Market-Consistent Valuation of Long-Term Insurance Contracts - Valuation Framework and Application to German Private Health Insurance

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Abstract In this paper we derive a market-consistent value for long-term insurance contracts, with a focus on long-term health insurance contracts as found, e.g., in the German private health insurance industry. To this end, we first set up a health insurance company model and, second, conduct a simulation study to calculate the present value of future profits and the time value of financial options and guarantees from a portfolio of private health insurance policies. Our analysis quantifies the impact of investment results and underwriting surpluses on shareholder profits with respect to profit sharing rules and premium adjustment mechanisms. In contrast to the valuation of life insurance contracts with similar calculation techniques the results indicate that the time value of financial options and guarantees of German private health insurance contracts is not substantial in typical parameter settings.

Keywords Private health insurance, Market-consistent embedded value, Long-term insurance contracts, Valuation

1 Introduction

Interest in market-consistent valuation in the insurance industry has increased significantly in recent years. Academics, insurance companies, and financial analysts all have demonstrated high interest in evaluating insurance cash flows, contracts, liabilities, and companies in light of pricing theory from the financial mathematics and economics fields. Market-consistent valuation is a frequent topic in academic literature. Some authors focus on the fair (or, equivalently, market-consistent) pricing of insurance cash flows and liabilities from single insurance policies (e.g., Grosen and Jørgensen, 2002; Malamud et al, 2008); others analyze the value associated with single insurance contracts using reduced balance sheet models (e.g., Bacinello, 2003; Coppola

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et al, 2011). A third line of research applies market-consistent valuation to insurance companies (e.g., Diers et al, 2012; Sheldon and Smith, 2004; Castellani et al, 2005; Wüthrich et al, 2010).

In this paper, we contribute to the latter approach and evaluate portfolios of insurance contracts. We focus on the market-consistent embedded value (MCEV) methodology as proposed by the European Chief Financial Officer Forum (see CFO Forum, 2009). Market-consistent embedded value calculations are the only recognized format of embedded value reporting for the largest insurance groups in Europe since December 31, 2011 (see CFO Forum, 2009). In addition, several insurance groups in the United States already calculate embedded values (see Frasca and LaSorella, 2009). MCEV calculations support the value- and risk-based management of insurance groups. They may be also used in the internal model approach of Solvency II. The paper extends the valuation literature with an analysis of German private health insurance, which offers interesting contract features for companies as well as for policyholders (e.g., whole-life contracts, adjustable premiums, no cancellation rights for insurance companies).

A market-consistent embedded value is based on three building blocks: future shareholder profits resulting from covered business, charges for the risk associated with realization of future profits, and the value of assets not linked to policyholder accounts at the valuation date (CFO Forum, 2009). Computation of the first two components is challenging, as different aspects need to be considered; e.g., the regulatory system, contract properties (policyholder options and guarantees), profit sharing mechanism, time horizon of the projection, and various assumptions about external factors (Schmidt, 2012). Due to the complexity of the profit sharing mechanism, we do not apply closed-form formulas for future returns, thus necessitating a projection algorithm. To this end, the valuation of future profits is essentially based on projection methods for insurance portfolios (similar to Kling et al, 2007; Gerstner et al, 2008).

Uncertainty of external factors is covered in a stochastic model of the capital market in which inflation plays a predominant role. Inflation is an important aspect of modeling health insurance claim sizes, as an empirical analysis shows that the development of health care costs is linked to observed inflation (see also Mehrotra et al, 2003; Drees and Milbrodt, 1995). We rely on the capital market modeling approach introduced in Jarrow and Yildirim (2003), a setup that enables a risk-neutral valuation of financial risks.

Typically, contract characteristics as options and guarantees result in a non-zero time value of financial options and guarantees (TVFOG). The TVFOG not only captures contract characteristics as options and guarantees, but also measures the impact of all asymmetric contract properties on profits resulting from the financial market. In life insurance, the TVFOG is in general a substantial component in relation to a full MCEV (e.g., Allianz Group (2012); Kochanski and Karnarski (2011)). In this paper we analyze the role of the TVFOG in a MCEV calculation for German private health insurance (health insurance similar to life insurance techniques) and quantify its impact on the MCEV. MCEV publications for German private health insurance contracts until now report a TVFOG of zero (e.g., Allianz Group (2012)). Our research reveals that the TVFOG may be non-zero, but with respect to our assumptions, its impact on MCEV is rather small due to special features of German private health insurance.

This paper is outlined as follows: In Section 2 we introduce the model for the financial market covering the uncertainty of the private health insurance company and describe the market-consistent valuation approach. Section 3 introduces a valuation framework for a private health insurance company and Section 4 shows the results of our calculations. Finally, Section 5 concludes.

2 Market-Consistent Valuation

2.1 Financial Market

Health insurance companies around the world face the risk of rising health care expenditure. There are several determinants identified in academic literature for rising health care expenditure; e.g., technological changes, aging populations, innovations in health care provision, and further long-term trends (Newhouse, 1992; Buchner and Wasem, 2006; Drees and Milbrodt, 1995). There is still debate in health economics literature about identifying the main drivers of health care expenditure. Overall health care expenditure usually leads to a rising amount of medical reimbursement (claims) of the policyholders in health insurance contracts reimbursing medical expenses.

We argue that the claim development is linked to the financial market via inflation. Figure 1 shows the percentage annual increase in the consumer price index (CPI) in Germany and the percentage annual increase of outpatient health care expenditure for all German private health insurance (PHI) companies from 1993 to 2008.[1] Empirically, we find that the annual increase in health expenditure from 1993 to 2008 always exceeded the annual increase in the consumer price index.[2]

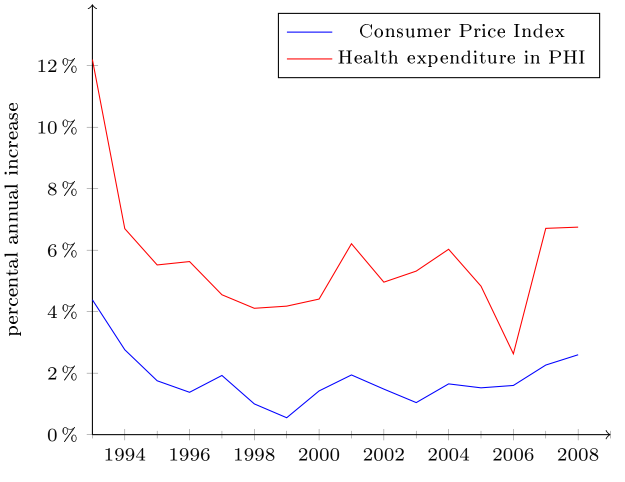

Fig. 1: Annual increase in CPI, Health Expenditure. Correlation: 0.87

Based on this observation, we use the financial market model of Jarrow and Yildirim (2003) (JY-model) which includes inflation as a separate stochastic process. This finan-

[1] The outpatient health care expenditure data were obtained from the information system of the German federal health monitoring.

[2] Note that in this illustration no adjustment is made for the aging portfolio of private health insurance companies (data not available). This adjustment would shift the upper curve downward.

cial market model is typically used in the pricing of inflation-linked derivatives; e.g., inflation swaps, inflation futures and inflation options (Dodgson and Kainth, 2006; Brigo and Mercurio, 2006). The main reason for adopting this model in our valuation of private health insurance contracts is the fact that it allows for a risk-neutral valuation while simultaneously modeling the two important risk factors of long-term health insurance contracts – interest and inflation. Assumptions on future expected claim sizes typically neglect the inflation rate; however, claim sizes are influenced to some extent by changes in the general price level. Thus, the development of the inflation rate represents a major risk in health insurance contracts in contrast to life insurance.

In our model, we link the average claim per capita of a private health insurance company to a stochastic inflation process. In addition, the amount of health expenditure exceeding the inflation process is captured by a deterministic additive spread on top of the inflation process. In German private health insurance, prudent assumptions on average claim per capita (Grundkopfschaden) for premium and reserve calculation are adjusted in the case of significant variation in average claim per capita. Our insurance company framework allows for adjustments based on the random development of the average claim per capita.

A short review of the JY-model is presented in the appendix of this paper describing the three stochastic processes for the nominal interest rate, real interest rate, and an inflation index. In the following we always consider the risk-neutral measure.

The JY-model has some obvious drawbacks: There are no prices of the real economy quoted in the market, such that proper calibration of the model constitutes a difficult task. Moreover, eight parameters need to be determined in this approach (Cipollini and Canty, 2010). We rely on this concept as it constitutes a standard approach in academic literature and practice and, furthermore, it covers the main financial risks of a private health insurance company in Germany.

2.2 Valuation Methodology

The MCEV calculation provides shareholders and investors with information on the expected value and drivers of change in value of companies' in-force business as well as a quantification of the risks associated with the realization of that value (CFO Forum, October 2009). Based on the stochastic processes from the financial market, i.e., n_k for the nominal interest rate and I_k for inflation index at time $k = 0, \dots, T$ (T is the projection horizon), we define the stochastic present value of future profits $\widetilde{\mathrm{PVFP}}$ (similar to Balestreri et al, 2011).

