Allowance for Surplus Funds under Solvency II: Adequate reflection of risk sharing between policyholders and shareholders in a risk-based solvency framework?

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Abstract In several member states of the European Union, collective bonus reserves are set up as part of the statutory reserves backing traditional participating life insurance business. Although primarily reserved for policyholders' future surplus participation, national law (for example in Germany and Austria) allows the insurance companies to use these funds to (partly) cover future losses. Under the risk-based solvency framework Solvency II, the loss absorbency of these buffer reserves is explicitly recognized by so-called Surplus Funds which are classified as Basic Own Funds. This paper performs a profound analysis of the approach currently used in Germany to reflect this type of risk sharing between policyholders and shareholders in the Solvency II framework. The comprehensive methodology developed in this paper can be used to determine the economic value of Surplus Funds and ensures that no double-counting of future cash flows occurs. It can easily be adapted to other countries, in particular Austria. Based on a stochastic balance sheet and cash flow projection model, we present numerical results that illustrate how the allowance for Surplus Funds affects Basic Own Funds, Solvency Capital Requirement, Risk Margin and deferred taxes under Solvency II. We conclude that the current valuation approach appears to be internally consistent, but some of the underlying assumptions are questionable. In particular, the valuation approach should be refined in order to better reflect local statutory requirements, including both, accounting rules and other regulatory constraints for participating business.

Keywords Surplus Funds · Participating Life Insurance · Risk Sharing · Solvency II

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1 Introduction

With Solvency II a new risk-based solvency framework for insurance companies in the European Union entered into force on 1 January 2016. Under Solvency II, each entity has to determine its solvency situation based on a market consistent valuation of all assets and liabilities that reflects the features and mechanisms inherent in the underlying insurance business. In this regard, an important aspect is the valuation of traditional participating life insurance business, which plays a major role in old-age provision in Continental Europe.

The holder of a traditional participating life insurance policy is entitled to participate in the surplus of the company in addition to the contractually guaranteed benefits. In several member states of the European Union, the surplus participation process includes a collective bonus reserve in which surplus, derived on an aggregated (portfolio) level, is pooled first and allocated to individual contracts in subsequent years. For example, such buffer reserves (called Reserve for Bonuses and Rebates (RfB)) are a relevant component of statutory reserves for participating business in Germany and Austria. The accumulated profits in the RfB are primarily reserved for policyholders' future surplus participation. However, national law allows the insurance companies to use these funds to (partly) cover future losses. Typically, the loss coverage is subject to specific constraints and requires approval by the local regulatory authority.

Solvency II explicitly recognizes the loss absorbency of such reserves based on the amount shown in the statutory balance sheet at the valuation date. The benefit payments resulting from those funds are not included in Technical Provisions (TP). Instead, they are recognized as so-called Surplus Funds (SF), which are part of the Basic Own Funds (BOF) under Solvency II. This raises the question what implications such a classification of SF has on capital requirements under Solvency II and whether it appropriately reflects the risk sharing via such type of buffer reserves.

Risk sharing between policyholders and shareholders is inherent in traditional participating life insurance business and includes several aspects. Therefore, it has been broadly discussed in scientific literature, especially since the beginning of the 21st century.

Literature on the impact of collective reserves include e. g. Hansen and Miltersen (2002), Kling et al. (2007) and Goecke (2013). Applying a lagged surplus distribution system similar to the one present in Germany, Hansen and Miltersen (2002) study the pooling effects due to a common bonus reserve for a simplified two-customer case. Kling et al. (2007), who explicitly consider the specific regulatory framework and business practice in Germany, analyze the risk resulting from traditional minimum interest guarantees under the "real-world" probability measure. Goecke (2013) analyzes the return smoothing effects of intergenerational risk transfer in pension schemes. Allowing for a collective buffer reserve, he finds how inter-generational risk transfer improves the risk-return profile for policyholders. However, this branch of literature does not address SF under Solvency II.

Explicitly addressing SF, Wagner (2013) gives a short overview of the motivation behind the allowance for SF. Further, the Federal Financial Supervisory Authority (BaFin) has published interpretative decisions concerning the valuation of SF under Solvency II (BaFin, 2015).

A different aspect of risk sharing between policyholders and shareholders in the context of Solvency II is covered by Burkhart et al. (2015). They analyze the so-called Going Concern Reserve, which is a special balance sheet item that reflects the risk reducing impact of inheritance effects caused by the pre-financing of acquisition cost of new business via cost surplus of existing business.

In total, to our knowledge, scientific literature lacks a profound analysis of the reflection of risk sharing between policyholders and shareholders via SF under Solvency II and the interaction of SF with other statutory requirements.

This paper provides such an analysis based on the Solvency II provisions concerning SF and the transposition of *Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II)* (*Solvency II Directive*) into German and Austrian law. As a basis, we present an overview of existing risk sharing mechanisms between policyholders and shareholders via the RfB. In order to illustrate the impact of SF on capital requirements under Solvency II, a stochastic balance sheet and cash flow projection model is set up for a stylized life insurance company. The model covers the relevant features of the German market, considering statutory requirements as well as typical management rules. In particular, it includes management rules for withdrawals from the RfB to cover losses.

We develop a comprehensive methodology that can be used to determine the economic value of SF and ensures that no double-counting of future cash flows occurs. Our method is consistent with the approach currently applied in the German standard valuation model for Solvency II, developed by the German Association of Insurance Companies (GDV) – the so-called Branchensimulationsmodell (BSM) (cf. GDV, 2016). This approach not only allows for SF but in addition includes explicit management rules regarding withdrawals of RfB funds to absorb losses in case of emergency. We also consider natural alternatives to this method that limit the reflection of the loss absorbency either to the explicit modeling of emergency withdrawals (based on management rules) or the allowance for SF. All alternative approaches ensure that no double-counting of cash flows occurs. We analyze those different approaches, also presenting numerical results based on our projection model. To clarify the impact on capital requirements under Solvency II, our analysis not only addresses implications on the BOF but also on other important components of the quantitative requirements, in particular Solvency Capital Requirement (SCR), Risk Margin (RM) and deferred taxes (DT).

The remainder of the paper is organized as follows. Focusing on Germany and Austria, Sect. 2 discusses how SF are supposed to reflect the risk sharing between policyholders and shareholders in the Solvency II framework, based on local statutory requirements for surplus participation. Sect. 3 provides the valuation framework underlying the analysis and Sect. 4 precisely describes the methodology used to determine the economic value of SF. The results of our analysis are presented and discussed in Sect. 5. Sect. 6 summarizes our findings and concludes.

2 The recognition of collective reserves under Solvency II

Using the RfB in Germany as an example, this section describes the recognition of the risk sharing between policyholders and shareholders via collective reserves under Solvency II. We first discuss the main features of the surplus participation process currently in place in Germany, which leads us to the dual character of the RfB and its crucial role for traditional life insurance business. Although details may vary, the mechanisms in place in other European countries, in particular Austria, are quite similar. Next, we describe the incorporation of such collective reserves into the Solvency II valuation framework via the allowance for SF. Concentrating on Germany, we outline the transposition of the European guidelines into national law, also taking into account the interpretative decisions of the BaFin. We show that the allowance for SF constitutes an exception from the market consistent valuation approach of Solvency II, which is based on expected cash flows to policyholders.

2.1 Dual character of the RfB in Germany

In addition to the (annual) cliquet-style interest guarantee, the holder of a traditional participating life insurance contract is entitled to participate in the company's annual surplus. As shown in Fig. 1, surplus participation is not implemented as a direct and individual process for each policyholder. Instead, a multi-stage mechanism with collective elements is applied to achieve a stable surplus participation for all policyholders.

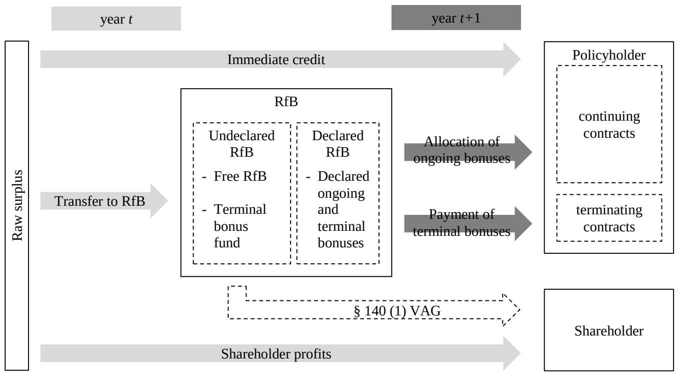


Fig. 1 Surplus participation process

The entire surplus participation process is based on local accounting rules and other regulatory constraints. In the first step, company's so-called raw surplus is split between policyholders and the insurance company. Hereby, minimum participation rules prescribed in the Executive Guidance Order on Minimum Surplus Participation (MindZV) have to be observed. Although policyholders may receive some portion of their share in raw surplus directly in the year of occurrence (immediate credit), the main part is transferred to a collective reserve, called RfB, first. From there, it is only allocated to individual contracts in subsequent years, either via ongoing or terminal bonuses. In both cases, the surplus to be allocated to policyholders in a certain year is always determined in advance, usually at the end of the previous year.[1] Based on this mechanism, the RfB can be divided into different parts:

– *Declared RfB*: It contains the funds bindingly designated for policyholders surplus participation, either via crediting of ongoing bonuses or payout of terminal bonuses. As mentioned above, those funds are declared one year in advance and already reserved on an individual contract basis. They will be allocated to policyholders during the next year.

[1] Notice that also longer declaration periods are possible, i. e. policyholders' surplus participation is declared several years in advance.

- *Terminal Bonus Funds (TBF)*: It contains funds accrued to finance terminal bonuses of future years. Similar to the declared RfB, those funds are already reserved on individual contract basis. Specific reserving rules apply and in principle, until those funds are due at maturity of the contract or in case of death or surrender, policyholders do not have a claim on them.
- *Free RfB*: The remaining funds are denoted by free RfB. It contains funds reserved for policyholders' future surplus participation but not yet allocated to individual contracts. Similar to the TBF, policyholders do not have a claim on the free RfB. Therefore, free RfB and TBF form the so-called undeclared RfB.

Note that in principle, policyholders' immediate credit is directly financed by raw surplus of the current year, such that no funds have to be reserved in the RfB for this type of surplus participation. However, if raw surplus is insufficient, funds of the free RfB may be used to (partly) finance the immediate credit.

Overall, the RfB separates origin and distribution of surplus. The lagged surplus participation allows the insurer to smooth returns for policyholders and to buffer fluctuations in raw surplus, especially fluctuations of the investment returns. In years with high raw surplus, more funds are transferred to the reserve than withdrawn for individual surplus allocation. In consequence, the RfB funds are built up to offset years with lower returns.

