Making use of netting effects when composing life insurance contracts

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Abstract: In this paper, we discuss netting effects in life insurance policies provided by the natural hedge between payments that are due when sojourning in a state and when leaving a state. We uncover potentials for such netting effects with the help of a sensitivity analysis, and we quantify the effect on solvency reserves with the help of a worst-case analysis. The paper discusses a number of examples where netting effects occur and shows for which ratios between different benefit types the netting effects are strongest.

Keywords: life insurance; netting effects; Solvency II; sensitivity analysis; worst-case analysis

1 Introduction

Life insurance policies are typically long term contracts whose actuarial assumptions may undergo significant and unforeseeable changes within the time horizon, thus exposing the insurer to a risk that is non-diversifiable. Theoretically, this risk can be eliminated by letting the contractual payments depend on the actual development of the actuarial parameters, e.g. defined contribution plans. However, in practice insurers often offer policies that also include guarantees in nominal amounts, e.g. defined benefit plans, since they are - especially in Germany - frequently demanded by customers, thus exposing the insurer to a considerable part of the non-diversifiable risk.

Though the non-diversifiable risk could theoretically be completely transferred to a third party by reinsurance or securitisation, most of it is practically still covered by the insurers themselves. Therefore it is worthwhile to start the risk management already at the earliest stage, the designing of the contracts, and to make use of netting effects provided by the natural hedge between payments that are due when sojourning in a state and when leaving a state. The latter method is not a panacea that solves all the problems that life insurers have with non-diversifiable risks, since the design of life insurance policies still has to take respect of customer demand. But by optimizing the ratios between different benefit types within the scope that is left, an insurance company can significantly reduce its risk load and gain a strategic advantage.

In order to find potential netting effects, we apply the sensitivity analysis concept of Christiansen (2008). The approach generates graphic images of the fundamental risk structure of a life insurance product that can be easily interpreted. As the concept is based on derivatives, we have some form of linearity of the risk structures with respect to benefit and premium payments, and therefore netting effects can be easily produced by linearly combining insurance contracts with opposite risk structures.

In order to measure the impact of netting effects on the risk load of an insurer, we quantify the impact on solvency reserves. It is tempting to use the standard formula of Solvency II, but the problem is that it produces very poor results just then when policies have strong netting effects. Therefore, we apply the worst-case concepts of Christiansen (2010) and Christiansen and Denuit (2010), which can be seen as further developments of the standard formula.

The structure of the paper is as follows: Section 2 yields a brief introduction into the sensitivity analysis concept and the worst-case concepts mentioned above. Algorithm 2.1 generalizes an idea of Christiansen and Denuit (2010) from a two-state model to a multi-state model. Section 3 studies typical life insurance contracts that show a netting effect. We analyze the risk structures and study which ratios between different benefit types lead to the strongest netting effects.

2 Methods

Consider a multi-state policy that is issued at time x, terminates at a fixed finite time ω, and is driven by a right-continuous Markovian jump process $(X_t)_t$ with finite state space $\mathcal{S}$. Let $J = \{(i,j) \in \mathcal{S} \times \mathcal{S} \,|\, i \neq j\}$ be the space of possible transitions. The cash-flows of the contract are described by the following functions:

(i) The lump sum $b_{ij}(t)$ is payable upon transition $(i,j) \in J$ at time t.

(ii) $B_i(t)$ gives the accumulated annuity benefits minus premiums that fall due due to a sojourn in state i up to and including time t.

We write $v(s,t)$ for the value at time s of a unit payable at time $t \geq s$ and assume that

$$v(s,t) = e^{-\int_s^t \varphi(u)\,\mathrm{d}u}\,.$$

Function φ is the so-called interest intensity. For the modeling of the probability distribution of the random pattern of states $(X_t)_t$, in our examples we only have information about the yearly transition rates

$$q_{x+n}^{ij} = P(T_{x+n}^{ij} \leq n+1)\,, \quad n \in \{0, ..., \omega - x - 1\}\,, (i,j) \in J\,,$$

where T_{x+n}^{ij} is the first time to reach state j after starting from state i at time $x+n$. In order to obtain a continuous time multi-state model, we assume that $(X_t)_t$ has the transition intensities

$$\mu_{ij}(t) := -\ln(1 - q_{\lfloor t \rfloor}^{ij})\,, \quad t \in [0, \omega)\,, (i,j) \in J\,.$$