Definition (Stochastic Present Value of Future Profits $\widetilde{\mathbf{PVFP}}$) *The stochastic present value of future profits is defined as*

$$\widetilde{PVFP} := \sum_{k=1}^{T} v_k Y_k,$$

where Y_k represents the cash flow between shareholders and insurance company and v_k the discount factor at time k.

Due to the stochasticity of the processes, $\widetilde{\mathrm{PVFP}}$ is a random variable. Due to profit sharing and non-linear contract characteristics, a closed form representation for Y_k in dependence of the stochastic processes is usually not studied analytically.

Definition (Certainty Equivalent Scenario) *Given J realizations of the stochastic processes. For $1 \leq k \leq T$ we set $n_k^* := \frac{1}{J}\sum_{j=1}^{J} n_k^j$ and $I_k^* := \frac{1}{J}\sum_{j=1}^{J} I_k^j$. The sequences n_k^* and I_k^* $(k = 1, \ldots, T)$ are called certainty equivalent scenario.*

The certainty equivalent scenario represents the scenario in which at each time k a best estimate of the stochastic process is considered.

Definition (Present Value of Future Profits) *The present value of future profits* $\mathrm{PVFP}^{\mathrm{CE}}$ *is defined by the present value of future profits of the certainty equivalent scenario:*

$$\mathrm{PVFP}^{\mathrm{CE}} := \sum_{k=1}^{T} v_k^* Y_k^*,$$

with v_k^* the discount factor at time k and Y_k^* the corresponding cash flow of the certainty equivalent scenario.

The $\mathrm{PVFP}^{\mathrm{CE}}$ does not fully measure the impact of the contract features (e.g., premium adjustments, profit sharing) as it only considers the development in the certainty equivalent scenario. The expected value $\mathbb{E}^Q\left(\widetilde{\mathrm{PVFP}}\right) = \mathbb{E}^Q\left(\sum_{k=1}^{T} v_k Y_k\right)$ with respect to the risk-neutral measure would consider the stochasticity more appropriately. One way to estimate the expected value is by Monte Carlo simulation. Thus we define the present value of future profits from a Monte Carlo simulation based on J realizations of the stochastic processes:

Definition (Present Value of Future Profits from a Monte Carlo simulation) *The present value of future profits from a Monte Carlo simulation* $\mathrm{PVFP}^{\mathrm{MC}}$ *is defined as*

$$\mathrm{PVFP}^{\mathrm{MC}} := \frac{1}{J}\sum_{j=1}^{J}\sum_{k=1}^{T} v_k^j Y_k^j,$$

with v_k^j discount factor at time k and Y_k^j the corresponding cash flow of the j-th scenario.

By the law of large numbers, $\mathrm{PVFP}^{\mathrm{MC}}$ converges in probability to $\mathbb{E}^Q\left(\widetilde{\mathrm{PVFP}}\right)$ for an increasing number of scenarios. In comparison to the MCEV methodology (CFO Forum, 2009), the term $\mathrm{PVFP}^{\mathrm{MC}}$ corresponds in our model to a full MCEV without adjustment for Cost of Residual Non-Hedgeable Risks.

Definition (Time Value of Financial Options and Guarantees) *The time value of financial options and guarantees* TVFOG *is defined by*

$$\mathrm{TVFOG} := \mathrm{PVFP}^{\mathrm{CE}} - \mathrm{PVFP}^{\mathrm{MC}}.$$

The TVFOG measures the difference between the present value of cash flows of the certainty equivalent scenario and the average of the present values of the risk-neutral scenarios from the Monte Carlo simulation.

3 Private Health Insurance Company Framework

In this paper we analyze German private health insurance contracts that substitute for German statutory health insurance (substitutive Krankenversicherung). These contracts usually cover medical costs due to inpatient, outpatient, and dental treatment. In this line of business, the pricing and reserving is similar to life insurance techniques (Schneider, 2002): Contracts are typically whole-life, policyholders pay a level premium based on the principle by equivalence, and consequently insurance companies set up actuarial reserves to finance differences between level premiums and expected claims. In particular, increasing age and deteriorating state of health do not initiate premium increases. However, German private health insurance contracts demonstrate important differences to life insurance contracts: If the average claim per capita in a portfolio of health insurance contracts differs from prudent assumptions (first-order basis), the insurance company checks the whole technical basis of first-order (mortality and lapse rates, technical interest for premium and reserve calculation, average claim per capita, ...). If changes are significant and not temporary, the technical basis of first-order is adjusted and consequently the level premium as well ("adjustable premium contract"). The necessity of adjustments has to be verified by an independent trustee.

Premium adjustments are solely initiated by the development of the average claim per capita.[3] Note that poor development of the assets return rate and problems in crediting the technical interest may not result in premium adjustments. However, in the course of a premium adjustment, the technical interest may be lowered such that the risk from crediting a technical interest is small and short-term compared to the guaranteed minimum interest rates in life insurance contracts.

Premium adjustments represent a crucial property of German private health insurance contracts. The possibility to adjust the technical basis of first-order and thus level premiums results from the fact that insurance companies neglect inflation in the underwriting process. At the same time, insurance companies waive the right to cancel contracts. Neglecting inflation and waiving cancellation rights necessitates adjustments to guarantee the whole-life coverage. However, adjustments in general lead to increasing premiums, which are disadvantageous for policyholders, especially in retirement ages. Insurance regulation sets several legal requirements on premium calculation, premium adjustments, and profit sharing to protect policyholders against unaffordable premiums (Drees and Milbrodt, 1995; Drees et al, 1996). To this end, the private health insurance market in Germany is strongly regulated compared to other lines of business.[4] For instance, policyholders have to pay an additional premium (statutory ten percent loading) on top of the actuarial fair premium to accumulate an additional reserve. The additional reserve solely serves for curbing premium increases in old ages (65+). Moreover, profit sharing obeys multiple rules and aims primarily at ensuring affordable premiums for those of old age.

We proceed with a technical description of our projection algorithm.

[3] The development of the mortality rates may initiate an adjustment in practice as well. However, the development of the mortality rates typically did not initiate premium adjustments in recent years. Thus, we neglect the mortality as an initiating factor here.

[4] In the following we will cite sections from the insurance supervision act (VAG), the insurance contract act (VVG), calculation act (KalV), capital adequacy act (KapAusstV), surplus act (ÜbschV), and corporate tax act (KStG).

3.1 Balance Sheet

In our model, all assets and liabilities are represented in a simplified balance sheet. Similar balance sheet models were applied in the literature for life insurance companies (e.g. Gerstner et al, 2008; Kling et al, 2007).

Table 1: Simplified Balance Sheet at the End of Period k

Assets		Liabilities	
Assets	A_k	Free surplus	F_k
		Required capital	R_k
		Actuarial reserve	D_k
		Additional reserve	Z_k
		Surplus funds	B_k

All accounts show book values at the end of period k. A_k denotes the assets value. On the liability side, we consider five positions. The first two positions capture the equity; F_k contains the free surplus and the required capital R_k is the amount such that the company satisfies external and internal solvency requirements. The actuarial reserve D_k compromises the prudent reserve of the policyholders (local GAAP reserve). Z_k represents the additional reserve. Surplus assigned to policyholders is partly credited to Z_k (investment surplus). Other underwriting surplus is stored in the surplus funds B_k (Rückstellung für Beitragsrückerstattung).[5] Z_k aims at curbing premiums increases for those of old age (Drees and Milbrodt, 1995; Drees et al, 1996) and B_k aims at short-term profit sharing. We do not consider the existence of unrealized gains and losses. At the end of period k it holds

$$F_k = A_k - R_k - D_k - Z_k - B_k.$$

In case of an insolvency, we assume that the shareholders do not exercise their limited liability option (Doherty and Garven, 1986; Gatzert and Schmeiser, 2008). The shareholders raise capital such that solvency capital requirements are satisfied (CFO Forum, 2009).

3.2 Portfolio

We focus our analysis on a closed insurance portfolio of identical risks. Thus, the number of policyholders in the company at the end of period k depends on the policyholders at the beginning of period k and on the mortality and lapse rates. We assume that actual and prudent mortality rates coincide. However, the actual lapse rates may differ from prudent lapse rates. Let q_k denote the actual as well as prudent mortality rate of policyholders in the model point for period k. Moreover, let w_k denote the prudent lapse rate in period k, with w_k^* the actual rate, respectively. Then the following relationship between the number of contracts at the beginning ℓ_k^* and at the end of the period ℓ_{k+1}^* holds $\ell_{k+1}^* = (1 - q_k - w_k^*)\,\ell_k^*$. In our model, w_k^* is deterministic and only depends on the policyholders' age and sex.

[5] We do not distinguish between a "Rückstellung für erfolgsabhängige Beitragsrückerstattung" and a "Rückstellung für erfolgsunabhängige Beitragsrückerstattung."