Although the RfB primarily acts as a buffer reserve for policyholders' future surplus participation, under certain circumstances the insurer may, subject to approval by the regulator, use those funds for other purposes. In particular, § 140 (1) of the Insurance Supervision Law (VAG) allows the insurer to withdraw funds from the RfB that have not been allocated to individual policyholders yet to avoid a pending financial imbalance of the company and to cover unforeseeable losses due to a general change of the environment. Note that consistent with the surplus participation process, this refers to losses derived on a local accounting basis.

Since only funds not bindingly declared yet may be used for loss coverage, the buffer is limited to funds of the undeclared part of the RfB, i. e. the TBF and the free RfB. Hence, the undeclared RfB represents an emergency reserve for the insurer whose capacity he can in principle control by his surplus participation policy. However, legal restrictions have to be considered concerning both, regulatory and tax law. In detail, § 9 MindZV prescribes a maximum amount for the undeclared RfB (based on § 145 (3) VAG), which is supposed to ensure an adequate surplus participation. Also, German tax provisions basically limit the availability of the initial undeclared RfB to three years (cf. § 21 of the Corporate Tax Law (KStG)).[2] Further constraints may apply concerning the (permanent) availability of those funds for loss coverage, e. g. due to existing business plans (relevant for old business in force, written before 1994), which require to allocate surplus to policyholders in a timely manner. Especially with respect to the TBF, it also has to be checked to which extent the contractual conditions allow to reduce accrued terminal bonuses (cf. GDV, 2008). Further note that it is not explicitly specified which proportion of a loss may be covered by such withdrawals.

This section shows that the RfB plays a key role in traditional participating life insurance business. The lagged surplus distribution does not only lead to inter-generational inheritance of profits but also to an inter-generational risk transfer, since profits accumulated in the RfB in past periods represent a buffer to cover future losses. Therefore, it is important that a risk-

[2] Note that currently there exists a temporary exception to be applied for assessment years 2016 and 2017 which extends this period to five years (cf. § 34 KStG).

based solvency framework such as Solvency II adequately reflects the risk sharing between policyholders and shareholders via the RfB.

2.2 Surplus Funds under Solvency II

This section summarizes some aspects of the Solvency II valuation framework relevant for the subsequent analysis concerning the reflection of risk sharing between policyholders and shareholders via SF. We further outline the transposition of the European guidelines into German law.

One of the main objectives of the new Solvency II regime is the sufficient capitalization of all insurance undertakings in the European Union in order to ensure adequate protection of policyholders. Following an economic risk-based approach, the Solvency II framework prescribes a market consistent valuation of all assets and liabilities.

In particular, the valuation of insurance contracts under Solvency II is based on the concept of a transfer value as defined in art. 75 and art. 76 of the *Solvency II Directive*: The value of so-called Technical Provisions shall correspond to the current amount the insurance undertaking would have to pay if it was to transfer its insurance obligations immediately to another insurance undertaking. To fulfill the requirement of market consistency, the valuation especially has to reflect the features and mechanisms inherent in the insurance business, e. g. the RfB described in Sect. 2.

According to art. 77 of the *Solvency II Directive*, the Technical Provisions can be decomposed into the Best Estimate of Liabilities (BEL) and the RM. The BEL corresponds to the expected present value of all future cash in- and outflows required to settle the insurance obligations. The calculation of BEL is typically based on an explicit portfolio projection in an actuarial cash flow projection model.

The RM shall be calculated separately from the BEL (cf. *Solvency II Directive*, art. 77). Its value represents an add-on to the BEL and equals the expected cost of capital for a reference undertaking in case of a transfer of the company's entire portfolio of insurance obligations. The reference undertaking has to provide the regulatory capital required to cover all unhedgeable risks over the lifetime of the transferred insurance obligations (cf. *Solvency II Directive*, art. 78).

According to art. 15 of the *Commission Delegated Regulation (EU) 2015/35*, DT are determined based on the valuation differences between Solvency II and the tax balance sheet. Clearly, this includes valuation differences of all assets and liabilities (in particular Technical Provisions).

The BOF can be derived from the excess of assets over liabilities in the Solvency II balance sheet and correspond to the funds available to cover the company's risks. Those risks are quantified by the SCR, i. e. the economic capital to be held in order to ensure that the undertaking will still be able, with a probability of at least 99.5 %, to meet its obligations to policyholders over the following 12 months (cf. *Solvency II Directive*, recital 64). Accordingly, art. 101 of the *Solvency II Directive* defines the SCR as the Value-at-Risk of the BOF with a confidence level of 99.5 % over a one-year period.[3] The SCR calculation should also take into account the loss-absorbing capacity of Technical Provisions and DT (cf. *Solvency II Directive*, art. 103).

In case of participating life insurance contracts, the cash flows used to derive the BEL generally include the Future Discretionary Benefits (FDB), i.e. payments expected to be

[3]For more details concerning the definition of SCR see Christiansen and Niemeyer (2014).

made in excess of the contractually guaranteed benefits (cf. *Solvency II Directive*, art. 78). However, the European legislators chose to allow for the risk reducing character of the RfB via an additional own fund item. Therefore, art. 91 of the *Solvency II Directive* introduces so-called SF which are defined as accumulated profits that have not been made available for distribution to policyholders, yet. To the extent those funds fulfill the criteria for classification as Tier 1 capital, they shall not be considered as insurance liabilities but can be recognized via SF as part of the BOF. Accordingly, the respective cash flows are not to be taken into account for the calculation of Technical Provisions, i. e. FDB do not include policyholders' future surplus participation resulting from those funds (cf. *Solvency II Directive*, art. 78).

Note that recital 51 of the *Solvency II Directive* explicitly requires the valuation of SF to be in line with the economic approach. A mere reference to their valuation in the statutory accounts is not sufficient. However, since the extent to which those accumulated profits can be used in case of an emergency depends on the statutory requirements in each country, the *Solvency II Directive* does not provide details on the valuation methodology. The VAG transposes the *Solvency II Directive* into German law. The transposition into Austrian law is almost the same as in Germany.

As mentioned in Sect. 2, § 140 VAG specifies the usage of RfB funds in case of adverse events. Based on this article, § 84 VAG specifies that all cash flows to policyholders resulting from the undeclared RfB available at the valuation date make up the SF as defined in the *Solvency II Directive* and that their economic value is given by the expected present value of those cash flows.

This set of regulatory requirements implies that available collective reserves receive special treatment under Solvency II. Although these funds are for the most part expected to be used for policyholders' future surplus participation, they do not have to be counted as part of the FDB if national law allows to use them for loss coverage in case of an emergency. Hence, SF constitute an exception from the market consistent valuation approach of Solvency II which is based on expected cash flows to policyholders. Note that the impact of SF is not limited to an increase of BOF but it further affects SCR, RM and DT, since these items are calculated based on the BOF. Therefore, a comprehensive analysis of the reflection of risk sharing between policyholders and shareholders via the RfB includes several aspects:

1. Does the valuation approach ensure that no double counting of any cash flows occurs?
2. Does the allowance for SF adequately reflect the risk sharing between policyholders and shareholders via the initial RfB?
3. How should SF be reflected in the calculation of SCR, RM and DT?

Another crucial aspect is the reflection of local statutory requirements that remain in place under Solvency II. In particular, this perspective is relevant for the assessment of the criteria for classification of SF as Tier 1 capital (cf. § 91 VAG).

3 Analysis Framework

In this section, we introduce the asset-liability framework underlying our analysis. We use a stochastic balance sheet and cash flow projection model for a stylized life insurance company. The model is based on the work of Burkhart et al. (2015); adjustments were made for the calculation of SF, in particular the RfB and TBF mechanisms were incorporated. Due to the complexity of the stochastic future cash flows, the valuation is based on Monte

Carlo simulations. Assets and liabilities are projected until the complete run off of the initial business in-force, following certain management rules.

Note that SF only refer to the RfB available at the valuation date, but do not address the risk sharing between policyholders and shareholders via future RfB funds. Hence, in addition to the allowance for SF, many valuation models in the German life insurance market also involve an explicit modeling of withdrawals from both, initial and future RfB funds based on § 140 VAG. This also holds for the BSM which was developed by the GDV and is the standard valuation model for Solvency II in Germany. In consequence, our model also allows for management rules concerning the application of § 140 VAG.

Since the focus of the paper is on the market-consistent valuation of insurance obligations, the model is introduced under the risk neutral measure.

3.1 Financial market model

We consider a risk-neutral, frictionless and continuous financial market. The insurer's assets are invested in two types of risky assets: coupon bonds and stocks. The short rate process $(r_t)_{t>0}$ is described by the Vasicek model and the stock price $(S_t)_{t>0}$ follows a geometric Brownian motion:

$$\begin{aligned} dr_t &= \kappa(\theta - r_t)\,dt + \sigma_r dW_t^{(1)}, \\ dS_t &= S_t\left(r_t dt + \rho\sigma_S dW_t^{(1)} + \sqrt{1-\rho^2}\sigma_S dW_t^{(2)}\right), \end{aligned}$$

with deterministic initial values r_0 and S_0. $W_t^{(1)}$ and $W_t^{(2)}$ denote two uncorrelated standard Wiener processes adapted to a filtration $\mathscr{F}$ on some probability space $(\Omega, \mathscr{F}, \mathbb{Q})$, satisfying the usual conditions, with a risk neutral measure $\mathbb{Q}$. The parameters κ, θ, σ_r, σ_S and ρ are deterministic and constant. From the discretely compounded yield curve at time t, we calculate par yields that determine the coupon rates of the considered coupon bonds.

The bank account $B_t = \exp\left(\int_0^t r_u du\right)$ at time t serves as the discount factor in the risk-neutral framework. Therefore, the market value at time $t = 0$ of a stochastic cash flow Y_t occurring at time t is given by $PV(Y_t) = \mathbb{E}^{\mathbb{Q}}\left(\frac{Y_t}{B_t}\right)$ where $\mathbb{E}^{\mathbb{Q}}(\cdot)$ denotes the expected value under the risk neutral measure $\mathbb{Q}$.

3.2 Liability model

The company's insurance portfolio consists of different cohorts of identical traditional participating life insurance contracts with a contract duration of n years and policyholder's age x at inception of the contract. The contract provides a guaranteed benefit G (sum insured) at maturity or death against annual premium payments P. The pricing is based on a guaranteed interest rate i.

Each cohort (k) consists of all contracts concluded at the start of year $k+1$. Based on the actuarial principle of equivalence, the annual premium ${}^{(k)}P$ is given by

$${}^{(k)}P = \frac{G \cdot \left({}^{(k)}A_{x:\overline{n}|} + \alpha^\gamma \cdot {}^{(k)}\ddot{a}_{x:\overline{n}|}\right)}{(1-\beta)\cdot {}^{(k)}\ddot{a}_{x:\overline{n}|} - \alpha \cdot n},$$

with ${}^{(k)}A_{x:\overline{n}|}$ and ${}^{(k)}\ddot{a}_{x:\overline{n}|}$ representing the present values of an n year endowment insurance and annuity-due of an x year old with guaranteed interest ${}^{(k)}i$, respectively.

