance risk is calculated so that dependencies are considered. That means no further correlation assumptions about premium and reserve risk have to be made, as the dependencies are automatically determined by the common estimation and re-reserving process.

4. Calculation of the Risk Margin

Next to the quantification of the non-life insurance risk based on best estimate reserves, for solvency purposes (Solvency II and Swiss Solvency Test) and in the context of the International Financial Reporting Standards (IFRS), insurance companies also have to calculate a risk margin. This is necessary because within an economic balance sheet the market-consistent value of liabilities is determined by the best estimate of liabilities (i.e., the expected value of future cash flows) and an additional allowance for uncertainty associated with the expected cash flows called risk margin (see IAA, 2009). The difference between the market value of assets and the market value of liabilities then yields available capital and thus defines the (solvency) coverage ratio between available capital and solvency capital requirements (SCR). An extra amount of available capital on top of the SCR is called free surplus (see Figure 2). The SCR includes, among others, the non-life insurance risk (reserve and premium risk). The calculation of the risk margin is thus another critical element of market-consistent valuation that we can analyze in a multi-year context.

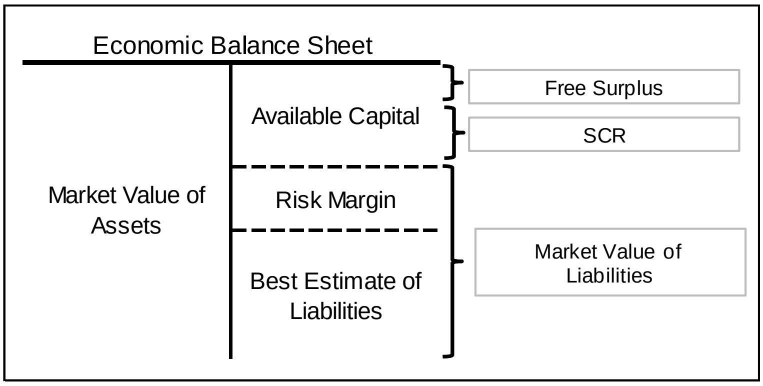

Figure 2: Economic Balance Sheet

There are several approaches to calculate the risk margin such as quantile-based methods, discount-related methods, and cost-of-capital methods (see IAA, 2009). Recent research uses an economic approach where the risk margin is related to the risk aversion of the owner/shareholder, modeled using probability distortion techniques (see Wüthrich et al., 2011). In the context of Solvency II, however, to calculate the risk margin a cost-of-capital approach is prescribed (see European Union, 2009). Using the cost-of-capital approach usually calls for simplification, since in most cases the risk margin is analytically not tractable and the use of numerical methods necessitates a large amount of nested simulations (see, e.g., Ohlsson and Lauzeningks, 2009; Salzmann and Wüthrich, 2010; Wüthrich et al., 2011). Thus approximation approaches, e.g., the duration approach, are necessary in order to be able to calculate the risk margin (see Ohlsson and Lauzeninkgs, 2009).

In this paper, however, we present a simulation-based model for calculating risk margin in a multi-year context so that approximation approaches are no longer needed. This approach can also be easily combined with the re-reserving model described in Section 3. The reason why both approaches can be integrated is that under Solvency II requirements, the risk margin (the so-called cost-of-capital margin CoCM_0) is defined as the product of a cost-of-capital rate coc and the sum of discounted future SCRs up to final settlement of the existing insurance business (see CEIOPS, 2010):

$$\mathrm{CoCM}_0 = \mathrm{coc} \cdot \sum_{t=1}^{\omega} \frac{\mathrm{SCR}_t}{(1+r_t)^t} \tag{7}$$

The SCR_t at each point in time is calculated by using some risk measure ρ (e.g., VaR) applied to the probability distribution of the one-year claims development result in year t (CDR_t) based on all past observations up to $t-1$ ($\mathfrak{D}^{t-1}$):

$$\mathrm{SCR}_t = \rho(\mathrm{CDR}_t | \mathfrak{D}^{t-1})$$

For simplification and illustration purposes we demonstrate the calculation of solvency capital requirements for reserve risk. The premium risk can be treated analogously. Moreover, following Ohlsson and Lauzeningks (2009), as a simplification we neglect the risk margin within our SCR calculations and the interest rate r_t is set equal to zero.

The real difficulty comes with the calculation of SCR_t at each point in time $t \in \{2, \dots, \omega\}$. In contrast to the multi-year claims development result $\mathrm{CDR}_{[0,t]}$ defined in Section 2, for the determination of the empirical probability distribution of the one-year claims development result in year t (CDR_t), nested simulations are necessary (see, e.g., Ohlsson and Lauzeningks, 2009). For example, for the calculation of the SCR_2 within each point of the respective individual simulation path (e.g., Z simulations) another set of Z simulations of the one-year claims development result based on the updated information $\mathfrak{D}^1$ is necessary to derive its corresponding empirical frequency distribution (see Figure 3). Overall, this process leads to Z^{t+1} iterations for SCR_t, $t \in \{2, \dots, \omega\}$ (see Ohlsson and Lauzeningks, 2009).

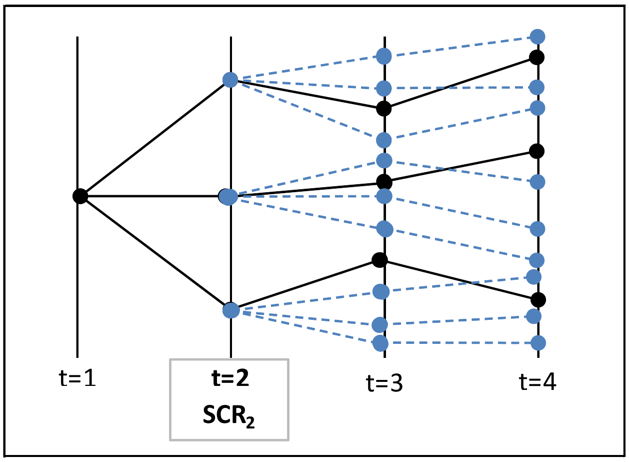

Figure 3: Nested Simulations

In our simulation-based modeling approach we avoid the problem of nested simulations by using the method of moments. Hereby, for an estimator of the first moment (mean) the best estimate reserve is used, and for an estimator of the second moment (variance) the MSEP of the one-year claims development result (see Merz and Wüthrich, 2008), based on the actual state of information, is used. Then the first and second moments can be applied to fit an appropriate probability distribution such as the normal, log-normal, or gamma distribution. Figure 4 describes the two modeling steps for calculating the future SCRs at each point in time and thus the corresponding risk margin.