3.3 Claims

The average claim per capita of a policyholder (Kopfschaden) is factorized into two components: an average claim per capita for a fixed reference age $\overline{C}_k$ (Grundkopfschaden, here: age 40) and a factor c_k (Profil) scaling $\overline{C}_k$ to the age of the policyholder.[6] Thus the average claim per capita of a policyholder is composed by an age-independent average claim per capita for a reference age and a time-independent profile for the age of the policyholder. Concerning the average claim amount for the reference age, we distinguish between prudent assumption $\overline{C}_k$ (first-order assumption and linked to actual value) and actual value $\overline{C}^*_k$. We assume that $\overline{C}^*_k$ is linked to the inflation index. The medical inflation is considered as a constant additive spread σ on top of the change in the inflation index:

$$\overline{C}^*_k = \overline{C}^*_{k-1}\left(\frac{I_k}{I_{k-1}} + \sigma\right).$$

It is $C^i_k = c_k\overline{C}_k$ and $C^{i,*}_k = c_k\overline{C}^*_k$.[7] On the portfolio basis, the total prudent and actual average claim per capita for the model point sum up to $C_k = \ell^*_k C^i_k$ and $C^*_k = \ell^*_k C^{i,*}_k$.

3.4 Premiums

The level premium of a policyholder is determined by the principle of equivalence:

$$P^i_k = \frac{K_k(z_k) - D^i_{k-1,+}}{(1-\lambda)\ddot{a}_k(z_k)}$$

- z_k is the technical interest rate in period k. It is bounded above by 3.5 % (§ 4 KalV). We restrict the possible rates to the set $Z = \{0.1\,\%, 0.2\,\%, \dots, 3.5\,\%\}$.
- $K_k(z_k) = \overline{C}_k \sum_{m\geq 0} c_{k+m}(1+z_k)^{-m}\prod_{n=0}^{m-1}(1-q_{k+n}-w_{k+n})$ is the present value of future average claim per capita. Note that inflation is not considered in $K_k(z_k)$.
- $\ddot{a}_k(z_k) = \sum_{m\geq 0}(1+z_k)^{-m}\prod_{n=0}^{m-1}(1-q_{k+n}-w_{k+n})$ is the present value of annual payments of 1 while the person is in the portfolio.
- $D^i_{k-1,+}$ is the actuarial reserve at the beginning of period k. The difference $D^i_{k-1,+} - D^i_{k-1}$ is the amount from the surplus funds or additional reserve transferred to the actuarial reserve at the beginning of period k (profit sharing).
- λ is the safety loading of at least 5 % of the premium (§ 7 KalV).

If $\overline{C}^i_k = \overline{C}^i_{k-1}$, $z_k = z_{k-1}$ and $D^i_{k-1,+} = D^i_{k-1}$, then $P^i_k = P^i_{k-1}$. Changes in $\overline{C}^i_k$ and z_k compared to $\overline{C}^i_{k-1}$ and z_{k-1} in general result in an adjusted premium.

The statutory ten percent loading $Q^i_k = 0.1P^i_k$ is paid by policyholders until age 60 (§ 12 (4a) VAG). Further cost parameters (e.g., acquisition costs) and premium loadings are neglected in our analysis. On a portfolio basis, we have $P_k = \ell^*_k P^i_k$ and $Q_k = \ell^*_k Q^i_k$.

[6] This factorization is motivated by the historical observation that the scaling factor is to a large extent time-independent.

[7] The superscript i always indicates that the value corresponds to an individual policyholder.

3.5 Adjustments

At the beginning of period k, the last three observations of the actual average claim per capita for reference age 40 ($\overline{C}^*_{k-2}$, $\overline{C}^*_{k-3}$ and $\overline{C}^*_{k-4}$) are extrapolated to $\overline{C}^{\text{extra}}_k$ by linear regression to estimate $\overline{C}^*_k$.[8] The relation between $\overline{C}^{\text{extra}}_k$ and the prudent average claim per capita from the previous period is defined as initiating factor q^C_k. It is

$$q^C_k = \frac{\overline{C}^{\text{extra}}_k}{\overline{C}_{k-1}} - 1.$$

If $\left|q^C_k\right| > \varepsilon$ for a fixed ε, then the technical basis of first-order is adjusted:[9] The prudent average claim per capita in period k is $\overline{C}_k = \overline{C}^{\text{extra}}_k$, and the technical interest rate is

$$z_k = \arg\min_{z \in Z} \left|\widehat{p}^*_k - z\right| - \zeta.$$

$\widehat{p}^*_k$ denotes an estimate of the portfolio return rate in period k. In practice, the next year's return on investment of the company is estimated with respect to a confidence level of 99.5 percent and z_k adjusted to this specific return rate (e.g. Maiwald et al, 2004). We consider the confidence level by a technical interest margin of ζ after rounding.

If $\left|q^C_k\right| \leq \varepsilon$, then there is no adjustment: $\overline{C}^i_k = \overline{C}^i_{k-1}$ and $z_k = z_{k-1}$. Thus $P^i_k = P^i_{k-1}$.

The technical interest is not guaranteed for the lifetime of the contract, and, in contrast to life insurance, the technical interest rate is adjusted in line with the company's investment results. Note that an adjustment of the technical interest is only initiated in the case of an adverse development of the actual claim per capita, but not due to poor investment results.

3.6 Projection of Reserve Accounts and Surplus Funds

The actuarial reserve is projected recursively by (Wolfsdorf, 1986)

$$D^i_k = \frac{1 + z_k}{1 - q_k - w_k} \left(D^i_{k-1,+} + (1-\lambda) P^i_k - C^i_k \right).$$

For the additional reserve Z^i_k, it holds that

$$\tilde{Z}^i_k = \frac{1 + z_k}{1 - q_k - w_k} \left(Z^i_{k-1,+} + Q^i_k \right).$$

The surplus funds B^i_k updates at the end of period k to

$$\tilde{B}^i_k = \frac{1}{1 - q_k - w_k} B^i_{k-1,+}.$$

[8] In practice, the extrapolation is proceeded during the previous period when C^*_{k-1} is unknown. We adopt this approach. In addition, this is in line with regulatory requirements (§ 14 KalV). Our approach is still a simplification as for instance the verification of an independent trustee is not considered. There are different approaches allowed to extrapolate average claim per capita.

[9] § 203 VVG. By § 12b (2) VAG, it is $\varepsilon \leq 0.10$.

$\tilde{Z}_k^i$ and $\tilde{B}_k^i$ will be adjusted due to profit participation. On an portfolio basis, we have that each account is multiplied by ℓ_{k+1}^* resulting in D_k, $\tilde{Z}_k$ and $\tilde{B}_k$.

3.7 Investment

All assets of the company are invested in zero coupon bonds with a fixed maturity of τ years. A buy and hold-management rule for the asset side applies; i.e., no bonds are sold before maturity. The following part is similar to Gerstner et al (2008); however, we do not consider stock investments.

The adjusted assets at the beginning of period k are

$$A_{k-1,+} = A_{k-1} + P_k + Q_k - C_k^* - \Delta B_{k-1}^{\text{refund}} - Y_k.$$

$\Delta B_{k-1}^{\text{refund}}$ denotes the policyholder refund; Y_k is the cash flow to shareholders.

$N_k = A_{k-1,+} - \sum_{j=1}^{\tau-1} A_{k-1}^j$ is the available part of the assets for new investment, where A_{k-1}^j denotes the value of the asset portfolio invested in zero bonds with maturity in j years. N_k is used to buy $n(k,\tau) = N_k b(k,k+\tau)^{-1}$ zero coupon bonds with maturity of τ years, price $b(k,k+\tau)$ and yield $r(k,\tau)$. Note that $N_k < 0$ leads to $n_k < 0$ and may be interpreted as short selling of bonds. In total, the number of bonds $n(k,j)$ in the bond portfolio at the beginning of period k having maturity in $j < \tau$ years is given by $n(k,j) = n(k-1,j+1)$.

The portfolio return rate p_k is given by

$$p_k = \frac{\sum_{j=1}^{\tau} n(k,j)(b(k+1,k+j) - b(k,k+j))}{A_{k-1,+}}.$$

The portfolio return rate with respect to book values is

$$p_k^* = \frac{\sum_{j=1}^{\tau} n(k,j) r(k-(\tau-j),\tau)}{A_{k-1,+}}.$$

The portfolio return rate is not known at the beginning of period k as it depends on premium adjustments. However, we can estimate the value by

$$\widehat{p}_k^* = \frac{\sum_{j=1}^{\tau} n(k,j) r(k-(\tau-j),\tau)}{A_{k-1} + \ell_k^*(P_{k-1}^i + Q_{k-1}^i) - C_k^* - R_k - Y_k}.$$

3.8 Surplus

The gross surplus G_k of period k is

$$G_k = S_k^{\text{invest}} + S_k^{\text{claim}} + S_k^{\text{lapse}} + S_k^{\text{loading}}.$$

The surplus resulting from investments and crediting of the technical interest is

$$S_k^{\text{invest}} = p_k^* A_{k-1,+} - z_k(D_{k-1,+} + Z_{k-1,+} + (1-\lambda)P_k + Q_k - C_k).$$

Differences between actual and prudent claim per capita result in a surplus $S_k^{\text{claim}} = C_k - C_k^*$. Moreover, the actual lapse rates of policyholders may differ from prudent lapse rates. In our framework, the surplus from the actuarial reserve, the additional reserve and the surplus funds associated with lapsed contracts is $S_k^{\text{lapse}} = (\ell_{k+1} - \ell_{k+1}^*)(D_k^i + Z_k^i + B_k^i)$. Furthermore, the surplus from the safety loading is $S_k^{\text{loading}} = \lambda P_k$.