Following the Zillmerisation procedure, the actuarial reserve ${}^{(k)}AR_t$ for the statutory balance sheet at the end of year $t = k+1,...,k+n$ can be calculated recursively by

$$ {}^{(k)}AR_t = \frac{\left({}^{(k)}AR_{t-1} + {}^{(k)}P\cdot(1-\beta) - G\cdot\alpha^{\gamma}\right)\cdot\left(1+{}^{(k)}i\right) - G\cdot q_{x+(t-k)-1}}{1-q_{x+(t-k)-1}}, $$

with initial value ${}^{(k)}AR_k = -\alpha\cdot n\cdot{}^{(k)}P$ and q_x being the first-order probability rate of an x year old to die within one year.

Besides the guaranteed annual interest ${}^{(k)}i$ on their actuarial reserve, policyholders participate in the company's annual raw surplus. The bonus ${}^{(k)}bon_t$ declared for a policyholder of cohort (k) at time t is credited in two ways.

- Ongoing bonuses (${}^{(k)}bon_t^{acc}$) are accumulated in the bonus reserve ${}^{(k)}BR_t$. In subsequent years, the guaranteed interest rate also applies to the bonus reserve such that the development is given by

$$ {}^{(k)}BR_t = {}^{(k)}BR_{t-1}\cdot\left(1+{}^{(k)}i\right) + {}^{(k)}bon_t^{acc} \quad (1) $$

  with ${}^{(k)}BR_k = 0$.
- The terminal bonus part (${}^{(k)}bon_t^{term}$) is allocated to the terminal bonus fund ${}^{(k)}TBF_t$. In contrast to the bonus reserve, the TBF do not earn the guaranteed interest and policyholders do not have a claim on them until they are declared for payout in the subsequent year. Hence, funds may be withdrawn from the TBF in line with § 140 VAG and we have that

$$ {}^{(k)}TBF_t = {}^{(k)}TBF_{t-1} + {}^{(k)}bon_t^{term} - {}^{(k)}CF_t^{\S\,140,TBF} $$

In case of the policyholder's death or at maturity of the contract, the bonus reserve and the terminal bonus funds are paid out in addition to the contractually guaranteed benefit. Hence, the benefits (liabilities) due add up to ${}^{(k)}L_t = G + {}^{(k)}BR_t + {}^{(k)}TBF_{t-1}$.[4]

All relevant figures determined for each cohort are summed up to determine the respective figures for the entire insurance portfolio.

We assume that the actual (best estimate) second-order mortality rates equal the first-order mortality rates q_x adjusted by a factor q^*. Hence, ${}^{(k)}l_t^* = {}^{(k)}l_{t-1}^*\cdot\left(1-q^*\cdot q_{x+(t-k)-1}\right)$ policyholders of cohort (k) remain in the portfolio at the end of year $t = k+1,...,k+n-1$, with ${}^{(k)}l_{k+n}^* = 0$ (maturity of contracts of cohort (k)). Since Solvency II requires a run-off valuation of the insurance portfolio existing at the valuation date, we do not consider new business. The overall size of the company's insurance portfolio at time t is given by

$$ l_t^* = \sum_{k=0}^{n-1} {}^{(t-k)}l_t^*. $$

For technical reasons, we do not take into account surrenders. Since benefit payments in case of surrender are similar to death benefit payments in our example, allowance for surrenders does not appear necessary in order to analyze the reflection of risk sharing between policyholders and shareholders under Solvency II. Of course, we acknowledge that lapse risk is a material risk for life insurance companies and that the modeling of surrenders is necessary for practical implementations.

[4] For technical reasons we assume that the death of a policyholder always occurs at the end of the year.

3.3 Asset model

The company's asset portfolio consists of stocks and coupon bonds yielding at par with fixed initial maturity T_B. Following a constant strategic asset allocation, it is rebalanced at the end of each year based on a constant target stock ratio q^{stock} (in terms of the market values). Thereby, the cash flow CF_t^+ at the beginning of the year (cash flow to shareholders X_t, premium payments P_t as well as administration costs AdC_t^* incurred) and the cash flows at the end of the year (coupon payments CP_{t+1}, nominal repayment $N_{t+1}^{(t+1-T_B)}$ of bonds at maturity and benefit payments L_{t+1}) are taken into account. During the year, CF_t^+ is invested in a risk-less bank account earning the interest rate $r_t(1)$.

If necessary, bonds are sold proportionally to their market values. Besides the Unrealized Gains or Losses (UGL) the company may realize due to the re-balancing, a certain portion d of the unrealized gains on stocks exceeding a limit q_+^{UGL} (in terms of the market values) is realized in order to stabilize the investment return. If necessary, the realization of unrealized gains is extended to reduce losses in the investment surplus as far as possible. Note that unrealized losses on stock exceeding the limit q_-^{UGL} have to be realized immediately.

Overall, the investment return rate is given by

$$i_{t+1}^* = \frac{CF_t^+ \cdot r_t(1) + CP_{t+1} + UGL_{t+1}^{real}}{BV_t^A + CF_t^+},$$

where UGL_{t+1}^{real} denotes the realized portion of the UGL.

3.4 Surplus distribution

The annual raw surplus Sp_{t+1} is based on local accounting rules and broken down into the relevant sources, i. e. cost surplus, risk surplus and investment surplus.

3.4.1 Sources of surplus

The cost surplus Sp_{t+1}^C at time $t+1$ equals the difference between charges included in the premium and actual expenses incurred (AdC_t^*) and also reflects the annual investment return i_{t+1}^*,

$$Sp_{t+1}^C = \left(1 + i_{t+1}^*\right) \cdot \left(\sum_{k=0}^{n-1} \left(\beta \cdot {}^{(t-k)}P + \alpha^\gamma \cdot G \right) \cdot {}^{(t-k)}l_t^* - AdC_t^* \right).$$

The risk surplus represents the difference between the actual mortality experience and mortality assumptions used for premium calculation. Hence,

$$Sp_{t+1}^R = \sum_{k=0}^{n-1} (1-q^*) \cdot q_{x+(t-k)-1} \cdot {}^{(t-k)}l_t^* \cdot \left(G - {}^{(t-k)}AR_{t+1} \right).$$

The investment surplus is defined as the difference between the investment return R^{*}_{t+1} and the guaranteed interest R^{gar}_{t+1} credited to the policyholders' accounts:

$$\begin{aligned} Sp^{I}_{t+1} = & \underbrace{\left(AR_t + BR_t + P_t \cdot (1-\beta) - G \cdot \alpha^{\gamma} \cdot l^{*}_t + fRfB_t + TBF_t\right) \cdot i^{*}_{t+1}}_{=R^{*}_{t+1}} \\ & \underbrace{- \sum_{k=0}^{n-1} \left({}^{(t-k)}AR_t + {}^{(t-k)}BR_t + {}^{(t-k)}P \cdot (1-\beta) - G \cdot \alpha^{\gamma} \right) \cdot {}^{(t-k)}l^{*}_t \cdot {}^{(t-k)}i}_{=R^{gar}_{t+1}} \, . \end{aligned} \tag{2}$$

3.4.2 Splitting of surplus

The splitting of surplus follows minimum surplus participation rules (based on MindZV). Hence, policyholders' share of surplus at time $t+1$ equals

$$\begin{aligned} PS_{t+1} = \max \big\{ & 0; \min \left[R^{*}_{t+1} - R^{gar}_{t+1} ; \max \left(90\,\% \cdot R^{*}_{t+1} - R^{gar}_{t+1} ; 0 \right) \right] \\ & + \max \left(90\,\% \cdot Sp^{R}_{t+1} ; 0 \right) + \max \left(50\,\% \cdot Sp^{C}_{t+1} ; 0 \right) \big\} \, . \end{aligned}$$

Note that losses originating from investment surplus can be offset by profits from other surplus sources to the extent that policyholders' share of surplus remains non-negative. The remaining part of raw surplus represents the insurer's profit X_{t+1} which results in a respective cash out-/inflow if the profit is positive/negative.

In case of a negative raw surplus, funds may be withdrawn from the undeclared RfB in line with § 140 VAG to partly cover the losses. Applying a moving average, the amount $CF^{\S\,140}_{t+1}$ withdrawn from the undeclared RfB depends on the split of raw surplus between shareholders and policyholders in the past $T_{\S\,140}$ years. Of course, the withdrawals are limited to the funds available and can be decomposed as follows. Funds are withdrawn from the free RfB ($CF^{\S\,140,fRfB}_{t+1}$) first and only if those funds are not sufficient, also funds from the TBF ($CF^{\S\,140,TBF}_{t+1}$) are withdrawn. Note that as a simplification, the terminal bonuses declared in the previous year to be paid out at the end of the current period (L^{term}_{t+1}), remain part of the TBF until paid out. Those funds are not available to cover losses. Hence,

$$\begin{aligned} CF^{\S\,140,fRfB}_{t+1} &= \min \left(fRfB_{t+1} + PS_{t+1} ; q^{PH}_{t+1} \cdot \max \left(-Sp_{t+1} ; 0 \right) \right) , \\ CF^{\S\,140,TBF}_{t+1} &= \min \left(TBF_t - L^{term}_{t+1} ; q^{PH}_{t+1} \cdot \max \left(-Sp_{t+1} ; 0 \right) - CF^{\S\,140,fRfB}_{t+1} \right) \end{aligned}$$

with $q^{PH}_{t+1} = \frac{\sum_{s=1}^{T_{\S\,140}} PS_{t+1-s}}{\sum_{s=1}^{T_{\S\,140}} \max\left(Sp_{t+1-s} ; 0\right)}$ representing the share of losses to be covered by policyholders.

The cash flow to shareholders further contains the investment income on the assets backing shareholders' equity under local accounting rules. We also reflect the release of shareholders' equity, i. e. its gradual reduction over time. For technical reasons, the cash flow occurs at the beginning of the next year and is given by

$$X_{t+1} = \left(Sp_{t+1} - PS_{t+1} \right) + CF^{\S\,140}_{t+1} + i^{*}_{t+1} \cdot Eq^{loc}_t + \left(Eq^{loc}_t - Eq^{loc}_{t+1} \right) . \tag{3}$$

3.4.3 Declaration of surplus

At the end of the year, the bonus declaration $bon_{t+1} = bon^C_{t+1} + bon^R_{t+1} + bon^I_{t+1}$ for the next year takes place, i. e. the insurer declares which portion of the RfB is allocated to policyholders in the next period via ongoing and terminal bonuses. In general, the amount is derived from the allocations of cost, risk and investment surplus to the RfB in the previous T_{bon} years. However, it also depends on the reserve ratio of the free RfB after declaration and the actuarial reserve at the end of the year. If the planned bonus declaration would result in a reserve ratio outside a certain corridor $\left[q^{fRfB}_{min}; q^{fRfB}_{max}\right]$, it is de-/increased, respectively. The bonuses declared are split into ongoing and terminal bonuses (bon^{acc}_{t+1}, bon^{term}_{t+1}) according to a fixed portion q^{TBF}. Note, that for simplification the ongoing bonuses declared are allocated to the policyholders' bonus reserves right away and not stored in the declared part of the RfB.