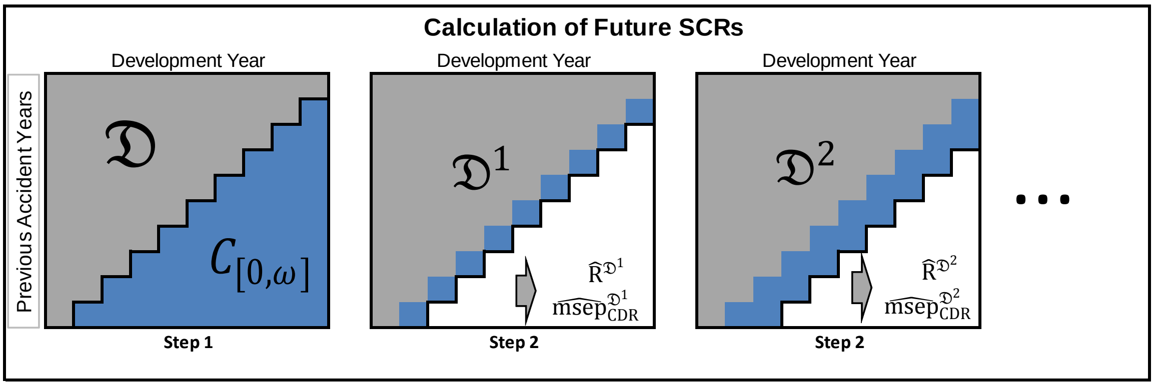

Figure 4: Calculating Future SCRs for Reserve Risk

In **Step 1** we use simulation techniques such as bootstrapping or Bayesian methods to derive simulated future claim payments until final settlement $t = \omega$ has been reached. This procedure equals Step 2 in our re-reserving model (Section 3) with m set equal to ω. Based on all past observations $\mathfrak{D}$ this delivers Z different possibilities of completing the upper claims development triangle into a quadrangle. In **Step 2** we now move one year ahead and only use the simulated payments for the next calendar year from Step 1 in order to derive the best estimate claims reserve $\widehat{R}^{\mathfrak{D}^1}$ and its corresponding mean squared error of prediction $\widehat{\mathrm{msep}}_{\mathrm{CDR}}^{\mathfrak{D}^1}$ using the analytical formula by Merz and Wüthrich (2008), both based on the (updated) observations $\mathfrak{D}^1$. Now, for each simulation step z, we are able to fit a distribution, e.g., the normal distribution with mean / variance given by $\widehat{R}^{\mathfrak{D}^1}$/ $\widehat{\mathrm{msep}}_{\mathrm{CDR}}^{\mathfrak{D}^1}$ and with the help of some risk measure ρ we calculate its corresponding SCR_2^z, without needing additional simulation. The overall SCR_2 is then approximated by calculating the expected value of SCR_2 ($\approx \frac{\sum_{i=1}^{Z} \mathrm{SCR}_2^i}{Z}$).

Step 2 delivers an empirical frequency distribution of different (possible) SCRs, given Z different real-world scenarios for the development from t to t + 1 and it is not quite clear which measure to consider for aggregation. We follow Stevens et al. (2010) and approximate the SCR in year t with the expected value of future values of SCR. We also might consider alternative measures for aggregation, e.g., the median. Step 2 is repeated until final settlement in $t = \omega$ has been reached, and within each repetition we have the following coherence:

$$\mathrm{SCR}_t \approx \frac{\sum_{i=1}^{Z} \mathrm{SCR}_t^i}{Z} \tag{8}$$

Using equation (7) we now are able to calculate the corresponding risk margin. Note that this procedure can be performed only if within Step 1 simulation techniques are chosen such that they are consistent with the Mack (1993) model and if the best estimate claims reserve is calculated using the chain-ladder method. This is because only for this case the analytical formula by Merz and Wüthrich (2008) delivers the standard deviation of the claims development result.

Next to the one-year view within Solvency II, in our paper we present multi-year risk capital, based on the multi-year non-life insurance risk. Along this line of reasoning we argue that the risk margin, following CEIOPS (2010), has a shape defined by the following equation:

$$\mathrm{CoCM_m} = \mathrm{coc} \cdot \left(\mathrm{SCR}_{[0,m]} + \sum_{t=m+1}^{\omega} \frac{\mathrm{SCR_t}}{(1+r_t)^t} \right)$$

The first term of the sum ($\mathrm{SCR}_{[0,m]}$) now represents the multi-year risk capital calculated using equation (6), and the remaining terms of the sum present a risk calculation for all the remaining years until final settlement $t = \omega$. Those SCRs can be derived using the same simulations steps described above.

5. Application of the Model to a Claims Development Triangle

5.1. Setup and Definitions

To illustrate the usefulness of modeling the multi-year non-life insurance risk for internal risk models we apply the stochastic re-reserving process to a typical claims development triangle used in academic literature. We show the development of risk capital in a multi-year context up to final settlement and compare our results with the one-year risk capital used for Solvency II purposes and the ultimo perspective used so far used in internal risk models for reserve risk and premium risk. The results presented in this paper are based on 100,000 simulations carried out using the simulation software EMB IGLOO™ Extreme. For quantification of the risk capital we use the value at risk (VaR) and tail value at risk (TVaR).