3.9 Profit Sharing

Profit sharing of investment results depends on the book value return rate p_k^* and the technical interest rate z_k. If the portfolio return rate p_k^* exceeds the technical interest z_k, a fraction $\xi \in [0.9; 1]$ of the portfolio return above the technical interest rate z_k (Überzins) earned on positive D_k and Z_k is credited to Z_k (Direktgutschrift, § 12a (1) VAG). In total, positive policyholder accounts D_k and Z_k receive the interest

$$z_k^* = \max \left\{ \xi(p_k^* - z_k);\, 0 \right\}.$$

The investment surplus

$$G_k^Z = z_k^* \max \left\{ D_{k-1,+} + Z_{k-1,+}^i + (1-\lambda)P_k + Q_k - C_k;\, 0 \right\}.$$

is directly credited to $Z_k = \tilde{Z}_k + G_k^Z$.

If the gross surplus is positive, then policyholders receive a fixed portion $1 - \pi \in [0.8; 1.0]$ (§ 4 (1) ÜbschV):

$$G_k^B = \max \left\{ (1-\pi)G_k - G_k^Z;\, 0 \right\}.$$

The amount is added to the surplus funds; thus $B_k = \tilde{B}_k + G_k^B$.

The surplus credited to policyholder accounts in total amounts to $G_k^P = G_k^Z + G_k^B$. The shareholders part G_k^S of the gross surplus is

$$G_k^S = G_k - G_k^P = \min \left\{ \pi G_k;\, G_k - G_k^Z \right\}.$$

The profit sharing is asymmetric, as shareholders participate with a fraction π if the gross surplus is sufficient to credit at least the investment surplus G_k^Z, but pay fully otherwise. In contrast to profit sharing in German life insurance, the surplus is aggregated at first. Profit sharing is based on the aggregated value of all surpluses, such that a negative surplus from the claim development may be offset by a positive surplus from the investment surplus (offsetting effect).

3.10 Profit Participation

A premium increase is typically disadvantageous for policyholders; thus, profit participation in German private health insurance primarily curbs premium increases. Curbing premium increases is technically a shift of capital from the additional reserve Z_k and the surplus funds B_k to the actuarial reserve D_k. If there is no change in the premium but the size of the surplus funds exceeds a limit, amounts of the surplus funds are refunded to policyholders.

In our model, we apply the following exemplary management rule: The amount taken from the surplus funds depends on the size of the surplus funds in relation to total premium income. The surplus funds quota is defined as $q_k^B = \frac{B_{k-1}}{P_{k-1}}$. Two parameters α and β represent lower and upper limits for the surplus funds quota. They determine the maximal and minimal amount available for shifting; i.e.,

$$\Delta B_{k-1}^{\max} = \begin{cases} B_{k-1} - \alpha P_{k-1} & \text{if } q_k^B > \alpha \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\varDelta B^{\min}_{k-1} = \begin{cases} B_{k-1} - \beta P_{k-1} & \text{if } q^B_k > \beta \\ 0 & \text{otherwise,} \end{cases}$$

such that lower and upper limits hold after shift and refund. In addition, we compute the required amount such that the premium from the previous year does not change. It is

$$\varDelta B^{\text{target}}_{k-1} = \Big(K_k(z_k) - D^i_{k-1} - (1-\lambda) P^i_{k-1} \ddot{a}_k(z_k) \Big)\, \ell^*_k.$$

If $\varDelta B_{k-1} \leq 0$ then the premium is lower than the premium from the previous period. In this case there is no shift to the actuarial reserve. The minimal amount $\varDelta B^{\min}_{k-1}$ is paid as a premium refund to policyholders. Otherwise, $\varDelta B_{k-1}$ is shifted to the actuarial reserve (but not more than $\varDelta B^{\max}_{k-1}$), and in the case of a positive difference between $\varDelta B^{\min}_{k-1}$ and the shifted amount, again this amount is refunded to policyholders.[10] To sum up, we have

$$\varDelta B^{\text{reserve}}_{k-1} = \begin{cases} \min\{\varDelta B^{\text{target}}_{k-1};\, \varDelta B^{\max}_{k-1}\} & \text{if } \varDelta B^{\text{target}}_{k-1} > 0 \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\varDelta B^{\text{refund}}_{k-1} = \begin{cases} \max\{\varDelta B^{\min}_{k-1} - \min\{\varDelta B^{\text{target}}_{k-1};\, \varDelta B^{\max}_{k-1}\};\, 0\} & \text{if } \varDelta B^{\text{target}}_{k-1} > 0 \\ \varDelta B^{\min}_{k-1} & \text{otherwise.} \end{cases}$$

A part of the additional reserve Z_{k-1} is shifted to the actuarial reserve if the policyholder is aged 65+ to curb premium increases (§ 12a (2a) VAG). A similar procedure as above takes place with the difference that no quota constraint and thus no refund are considered.[11] It is

$$\varDelta Z^{\text{target}}_{k-1} = \Big(K_k(z_k) - \Big(D^i_{k-1} + L^B_k \Big) - (1-\lambda) P^i_{k-1} \ddot{a}_k(z_k) \Big)\, \ell^*_k,$$

and

$$\varDelta Z^{\text{reserve}}_{k-1} = \begin{cases} \min\{\varDelta Z^{\text{target}}_{k-1};\, Z_{k-1}\} & \text{if } Z_{k-1} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

The adjusted actuarial reserve, additional reserve, and surplus funds are

$$\begin{aligned} D_{k-1,+} &= D_{k-1} + \varDelta B^{\text{reserve}}_{k-1} + \varDelta Z^{\text{reserve}}_{k-1} \\ Z_{k-1,+} &= Z_{k-1} - \varDelta Z^{\text{reserve}}_{k-1} \\ B_{k-1,+} &= B_{k-1} - \varDelta B^{\text{reserve}}_{k-1} - \varDelta B^{\text{refund}}_{k-1} \end{aligned}$$

[10] We do not model the fact that the maximal time period for capital in the surplus funds is three years due to tax reasons (§ 21 KStG) and we neglect special rules for policyholders aged 80+.

[11] If the full additional reserve is required to curb the premium, the full additional reserve is shifted to the actuarial reserve. Additional management rules may be applied in this process.

3.11 Cash Flow

In our framework, all cash flows are incurred at the beginning of a period. Cash flows between shareholders and insurance company are 1) the shareholders part of the gross surplus and 2) the adjustments of required capital. Cash flows between policyholders and insurance company are 1) premiums, 2) claims, and 3) premium refunds.

The required capital is $\overline{R}_k = \frac{1}{3}\rho \max\{0.26C_k^*;\, 0.18P_k\}$, where C_k^* denotes the claim amount and P_k the total premium income of period k, both incurred at the beginning of period k. The parameter ρ denotes the internal solvency capital requirement quota above the external solvency capital requirement. This management rule is deduced from § 1 (2),(3),(4) KapAusstV.

The cash flow at the beginning of a period depends on the shareholders' part of the gross surplus of the previous period and the adjustment of the required capital:

$$Y_{k+1} = \min\left\{\pi G_k;\, G_k - G_k^Z\right\} + (\overline{R}_{k-1} - \overline{R}_k).$$

The present value of future profits alone does not provide insights into the value drivers and risks of a private health insurance company. Therefore, we split the $\mathrm{PVFP}^{\mathrm{CE}}$ and the $\mathrm{PVFP}^{\mathrm{MC}}$ into several components such that the impact of the different surplus sources can be deduced. The following represents one approach to analyze various effects on MCEV and explains the effects analyzed in this paper.