3.4.4 Allocation of surplus to individual policyholders

The declared bonuses are allocated to individual contracts based on a so-called natural allocation system (cf. Wolfsdorf, 1997). Hence, all contracts of a certain cohort (k) receive the same bonus ${}^{(k)}bon_{t+1}$ but the amount may differ between contracts from different cohorts. Total bonuses are allocated to individual contracts as follows:[5]

- Cost bonuses bon^C_{t+1} are allocated based on the premium ${}^{(k)}P$.
- Risk bonuses bon^R_{t+1} are allocated based on the capital at risk $(G - {}^{(k)}AR_{t+1})$.
- The investment bonuses bon^I_{t+1} are distributed such that all policyholders receive the same total yield (sum of the guaranteed interest rate ${}^{(k)}i$ and the bonus rate ${}^{(k)}i^*_{t+1}$) on their accounts value $({}^{(k)}AR_t + {}^{(k)}BR_t)$. However, all policyholders receive at least the guaranteed interest rate. Hence, if investment bonuses are not sufficient for all policyholders to receive at least the guaranteed interest rate, the bonus rates of cohorts with a lower guaranteed interest rate are reduced accordingly.

Note that the surplus distribution can easily be adjusted to surplus participation mechanisms in other countries. E. g. in Austria, a minimum surplus participation rate of 85 % is applied, with no distinction between sources of surplus.

3.5 Statutory accounting balance sheet

At the end of each year, the statutory accounting balance sheet can be set up as shown in Table 1. The left-hand side contains the book value of the assets (stocks and bonds). On the right-hand side, the shareholders' equity equals a fixed percentage q^{loc} of the actuarial reserves. Free RfB and TBF at the end of the year are given by

$$\begin{aligned} fRfB_{t+1} &= fRfB_t + PS_{t+1} - CF^{\S 140, fRfB}_{t+1} - bon_{t+1} \\ TBF_{t+1} &= TBF_t - L^{term}_{t+1} - CF^{\S 140, TBF}_{t+1} + bon^{term}_{t+1} . \end{aligned}$$

[5] For details see Burkhart et al. (2015).

Table 1 Statutory accounting balance sheet at time $t+1$

Assets	Liabilities
BV_{t+1}^{A}	Eq_{t+1}^{loc}
	X_{t+1}
	$fRfB_{t+1}$
	TBF_{t+1}
	AR_{t+1}
	BR_{t+1}

4 Calculation of Surplus Funds

We now describe the method used to calculate the SF based on the asset-liability framework introduced in the previous section. Subsequently, we describe how to derive the economic balance sheet under Solvency II considering SF. The method implemented is consistent with the interpretative decisions of BaFin (2015), especially concerning the avoidance of double counting of cash flows. It also takes into account possible withdrawals for loss coverage in line with § 140 VAG.

4.1 Development of the initial RfB

In their interpretative decisions, BaFin requires the first in – first out (fifo) concept to be applied, i. e. the earliest funds added to the RfB are the first to be withdrawn. Hence, we have to track the withdrawals from the RfB until the total of those withdrawals (undiscounted) exceeds the undeclared RfB at the valuation date. Thereby, not only the withdrawals for the purpose of policyholders' surplus participation have to be considered, but also withdrawals to cover losses according to § 140 VAG. To identify the relevant cash flows, we have to distinguish between the free RfB and the TBF.

The ongoing bonuses (bon_t^{acc}) are declared for the next period and reduce the free RfB accordingly. Furthermore, the free RfB is reduced by withdrawals for future terminal bonuses (bon_t^{term}) which are transferred to the TBF. Hence, the initial free RfB evolves according to

$$fRfB_t^{SF} = \max\left(fRfB_{t-1}^{SF} - CF_t^{\S\,140,fRfB} - bon_t^{acc} - bon_t^{term}, 0\right),\ t > 0,$$

where $fRfB_0^{SF}$ equals the free RfB at the valuation date.

Concerning the TBF, the funds available to cover losses are reduced by withdrawals $trans_t^{term} = L_{t+1}^{term}$ for the terminal bonus payments at the end of the next period and by withdrawals in line with § 140 VAG ($CF_t^{\S\,140,TBF}$). On the other side, RfB funds assigned to individual policyholders for future terminal bonus payments bon_t^{term} are shifted from the free RfB to the TBF. Hence, we have that

$$TBF_t^{SF} = \max\left[\max\left(TBF_{t-1}^{SF} - CF_t^{\S\,140,TBF}, 0\right) + bon_t^{SF,term} - trans_t^{term}, 0\right],\ t > 0,$$

with

$$bon_t^{SF,term} = bon_t^{term} \cdot q_t^{SF,fRfB}.$$

Hereby, $q_t^{SF,fRfB}$ represents the share of the bonuses of year t originating from the initial free RfB, i. e.

$$q_t^{SF,fRfB} = \max\left[0, \min\left(1, \frac{fRfB_{t-1}^{SF} - CF_t^{\S\,140,fRfB}}{bon_t^{acc} + bon_t^{term}}\right)\right]. \tag{4}$$

As mentioned in Sect. 3.4.2, the terminal bonuses declared in advance for the next period are not shifted to the declared part of the RfB but remain part of the TBF until paid out at the end of the next period. Hence, the terminal bonus payments at the end of the first period are not to be considered in the calculation of SF since they have already been bindingly declared at the end of the previous period and are not available to cover future losses, i. e. $TBF_0^{SF} = TBF_0 - L_1^{term}$.

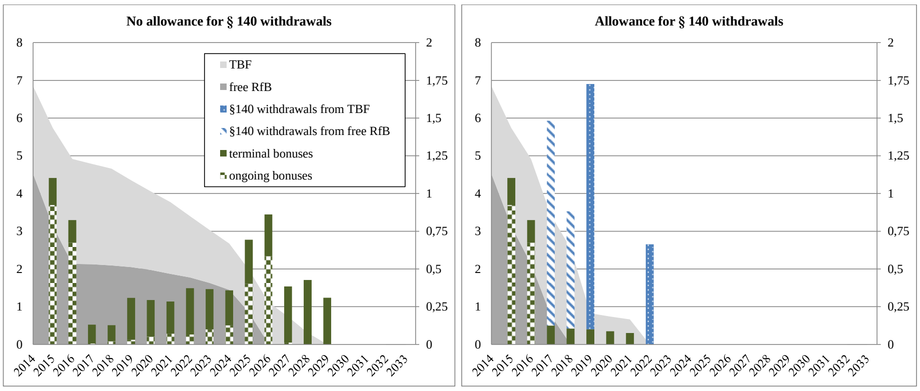
4.2 Identification of relevant cash flows

After the development of the initial RfB and the respective withdrawals have been identified, the economic value of SF can be derived. Since the FDB are not to be influenced by the modeling of SF, an ex-post approach is recommended in which the preliminary value of BEL (which includes FDB) is determined based on a valuation without consideration of SF and subsequently the amount of SF is deducted .

BaFin (2015) states that the approach applied for the valuation of SF has to ensure that no double-counting of any cash flows occurs, i. e. that the BEL is only reduced by the present value of policyholders' benefits resulting from the initial RfB. Conversely, cash flows which are already included future cash flows to shareholders are not to be considered again in the value of SF. In this regard, two aspects have to be considered.

First, as we have mentioned in Sect. 2.1, RfB funds may not be used for policyholders' surplus participation but to cover losses in line with § 140 VAG, therefore reducing shareholders' losses. Those cash flows are not reflected in the policyholders' preliminary FDB but lead to an increase of the cash flows to shareholders (cf. eq. 3). Hence, if such withdrawals refer to funds of the initial RfB, the corresponding cash flows are to be excluded from the calculation of SF to avoid double counting of BOF. However, we cannot neglect such emergency withdrawals, since they not only reduce the initial RfB funds remaining, but also affect the regular withdrawals to be made in future periods, as illustrated in Fig. 2. Both graphs show the development of the initial RfB and the respective withdrawals from it over time for a single (arbitrarily chosen) capital market scenario with low interest rates. Both graphs show a reduction of regular withdrawals for policyholders' (ongoing) surplus participation starting in year 2017 to buffer the adverse capital market. However, on the right-hand-side also § 140 withdrawals are made to partly cover the losses incurred in years 2017–2022 such that the funds of the initial RfB are already exhausted at the end of year 2022. On the left-hand side, further regular withdrawals are made in later years. In consequence, the cash flows relevant for the calculation of SF add up to € 7.0 m on the left-hand side, but only to € 2.4 m on the right-hand side.

Second, shareholders' cash flows (and therefore BOF) are influenced by the amount of RfB funds even without taking into account SF. Until withdrawn for policyholders' surplus participation, RfB funds are invested in the capital market and contribute to the investment surplus, which is again split between shareholders and policyholders according to the rules prescribed in the MindZV (cf. eq. 2). This implies that RfB funds create shareholders' cash flows which are already considered in the BOF in a preliminary calculation without



Notes: The primary vertical axis shows the development of TBF and free RfB. The secondary vertical axis shows withdrawals and bonuses.

Fig. 2 Development of the initial RfB (in € million (m))

allowance for SF. Regarding policyholders' FDB, the present value of future benefit payments due to initial RfB funds in a risk-neutral valuation setting is smaller than the nominal value since it is reduced by the respective shareholder part of future investment surplus. In consequence, allowing for the nominal value of the undeclared RfB to be counted as SF would cause a double counting of shareholder cash flows.

Third, it has to be considered that until paid out, surplus allocated to policyholders' accounts contributes to the investment income in subsequent years which is also shared between shareholders and policyholders. However, it also may create additional future obligations for the company, e. g. in form of guaranteed interest to be paid on previously allocated bonuses. Both aspects have to be taken into account in the calculation of SF. Technically speaking, this means that accumulation and discounting factors to be applied for the funds of the initial RfB are not the same. In consequence, we do not use the withdrawals from the RfB to determine the economic value of SF, but the future benefit payments resulting from those withdrawals.[6]

Fig. 3 illustrates this third aspect of the double-counting. It shows the benefit payments resulting from the regular withdrawals presented in the left-hand graph of Fig. 2. Compared to the withdrawals, the actual benefit payments are delayed by several years, which means a higher discounting for the valuation of SF. At the same time, due to the interest earned on the allocated bonuses (policyholder share on future investment surplus), the benefit payments exceed the respective withdrawals. In total, the benefit payments add up to € 7.5 m (whereas only € 7.0 m are withdrawn from the RfB). Taking into account the discounting, we end up with SF of € 5.9 m based on actual benefit payments compared to € 6.0 m based on withdrawals.