Definition 2 (Value at Risk) Let L be a real random variable on a probability space $\{\Omega, \mathcal{F}, \mathbb{P}\}$. The value at risk at confidence level $\alpha \in (0,1)$ is defined as:

$$\mathrm{VaR}_\alpha(L) = \inf\{x \in \mathbb{R} \colon F_L(x) \geq 1 - \alpha\}$$

Definition 3 (Tail Value at Risk) Let L be a real random variable on a probability space $\{\Omega, \mathcal{F}, \mathbb{P}\}$. The tail value at risk at confidence level $\alpha \in (0,1)$ is defined as:

$\text{TVaR}_\alpha(\text{L}) = \text{E}[\text{L}|\text{L} \geq \text{VaR}_\alpha(\text{L})]$

We use the claims development triangle presented in Mack (1993) and England and Verrall (2006) shown in Table 1. This kind of claims development triangle corresponds to a long-tail line of business such as third-party motor liability. The second column of Table 2 shows the chain-ladder reserve estimates, which are calculated using the deterministic chain-ladder algorithm. The prediction error according to Mack (1993) is estimated using Mack's formula and is presented in the third column of Table 2. The results shown in our paper slightly differ from those in Mack (1993). Since we do not have enough data, to calculate $\hat{\sigma}_8^2$, extrapolation techniques have to be used. We used the simplified extrapolation rule $\hat{\sigma}_{10}^2 = \min\{\hat{\sigma}_9^2, \hat{\sigma}_8^2, \hat{\sigma}_7^2\}$. This kind of extrapolation differs from the form used in Mack (1993).

	$C_{i,1}$	$C_{i,2}$	$C_{i,3}$	$C_{i,4}$	$C_{i,5}$	$C_{i,6}$	$C_{i,7}$	$C_{i,8}$	$C_{i,9}$	$C_{i,10}$
i=1	357,848	1,124,788	1,735,330	2,218,270	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
i=2	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
i=3	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
i=4	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
i=5	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311				
i=6	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
i=7	440,832	1,288,463	2,419,861	3,483,130						
i=8	359,480	1,421,128	2,864,498							
i=9	376,686	1,363,294								
i=10	344,014									

Table 1: Claims Development Triangle (Accumulated Figures)

	Chain-Ladder Reserves	Prediction Error Mack (1993)	Prediction Error in %
i=2	94,634	75,535	79.82
i=3	469,511	121,699	25.92
i=4	709,638	133,549	18.82
i=5	984,889	261,406	26.54
i=6	1,419,459	411,010	28.96
i=7	2,177,641	558,317	25.64
i=8	3,920,301	875,328	22.33
i=9	4,278,972	971,258	22.70
i=10	4,625,811	1,363,155	29.47
Total	18,680,856	2,447,095	13.10

Table 2: Estimated Reserves and Prediction Errors

The payments for the next m diagonals are simulated using bootstrapping techniques and Bayesian methods for the clearly defined Mack model (see England and Verrall, 2006). Because both techniques lead to very similar results, we present only those results derived by the bootstrap methodology (Bayesian results are available upon request). For the calculation of the reserve risk we consider 10 previous accident years as shown in Table 1. For the calculation of the premium risk we consider five future (upcoming) accident years. To simulate future accident years volume measures are needed. We estimated future premium income using loss ratios for a comparable claims development triangle presented in DAV-Arbeitsgruppe Interne Modelle (2008). By applying those ratios to our claims development triangle, we determine premium income for all past accident years. With linear projection we then obtain 6,943,622 (i=11), 7,055,884 (i=12), 7,234,379 (i=13), 7,417,390 (i=14), and 7,605,031 (i=15) as volume measures (details are available upon request).

We use parametric bootstrap to generate pseudo data and calculate pseudo development (= chain ladder) factors for measuring parameter uncertainty. We use the normal distribution for the simulation of pseudo data and the process error, which is considered in this application only for illustrative purposes. Relying on the normal distribution, however, has some disadvantages, e.g., it allows negative cumulative claim payments, which is not adequate. Furthermore, the symmetric form of the normal distribution is not adequate for the right-skewed claim distributions in non-life insurance. As a different process distribution the log-normal or the gamma distribution can be used (see, e.g., England and Verrall, 2002, 2006; Bjoerkwall et al., 2009).

100,000 repetitions of this process lead to 100,000 different claims development triangles. For the multi-year stochastic re-reserving process we use the cash flows generated with bootstrapping techniques for the next m calendar years. Hereby we get 100,000 new claims development triangles, differing only in the last m diagonals. We use the deterministic chain-ladder method on each of these new claims development triangles for the re-reserving process.

5.2. Results for Multi-Year Reserve Risk, Premium Risk and Insurance Risk

We repeat the re-reserving process until the final settlement of all claims has been reached. Figure 5 shows the empirical frequency density of the simulated multi-year claims development result for previous accident years ($\mathrm{CDR}_{[0,m]}$; reserve risk) and for future accident years ($\mathrm{CDR}_{[0,m]}^{\mathrm{NY}}$; premium risk). Corresponding descriptive statistics are presented in Table 3.