We set $\chi_k = 1$ if $\pi G_k < G_k - G_k^Z$ and $\chi_k = 0$ otherwise. Thus we have

$$\begin{aligned} Y_{k+1} &= \pi\, S_k^{\text{invest}}\, \chi_k + (S_k^{\text{invest}} - G_k^Z)(1-\chi_k) \\ &\quad + \pi\, S_k^{\text{claim}}\, \chi_k + S_k^{\text{claim}}(1-\chi_k) \\ &\quad + \pi\, S_k^{\text{lapse}}\, \chi_k + S_k^{\text{lapse}}(1-\chi_k) \\ &\quad + \pi\, S_k^{\text{loading}}\, \chi_k + S_k^{\text{loading}}(1-\chi_k) \\ &\quad + (\overline{R}_{k-1} - \overline{R}_k). \end{aligned}$$

With this representation of Y_{k+1} we split up $\widetilde{\mathrm{PVFP}}$ into five summands (corresponding to each line):

$$\widetilde{\mathrm{PVFP}} = \widetilde{\mathrm{PVFP}}^{\text{invest}} + \widetilde{\mathrm{PVFP}}^{\text{claim}} + \widetilde{\mathrm{PVFP}}^{\text{lapse}} + \widetilde{\mathrm{PVFP}}^{\text{loading}} + \widetilde{\mathrm{PVFP}}^{\text{RC}}.$$

In the same way, we split up the PVFP into five components. Analogous to the definition of the $\mathrm{PVFP}^{\mathrm{CE}}$, the certainty equivalent scenario is used instead of the stochastic processes. It is

$$\begin{aligned} \mathrm{PVFP}^{\mathrm{CE}} &= \mathrm{PVFP}^{\mathrm{CE,invest}} + \mathrm{PVFP}^{\mathrm{CE,claim}} + \mathrm{PVFP}^{\mathrm{CE,lapse}} \\ &\quad + \mathrm{PVFP}^{\mathrm{CE,loading}} + \mathrm{PVFP}^{\mathrm{CE,RC}}. \end{aligned}$$

The linearity of the expected value enables us to split up $\mathbb{E}^Q\left(\sum_{k=1}^T v_k Y_k\right)$ and its estimate from a Monte Carlo simulation $\mathrm{PVFP}^{\mathrm{MC}}$. It is

$$\begin{aligned} \mathrm{PVFP}^{\mathrm{MC}} &= \mathrm{PVFP}^{\mathrm{MC,invest}} + \mathrm{PVFP}^{\mathrm{MC,claim}} + \mathrm{PVFP}^{\mathrm{MC,lapse}} \\ &\quad + \mathrm{PVFP}^{\mathrm{MC,loading}} + \mathrm{PVFP}^{\mathrm{MC,RC}}. \end{aligned}$$

This decomposition allows us to illustrate the value drivers of the PVFP^{CE} and PVFP^{MC}. However, it does not fully capture the offsetting effect of the profit sharing rule.

In our analysis of the TVFOG, we will compute the following values: $\text{TVFOG}^{\text{invest}}$, $\text{TVFOG}^{\text{claim}}$, $\text{TVFOG}^{\text{lapse}}$, $\text{TVFOG}^{\text{loading}}$ and TVFOG^{RC}, each as a difference of the corresponding values above.

4 Results

4.1 Certainty Equivalent Scenario

In our simulation we perform a monthly discretization of the stochastic processes and an annual discretization of insurance company accounts. We generate 5,000 scenarios based on the following parameter configuration of the stochastic processes. Our simulation covers 30 years. At the end of year 30, the insurance company accounts are closed.[12]

Table 2: Summary of Parameters for the Stochastic Processes

Nominal:	$a_n = 0.03398$	$\sigma_n = 0.00566$	$n(0) = 0.04$
Real:	$a_r = 0.04339$	$\sigma_r = 0.00299$	$r(0) = 0.02$
Inflation:		$\sigma_I = 0.00874$	$I(0) = 100$
Correlations:	$\rho_{n,r} = 0.01482$	$\rho_{r,I} = -0.32127$	$\rho_{n,I} = 0.06084$

The values for the financial market (Table 2) are adopted from Jarrow and Yildirim (2003) (See appendix for further details). Assumptions on the yield curve and the starting values of the stochastic processes are arbitrary but varied in robustness tests in order to assess their relevance on MCEV. Here we assume a flat yield curve of 4 % for the nominal interest rate and 2 % for the real interest rate.

The actuarial reserve accumulates to $D_0 = 60,964,712$ and the additional reserve is $Z_0 = 9,821,259$. We assume the existence of surplus funds of size $B_0 = 4,000,000$. The required capital amounts to $R_0 = 750,000$ and the free surplus is $F_0 = 0$. This corresponds to assets valued $A_0 = 75,535,971$.

We are studying contracts of male policyholders at age 40 at the beginning of the first period having a health insurance contract for outpatient treatment. Previous to the start of the simulation, all policyholders are ten years insured within the company. The projection starts with $\ell_0^* = 5,000$ policyholders. Information about c_k, q_k, and w_k is adopted from the "Association of German private health care insurers." We assume that $w_k^* = 1.03w_k$. The premium in period 1 is $P_0^i = 1,960$. The average claim per capita amounts to $C_0^i = \overline{C}_0 = C_0^{i,*} = \overline{C}_0^* = 1,197$. The technical interest rate is $z_0 = 3.5\,\%$ (maximal allowed).

We assume a safety loading factor of $\lambda = 10\,\%$. The boundaries for the surplus funds quota are $\alpha = 20\,\%$ and $\beta = 50\,\%$. Zero bonds have a maturity of $\tau = 10$ years. Investment surplus is credited with $\xi = 90\,\%$. The margin on technical interest is $\zeta = 0.1\,\%$, and the solvency level is $\rho = 150\,\%$.

In the reference situation we assume the following values:

[12] All calculations are performed with the software R (R Development Core Team, 2011).

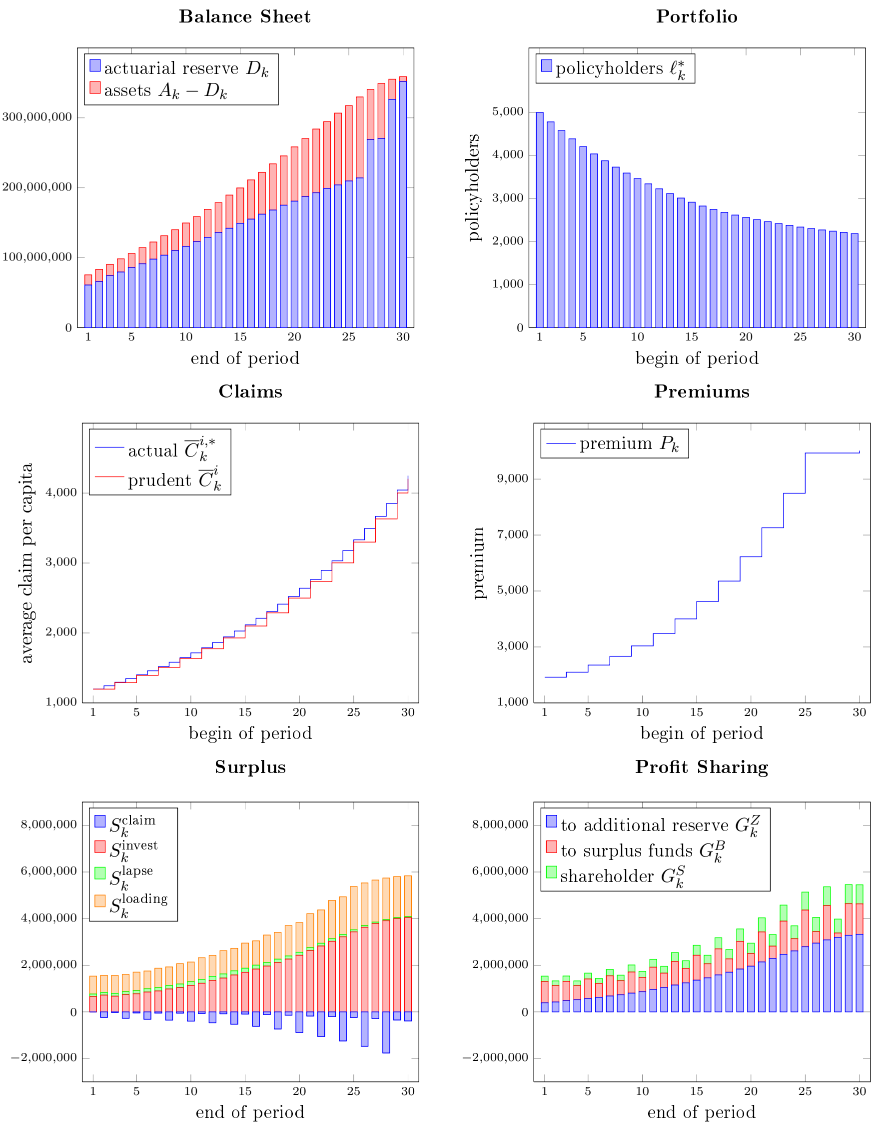

Fig. 2: Certainty Equivalent Scenario

– The additive spread on the inflation process is $\sigma = 2\,\%$.
– The shareholder participation rate for the gross surplus is $\pi = 15\,\%$.
– The boundary for the initiating factor is $\varepsilon = 5\,\%$.