Based on the reasoning described above, the economic value of SF is based on the present value of benefit payments

$$L_t^{SF} = L_t^{SF,acc} + L_t^{SF,term},$$

resulting from the relevant RfB funds initially available. Hereby,

[6] Note that BaFin allows for a calculation based on those withdrawals as an approximation for the amount of SF if certain requirements are fulfilled.

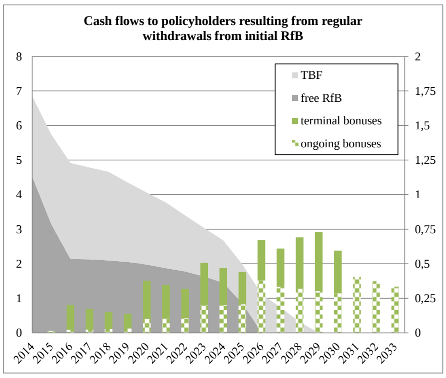


Fig. 3 Benefit payments due to initial RfB funds (in € m)

- $L_t^{SF,acc}$ denotes the benefit payments resulting from ongoing bonuses accumulated in the policyholders' bonus reserve *BR* and
- $L_t^{SF,term}$ denotes the benefits resulting from terminal bonuses accumulated in the TBF.

Ongoing bonuses withdrawn from the RfB and allocated to policyholders' bonus accounts remain within the company until the event of a claim, in the meantime earning the guaranteed interest (cf. eq. 1). So, when accounting for the respective benefit payments from the initial RfB also the amount of interest policyholders receive on those bonuses have to be taken into account. Therefore, the benefits ${}^{(k)}L_t^{SF,acc}$ to be paid to a policyholder of cohort (k) leaving the company at time t that result from the initial RfB can be calculated as

$$ {}^{(k)}L_t^{SF,acc} = \sum_{s=1}^{t} \left[q_s^{SF,fRfB} \cdot {}^{(k)}bon_s^{acc} \cdot \prod_{j=s+1}^{t} \left(1 + {}^{(k)}i \cdot q_j^{SF,fRfB} + {}^{(k)}i_j^* \cdot (1 - q_j^{SF,fRfB}) \right) \right]. $$

As long as ongoing bonuses (partly) come from the initial RfB ($q_j^{SF,fRfB} > 0$, cf. eq. 4), the amount ${}^{(k)}bon_j^{acc}$ allocated in the current year j also includes the bonus rate (${}^{(k)}i_j^* - {}^{(k)}i$) on bonuses allocated in previous years $s < j$ such that only the guaranteed interest ${}^{(k)}i$ on previously allocated bonuses has to be considered separately. Only if the funds of the initial free RfB are exhausted ($q_j^{SF,fRfB} = 0$), we have to explicitly take into account the entire investment return ${}^{(k)}i_j^*$ of the current year (including the guaranteed interest) policyholders receive on previously allocated bonuses. Again, this differentiation is necessary to avoid double counting.

The share of terminal bonus payments to a policyholder of cohort (k) leaving the company at time $t+1$ that refer to TBF^{SF} equals

$$ {}^{(k)}L_{t+1}^{SF,term} = {}^{(k)}L_{t+1}^{term} \cdot q_t^{SF,TBF} \ , \ t > 0 $$

with $q_t^{SF,TBF}$ representing the share of terminal bonus payments of year t originating from the initial TBF. It is given by

$$ q_t^{SF,TBF} = \min \left(1, \frac{\max \left(TBF_{t-1}^{SF} - CF_t^{\S 140,TBF}, 0 \right) + bon_t^{term}}{trans_t^{term}} \right). $$

So, depending on several parameters, the economic value of SF will differ from the nominal value. An allowance for the nominal value to be accounted as SF is not justified, but a valuation based on actual benefit payments to policyholders is required. However, note that the total value of SF is limited to the nominal value of the relevant part of the initial RfB.

4.3 Economic balance sheet under consideration of Surplus Funds

Based on the stochastic projection of the company until complete run off of the initial business in-force, we apply a risk neutral valuation to derive the economic balance sheet at time $t = 0$.

The left-hand side of the balance sheet contains the market value of the asset portfolio, i. e. $MV_0^A = MV_0^{stocks} + MV_0^{bonds}$, which is determined by the initial data of the financial market. Before consideration of RM and DT, the right-hand side contains the following items:

- the BOF, consisting of the expected present value of future cash flows to shareholders – decomposed into the Present Value of Future Profits (PVFP) and the initial amount of shareholders' equity Eq_0^{loc} – and the SF;
- the BEL, i. e. the expected present value of the insurer's future obligations towards policyholders. Note that the BEL has to be reduced by the value derived for SF.

Since the exact distribution of PVFP, SF and BEL cannot be determined analytically, we use Monte Carlo simulation to estimate the expected values. Based on J realizations of the stochastic capital market, the items are estimated by

$$\mathrm{PVFP} = \frac{1}{J}\sum_{j=1}^{J}\sum_{t=1}^{T}\frac{X_t^{[j]}}{B_t^{[j]}} - Eq_0^{loc}\,,$$

$$\mathrm{SF} = \min\left[fRfB_0 + TBF_0 - L_1^{term}, \frac{1}{J}\sum_{j=1}^{J}\sum_{t=1}^{T}\frac{L_t^{SF\,[j]}}{B_t^{[j]}}\right],$$

$$\mathrm{BEL} = \frac{1}{J}\sum_{j=1}^{J}\sum_{t=1}^{T}\left(\frac{AdC_{t-1}^{*} - P_{t-1}}{B_{t-1}^{[j]}} + \frac{L_t^{[j]}}{B_t^{[j]}}\right) - \mathrm{SF}\,,$$

where $(\cdot)^{[j]}$ denotes the respective values/cash flows in scenario j.[7] Note that the PVFP also includes the present value of expected § 140 withdrawals from the RfB (cf. eq. 3). Based on the fifo principle, this can be decomposed into withdrawals from initial and future RfB.

As mentioned in Sect. 2.2, for a proper analysis of the method used to reflect the risk sharing between policyholders and shareholders via the RfB under Solvency II, a holistic analysis in required, which also takes into account the impact on SCR, RM and DT. Therefore, the economic balance sheet must be set up under different stress scenarios. Details are presented in the next section.

5 Numerical results and discussion

This section analyzes the approach currently used in most of the German life insurance market to reflect the risk sharing between policyholders and shareholders via the RfB under

[7] Premiums and costs do not depend on the scenario.

Solvency II, in particular the allowance for SF. We perform a comparative analysis based on four natural alternatives for taking into account the risk reducing capacity of the RfB, varying both the allowance for SF and the explicit modeling of § 140 VAG (cf. Table 2). Although alt. 4 represents the current market standard (also applied in the BSM), some companies may use one of the other approaches as well.

Table 2 Alternatives for reflection of risk sharing between policyholders and shareholders (via the RfB)

Allowance for ...	**... Surplus Funds?**	
... § 140 VAG withdrawals?	No	Yes
No	Alternative 1	Alternative 2
Yes	Alternative 3	Alternative 4

First, we will show that the impact on the Solvency II capital significantly depends on how the risk sharing between policyholders and shareholders via the RfB but that all alternatives considered – especially alt. 4 – are internally consistent. Second, we discuss different assumptions and possible implications of the alternatives considered, with a focus on local statutory requirements.

5.1 Assumptions

5.1.1 Best Estimate assumptions

The valuation date is December 31, 2014. Except for the guaranteed interest rate ${}^{(k)}i$, which is chosen in accordance with the historic development in Germany (cf. Table 10 in the Appendix) and therefore depends on the year the contract was written, the insurance portfolio at the valuation date consists of life insurance contracts with identical contract parameters as given in Table 3.

Table 3 Insurance contract parameters

x	n	G	α	α^γ	β	mortality table
40	20 years	€ 20,000	4.0 %	0.1 %	4.0 %	DAV 2008 T[a]

[a] This is the German standard mortality table.

Since Solvency II requires a run-off valuation, we do not consider any new business and assume that in the past ${}^{(k)}l_k^* = 1{,}000$ contracts were sold at the beginning of each year $k+1$. The actual mortality rates are assumed to equal $q^* = 70\,\%$ of the first-order rates q_x.

At the valuation date $t = 0$, the portfolio contains 19 cohorts of policyholders with time to maturity from 1 to 19 years. Therefore, the time horizon for the projection is $T = 19$ years. The actuarial reserve AR_0 and the bonus reserve BR_0 as well as $fRfB_0$ and TBF_0 are derived from a projection in a deterministic scenario which is based on historic data from the German life insurance market concerning net investment return and cost parameters (cf. Table 10 in the Appendix).

The best estimate cost parameters AdC_t^* are assumed to be constant over time and chosen such that the administration cost rate coincides with the average of the German life insurance market of 2.2 % of the gross written premium income in 2014 (cf. GDV, 2015). No cost inflation is assumed for the best estimate.

At time $t = 0$, the book value of the asset portfolio coincides with the book value of liabilities (including shareholders' equity). We assume a stock ratio of $q = 10\,\%$ with unrealized gains on stocks of 25 % of the book value of stocks. The coupon bond portfolio at $t = 0$ consists of bonds with coupon derived from historic data where the time to maturity is equally split between 1 and $T_B = 12$ years.

The parameters for the management rules are shown in Table 4. The management rules are consistent with current regulation and practice in the German life insurance market.

Table 4 Parameters for management rules

q^{stock}	T_B	d	q_+^{UGL}	q_-^{UGL}	$T_{\S\,140}$	T_{bon}	q_{min}^{fRfB}	q_{max}^{fRfB}	q^{TBF}	q^{loc}
10 %	12 years	50 %	15 %	15 %	5 years	10 years	1.5 %	4.5 %	$\frac{1}{3}$	2.0 %

The financial market parameters for the projection are shown in Table 5. The parameters are adopted from Reuß et al. (2015). However adjustments were made for r_0 and θ to better fit the Solvency II interest rate term structure at the end of 2014.

Table 5 Financial market parameters

r_0	θ	κ	σ_r	σ_S	ρ
0.0 %	2.5 %	30.0 %	2.0 %	20.0 %	15.0 %

The stochastic projection is performed for 5,000 scenarios of the financial market. Further analysis showed that this allows for a precise estimation of the relevant figures (cf. Glasserman, 2010).