The prospective reserve $V_i(s)$ at time s in state i is defined as the expected present value of future benefits minus the expected present value of future premiums given that the policyholder is at present time s in state i. (We only include payments that occur *strictly past* time s.) It can be calculated by solving Thiele's integral equation system,

$$V_i(s) = B_i(\omega) - B_i(s) - \int_s^\omega V_i(t-)\,\varphi(t)\,\mathrm{d}t + \sum_{j:j\neq i} \int_s^\omega R_{ij}(t)\,\mu_{ij}(t)\,\mathrm{d}t$$

with starting values $V_i(\omega) = 0$, where $R_{ij}(t) = b_{ij}(t) + V_j(t-) - V_i(t-)$ is the so-called sum at risk for transition (i,j) at time t. In order to find the unique solution, we decompose the prospective reserve into

$$V_i(s) = \big(V_i(s) + B_i(s)\big) - B_i(s) =: W_i(s) - B_i(s)$$

and calculate the differentiable functions W_i by solving the following differential equation system numerically (e.g. with the Runge-Kutta method),

$$\frac{\mathrm{d}}{\mathrm{d}s} W_i(s) = \big(W_i(s) - B_i(s-)\big)\,\varphi(s) - \sum_{j:j\neq i} \Big(b_{ij}(s) + W_j(s) - B_j(s-) - W_i(s) + B_i(s-)\Big)\,\mu_{ij}(s)$$

with starting values $W_i(\omega) = B_i(\omega)$.

Suppose for the moment that the future transition probabilities q^{ij}_{x+n} and discounting factors $v(s,t)$ are perfectly known. Then the law of large numbers yields that the prospective reserve equals approximately the average future liabilities per policy for a large homogeneous portfolio of independent contracts. The only risk is in the approximation error, which can be diversified by increasing the size of the portfolio. It is a core competence of life insurers and reinsurers to deal with this diversifiable risk (or unsystematic biometric risk). In this paper we are interested in the risk that the future transition probabilities q^{ij}_{x+n} evolve differently than anticipated. This risk – usually denoted as systematic biometric risk – is not diversifiable by just increasing the size of the portfolio since a change of the q^{ij}_{x+n} has the same effect on all policies. Policy design can have a great influence on the load of systematic biometric risk that is on the account of the insurer, and it is the aim of this paper to find policy designs which reduce that risk load. The following subsection presents a sensitivity analysis concept that helps to study the effect that fluctuations of the transition probabilities q^{ij}_{x+n} have on prospective reserves $V_i(s)$.

2.1 Sensitivity Analysis

Assume that the transition intensities $\mu_{ij}(t) = -\ln(1-q^{ij}_{\lfloor t \rfloor})$ are shifted by h_{ij} to $\mu_{ij}+h_{ij}$. According to Christiansen (2008), in obvious notation we have a generalized first-order Taylor expansion of the form

$$V_i(s, \mu + h) = V_i(s, \mu) + \sum_{(j,k)\in J} \int_s^\omega h_{jk}(t)\, \mathrm{grad}_{\mu_{jk}}\big(V_i(s,\mu)\big)(t)\, \mathrm{d}t \;+\; o(\|h\|) \tag{2.1}$$

with generalized gradients

$$\mathrm{grad}_{\mu_{jk}}\big(V_i(s,\mu)\big)(t) = v(s,t)\, P(X_t = j | X_s = i)\, R_{jk}(t)\,. \tag{2.2}$$

The quantity $\mathrm{grad}_{\mu_{jk}}\big(V_i(s)\big)(t)$ can be seen as sensitivity of $V_i(s,\mu)$ to changes of the transition intensity μ_{jk} at time t. Because the prospective reserve is linear with respect to the payment functions $b_{jk}(t)$ and $B_j(t)$, the sum at risk $R_{jk}(t)$ is as well, and hence we have that also the sensitivities $\mathrm{grad}_{\mu_{jk}}\big(V_i(s,\mu)\big)(t) = v(s,t)\, P(X_t = j|X_s = i)\, R_{jk}(t)$ are linear with respect to the payment functions. This property is useful when combining different types of insurance policies: The sensitivity of a linear combination of different insurance contracts is equal to the linear combination of the sensitivities of the single insurance contracts.

Netting effects occur there where different types of benefit schemes have corresponding sensitivities (2.2) with opposite signs which partly cancel out each other. Hence, in order to create strong netting effects, we can use the following concept, which makes use of the linearity of the sensitivities (2.2) with respect to the payment functions:

1. For each type of benefit scheme (e.g., death benefit, disability annuity, old-age pension, ...) that could be part of a combined insurance contract calculate the sensitivities (2.2) separately.

2. Look for linear combinations of the separate sensitivities that have strong cancelation effects between positive and negative values.