Figure 2 shows the development of the private health insurance company in the certainty equivalent scenario. The actuarial reserve is increasing during the simulation time as well as the book value of assets. The assets value grows steeper due to surpluses kept in the additional reserve and surplus funds. Increases in the actuarial reserve in periods 27, 29, and 30 are due to premium adjustments. The additional reserve and

Table 3: Results for Certainty Equivalent Scenario (in Relation to A_0)

$\mathrm{PVFP}^{\mathrm{CE}}$	$\mathrm{PVFP}^{\mathrm{invest}}$	$\mathrm{PVFP}^{\mathrm{claim}}$	$\mathrm{PVFP}^{\mathrm{lapse}}$	$\mathrm{PVFP}^{\mathrm{loading}}$
8.61 %	5.54 %	-1.14 %	0.45 %	4.45 %

the surplus funds are shifted to the actuarial reserve to limit premium increases. The figure displaying the development of the premium indicates that no premium increase occurs in periods 25 to 29 despite the adjustments of average claim per capita in these periods. The inflation and medical inflation directly influences the initiating factor. If the initiating factor exceeds the boundary, the prudent average claim per capita is adjusted. In the certainty equivalent scenario, an adjustment occurs every second year. Analogously, the premium increases, corresponding to the adjustment of the average claim per capita (with the exception of periods 25 to 29). The technical interest rate is not adjusted in the certainty equivalent scenario. The two figures at the bottom indicate the size of surplus and the profit sharing mechanism. Investment surplus and surplus from the safety loading dominate the gross surplus in our model. The investment surplus increases along with a higher value of assets while surplus from the safety loading increases as the premium increases due to inflation and medical inflation. The surplus from the claim development is negative in each period in the certainty equivalent scenario as the adjustment of the average claim per capita by a linear regression underestimates the exponential growth in average claim per capita. Correspondingly, the investment surplus and the surplus from the safety loading dominate the $\mathrm{PVFP}^{\mathrm{CE}}$ result (cf. Table 3). Due to the simple assumption on lapse rates, the surplus from lapsed contracts is rather small. The disadvantageous claim surplus diminishes the $\mathrm{PVFP}^{\mathrm{CE}}$ slightly.

4.2 Market Consistent Embedded Value

We analyze the $\mathrm{PVFP}^{\mathrm{CE}}$, the $\mathrm{PVFP}^{\mathrm{MC}}$, and TVFOG with respect to safety loadings ranging from 5 % to 10 % (Figure 3). We distinguish between the situation in which the technical interest is adjusted, and the situation in which the technical interest is not adjusted. The latter case allows to compare results to the valuation of life insurance with a guaranteed minimum interest.

We observe that the present value of future profits of the certainty equivalent scenario and of the Monte Carlo simulation increases according to an increasing safety loading. A higher safety loading increases the gross surplus of a private health insurance company and consequently future profits. The time value of financial options and guarantees is small and slightly negative if the technical interest rate is adjusted due to premium adjustments. If the technical interest is a guaranteed minimum interest as in life insurance, the valuation by the certainty equivalent scenario does not change, as there is no adjustment of the technical interest in the certainty equivalent scenario. However, the valuation by the Monte Carlo simulation leads to significant smaller present values of future profits. The TVFOG has a significant positive size. Higher safety loading factors diminish the absolute size of the TVFOG.

In the design of German private health insurance contracts, we are facing an asymmetry in profit sharing resulting from regulatory profit sharing rules; the size of the

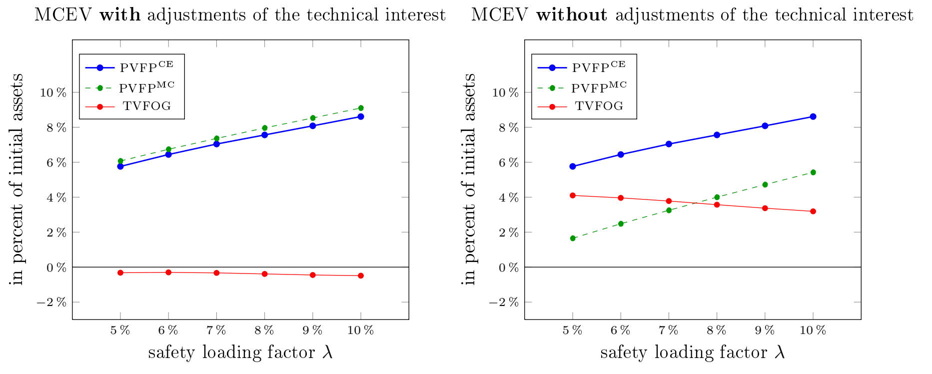

Fig. 3: Results for the Reference Situation

shareholder profits is asymmetric with respect to the gross surplus. Shareholders participate with a participation rate of $\pi \leq 20\,\%$ (§ 4 ÜbschV) from a positive gross surplus; however, a negative gross surplus is fully covered by shareholder accounts. This profit sharing rule is similar to profit sharing in life insurance and typically results in life insurance in a positive time value of financial options and guarantees (see, e.g., Allianz Group (2012)).

If the technical interest rate is adjusted as required in German private health insurance, the asymmetry does not result in a positive time value of financial options and guarantees. We even find a small negative time value of financial options and guarantees as the average gross surplus in the Monte Carlo simulation is higher than the gross surplus in the certainty equivalent scenario. Here, particular offsetting effects arise in the determination of the gross surplus as all surpluses are aggregated at the end of a period. In contrast to profit sharing in German life insurance, a negative surplus from the claim development for instance may be fully balanced by a positive surplus from investment results and vice versa.

However, if the technical interest is not adjusted, the asymmetry in profit sharing induces in our calculations the positive and substantial TVFOG. Valuation of future profits by the certainty equivalent scenario results in a positive gross surplus in all projected periods. In contrast, the Monte Carlo simulation incorporates scenarios with negative gross surplus resulting from investment results not sufficient to credit the technical interest. Consequently, the asymmetry of the profit sharing rule emerges. Therefore, these results are similar to the MCEV results of life insurance portfolios.

Studying the components of the gross surplus separately (investment, claim, safety loading, and lapse surplus) and the corresponding decomposition of the $\mathrm{PVFP}^{\mathrm{MC}}$ and TVFOG gives further explanations for the observed deviations between a valuation by the certainty equivalent scenario and by the Monte Carlo simulation.

In Figure 4 we observe that the investment surplus and the safety loading surplus are the dominating drivers in the $\mathrm{PVFP}^{\mathrm{MC}}$. The increasing $\mathrm{PVFP}^{\mathrm{MC}}$ mainly results from the increasing safety loading surplus. Secondly, we present $\mathrm{TVFOG}^{\mathrm{invest}}$, $\mathrm{TVFOG}^{\mathrm{loading}}$, and $\mathrm{TVFOG}^{\mathrm{claim}}$. With increasing surplus from safety loading, deviations between values from the certainty equivalent scenario and the Monte Carlo simulation decrease. The impact of the profit sharing asymmetry is low, especially due

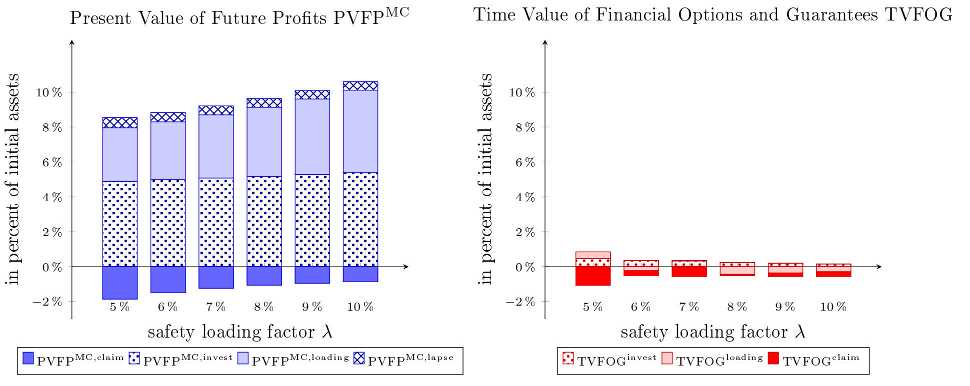

Fig. 4: Composition of $PVFP^{MC}$ and TVFOG **with** technical interest adjustments

to the offsetting effects in the determination of the gross surplus. We observe that the impact of the investment surplus on future profits in the Monte Carlo simulation is slightly overestimated by the certainty equivalent scenario. The small positive $TVFOG^{invest}$ shows the asymmetric impact of investment surplus on future profits. Moreover, the impact of the surplus from the claim development in the Monte Carlo simulation is underestimated by certainty equivalent scenario; the claim surplus is on average higher than indicated by the certainty equivalent scenario. The certainty equivalent scenario overestimates the impact of the safety loading surplus for some safety loading factors ($\lambda = 5\,\%$ and $\lambda = 7\,\%$) and underestimates it for the other values of the safety loading factor. Theses contrary effects sum up to a small negative time value of financial options and guarantees.