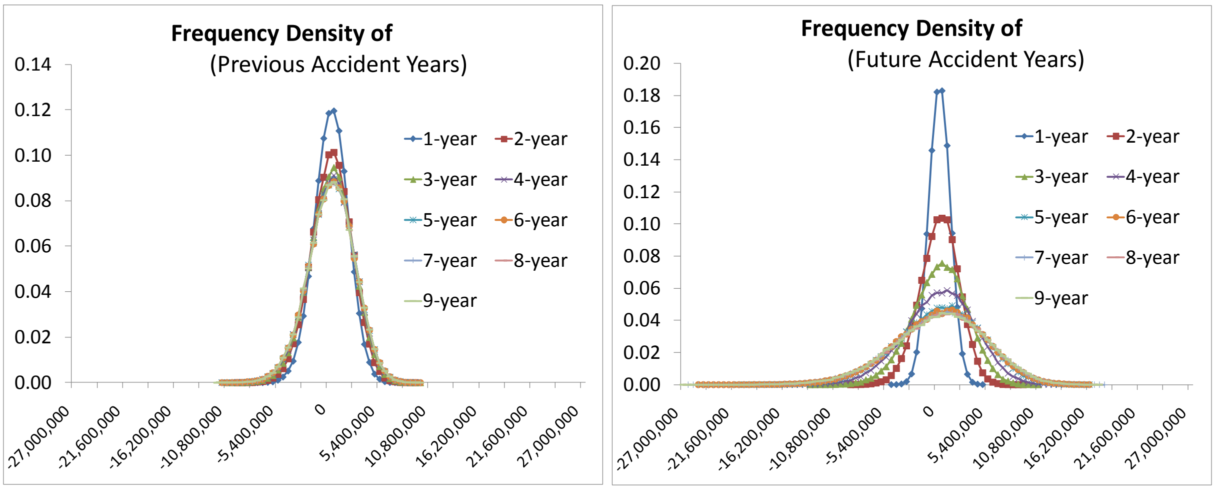

Figure 5: Frequency Density

	Statistics	1-year	2-year	3-year	4-year	5-year	6-year	7-year	8-year	9-year
Previous Accident Years	Minimum	-8,476,694	-10,090,542	-11,240,089	-11,797,525	-11,636,995	-11,535,298	-11,727,293	-11,833,367	-11,937,955
	Maximum	7,053,536	9,033,016	9,474,044	8,874,613	9,119,273	9,420,865	9,385,580	9,310,110	9,297,364
	Median	23,279	43,272	61,624	62,010	64,535	68,196	66,576	67,636	66,851
	Mean	41	444	-136	-406	-238	-353	-422	-484	-495
	Std.Dev.	1,777,576	2,128,792	2,310,305	2,393,617	2,430,902	2,445,167	2,448,778	2,451,074	2,451,642
	Skewness	-0.0824	-0.1313	-0.1526	-0.1678	-0.1744	-0.1769	-0.1779	-0.1785	-0.1785
	Kurtosis (Excess)	0.0036	0.0164	0.0476	0.0626	0.0692	0.0697	0.0708	0.0720	0.0713
Future Accident Years	Minimum	-5,300,287	-9,713,775	-13,870,920	-18,103,486	-21,168,624	-25,785,345	-26,662,522	-27,986,612	-25,999,204
	Maximum	4,827,589	8,844,468	10,560,766	12,841,581	15,757,742	16,181,548	17,312,924	17,206,025	17,224,165
	Median	5,118	68,213	120,669	161,609	207,785	224,717	240,823	235,955	246,059
	Mean	-161	503	550	954	1,148	621	728	339	838
	Std.Dev.	1,134,309	2,064,130	2,887,956	3,706,287	4,491,062	4,744,158	4,855,871	4,925,736	4,957,674
	Skewness	-0.0295	-0.1890	-0.2267	-0.2456	-0.2555	-0.2649	-0.2718	-0.2759	-0.2792
	Kurtosis (Excess)	0.0264	0.0761	0.0691	0.0761	0.0911	0.1101	0.1065	0.1054	0.1061
Aggregated Accident Years	Minimum	-11,521,278	-17,093,449	-21,881,676	-27,005,804	-27,005,376	-29,147,608	-29,164,262	-29,409,450	-28,481,491
	Maximum	8,904,549	13,107,117	15,119,426	17,445,809	20,357,457	21,063,225	21,967,108	22,256,138	22,587,783
	Median	34,420	92,687	139,172	175,497	244,604	255,686	262,232	279,466	279,566
	Mean	-119	947	414	551	907	260	296	-153	334
	Std.Dev.	2,257,751	3,263,050	4,120,271	4,925,328	5,684,727	5,897,981	5,992,552	6,052,526	6,079,577
	Skewness	-0.0985	-0.1685	-0.2070	-0.2213	-0.2361	-0.2426	-0.2486	-0.2492	-0.2508
	Kurtosis (Excess)	0.0113	0.0788	0.0912	0.0794	0.0916	0.1053	0.1037	0.0994	0.1000
	Pearson's correlation	0.1614	0.2111	0.2472	0.2701	0.2857	0.2716	0.2664	0.2635	0.2621
	Spearman´s correlation	0.1542	0.1987	0.2338	0.2569	0.2724	0.2588	0.2540	0.2514	0.2500

Table 3: Descriptive Statistics

The figures show that the greater the time horizon, the greater the risk exposure since the standard deviation increases. Furthermore, with increasing time horizon, the variation, and thus the risk exposure, becomes greater for the simulation process of future accident years (premium risk) than for previous accident years (reserve risk). For example, for a one-year time horizon the standard deviation of $\text{CDR}_{[0,1]}$ is 1,777,576 whereas for the $\text{CDR}_{[0,1]}^{\text{NY}}$ it is 1,134,309, but for a nine-year time horizon the standard deviation of $\text{CDR}_{[0,9]}$ is 2,451,642 and 4,957,674 for the $\text{CDR}_{[0,9]}^{\text{NY}}$. To emphasize the benefit of our integrated simulation-approach, we also present the descriptive statistics for the aggregated multi-year non-life insurance risk and correlation coefficients between premium and reserve risk. The results show a diversification effect between premium and reserve risk. For example, the sum of the standard deviation of $\text{CDR}_{[0,1]}$ and $\text{CDR}_{[0,1]}^{\text{NY}}$ is 2,911,885 (= 1,777,576 + 1,134,309) whereas the standard deviation of $\text{CDR}_{[0,1]}^{\text{PY+NY}}$ only is 2,257,751. The respective correlation parameters are derived automatically and no correlation assumptions for modeling the dependencies between reserve risk and premium risk are necessary. This is a major advantage since, usually, for the combined non-life insurance risk the two empirical frequency distributions of the m-year claims development result for previous accident years $\text{CDR}_{[0,\text{m}]}$ and for future accident years $\text{CDR}_{[0,\text{m}]}^{\text{NY}}$ have to be aggregated with appropriate correlation assumptions (e.g., within the standard formula of Solvency II or within internal risk models). For this purpose appropriate methods for modeling dependencies between the different stochastic variables have to be found (see e.g., Kaufmann et al., 2001) which is difficult and so very different assumptions can be found in practice. For example, in case of normally distributed risks, this can be done using a square root aggregation formula and predefined correlation parameters (for a critical discussion of the square root aggregation formula see Pfeifer and Strassburger, 2008). Within internal risk models, very often independence between premium risk and reserve risk is assumed, whereas in Solvency II the correlation coefficient used within the standard formula is 50% (see CEIOPS, 2010). In our case, both assumptions don't match. The first choice (i.e., 0%) seems relatively low, whereas the second choice (i.e., 50%) seems relatively high, since the correlation coefficient for the combination of the one-year reserve and premium risk in our model is 16.14%.