2.2 Worst-case analysis with bounded transition intensities

The sensitivity analysis concept of above helps to find or create strong netting effects, but it does not yield a risk measure (for the systematic biometric risk of a policy) that would allow to quantify the advantage that we gained from netting effects. For several reasons it seems to be convenient to use (parts of) the standard formula of Solvency II as risk measure. That means we measure the systematic biometric risks as the change in the net value of future assets and liabilities due to transition intensities μ_{ij} that are much worse than anticipated. Unfortunately, the standard formula produces very poor results just then when we have strong netting effects, because it takes only scenarios into consideration where the transition intensities are throughout higher and throughout lower than anticipated. While this upper und lower scenarios are worst-cases if the policy has throughout positive and negative sensitivities, respectively, they are mostly not the crucial scenarios to look at if the sensitivity functions $\mathrm{grad}_{\mu_{jk}}(V_i(s,\mu))(t)$ have sign switches, which is usually the case for policies with strong netting effects. The inventors of the standard formula seemed to be aware of that problem when they offered an alternative calculation method (option 2) where policies with mixed character are separated into two contracts. But then we loose the netting effects that we are looking for in this paper.

A possible solution is to modify the standard formula for policies with mixed character: We interpret the throughout higher and throughout lower scenarios of the standard formula just as upper and lower bounds and take as risk measure the maximal change in net value of assets and liabilities that can occur with respect to *all* scenarios within the upper and lower bound. With writing

$$l_{ij}(t) \le \mu_{ij}(t) \le u_{ij}(t)\,, \quad (i,j) \in J\,, \tag{2.3}$$

for the lower and upper bounds of the μ_{ij} , our risk measure is now defined as

$$\max_{l \le \mu+h \le u} V_i(s, \mu+h) - V_i(s,\mu)\,. \tag{2.4}$$

In order to find that maximum, we can use a method of Christiansen (2010): The maximum $\overline{V}_i(s) := \max_{l\le\mu+h\le u} V_i(s,\mu+h)$ can be obtained as the unique solution of an integral equation system similar to Thiele's integral equation system,

$$\begin{aligned}\overline{V}_i(s) = B_i(\omega) - B_i(s) - \int_s^\omega \overline{V}_i(t-)\,\varphi(t)\,\mathrm{d}t \\ + \sum_{j:j\neq i} \int_s^\omega \left(\overline{R}_{ij}(t)\,\frac{u_{ij}(t)+l_{ij}(t)}{2} + |\overline{R}_{ij}(t)|\,\frac{u_{ij}(t)-l_{ij}(t)}{2}\right) \mathrm{d}t\end{aligned}$$

with starting values $\overline{V}_i(\omega) = 0$. In order to solve the integral equation system, we decompose the maximal prospective reserve into

$$\overline{V}_i(s) = \big(\overline{V}_i(s) + B_i(s)\big) - B_i(s) =: \overline{W}_i(s) - B_i(s)$$

and calculate the differentiable functions $\overline{W}_i$ by solving the following differential equation system numerically,

$$\frac{\mathrm{d}}{\mathrm{d}s}\overline{W}_i(s) = \big(\overline{W}_i(s) - B_i(s-)\big)\,\varphi(s) - \sum_{j:j\neq i}\left(\overline{R}_{ij}(s)\,\frac{u_{ij}(s)+l_{ij}(s)}{2} + |\overline{R}_{ij}(s)|\,\frac{u_{ij}(s)-l_{ij}(s)}{2}\right)$$

with starting values $\overline{W}_i(\omega) = B_i(\omega)$ and $\overline{R}_{ij}(s) = b_{ij}(s) + \overline{W}_j(s) - B_j(s-) - \overline{W}_i(s) + B_i(s-)$. With defining $\overline{\mu}_{ij}$ by

$$\overline{\mu}_{ij}(t) := \begin{cases} u_{ij}(t) & : \overline{R}_{ij}(t) > 0 \\ \frac{1}{2}u_{ij}(t) + \frac{1}{2}l_{ij}(t) & : \overline{R}_{ij}(t) = 0 \\ l_{ij}(t) & : \overline{R}_{ij}(t) < 0 \end{cases}$$

we have a scenario that corresponds to the maximum $\overline{V}_i(s) = V_i(s,\overline{\mu})$ for all s and all i.

2.3 Worst-case analysis with bounded transition intensity increments

The worst-case scenario $\overline{\mu}$ of the previous section looks not very realistic as it jumps between extremes, in particular if we think of mortality rates. However, such a scenario makes sense in risk management if one is interested not only in usual but also in extreme future developments. But still we can ask if it is possible to restrict the worst-case calculation to scenarios which do not have extreme jumps. Christiansen and Denuit (2010) present an idea to that question for two-state policies where the only transition is from active to dead. Here we generalize their idea to a multi-state model:

Instead of bounding the μ_{ij} by (2.3) for all times t, we bound the μ_{ij} only at present time s but in addition limit the increases $\Delta\mu_{ij}(x+n) := \mu_{ij}(x+n)-\mu_{ij}(x+n-1) = -\ln(1-q^{ij}_{x+n})+\ln(1-q^{ij}_{x+n-1})$ at integer times $x+n>s$,

$$\begin{aligned} l_{ij}(s) \leq \mu_{ij}(s) \leq u_{ij}(s)\,, \quad & (i,j)\in J\,, \\ \Delta l_{ij}(x+n) \leq \Delta\mu_{ij}(x+n) \leq \Delta u_{ij}(x+n)\,, \quad & (i,j)\in J\,, x+n=\lfloor s\rfloor+1,...,\omega-1\,. \end{aligned} \tag{2.5}$$

In order to simplify the notation, we set $\Delta\mu_{ij}(\lfloor s\rfloor) := \mu_{ij}(\lfloor s\rfloor)$, $\Delta l_{ij}(\lfloor s\rfloor) := l_{ij}(\lfloor s\rfloor)$, $\Delta u_{ij}(\lfloor s\rfloor) := u_{ij}(\lfloor s\rfloor)$ and use the equivalent condition

$$\Delta l_{ij}(x+n) \leq \Delta\mu_{ij}(x+n) \leq \Delta u_{ij}(x+n)\,, \quad (i,j)\in J\,, x+n=\lfloor s\rfloor,...,\omega-1\,.$$

(Note that Christiansen and Denuit (2010) assume that the mortality rate is fixed at time s, whereas we allow here that the $\mu_{ij}(\lfloor s\rfloor)$ may vary.) The idea is now to measure the systematic biometric risk by

$$\max_{\Delta l \leq \Delta(\mu+h)\leq \Delta u} V_i(s,\mu+h) - V_i(s,\mu)\,. \tag{2.6}$$

How can we find that maximum? By replacing $h_{jk}(t)$ in (2.1) with $h_{jk}(t) = \sum_{m=[s]}^{\lfloor t\rfloor} \Delta h_{jk}(m)$ and changing the order of summation and integration, we obtain the following first-order Taylor expansion

$$V_i(s,\mu+h) = V_i(s,\mu) + \sum_{(j,k)\in J}\sum_{m=[s]}^{\omega-1} \Delta h_{jk}(m)\,\frac{\partial V_i(s,\mu)}{\partial(\Delta\mu_{jk}(m))} \;+\; o(\|h\|) \tag{2.7}$$

with partial differentials of the form

$$\frac{\partial V_i(s,\mu)}{\partial(\Delta\mu_{jk}(m))} = \int_{s\vee m}^{\omega} \mathrm{grad}_{\mu_{jk}}\big(V_i(s,\mu)\big)(t)\,\mathrm{d}t\,.$$

In the following we write

$$\mathrm{grad}_{\Delta\mu}\big(V_i(s,\mu)\big) := \Big(\frac{\partial V_i(s,\mu)}{\partial(\Delta\mu_{jk}(m))}\Big)_{m=[s]...\omega-1,\,(j,k)\in J}\,. \tag{2.8}$$

With the first-order Taylor expansion (2.7) we can formulate a gradient ascent method in order to find the maximum in (2.6):

Algorithm 2.1.

1. Choose a starting scenario $\Delta\mu^{(0)} = \Big(\Delta\mu^{(0)}_{jk}(m)\Big)_{m=[s]...\omega-1,\,(j,k)\in J}$.

2. Calculate a new scenario by using the iteration

$$\Delta\mu^{(l+1)} := \Delta\mu^{(l)} + K \operatorname{grad}_{\Delta\mu^{(l)}}(V_i(s,\mu^{(l)}))\,,$$

   where $K > 0$ is some step size that has to be chosen. If $\Delta\mu^{(l+1)}$ is above or below the bounds in (2.5), we cut it off at the Δu_{ij} or Δl_{ij}, respectively.

3. Repeat step 2 until $\left|V_i(s,\mu^{(l+1)}) - V_i(s,\mu^{(l)})\right|$ is below some error tolerance.

3 Examples

Here we focus only on the systematic mortality risk part of the systematic biometric risk, that is the risk that mortality rates may increase (in Solvency II denoted as 'mortality risk') or decrease (in Solvency II denoted as 'longevity risk') or even both at the same time (but not at the same age). We calculate the following risk measures with respect to the ratio between (opposite) benefit types:

(R1) systematic mortality risk according to option 1 of the standard formula

$$\sqrt{SCR_{mort}^2 + SCR_{long}^2 + 2\,(-0.25)\,SCR_{mort}\,SCR_{long}}\,,$$
$$SCR_{mort} = \max\{V_i(s,u),0\}\,, \quad SCR_{long} = \max\{V_i(s,l),0\}\,,$$

(R2) systematic mortality risk according to option 2 of the standard formula

$$\sqrt{SCR_{mort}^2 + SCR_{long}^2 + 2\,(-0.25)\,SCR_{mort}\,SCR_{long}}\,,$$
$$SCR_{mort} = V_i^{death}(s,u)\,, \quad SCR_{long} = V_i^{survival}(s,l)\,,$$

where $V_i(s,\cdot) = V_i^{death}(s,\cdot) + V_i^{survival}(s,\cdot)$ is a decomposition of the insurance contract into two separate components: one contingent, on the death and the other contingent on the survival of the insured person,

(R3) systematic mortality risk according to risk measure (2.4),

(R4) systematic mortality risk risk measure (2.6).