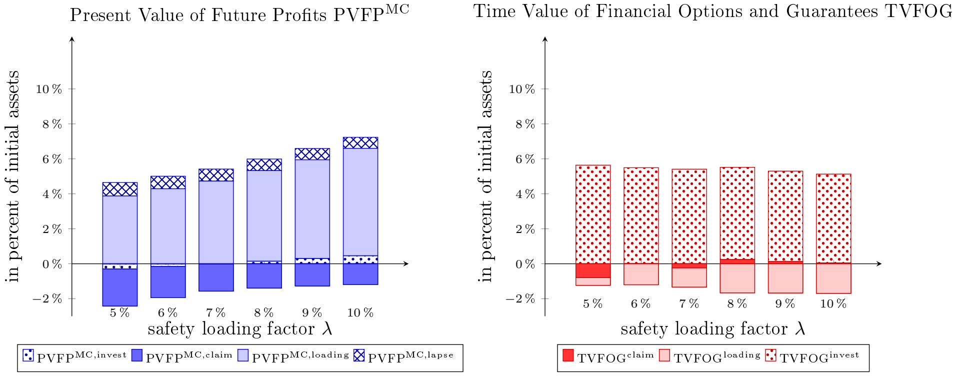

Fig. 5: Composition of $PVFP^{MC}$ and TVFOG **without** technical interest adjustments

Analyzing the composition of the $PVFP^{MC}$ for the situation without adjustments of the technical interest, we find in analog to the previous result that the surplus from the loading factor leads to increasing $PVFP^{MC}$. In this case, the investment surplus does not contribute to future profits as in the previous situation. The large positive TVFOG mainly results from poor investment surpluses, as in this case the technical interest corresponds to a guaranteed minimum interest. In adverse developments of the interest rates, the insurance company often faces negative gross surpluses due to the technical interest guarantee. As a consequence, the asymmetry in gross surplus emerges and diminishes the present value of future profits. The $\text{TVFOG}^{\text{invest}}$ is only diminished by the negative $\text{TVFOG}^{\text{loading}}$. Here, the certainty equivalent scenario underestimates the impact of the loading surpluses on future profits. The impact from claim surplus is rather small.

In the following, we analyze the impact of each surplus source on the TVFOG.

Impact of investment surplus On the one hand, the investment surplus entering the gross surplus is asymmetric with respect to the total investment surplus. A fraction $\xi \leq 10\,\%$ (§12a (1) VAG) from a positive investment surplus increases the gross surplus; however, if the investment surplus does not suffice to credit the technical interest, the gross surplus is fully affected. This may have a negative impact on shareholder profits and is captured in a positive $\text{TVFOG}^{\text{invest}}$ (cf. Figure 5).

On the other hand, the technical interest is asymmetric with respect to investment results. Insufficient investment results induce an adjustment of the technical interest rate; however, the technical interest rate is bounded above; i.e., $z_k \leq 3.5\,\%$ (§ 4 KalV). Due to the possibility of adjustments of the technical interest rate with a safety margin, the average investment surplus in the Monte Carlo simulation may be higher than the investment surplus in the certainty equivalent scenario. This may infer a negative $\text{TVFOG}^{\text{invest}}$.

Impact of claim surplus The surplus resulting from the actual claim per capita development in the certainty equivalent scenario may differ from the average surplus from claim per capita development of the Monte Carlo simulation. For instance, if the adjustment frequency of the prudent claim per capita in the Monte Carlo simulation is on average higher (smaller) than that of the certainty equivalent scenario, the claim surplus in the Monte Carlo simulation tends to be smaller (higher). This observation explains a non-zero $\text{TVFOG}^{\text{claim}}$ (cf. Figure 4 and Figure 5).

Impact of safety loading surplus If the technical interest rate is adjusted to a lower value due to poor investment results, then the premium increases. A premium increase induces a higher safety loading surplus (as the safety loading is a percentage of the premium) and therefore a higher shareholder profit. Thus, the investment result acts asymmetrically on safety loading surpluses and, consequently on shareholder profits. The initial technical interest of 3.5 % is not adjusted in the certainty equivalent scenario as the investment results suffice to credit the technical interest. However, the Monte Carlo simulation incorporates scenarios with technical interest adjustments below 3.5 % such that the asymmetric impact of safety loading surplus emerges. In the Monte Carlo simulation the asymmetric safety loading surplus increases the present value of future profits compared to the valuation by the certainty equivalent scenario; i.e., a negative $\text{TVFOG}^{\text{loading}}$ (cf. Figure 5). In general, a non-zero $\text{TVFOG}^{\text{loading}}$ (cf. Figure 4) indicates that the average surplus from the safety loading of the Monte Carlo simulation differs from the safety loading surplus generated in the certainty equivalent scenario.

Impact of lapse surplus Due to the static modeling of the lapse rates, the impact of lapse surplus in our model is not substantial and thus neglected in Figure 4 and Figure 5.

4.3 Sensitivity Analyses

In the following section we analyze the impact of different parameter settings for the external parameter σ (spread on inflation) and the management parameters π (profit participation) and ε (boundary for initiating factor) on the results. In this sensitivity analysis the technical interest rate is adjusted if it is initiated by the development of the average claim per capita.

In the first row of Figure 6 we observe that a lower (higher) spread compared to the reference situation decreases (increases) the present value of future profits. This is expected, as a lower (higher) medical inflation decreases (increases) the insurance coverage compared to the reference situation; i.e., the policyholder average claim per capita, premiums, and loadings. Higher premiums increase the gross surplus, especially due to higher investment results and surpluses from the loading factor. The time value of financial options and guarantees does not vary significantly, but is in this calculation closer to zero.

In contrast to the external parameter for the medical inflation, we conduct a sensitivity analysis for the shareholders' participation π in gross surpluses in the second row of Figure 6. A lower (higher) shareholder participation compared to the reference situation goes along with a lower (higher) present value of future profits. The time value of financial options and guarantees gets closer to zero if the shareholder participation rate is decreased compared to the reference situation. For all loading factors, the time value of financial options and guarantees is again small and not substantial.

The figures at the bottom of Figure 6 reveal that the ε parameter directly influences the frequency of adjustments of average claim per capita and technical interest. Our results show that, if only large deviations between prudent and actual average claim per capita result in adjustments, then the present value of future profits is smaller. So a decreasing adjustment frequency significantly increases the impact of the profit sharing asymmetry. For $\varepsilon = 10\,\%$, the time value of financial options and guarantees is positive and the highest compared to all considered parameter settings. The TVFOG is in the range from $35\,\%$ ($\lambda = 5\,\%$) to $12\,\%$ ($\lambda = 10\,\%$) in relation to the size of the PVFP^{CE}. If $\varepsilon = 10\,\%$, then adjustments of the technical interest in the Monte Carlo simulation are less often possible compared to the reference case with $\varepsilon = 5\,\%$ as only large deviations of the average claim per capita initiate adjustments. The positivity of the TVFOG results from the fact that, in some scenarios of the Monte Carlo simulation, adjustments of the technical interest are not always permitted in time (as deviations between prudent and actual average claim per capita are not above the $10\,\%$ boundary) even if investment results do not suffice to credit the technical interest. This induces a substantial negative result of the investment surplus and, consequently, decreases gross surplus. Offsetting effects in the determination of the gross surplus do not relax or even eliminate this effect. Thus, the technical interest is a short-term guaranteed minimum interest in these scenarios (up to the next adjustment initiated by the claim development).

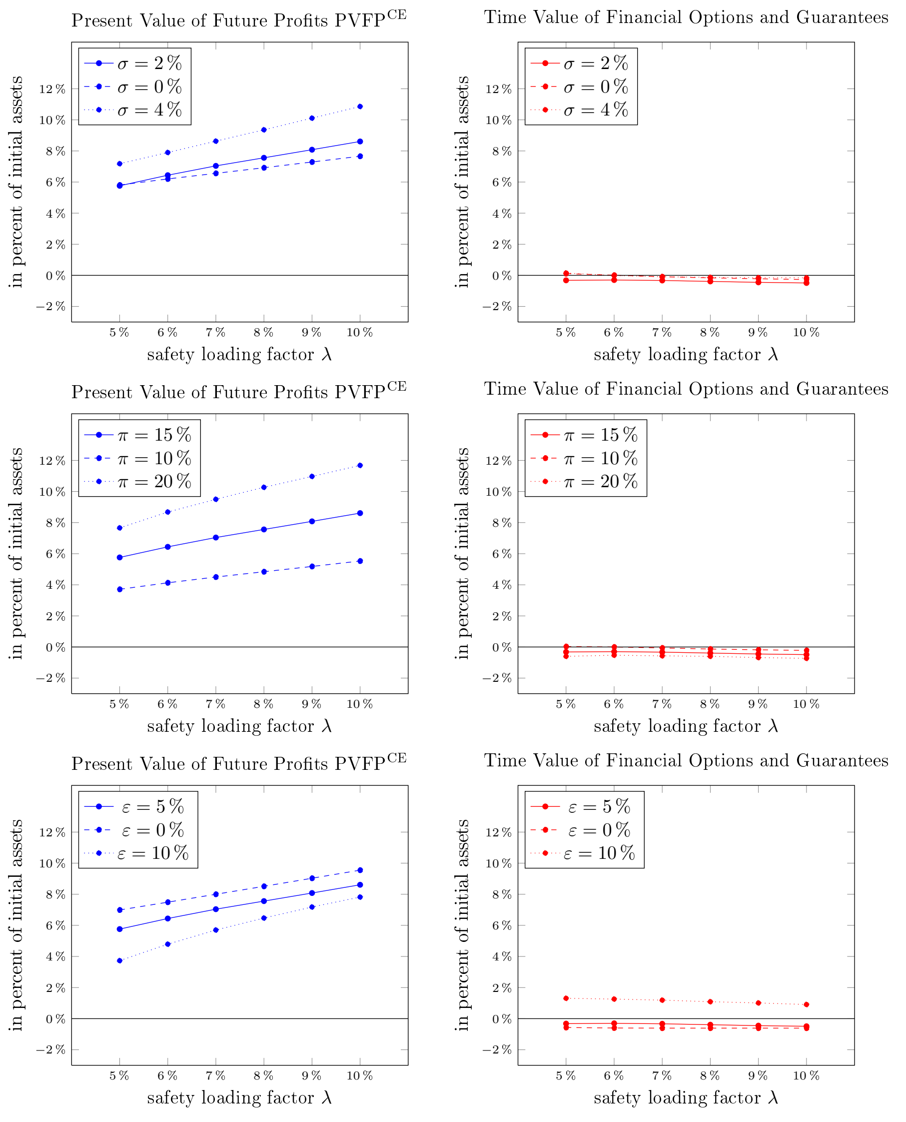

Fig. 6: Results for Varying Parameters

To sum up the results of the sensitivity analysis, we find that, if a parameter setting induces a high number of premium adjustments (e.g., due to a higher σ or smaller ε compared to the reference situation), then adverse scenarios of the investment results and the average claim per capita development do not substantially affect the shareholders' position. On the other hand, if premium adjustments are more infrequent (e.g., due to a smaller σ or higher ε compared to the reference situation), then the impact of the asymmetry in gross surplus increases. This is especially due to the impact of negative investment results as the technical interest corresponds to a short-term minimum interest guarantee (up to the next adjustment). Then the technical interest substantially affects the shareholders' position.