5.1.2 SCR stress assumptions

Fur the purpose of calculating the SCR, we take into account four different stresses for which we repeat the cash flow projection and valuation. Following the standard formula, the stressed parameters are (ceteris paribus) given as follows:

- Mortality stress: $q^*(\text{stress}) = q^* \cdot (1 + 15\,\%)$
- Life expense stress: $AdC_t^*(\text{stress}) = AdC_t^* \cdot 1.1 \cdot (1 + 1\,\%)^t$
- Equity stress: $MV_0^{stocks}(\text{stress}) = MV_0^{stocks} \cdot (1 - 39\,\%)$
- Interest rate down stress: $\theta(\text{stress}) = \theta - 0.5\,\%$ (as an approximation for the relative stress factors depending on maturity)

5.2 Impact on Solvency II balance sheet

Before presenting the numerical results, notice that for alt. 1 and 2 as well as for alt. 3 and 4 the results of each pair are based on the same cash flow projection. Before taking into account SF, we have the same PVFP and same BEL for those pairs since SF are just an Solvency II accounting item derived ex-post with no impact on projected cash flows.

5.2.1 Basic Own Funds

Fig. 4 shows the BOF before RM and DT. For alt. 1, BOF consist of shareholders' equity Eq_0^{loc} and PVFP.

In alt. 2, which allows for SF but not for any withdrawals from the RfB for the purpose of loss absorbency, the initial RfB leads to SF of € 6.5 m, increasing BOF by 158 % with a corresponding reduction of BEL. This valuation approach assumes that funds of the initial RfB may be completely used to cover losses in a worst case scenario. Hence, it does not consider any adjustment to allow for the (minimum) shareholder part of future losses.

In contrast, the present value of § 140 withdrawals calculated in alt. 3 is based on expected cash flows due to management rules that imply usage of RfB funds to cover losses under certain conditions. Hence, only RfB funds that are not used for policyholders' surplus participation increase the company's BOF with a respective decrease of BEL. Although those withdrawals are not limited to the initial RfB (which is the basis for SF) but can be applied to all RfB funds available during the projection, the increase of BOF for alt. 3 only amounts to € 2.5 m. Hence, most of the cash flows arising from the RfB are in fact reflected in the BEL. In particular, the portion relating to the initial RfB is only € 1.0 m, much less than SF in alt. 2.

Fig. 4 also shows that the impact of the § 140 withdrawals depends on the allowance for SF. If no allowance for SF is made (alt. 1 vs. 3), each § 140 withdrawal, no matter whether from initial or future RfB funds, increases the company's BOF. In this case, cash flows which are used for policyholders' surplus participation in alt. 1 (and increase BEL), are used to reduce shareholders' losses in alt. 3 (increasing BOF). If we compare alt. 2 and 4, the impact of § 140 withdrawals on BOF is less significant. The allowance for § 140 withdrawals reduces SF, which represents one aspect of the double counting described in Sect. 4: Funds withdrawn from the initial RfB to reduce shareholders' losses are not included in the calculation of SF. In consequence, only § 140 withdrawals that do not come from the initial RfB significantly increase BOF. Fig. 4 shows that in expectation, initial RfB funds of about € 1.0 m are withdrawn to reduce the shareholders' losses, such that the increase of BOF between alt. 2 and 4 only amounts to € 1.5 m, which is mainly the result of § 140 withdrawals from future RfB funds.

Compared to alt. 2, the amount of additional BOF due to § 140 withdrawals from the initial RfB in alt. 4 is slightly higher than the respective reduction of SF . This is mainly due to the fact that the § 140 withdrawals result in a respective cash flow to shareholders right away, whereas regular withdrawals for policyholders' surplus participation lead to benefit payments mostly in later years, which implies a higher discounting of such cash flows for the calculation of SF.

This analysis shows SF have a major impact on BOF. The impact of § 140 withdrawals is significantly smaller but still material. Overall, the highest BOF are observed for the combination in alt. 4.

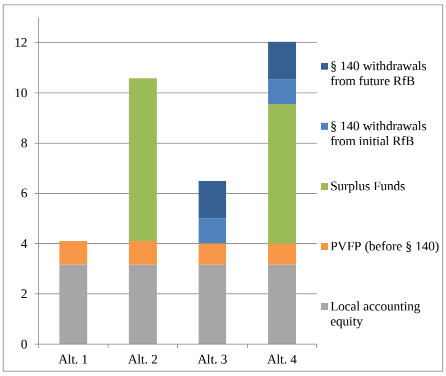


Fig. 4 Basic Own Funds before Risk Margin and deferred taxes (in € m)

5.2.2 Solvency Capital Requirements

The calculation of SCR itself is based on BOF and therefore, significantly affected by the BEL. The SCR modules in the standard formula are based on prespecified stresses that are assumed to occur at the valuation date. The BEL after stress is determined by applying the same valuation model but using stressed assumptions; the BOF are recalculated accordingly (based on the assumption that the Risk Margin does not change). The change of the BOF due to a certain risk (stress) represents the SCR:

$$\mathrm{SCR} = \max\left(\mathrm{BOF} - \mathrm{BOF}^{(\mathrm{stress})}; 0\right).$$

In our setting, BOF include SF and § 140 withdrawals, which implies that they have to be recalculated accordingly.

The amount of SF may vary between the Best Estimate and the stresses considered in the SCR. Adverse events cause different management decisions than assumed in the Best Estimate case which also influences the surplus participation process. E. g. policyholders' surplus participation may be reduced causing a delayed payout of RfB funds for policyholders' surplus participation. Hence, policyholders' and shareholders' present value of cash flows due to the initial RfB funds changes with the stresses considered. Without the allowance for SF, the delayed benefit payments to policyholders would reduce FDB in the stress and in consequence reduce the SCR. However, when allowing for SF those benefit payments (i. e. the loss absorbency of the initial RfB) is completely reflected in the BOF and in consequence, the insurer's option to delay those benefit payments should not be considered in the SCR. Consequently, to avoid double counting and to preserve consistency concerning the SCR, BaFin, 2015 requires SF to be recalculated for each stress. Otherwise the stress scenario projections would only include the changes in the shareholders' PVFP (e. g. due to changed investment returns on the RfB funds considered).

For alt. 2, Fig. 5 shows a slight increase of total SCR compared to alt. 1. This is due to the change of the economic value of SF in the respective stresses compared to the Best Estimate, which is a combination of several effects.

First, the benefit payments resulting from the initial RfB are delayed. All of the initial stresses considered reduce the amount of profits generated in future periods compared to the Best Estimate and due to that less funds are transferred to the RfB. However, the amount of funds withdrawn from the undeclared RfB for policyholders' surplus participation depends on the inflows to the RfB of past years (cf. parameters T_{bon} and q_{min}^{fRfB}). The less funds were transferred to the RfB in the past, the less surplus is distributed to policyholders in following years. Hence, the benefit payments resulting from the initial RfB and the resulting reduction of the initial RfB over time are delayed in the stress scenarios.

Second, the market stresses reduce the company's investment income. Hence, policyholders may receive lower interest on their accounts, especially on previously allocated bonuses due to initial RfB funds. In combination, we observe a reduction of benefit payments to policyholders relevant for calculation of SF combined with a greater discounting due to a delayed payment pattern, which in total leads to a decrease in the value of SF for the mortality, expense and equity stress.

Third, in case of the interest rate stress, the decrease of discount factors used for the economic valuation of cash flows dominates the other effects described above and causes an increase of SF. In total, the increase in the company's life and equity risk SCR for alt. 2 is almost leveled by the decrease of SCR for the interest rate down stress.

Alt. 3 shows the smallest SCR of all alternatives considered. Compared to alt. 1, additional losses in the stress scenarios can partly be covered by additional § 140 withdrawals from the RfB, in particular in the equity stress. In total, SCR is reduced by about € 1.6 m.

For alt. 4, higher stand-alone SCR for all four stress scenarios can be observed compared to alt. 3. The total SCR increases by € 1.7 m, mostly caused by higher equity risk. In particular, SF partly neutralize the risk reducing effect of the RfB from additional § 140 withdrawals. § 140 withdrawals from the initial RfB, which on the one hand reduce shareholders' losses, are on the other hand leveled by an according decrease of SF. This is necessary to avoid double counting since initial RfB funds used for additional § 140 withdrawals in the stresses are already reflected via additional BOF. Therefore, it is economically appropriate that the risk reducing impact of those additional withdrawals from the initial RfB is not reflected in the SCR of alt. 4.

Compared to alt. 2, the total SCR in alt. 4 is slightly increased, although the allowance for § 140 VAG means that in addition to SF, future RfB funds may be used to cover additional losses. However, the impact of the additional allowance for § 140 withdrawals on the SCR differs, depending on the stress scenario. The increase of SCR for the mortality and equity stress can be explained by the fact that in those stresses more § 140 withdrawals from the initial RfB are observed and less § 140 withdrawals from future RfB funds (compared to the Best Estimate, cf. Table 6). Since only § 140 withdrawals from future RfB funds (not leading to SF anyway) increase BOF, a reduction of those withdrawals leads to a stronger decrease of BOF and a corresponding increase of the SCR.

Table 6 Increase of § 140 withdrawals due to stress compared to Best Estimate

(in € 1,000)	in total	from initial RfB	from future RfB
mortality	-2	47	-49
expenses	41	33	8
equity	1,162	1,646	-484
interest rate	693	268	425

Overall, allowance for SF increases SCR (alt. 2 and 4). In contrast, additional § 140 withdrawals in the stress scenarios reduce SCR (alt. 3). However, in combination with the al-

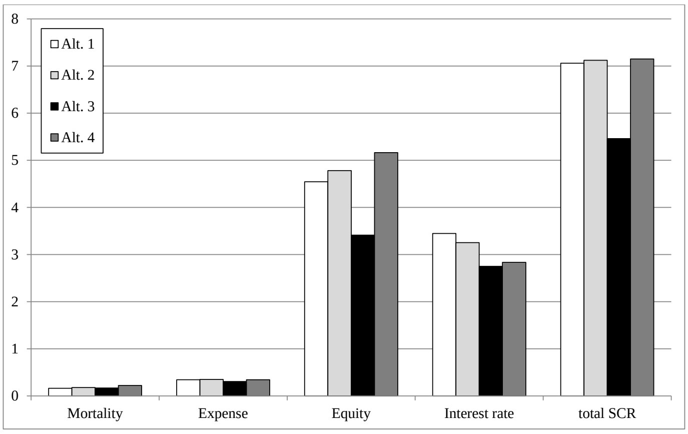


Fig. 5 Solvency Capital Requirements for individual risks (in € m)

lowance for SF (alt. 4), their risk reducing effect is lower since it is limited to withdrawals from future RfB funds.