In all examples we choose as reference point the beginning of the contract period $s = x$ and the initial state $i = a =$'active' The definition of the standard formula is the latest version given by the technical specifications of Quantitative Impact Study 4 and Consultation Paper no. 49 of the Committee of European Insurance and Occupational Pensions Supervisors. (R1) often underestimates the systematic biometric risk because it considers only throughout higher or throughout lower biometric scenarios and does not take respect of mixed scenarios. (R2) allows for netting effects only via fixed correlation assumptions between different types of systematic biometric risks and therefore is not able to describe the real netting effects. (R3) and (R4) both allow for mixed scenarios and take fully respect of netting effects, thus giving a more realistic picture of the true risks. (R3) is generally greater than (R4) because it considers a wider range of scenarios, in particular scenarios that change very rapidly (e.g. direct jumps from the lower to the upper bound in (2.3) and vice versa).

We generally assume that interest is paid with intensity $\varphi(t) = \ln(1.0225)$, which corresponds to a yearly interest rate of 2.25%.

3.1 Pure endowment insurance and temporary life insurance

Consider a $x = 30$ year old male who contracts a combination of a pure endowment insurance and a temporary life insurance. The policy shall terminate at age 65. A lump sum of 1 is payable in case of survival, and a lump sum of $c > 0$ is payable in case of death before age 65. A constant premium is paid yearly in advance. The mortality rates q_{x+n}^{ad} are taken from the life table 2008 of the German Federal Statistical Office. The premium level is chosen in such a way that the equivalence principle holds. Figure 3.1 shows the mortality sensitivities $t \mapsto \operatorname{grad}_{\mu_{ad}}\big(V_a(30, \mu)\big)(t)$ of the pure endowment insurance part and of the temporary life insurance part with $c = 1$. As the sensitivities have opposite signs, we expect some netting effect here. As lower and upper bound in (2.3) we take the longevity shock (-25%) and the mortality shock ($+15\%$) of Solvency II (cf. Consultation Paper no. 49 of the Committee of European Insurance and Occupational Pensions Supervisors),

$$l_{ad}(t) = -0.75 \cdot \ln(1 - q_{\lfloor t \rfloor}^{ad}), \quad u_{ad}(t) = -1.15 \cdot \ln(1 - q_{\lfloor t \rfloor}^{ad}),$$

and their increments at integer times are used as limits in (2.5). Figure 3.2 shows the systematic mortality risk measured by (R1) to (R4) with respect to the level c of the death benefit. The risk measures (R3) and (R4) both yield that the netting effect between survival and death benefits is strongest for c near to 70%.

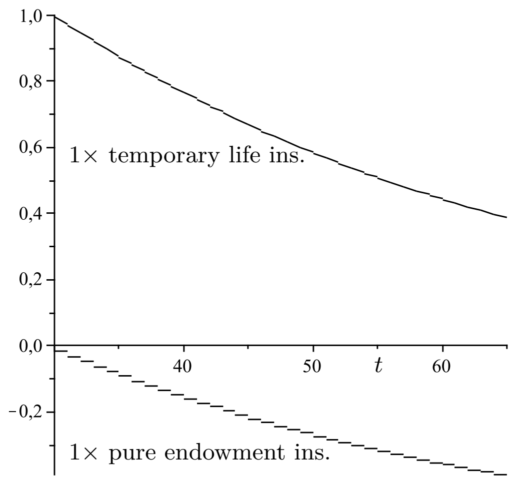

Figure 3.1: Sensitivity of $V_a(30-)$ with respect to the mortality intensity $\mu_{ad}(t)$