Because the claims development triangle in Table 1 is completely settled after nine years, we have $\mathrm{CDR}_{[0,9]} = \mathrm{CDR}_{[0,\omega]}$. This means for previous accident years the ultimate claims development result is equal to the nine-year claims development result. This is achieved by repeating the re-reserving method for nine future development years. For future accident years, however, the state of final settlement has only been reached after 14 years ($14 = 9 + 5$), i.e., $\mathrm{CDR}^{\mathrm{NY}}_{[0,14]} = \mathrm{CDR}^{\mathrm{NY}}_{[0,\omega]}$. Nevertheless, for the reason of comparability between reserve risk and premium risk, we decided to only present nine future development years also.[4]

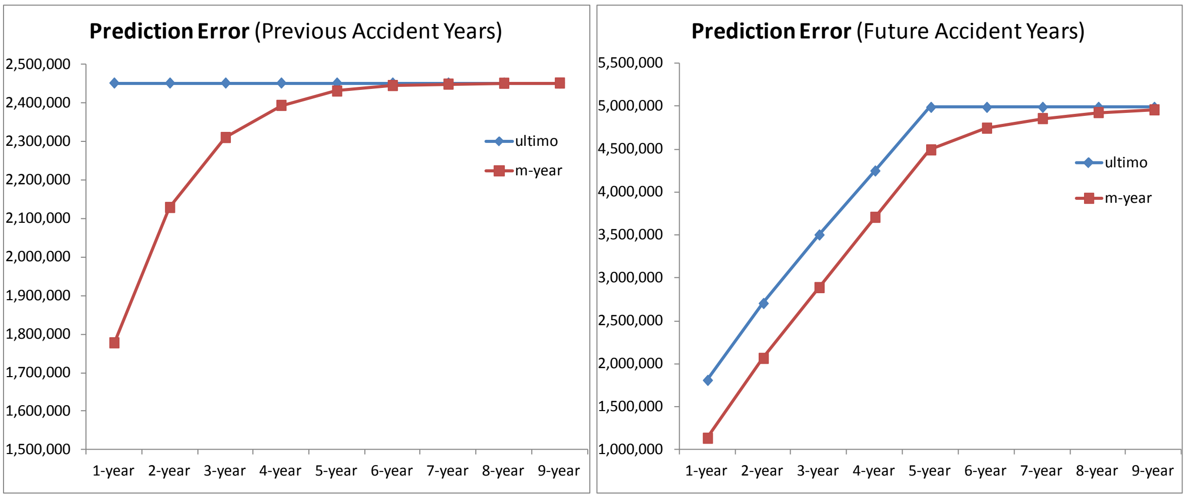

Figure 6: Prediction Error (Ultimo versus Multi-Year)

To illustrate the mechanism of the re-reserving process, in Figure 6 we show the development of the prediction error (standard deviation, see Table 3) of the one-year claims development result for previous accident years (and for future accident years, respectively) up to the nine-year claims development result for previous accident years (and for future accident years, respectively). Here we find that the greater the time horizon the higher the variability that comes from the claims development result. This is because the greater the time horizon, the more future claim payments are simulated via stochastic simulation methods and thus the variability increases. Moreover – since we only consider five future accident years – the

[4] This is not a critical assumption, since later considerations will show that, also for future accident years, after nine development years the ultimate has almost been reached. For example in Table 4 we show the development of risk capital for reserve risk and premium risk. Here the risk capital for the nine-year CDR, at 99.5% confidence level using VaR, equals 14,087,283. The risk capital for the 14-year CDR, at 99.5% confidence level using VaR, equals 14,127,872. This only represents a difference of 0.29%.

ultimate prediction error of future accident years (right part of Figure 6) increases within the first five accident years on an annual basis whereas the final ultimate prediction error is only reached, when no further accident years are considered (starting in year 5). It is interesting to note, that especially for the first new accident year the one-year premium risk is below the ultimate risk.

By means of the risk measures VaR and TVaR we now calculate the risk capital needed to survive at a given confidence level α. For this purpose we use the empirical frequency density shown in Figure 5 where we have the negative multi-year claims development result for previous accident years ($-\mathrm{CDR}_{[0,m]}$) as well as the negative multi-year claims development result for future accident years ($-\mathrm{CDR}_{[0,m]}^{\mathrm{NY}}$) as random variables of losses L for the respective time horizon (m-year). The result, i.e., the development of risk capital over time from a one-year time horizon up to the ultimo time horizon, for different confident levels, is shown in Table 4. Here we first apply ρ on $-\mathrm{CDR}_{[0,m]}$ and $-\mathrm{CDR}_{[0,m]}^{\mathrm{NY}}$ (see equation (4)), whereas in the following step (see Table 5) we apply ρ on the maximum function of $-\mathrm{CDR}_{[0,m]}$ and $-\mathrm{CDR}_{[0,m]}^{\mathrm{NY}}$ (see equation (6)).