5.2.3 Risk Margin

Previous sections have shown that the approach used to consider the risk sharing between policyholders and shareholders via the RfB affects the BEL of a company. To ensure that the Technical Provisions correspond to the transfer value, allowance needs to be made for a RM.

Using a cost-of-capital approach as stated in art. 37 of the *Solvency II Delegated Act*, the RM is given by:

$$\mathrm{RM} = \mathrm{CoC} \cdot \sum_{t \geq 0} \frac{\mathrm{SCR}^{RU}(t)}{(1 + r_0^{CE}(t+1))^{t+1}}\,, \tag{5}$$

with CoC $= 6\,\%$ being the Cost-of-Capital rate (cf. art. 39 of the *Solvency II Delegated Act*) and $\mathrm{SCR}^{RU}(t)$ denoting the reference undertaking's SCR after t years.

According to art. 38 of the *Solvency II Delegated Act*, the calculation of the RM is subject to the assumptions that before the transfer, the reference undertaking does not have any (re)insurance obligations or own funds. After the transfer, it raises eligible own funds equal to the necessary SCR^{RU}, i. e. such that the solvency ratio is $100\,\%$. Although it does not write any new business, its future management actions are consistent with the assumed future management actions of the original undertaking. Its assets amount to the sum of SCR^{RU} and BEL.

In particular, the reference undertaking does not receive any assets covering own funds resulting from SF since they are the property of the original entity (cf. *Solvency II Directive*, recital 50). Furthermore, the reference undertaking cannot expect any future profits after the transfer of the insurance obligations, since the assets covering the expected shareholder cash flows resulting from the insurance portfolio remain with the original company. Those cash flows include future profits as well as the expected § 140 withdrawals, but also the shareholder's investment income on the initial RfB (cf. eq. 3). However note that despite the assumption of $100\,\%$ participation of policyholders on future profits, it is also assumed that the reference undertaking can make use of § 140 VAG to cover additional risks (SCR) to the same extent as the original company.

First, we discuss what is the adequate SCR^{RU} to be used for the calculation of the RM, depending on the alternative considered. In this regard, we analyze the appropriateness of the assumption of consistent future management actions between the reference and the original undertaking with respect to the the risk reducing character of the RfB.

Alt. 1 does not consider the risk reducing character of the RfB at all, neither due to initial nor future RfB funds. It is assumed that those funds are used for policyholders' surplus participation only. Therefore, BEL includes all benefits expected to be made to policyholders due to future surplus participation and in case of a transfer of the obligations, the reference undertaking also receives assets covering those additional (non-guaranteed) benefits. However, since the calculation of the RM assumes the reference undertaking to adopt future management actions of the original undertaking, we may not assume that the overtaking company will withdraw any funds from the RfB in line with § 140 VAG. In consequence, the SCR of the reference undertaking should not contain any reduction due to additional § 140 withdrawals from the RfB in a stress scenario and in conclusion, the SCR of the original undertaking is also appropriate for the calculation of the RM.

Under alt. 2, the BEL does not contain the present value of benefit payments resulting from the initial RfB. As we have seen, compared to alt. 1, the allowance for SF also affects the company's SCR due to the missing option to reduce the insurer's risk by delaying the payments to policyholders resulting from the initial RfB. A reference undertaking does not have this possibility either since it does not receive any assets backing SF. Therefore, the reference undertaking also lacks the possibility of risk reduction via the initial RfB which means that also for alt. 2 the SCR derived for the original undertaking is appropriate for the calculation of the RM.

In alt. 3, the BEL is reduced by the § 140 withdrawals expected to be made under Best Estimate assumptions. Thereby, it does not matter whether those are withdrawals from the initial or future RfB. In consequence, a reference undertaking may use the option to increase withdrawals in stress scenarios, therefore reducing its risk in the same way the original undertaking is able to. In conclusion, the SCR of the original undertaking, which reflects this risk reducing option, is also appropriate for the calculation of the RM.

Finally, the BEL of alt. 4 contains the benefit payments expected to be made due to policyholders' future surplus participation under Best Estimate assumptions, but not the present value of cash flows resulting from the initial RfB. The latter is part of the company's BOF, either via SF or via the present value of § 140 withdrawals expected to be made under Best Estimate assumptions. Therefore, it is not included in the assets a reference undertaking would receive. In contrast to alt. 2, the BEL is further reduced by withdrawals of future RfB funds in line with § 140 VAG, expected to be made under Best Estimate assumptions. However, the BEL still contains policyholders' future surplus which is only expected to be used for loss coverage in line with § 140 VAG in the stress scenarios. In consequence, a reference undertaking can reduce its risk by increasing the § 140 withdrawals from future RfB but not from the initial RfB. Hence, the risk reducing effect of those increased withdrawals should be reflected in the SCR and the SCR of the original undertaking can be used for calculation of the RM.

Formula (5) implies that the calculation of the RM requires a projection of the reference undertaking's SCR. However, art. 58 of the *Solvency II Delegated Act* allows for a simplified calculation of the RM, which we also apply in this paper. In particular, we assume the SCR over time to be proportional to the run-off pattern of the insurance obligations in the

Table 7 Risk Margin

(in € 1,000)	Alternative 1	Alternative 2	Alternative 3	Alternative 4
SCR^{life}	418	433	387	455
Duration factor	68 %	68 %	68 %	68 %
RM	287	297	265	312

Certainty Equivalent (CE) scenario. Hence,

$$\text{RM} = \text{CoC} \cdot \text{SCR}^{life} \cdot \underbrace{\sum_{t=0}^{T} \frac{1}{(1+r_0^{CE}(t+1))^{t+1}} \cdot \frac{\text{BEL}_t^{CE}}{\text{BEL}_0^{CE}}}_{:=\text{Duration factor}},$$

with BEL_t^{CE} being the Best Estimate of Liabilities at time t derived in the CE scenario.

According to art. 38 of the *Solvency II Delegated Act*, the calculation of the RM assumes that the reference undertaking's assets are selected in such a way that they minimize the required regulatory capital for market risk. In particular, we assume that the reference undertaking is able to choose its assets in such a way that market risk can be completely eliminated. We also ignore SCR for operational risks. Hence, we only take into account the reference undertaking's life underwriting risk, i. e. SCR^{life}, which we derive from the SCR for mortality and life-expense risk.

Following the simplified calculation method, we have two main factors, the relevant SCR at time $t = 0$ and a duration factor. Of course, the SCR to be used for the calculation varies between the alternatives considered. The duration factor is given by run-off pattern of the BEL under the CE scenario before deduction of SF.[8] Hence, it depends on the allowance for § 140 withdrawals and may differ between alt. 1/2 and alt. 3/4. Note, that in our case, the duration factor is the same for all four alternatives, since no § 140 withdrawals are expected for the CE scenario. Hence, compared to alt. 1, the RM in alt. 3 is decreased, whereas it is increased in alt. 2 and 4.

Overall, a change of SCR for unhedgebale risks is counted twice: in the SCR and partially in the RM (which reduces BOF).

5.2.4 Deferred taxes

Keeping in mind the motivation behind SF, the cash flows recognized as SF can be expected to either be used for policyholders' future surplus participation or to cover unexpected losses in a worst case scenario. Hence, even though FDB and Technical Provisions are reduced, the allowance for SF does not imply that additional (taxable) profits emerge from these funds. In consequence, when deriving deferred tax assets/liabilities based on the valuation differences between the Solvency II and the tax balance sheet, Technical Provisions before deduction of SF and the initial undeclared RfB, respectively should be used.

Based on a tax rate of $q^{tax} = 33\,\%$, Table 8 shows the net deferred tax liabilities (DTL) derived from the valuation differences regarding assets and liabilities, applying the valuation approach described above. Thereby, we assume that the tax balance sheet coincides with the

[8] Note that our approach to use the BEL before deduction of SF is in line with the calculation of the duration factor in the BSM. We are aware of the fact that the choice of drivers for SCR projection is an important aspect in the calculation of the RM. However, an in-depth analysis is beyond the scope of this paper and is left for future research.

statutory balance sheet. First, we can see a significant increase of DTL due to the explicit modeling of § 140 withdrawals in alt. 3 and 4. Second, the allowance for SF has no material impact on tax calculations (cf. alt. 1/2 and alt. 3/4). Further note that for all four alternatives, the value derived for DTL also equals the SCR adjustment for the loss-absorbency of deferred taxes. Thereby, we assume that the adjustment cannot exceed the DTL before stress.

Table 8 Calculation of deferred taxes

(in € 1,000)	Stat. acc.	Alternative 1	Alternative 2	Alternative 3	Alternative 4
Assets	174,534	196,440	196,440	196,440	196,440
Technical Provisions[a]	164,526	192,654	186,192	190,235	184,742
SF / initial undecl. RfB	6,842	-	6,473	-	5,540
Net diff. to stat. acc.		620	609	3,039	2,991
(BaFin approach)		(-6,222)	(240)	(-3,803)	(1,689)
DTL		205	201	1,003	987
(BaFin approach)		(-2,053)	(79)	(-1,255)	(557)

[a] It is the value after deduction of SF / initial undeclared RfB.

Note that in its interpretative decisions, BaFin prescribes that for valuation of deferred taxes, Technical Provisions after deduction of SF and the initial undeclared RfB, respectively should be used (cf. BaFin, 2016). Since by definition the SF never exceeds the initial undeclared RfB, this implies additional valuation differences and in consequence additional taxable losses for shareholders with a respective increase/decrease of deferred tax assets/liabilities (cf. BaFin approach in Table 8).

5.2.5 Solvency ratio under Solvency II

The allowance for risk sharing between policyholders and shareholders via the RfB has a significant impact on the solvency ratio under Solvency II. The allowance for SF in alt. 2 and 4 creates a large amount of additional BOF. At the same time, it also increases SCR and RM. In contrast, an economic valuation based on § 140 withdrawals in alt. 3 leads to a significant reduction of SCR and RM, but also to a smaller increase of BOF compared to the allowance for SF. Further, the allowance for SF in alt. 4 reduces the positive impact of § 140 withdrawals on BOF, SCR and RM. In addition, the explicit modeling of § 140 withdrawals in alt. 3 and 4 increases DTL.

Overall, alt. 4 leads to the highest amount of Excess Capital (ExC), i. e. the BOF not required to cover the SCR, as shown in Table 9. Due to the positive effect of § 140 withdrawals on SCR and RM, the difference in the resulting ExC between alt. 2/4 and alt. 3 is less pronounced than the difference of BOF/BEL. However, solvency ratio differ significantly.