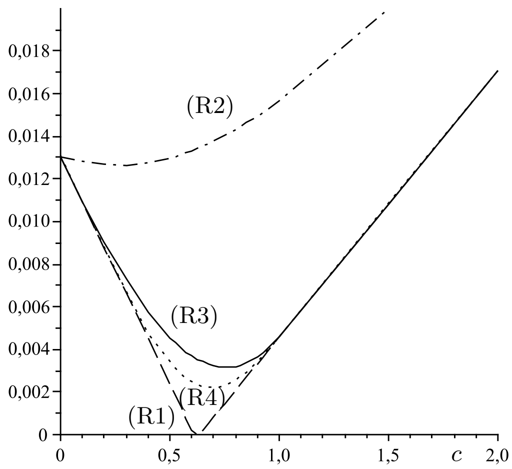

Figure 3.2: Systematic mortality risk for a combination of 1×(pure endowment insurance) and c×(temporary life insurance)

3.2 Annuity insurance and whole or temporary life insurance

Consider a $x = 30$ year old male who contracts a combination of an annuity insurance and a whole or temporary life insurance. A constant annuity of 1 is paid yearly in advance from age 65 on till death. A lump sum of $c > 0$ is payable

(a) in case of death at any age,

(b) in case of death before age 65.

A constant premium is paid yearly in advance. Again, the mortality rates q_{x+n}^{ad} are taken from the life table 2008 of the German Federal Statistical Office. The premium level is chosen in such a way that the equivalence principle holds. We take the same bounds for (2.3) and (2.5) as in the previous example.

Figures 3.3 and 3.5 show the mortality sensitivities $t \mapsto \text{grad}_{\mu_{ad}}\big(V_a(30,\mu)\big)(t)$ of the annuity insurance part and of the whole/temporary life insurance part of the policy. For example (a) we can have netting effects between survival and death benefits for the whole contract period, whereas example (b) can only have netting effects till age 65. Therefore we expect that example (a) shows stronger netting effects.

Figures 3.4 and 3.6 show the systematic mortality risk measured by (R1) to (R4) with respect to the level c of the death benefit. Indeed, example (a) has much stronger netting effects than example (b). Interestingly, in example (b) the risk measures (R3) and (R4) are greater than (R2) for c greater than 26 and 76. That means that the Solvency II correlation assumption of -25% between the risks of rising and falling mortality rates exaggerates the netting effects here.

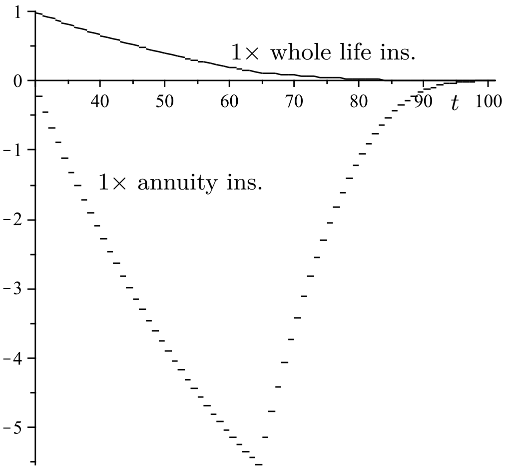

Figure 3.3: Sensitivity of $V_a(30-)$ with respect to the mortality intensity $\mu_{ad}(t)$

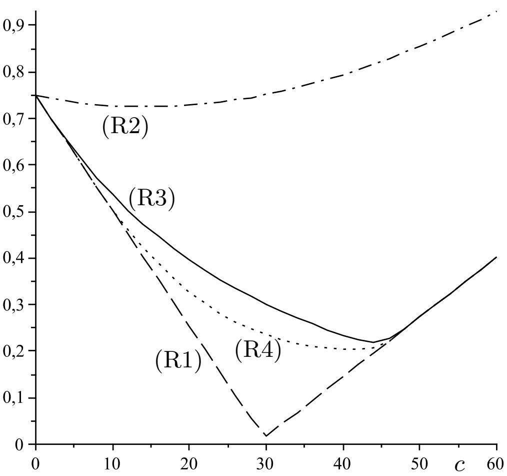

Figure 3.4: Systematic mortality risk for a combination of $1\times$(annuity insurance) and $c\times$(whole life insurance)

3.3 Disability insurance and temporary life insurance

Consider a $x = 30$ year old male who contracts a combination of a disability insurance and a temporary life insurance. In case of disability a constant annuity of 1 is paid yearly in advance till age 65 or till death, whichever occurs first. A lump sum of $c > 0$ is payable in case of death before age 65. A constant premium has to be paid yearly in advance in both states, active and disabled. The mortality rates q_{x+n}^{ad} and q_{x+n}^{id} are taken from the life table 2008 of the German Federal Statistical Office and the life table DAV 1997 TI of the German Actuarial Association. For the transition intensities q_{x+n}^{ai} and q_{x+n}^{ia} we use the finite tables of DAV 1997 I and DAV 1997 RI. The premium level is chosen in such a way that the equivalence principle holds. We focus only on the systematic mortality risk part of the systematic biometric risk by letting $\mu_{ai}(t)$ and $\mu_{ia}(t)$ be