Year	**Previous Accident Years**				**Future Accident Years**			
	$\mathrm{VaR}_{99.5\%}$	$\mathrm{VaR}_{99.8\%}$	$\mathrm{TVaR}_{99.5\%}$	$\mathrm{TVaR}_{99.8\%}$	$\mathrm{VaR}_{99.5\%}$	$\mathrm{VaR}_{99.8\%}$	$\mathrm{TVaR}_{99.5\%}$	$\mathrm{TVaR}_{99.8\%}$
1-year	4,749,386	5,316,952	5,286,335	5,823,192	2,977,628	3,321,919	3,350,394	3,691,877
2-year	5,792,383	6,507,259	6,472,509	7,155,006	5,716,118	6,498,808	6,506,921	7,181,772
3-year	6,327,244	7,156,092	7,135,439	7,900,373	8,003,254	9,059,710	9,107,288	10,077,765
4-year	6,581,494	7,468,877	7,462,006	8,254,423	10,349,252	11,664,837	11,760,559	13,004,729
5-year	6,677,161	7,618,148	7,580,640	8,450,888	12,588,903	14,243,983	14,333,964	15,943,355
6-year	6,734,002	7,660,221	7,623,855	8,491,308	13,428,579	15,168,296	15,270,085	16,916,089
7-year	6,741,888	7,669,861	7,602,873	8,499,022	13,696,218	15,435,701	15,588,639	17,318,276
8-year	6,741,053	7,681,002	7,635,883	8,521,712	13,965,029	15,754,658	15,829,143	17,463,571
9-year	6,737,416	7,680,650	7,608,386	8,527,169	14,087,283	15,901,677	15,948,394	17,583,110

Table 4: Risk Capital (Non-Maximum Function)

Table 4 shows that the one-year risk capital for the reserve risk measures is around 70% of the ultimo risk capital (e.g., at 99.5% confidence level using VaR, 4,749,386 are needed at a one-year horizon and 6,737,416 at a nine-year horizon). The one-year risk capital for the premium risk, however, is around 62% of the ultimo risk capital. Moreover, the development of risk capital from a one-year perspective to a nine-year perspective shows that after approximately five years we have almost reached the ultimate.

In a multi-year context management also risks of running out of capital before the end of period $t = m$. Hence, we have to take into account the fact that the negative multi-year claims development result for previous accident years $(-\mathrm{CDR}_{[0,m]})$ and for future accident years $(-\mathrm{CDR}_{[0,m]}^{\mathrm{NY}})$ at the end of period $t = m$ can be lower than any negative multi-year claims development result before the end of this period. Thus, we take the maximum loss of all negative multi-year claims development results as a random variable of losses L for the respective future development years $\{1, \ldots, m\}$. To illustrate this effect we picked one randomly chosen scenario out of the 1,000,000 simulations and compared the development of the multi-year claims development result for previous and future accident years using the maximum function defined within equation (5) (see Figure 7). For the simulated previous accident years (starting from year 3) and for the simulated future accident years (starting from year 2) we have a different development with and without the use of the maximum function.

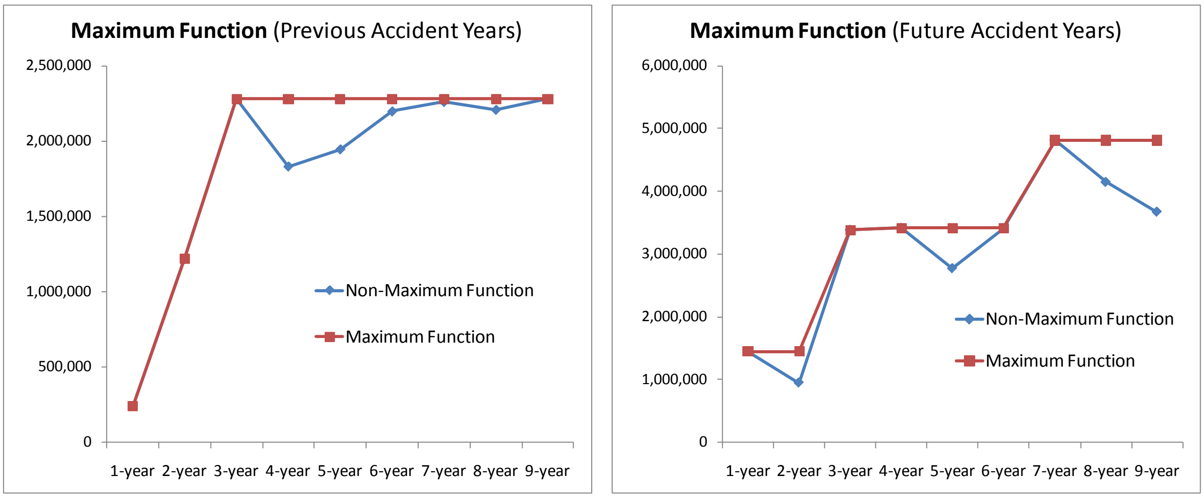

Figure 7: Maximum Function Versus Non-Maximum Function

This process leads to different empirical frequency distributions for the multi-year claims development result of previous accident years and of future accident years and thus to a different need for risk capital. The results for the risk capital in the case of using the maximum function are shown in Table 5. Hereby, risk capital for the one-year claims development result exactly equals the case of not using the maximum function (see Table 4).

For all the other claims development results, however, the demand for risk capital is slightly higher than before (e.g., for a five-year time horizon at 99.5% confidence level using VaR 6,677,161 are needed not using the maximum function, whereas 6,927,992 are needed using the maximum function).