5 Conclusion and Outlook

We developed a mathematical model to measure the value of private health insurance business in Germany and obtained evidence on how assumptions about different management rules influence the shareholder value of a company. To our knowledge, this is the first paper to present a stochastic valuation framework for German private health insurance companies that takes into consideration the future development of financial markets and health insurance claims at the same time. The market-consistent embedded value methodology measures options and guarantees of insurance contracts within the company. We examine the impact of the profit sharing on future profits among the most important contract specifics of German private health insurance contracts. We quantify the impact of technical interest rate adjustments and show that this contract feature is essential in the MCEV for German private health insurance.

The profit sharing in German private health insurance has an asymmetric impact on future profits; a positive gross surplus is only partly credited, whereas a negative gross surplus fully affects shareholder accounts. We found that the possibility to adjust the technical interest in the context of a premium adjustment diminishes the impact of this asymmetry on future profits such that the time value of financial options and guarantees in German private health insurance is less substantial as in life insurance portfolios. We even identified several situations resulting in small negative time values of options and guarantees. This result indicates that the valuation by a Monte Carlo simulation systematically yields a higher value for the present value of future profits compared to the present value of future profits from a certainty equivalent scenario.

Life insurance companies generally have a substantial positive time value of financial options and guarantees due to a long-term minimum interest rate guarantee and further contract specifics. Private health insurance companies in Germany, however, report a time value of financial options and guarantees of zero. For example, in their MCEV report of 2011, the Allianz Group argues that the possibility of premium adjustments "[...] is sufficient to fully cover the financial guarantees" (Allianz Group, 2012). Our study indicates that the argument is valid, but that under certain circumstances a time value of financial options and guarantees is non-zero. Our decomposition of the present value of future profits shows that the certainty equivalent scenario may underestimate to some extent surpluses resulting from the claim development and the safety loading.

Future work will address the impact of dynamic policyholder behavior on shareholders and policyholders accounts. Increasing premiums change policyholders' attitudes regarding their health insurance contract and thus contracts are revised, changed (e.g., lower coverage through higher deductible), or even lapsed. However, it is uncertain how deviations from prudent lapse rates affect the shareholder value. Another line of research will focus on the effects of medical inflation. Our stochastic model of inflation enables us to analyze how medical inflation influences shareholder profits. Furthermore, we will measure the risks associated with medical inflation and assert in our stochastic environment whether the regulatory rules ensure whole-life affordable premiums.

Remarks on the Stochastic Environment

In the following we give a short review of the JY-model. The review is similar to the description of the model in Jarrow and Yildirim (2003); Brigo and Mercurio (2006); Dodgson and Kainth (2006); Cipollini and Canty (2010).

Consider a financial market with finite horizon T described by the probability space $(\Omega, \mathcal{F}, P)$ and filtration $(\mathcal{F}_t)_{0 \leq t \leq T}$. The probability measure P is the real-world measure. The model is based on the assumption that there exist nominal as well as real prices in the financial market. The inflation (i.e. the development of the consumer price index) explains the difference between the corresponding nominal and real economy. The JY-model is an analog to a two-currency interest rate model, whereas the inflation rate in the JY-model corresponds to the spot exchange rate in the two-currency analog.

The following two equations constitute a Heath-Jarrow-Morton framework for the instantaneous forward rates $f_n(t,T)$ (nominal economy) and $f_r(t,T)$ (real economy). The instantaneous forward rates under the real-world probability measure P satisfy the following stochastic differential equations for $t \in [0,T]$:

$$
\begin{aligned}
df_n(t,T) &= \alpha_n(t,T)dt + \varsigma_n(t,T)dW_n^P(t), \\
df_r(t,T) &= \alpha_r(t,T)dt + \varsigma_r(t,T)dW_r^P(t)
\end{aligned}
$$

with initial conditions $f_n(0,T) = f_n^M(0,T)$ and $f_r(0,T) = f_r^M(0,T)$. $\alpha_n(t,T)$ and $\alpha_r(t,T)$ are adapted processes; $\varsigma_n(t,T)$ and $\varsigma_r(t,T)$ are deterministic functions; $W_n^P(t)$ and $W_r^P(t)$ are Brownian Motions. $f_n^M(0,T)$ and $f_r^M(0,T)$ denote the observed instantaneous forward rates in the market at time 0 for maturity T; i.e.,

$$
f_n^M(0,T) = -\frac{\partial \log P_n^M(0,T)}{\partial T} \quad \text{and} \quad f_r^M(0,T) = -\frac{\partial \log P_r^M(0,T)}{\partial T}.
$$

$P_n^M(0,T)$, $P_r^M(0,T)$ are the bond prices in the nominal and real market for maturity T.

The development of the consumer price index $I(t)$ is explained in terms of a Geometric Brownian Motion, i.e.

$$
dI(t) = I(t)\mu(t)dt + I(t)\sigma_I dW_I^P(t),
$$

with initial condition $I(0) = I_0 > 0$, an adapted process $\mu(t)$, and a positive constant volatility parameter σ_I.

The three Brownian motions $W_n^P(t)$, $W_r^P(t)$, and $W_I^P(t)$ are correlated with correlation coefficients $\rho_{n,r}$, $\rho_{n,I}$ and $\rho_{r,I}$. It is

$$
dW_n^P(t)dW_r^P(t) = \rho_{n,r}dt, \quad dW_n^P(t)dW_I^P(t) = \rho_{n,I}dt, \quad dW_r^P(t)dW_I^P(t) = \rho_{r,I}dt.
$$

Following Jarrow and Yildirim (2003) we assume a decaying volatility structure. For $t \in [0,T]$ we let

$$
\varsigma_n(t,T) = \sigma_n \exp\left(-a_n(T-t)\right) \quad \text{and} \quad \varsigma_r(t,T) = \sigma_r \exp\left(-a_r(T-t)\right),
$$

with positive constants a_n, a_r, σ_n and σ_r.

A change of measure from the real-world measure P to the risk-neutral measure Q^n (corresponding to the nominal economy) and a restatement of the stochastic differential equations in terms of short rates yields

$$
\begin{aligned}
dn(t) &= (\vartheta_n(t) - a_n n(t))dt + \sigma_n dW_n(t), \\
dr(t) &= (\vartheta_r(t) - \rho_{r,I}\sigma_r\sigma_I - a_r r(t))dt + \sigma_r dW_r(t), \\
dI(t) &= I(t)(n(t) - r(t))dt + I(t)\sigma_I dW_I(t).
\end{aligned}
$$

Again the three Brownian motions W_n, W_r, and W_I are correlated with the parameters $\rho_{n,r}$, $\rho_{n,I}$, and $\rho_{r,I}$, and we have

$$
\begin{aligned}
\vartheta_n(t) &= \frac{\partial f_n^M(0,t)}{\partial t} + a_n f_n^M(0,t) + \frac{\sigma_n^2}{2a_n}\left(1 - \exp(-2a_n t)\right) \\
\vartheta_r(t) &= \frac{\partial f_r^M(0,t)}{\partial t} + a_r f_r^M(0,t) + \frac{\sigma_r^2}{2a_r}\left(1 - \exp(-2a_r t)\right),
\end{aligned}
$$

to fit the observed term structure at the initial date. $\frac{\partial f_n^M(0,t)}{\partial t}$ and $\frac{\partial f_n^M(0,t)}{\partial t}$ denote the partial derivatives of $f_n^M(0,t)$ and $f_r^M(0,t)$ with respect to the second argument. The equations for the nominal and real interest rate under the risk-neutral measure Q^n are referred to in the literature as the “Hull-White Extended Vasicek” model (Brigo and Mercurio, 2006). Note that the drift term of the inflation process after the measure change is described by the difference of the nominal and real short rate. In economic literature, other authors denote this relation as the Fisher equation.

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