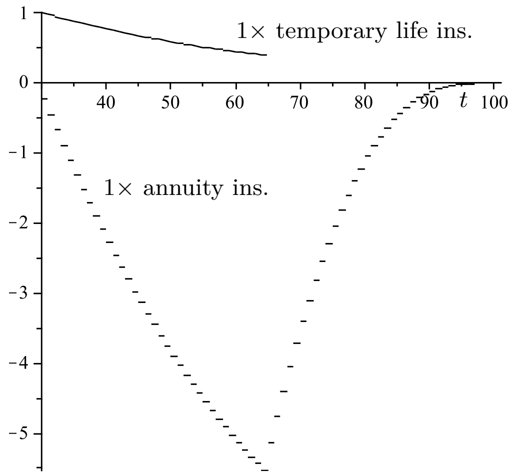

Figure 3.5: Sensitivity of $V_a(30-)$ with respect to the mortality intensity $\mu_{ad}(t)$

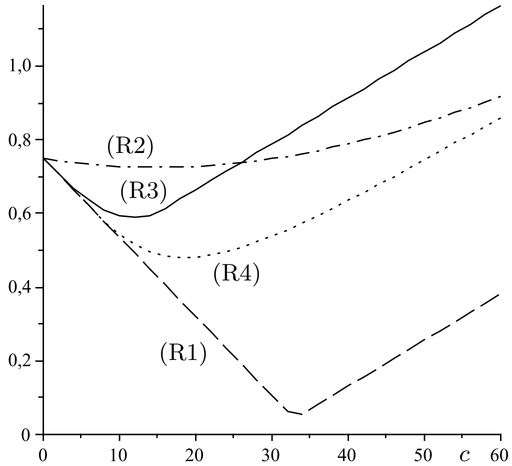

Figure 3.6: Systematic mortality risk for a combination of 1×(annuity insurance) and c×(temporary life insurance)

fixed and allowing for variations of $\mu_{ad}(t)$ and $\mu_{id}(t)$ within the bounds

$$l_{ad}(t) = -0.75 \cdot \ln(1 - q^{ad}_{\lfloor t \rfloor}) , \quad u_{ad}(t) = -1.15 \cdot \ln(1 - q^{ad}_{\lfloor t \rfloor}) ,$$
$$l_{id}(t) = -0.75 \cdot \ln(1 - q^{id}_{\lfloor t \rfloor}) , \quad u_{id}(t) = -1.15 \cdot \ln(1 - q^{id}_{\lfloor t \rfloor})$$

for risk measure (R3) and

$$\Delta l_{ad}(t) = -0.75 \cdot \Delta \ln(1 - q^{ad}_{\lfloor t \rfloor}) , \quad \Delta u_{ad}(t) = -1.15 \cdot \Delta \ln(1 - q^{ad}_{\lfloor t \rfloor}) ,$$
$$\Delta l_{id}(t) = -0.75 \cdot \Delta \ln(1 - q^{id}_{\lfloor t \rfloor}) , \quad \Delta u_{id}(t) = -1.15 \cdot \Delta \ln(1 - q^{id}_{\lfloor t \rfloor})$$

for risk measure (R4). Practical experience shows that q^{ad}_{x+n} and q^{id}_{x+n} can evolve differently but are to some extent correlated. We study here the two extreme cases where

(i) variations of $\mu_{ad}(t)$ and $\mu_{id}(t)$ are completely independent,

(ii) relative variations of $\mu_{ad}(t)$ and $\mu_{id}(t)$ are always equal, that is, $\frac{h_{ad}(t)}{\mu_{ad}(t)} = \frac{h_{id}(t)}{\mu_{id}(t)}$ for risk measure (R3) and $\frac{\Delta h_{ad}(t)}{\Delta \mu_{ad}(t)} = \frac{\Delta h_{id}(t)}{\Delta \mu_{id}(t)}$ for risk measure (R4).

The maximization methods in sections 2.2 and 2.3 deal only with case (i). In order to find the maxima (2.4) and (2.6) for case (ii), we use gradient ascent methods analogously to algorithm 2.1 but here with generalized gradient

$$(x, \omega) \ni t \mapsto \mu_{ad}(t) \operatorname{grad}_{\mu_{ad}}\big(V_a(x, \mu)\big)(t) + \mu_{id}(t) \operatorname{grad}_{\mu_{id}}\big(V_a(x, \mu)\big)(t)$$

instead of (2.2) and gradient

$$\Big(\Delta\mu_{ad}(m) \frac{\partial V_a(x, \mu)}{\partial(\Delta\mu_{ad}(m))} + \Delta\mu_{id}(m) \frac{\partial V_a(x, \mu)}{\partial(\Delta\mu_{id}(m))}\Big)_{m=x\ldots\omega-1}$$

instead of (2.8). Figures 3.7 and 3.8 show the mortality sensitivities from state active and state invalid/disabled. Figures 3.9 and 3.10 show the systematic mortality risks for the extreme cases