Year	Previous Accident Years				Future Accident Years			
	$\text{VaR}^{max}_{99.5\%}$	$\text{VaR}^{max}_{99.8\%}$	$\text{TVaR}^{max}_{99.5\%}$	$\text{TVaR}^{max}_{99.8\%}$	$\text{VaR}^{max}_{99.5\%}$	$\text{VaR}^{max}_{99.8\%}$	$\text{TVaR}^{max}_{99.5\%}$	$\text{TVaR}^{max}_{99.8\%}$
1-year	4,749,386	5,316,952	5,286,335	5,823,192	2,977,628	3,321,919	3,350,394	3,691,877
2-year	5,829,230	6,535,172	6,487,012	7,168,015	5,716,118	6,498,808	6,506,921	7,181,772
3-year	6,453,611	7,258,944	7,226,344	7,972,264	8,020,660	9,060,623	9,118,491	10,079,486
4-year	6,762,882	7,636,010	7,628,222	8,397,742	10,385,040	11,688,567	11,783,031	13,017,758
5-year	6,927,992	7,850,977	7,889,427	8,642,606	12,639,356	14,273,330	14,380,954	15,991,365
6-year	7,027,061	7,941,547	7,950,906	8,737,128	13,698,378	15,515,873	15,538,528	17,161,237
7-year	7,057,542	7,969,071	7,980,294	8,763,660	14,175,366	16,036,263	16,074,499	17,734,383
8-year	7,069,649	7,986,028	7,987,464	8,784,452	14,519,945	16,312,349	16,414,612	18,051,804
9-year	7,072,591	7,993,424	7,992,291	8,795,100	14,739,488	16,440,857	16,602,097	18,224,937

Table 5: Risk Capital (Maximum Function)

Finally, in Figure 8 we illustrate the development of risk capital for all risk measures and confidence levels we used overall. This Figure shows that the higher the confidence level, the higher the demand for risk capital. Furthermore, the use of TVaR instead of VaR leads to a higher demand as well. Moreover, the use of the maximum function also leads to a different demand on risk capital. The use of the maximum function yields a slightly higher demand than the non use of the maximum function.

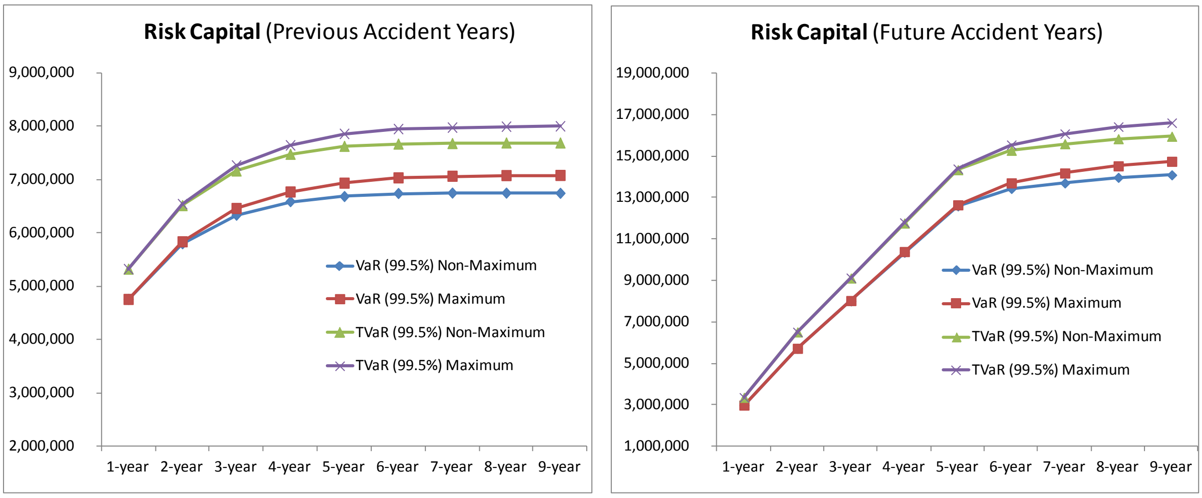

Figure 8: Risk Capital (Ultimo versus Multi-Year)

5.3. Discussion of Long-Tail versus Short-Tail Business

In this application we use a claims development triangle for a long-tail line of business. To complement the results of this analysis, we have also used a claims development triangle for a short-tail line of business. In this case the one-year risk capital for the reserve risk is around 90% of the ultimo risk, and for the premium risk it is around 85% of the ultimo risk (detailed results are available upon request). Comparing the results for the short- and long-tail line, we can see that the risk capital for the long-tail line is strongly underestimated in the one-year view compared to the ultimo view – a problem which was discussed several times in the literature (see Ohlsson and Lauzeningks, 2009; Dhaene et al., 2008). Hence, in the context of Solvency II, the use of an additional risk margin is supposed to prevent the underestimation of risk in the one-year view.

For this phenomenon the multi-year view introduced in this paper can provide valuable managerial information as it adequately takes into account the long-term nature of some insurance contracts and provides a more complete picture of the development of the risk situation over time. The integration of the multi-year view in internal risk models might also serve as a solution to the dilemma outlined by Ohlsson and Lauzeningks (2009) that "an ultimo perspective for liabilities with a one-year perspective for assets is not an alternative if we are interested in the combined total risk of the company." Both perspectives can be well integrated in the multi-year analysis of assets and liabilities, e.g., using a five-year planning horizon both for assets and liabilities. We thus believe that the multi-year approach can create a better sense for risk exposure and enriches the one-year and ultimo perspective.

5.4. Results for the Risk Margin

In a last step we calculate the risk margin used within Solvency II (see Section 4). For this purpose we first have to determine the future SCR_t at each point of time $t \in \{2, \ldots, 9\}$. We use the method of moments within each simulation step z (i.e. 100,000 simulation steps) to fit a normal distribution and then use VaR at 99.5% confidence level (see CEIOPS, 2010) to derive the corresponding SCR_t^z (we select the normal distribution to derive consistent results with premium risk and reserve risk as presented in Section 5.2). Hence, within each point of time $t \in \{2, \ldots, 9\}$ we get an empirical frequency distribution of future SCR_t (see Figure 9).