(i) and (ii) with respect to the level of the death benefit. We see that the netting effect is much stronger if the fluctuations of the mortality rates μ_{ad} and μ_{id} are similar in terms of (ii). Note that in figure 3.10 risk measure (R3) is not always greater than (R4) anymore because the additional conditions $\frac{h_{ad}(t)}{\mu_{ad}(t)} = \frac{h_{id}(t)}{\mu_{id}(t)}$ and $\frac{\Delta h_{ad}(t)}{\Delta \mu_{ad}(t)} = \frac{\Delta h_{id}(t)}{\Delta \mu_{id}(t)}$ are not equivalent.

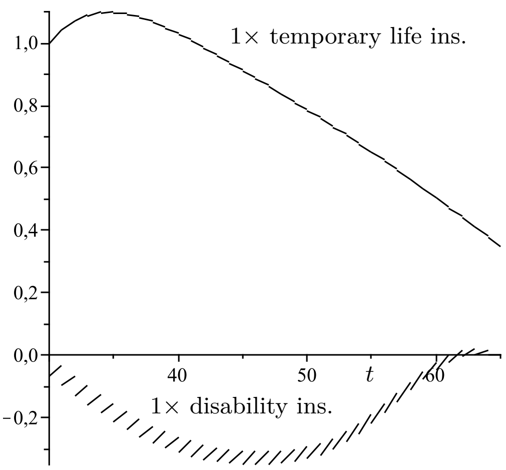

Figure 3.7: Sensitivity of $V_a(30-)$ with respect to the mortality intensity $\mu_{ad}(t)$

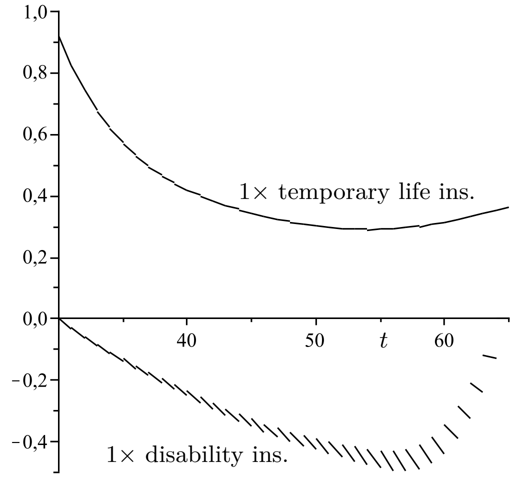

Figure 3.8: Sensitivity of $V_a(30-)$ with respect to the mortality intensity $\mu_{id}(t)$

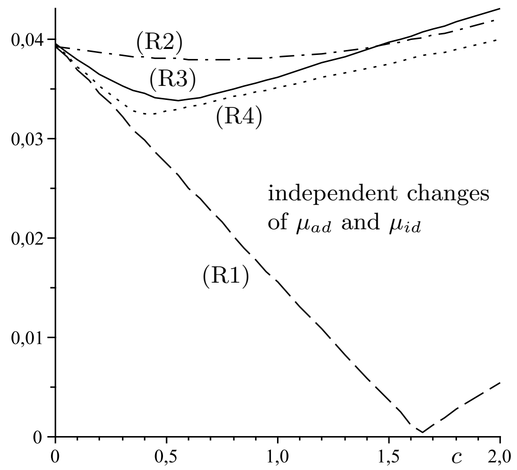

Figure 3.9: Systematic mortality risk for a combination of 1×(disability insurance) and c×(temporary life insurance)

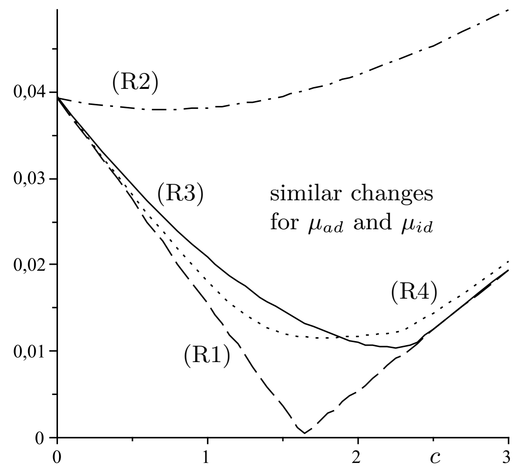

Figure 3.10: Systematic mortality risk for a combination of 1×(disability insurance) and c×(temporary life insurance)

4 Acknowledgement

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