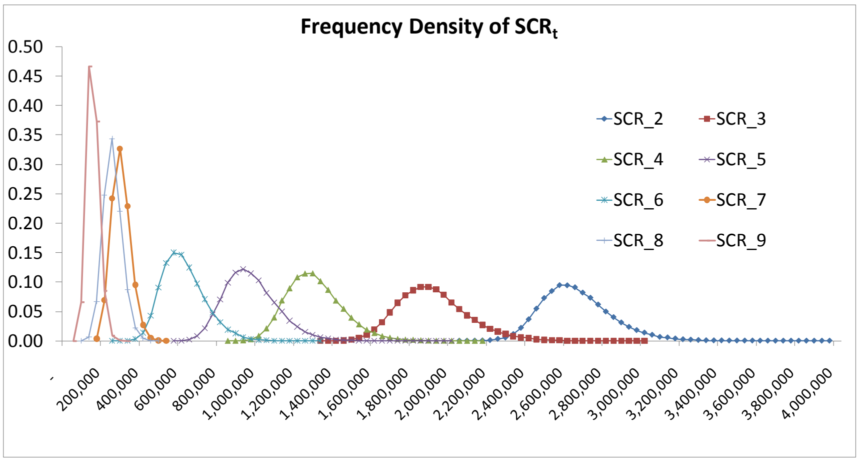

Figure 9: Frequency Density of $\mathrm{SCR_t}$

The greater the time horizon, the smaller the variation and thus the risk exposure, since the standard deviation decreases. The reason for this effect comes from the fact that for later years the claims in our application are almost completely settled and thus not much variation is left. As a consequence thereof, we can also see that the greater the time horizon the smaller the mean, since the different frequency distributions move to the left. We can use equation (8) to derive the overall $\mathrm{SCR_t}$ at each point in time and then use equation (7) to calculate the corresponding risk margin. The results are shown in Table 6.

	$\mathrm{SCR_t}$	Aggregated $\mathrm{SCR_{[0,m]}}$	Multi-Year $\mathrm{SCR_{[0,m]}}$
1-year	4,749,386	4,749,386	4,749,386
2-year	2,628,209	7,377,595	5,792,383
3-year	1,883,095	9,260,690	6,327,244
4-year	1,269,995	10,530,686	6,581,494
5-year	939,482	11,470,168	6,677,161
6-year	590,827	12,060,995	6,734,002
7-year	265,374	12,326,369	6,741,888
8-year	223,263	12,549,632	6,741,053
9-year	120,191	12,669,823	6,737,416
Risk Margin	760,189		

Table 6: Aggregated SCR versus Multi-Year SCR

The second column of Table 6 shows the $\mathrm{SCR_t}$ at each point in time $\mathrm{t} \in \{2, \dots, 9\}$. The $\mathrm{SCR_t}$ is calculated by quantifying the expected value of the corresponding random variables shown in Figure 9. The risk margin is then derived by the product of the sum of future $\mathrm{SCR_t}$ at each point in time, and a cost-of-capital rate (see equation (7)). We choose the cost-of-capital rate of 6% (see CEIOPS, 2010). The third and fourth column of Table 6 show a comparison between the aggregated $\mathrm{SCR_{[0,m]}}$ derived by summing up the one-year $\mathrm{SCR_t}$ in year t and the multi-year $\mathrm{SCR}_{[0,\mathrm{m}]}$ taken from Table 5 of the multi-year internal risk model. The comparison reveals a tremendous diversification effect by using multi-year risk capital instead of summing up the one-year risk capital of each future calendar year t. This is because calculating the one-year $\mathrm{SCR}_{\mathrm{t}}$ at each point of time means that some risk measure ρ is applied every year, whereas within the multi-year internal risk model, the risk measure ρ is only applied once over the whole time horizon of m years; we thus see diversification over time.

6. Conclusion

The aim of this paper was to present a modeling approach for determining the non-life insurance risk in a multi-year context. Multi-year non-life insurance risk can be analyzed by simulating the probability distributions of the random variables of the claims development result for previous accident years (reserve risk) and for future accident years (premium risk). We quantified the corresponding risk capital using risk measures such as VaR and TVaR. Furthermore, based on the cost-of-capital approach used within Solvency II, we presented an integrated simulation model for determining the corresponding risk margin in a multi-year context. Next to the traditional view (ultimo perspective) of non-life insurance risk, academic literature has so far focused only on a one-year perspective (see, e.g., Merz and Wüthrich, 2008; Ohlsson and Lauzeningks, 2009; Gault et al., 2010). We extend those recent contributions by illustrating how the one-year perspective can be transferred into an ultimo perspective using a step-by-step multi-year perspective. We believe the multi-year approach can improve our sense of risk exposure and thus enrich the one-year and ultimo perspective.

The three main contributions of this paper are the following. (1) Strategic management and decision making of insurance companies require a multi-year risk horizon; the model presented in this paper offers the benefit of a multi-year risk perspective on reserve risk and premium risk that can be used in the context of internal risk models. (2) The one-year risk perspective within Solvency II does not take into account the long-term nature of especially long-tail lines of business. The simulation model presented here provides a good understanding of how non-life insurance risk evolves over time. (3) For the calculation of the Solvency II risk margin a cost-of-capital approach is used. We present an integrated way of simulating future SCRs that can be used for calculating the risk margin in a one-year economic perspective as well as in a multi-year economic perspective.

The risk model presented here opens various future research options. First, the use of the Mack (1993) model as the underlying stochastic re-reserving model and the corresponding chain-ladder claims reserving algorithm can be replaced by different stochastic claims reserving models such as the over-dispersed Poisson model or the over-dispersed negative binomial model (see England and Verrall, 2006). Second, the underlying claims development triangle usually ends before the claims are completely settled, thus extrapolation techniques can be used to analyze the tail behavior by estimating corresponding tail factors. Third, the market-consistent valuation of best estimate claims reserve and the corresponding risk margin usually includes discounting by an adequate risk-free yield curve (see Ohlsson and Lauzeningks, 2009), hence future research can extend the present risk model by discounting aspects.

Another avenue of future research might be to evaluate whether the multi-year view can also be transferred to analytical reserving methods. For this purpose one could try to extend the analytical approach for quantifying the MSEP of the one-year claims development result – based on the classic chain-ladder method (see Merz and Wüthrich, 2008) or based on the additive loss reserving method (see Merz and Wüthrich, 2010) – from a one-year perspective to a multi-year context. The results for risk capital based on analytical and simulation-based methods could then be compared.

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