5.3 Reflection of local statutory requirements

So far, our analysis mainly focused on the Solvency II valuation perspective. However, local statutory requirements including both, accounting rules and other regulatory constraints for participating business remain in place. In consequence, they represent a binding condition for a life insurer and need to be properly reflected in the valuation under Solvency II.

Table 9 Summary of Solvency II key figures

(in € 1,000)	Alternative 1	Alternative 2	Alternative 3	Alternative 4
BOF	3,614	10,080	5,226	10,734
– ExC	-3,240	3,160	770	4,571
– SCR	6,854	6,920	4,456	6,163
Technical Provisions	192,654	186,192	190,235	184,742
– BEL	192,368	185,895	189,970	184,430
– RM	286	297	265	312
DTL	205	201	1,003	987
Solvency ratio	53%	146%	117%	174%

5.3.1 Loss absorbency of the initial RfB

The previous section showed that the overall solvency ratio significantly depends on the way the risk sharing between policyholders and shareholders is recognized in the valuation. As discussed in Sect. 2.2, SF are an exception to the market consistent valuation approach of Solvency II based on expected cash flows. Assuming that an insurance company may use all RfB funds available at the valuation date for loss coverage instead of policyholders' surplus participation, SF are supposed to value the option an insurer has for a worst case scenario to ensure that he can fulfill his obligations against policyholders. Consistently, SF are fully recognized as part of the BOF. More precisely, art. 96 of the *Solvency II Directive* explicitly classifies SF as Tier 1 capital without any restrictions. Hence, in principle there is no need to check SF against the Tier 1 criteria laid down in art. 93 of the *Solvency II Directive*. However, we analyze whether SF substantially possess the Tier 1 characteristics to see if SF represent an exception from the Tier 1 classification as defined in art. 94 of the *Solvency II Directive*. Further we show implications and possible drawbacks of such a recognition. Focusing on Germany, we take into account local accounting rules and other regulatory constraints.

Absence of incentives to redeem, mandatory servicing costs and encumbrances The undeclared RfB does not have any fixed costs. As stated before, although primarily reserved for policyholders' future surplus participation, policyholders may not specify any claim on the funds, especially concerning any interest payment. In addition, they have no right to redeem the nominal value.

Sufficient duration In principle, the undeclared RfB is not dated and available for loss coverage since it is not before the binding declaration that the insurer is deprived of the right to use those funds in case of emergency. Therefore, the use of those funds is subject to the discretion of the undertaking's managing board. However, although policyholders may not specify any claim on the funds of the undeclared RfB, the insurer must ensure their appropriate use (§ 140 (2) VAG). In addition, the legal constraints presented in Sect. 2.1 have to be considered which prescribe a maximum size of the undeclared RfB and basically limit its availability to three years. However, all these restrictions apply in a going-concern perspective including new business which is in contrast to the run-off projection required for Solvency II.

Hence, the *"sufficient duration"* of the initial RfB is questionable and the actual loss-absorbing capacity of the initial RfB depends on the impact of the stress scenario on the Profit and Loss Statement (P&L) based on local accounting rules. To illustrate different effects of stress scenarios, Fig. 6 shows the funds to be withdrawn from the RfB for loss

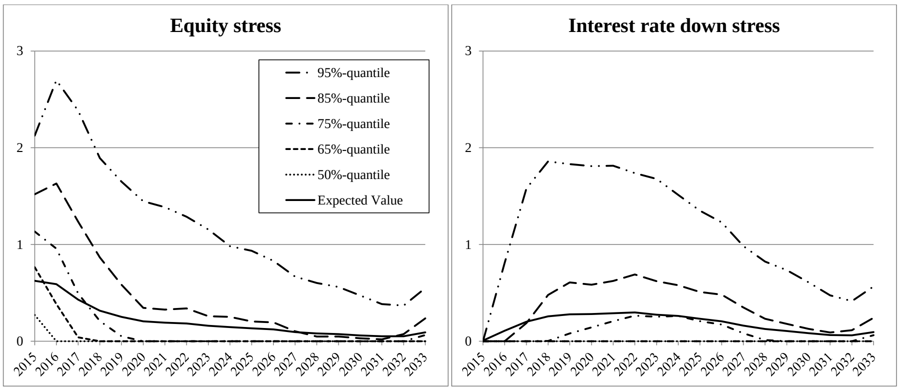


Fig. 6 § 140 withdrawals for equity and interest rate down stress (in € m)

coverage in the equity and interest rate down stress scenario. As expected, the equity stress leads to withdrawals right away, especially, since management rules only allow for unrealized losses on stocks up to $\underline{q}^{UGL} = 15\,\%$. The exceeding part requires a write-off which leads to a respective loss in the P&L. In contrast, the negative impact of a interest rate down stress on the P&L mainly occurs in later years. In consequence, to a great extent § 140 withdrawals only become necessary in the interest rate stress scenario when the initial RfB funds may already be depleted.

Permanent availability and subordination Surplus participation (based on the undeclared RfB) is generally subordinated to withdrawals to be made in order to guarantee fulfillment of the obligations against policyholders. More specific, although primarily reserved for policyholders' future surplus participation, § 140 VAG generally allows those funds to be used for loss coverage in case of adverse events, but also states that the application of this emergency measure is subject to approval by the local regulator (BaFin). Also the GDV stresses this fact in a newsletter concerning the application of regulatory requirements during the 2008 financial crisis (cf. GDV, 2008). According to that, unpredictable losses due to this global crisis, which BaFin agrees to represent a general change of economic circumstances, do not imply any automatism concerning the approval for emergency withdrawals from the RfB. Instead, each case has to be considered individually.

Even if the approval in such a worst case scenario is taken for granted, we are still confronted with the question how losses would be shared between shareholders (by injection of additional equity) and policyholders (by withdrawals from the RfB). Statutory provision do not specify what proportion may be withdrawn in such a situation and it is most questionable, whether regulators will allow the insurer to completely use the available funds of the undeclared RfB without shareholders taking a certain share in the rescue of the company. A reasonable estimation is that losses are split in the same proportion as profits were shared in the past (cf. parameter $T_{\S\,140}$ in our model). Not only the management rules for the application of § 140 VAG implemented in the BSM follow this recommendation, but also the working group of the German Actuary Association (DAV) in charge for the Market Consistent Embedded Value (MCEV) suggests it (cf. DAV, 2011). Furthermore, the DAV paper specifies that before allowance for § 140 withdrawals other options like the realization of unrealized gains on the company's assets have to be checked. Hence, the assumption that SF

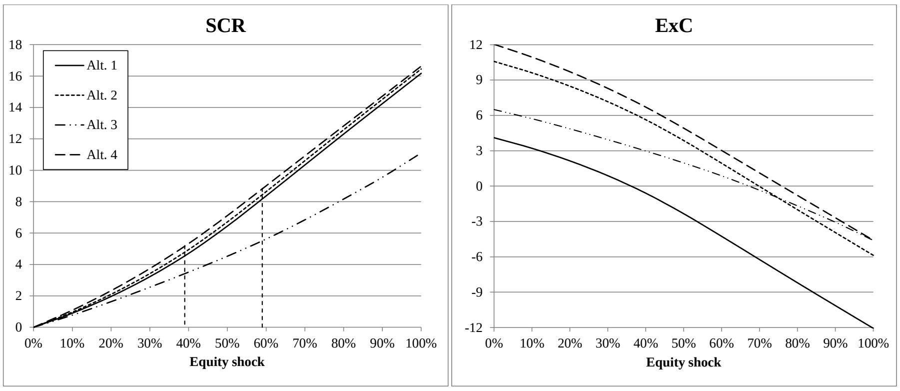


Fig. 7 Sensitivity concerning equity stress (in € m)

"can be called up on demand, to fully absorb losses" (*Solvency II Directive*, art. 93) may be not justified under all circumstances.

In contrast, the actual risk reducing capacity clearly depends on the amount of losses incurred. The higher the losses the more RfB funds may be withdrawn for loss coverage. Eventually, all initial RfB funds may be used which sets the upper limit for the loss-absorbing capacity. Hence, SF to be fully recognized as BOF without any adjustment for these restrictions tends to overestimate the solvency ratio.

Fig. 7 illustrates this effect for the equity risk, in which we increase the size of the equity stress from $0-100\,\%$ (causing a greater loss of the initial market value of stocks). The left-hand side contains the SCR for equity risk and the right-hand side shows the resulting ExC for the equity stress. Since only the stress projection is concerned, BOF do not change but the reduction in ExC is caused by an increase of the SCR for equity risk. As long as RfB funds are not exhausted in alt. 3, more and more RfB funds are withdrawn to (partly) absorb additional losses caused by the equity stress. E. g. increasing the stress from 39 % to 59 %, the slope of the curves on the left-hand side of Fig. 7 show that for alt. 3 only 52 % of the additional loss is borne by the shareholders, resulting in a higher SCR (compared to more than 90 % for the other alternatives). There are several reasons why significantly less than 100 % of the additional market value loss is absorbed by additional withdrawals. First, losses are split between shareholders and policyholders as mentioned above. Second, the loss to be covered is derived on a book values basis following local accounting rules, which in general differs from the market value perspective. More specifically, the allowance for unrealized losses in our model spreads the market value loss over several years. In conclusion, *"subordination"* concerning the initial RfB can not be taken for granted in each stress scenario, but the impact of such a scenario on the P&L has to be considered.

A further drawback of the assumption of the full loss-absorbency of SF is that SCR and ExC, respectively become quite volatile compared to alt. 3. Our cash flow projections indicate that in expectation, funds of the initial RfB mostly lead to future benefit payments to policyholders, but are not used for loss coverage (even in stress scenarios). Hence, the amount of additional BOF due to the allowance for SF exceeds the combined impact of projected § 140 withdrawals on BOF (increase due to withdrawals under Best Estimate assumptions) and on SCR (decrease due to additional withdrawals in the stresses) in alt. 3. In consequence, higher ExC can be observed for alt. 4 and, up to an equity shock of 75 %, also for alt. 2. However, the solvency ratio of alt. 3 contains further emergency buffers which are not used (and not visible on the balance sheet) before management rules imply additional § 140 withdrawals. This is revealed in a reduced slope of the SCR curve for alt. 3 compared to the other three alternatives (cf. left-hand side of Fig. 7). Only if the equity shock approaches 100%, the RfB funds are more and more exhausted such that additional losses cannot (partly) be covered by additional § 140 withdrawals anymore which results in an increasing slope of the SCR curve for alt. 3. At the same time, the ExC curves of alt. 2 to 4 on the right-hand side converge. In particular, the combined impact on BOF (before RM) and SCR for equity risk is the same for alt. 3 and 4 since almost all funds of the initial RfB are used for loss coverage in the stress. In comparison, the ExC of alt. 2 remains slightly smaller since for this alternative future RfB funds cannot be counted in favor of shareholders.

Note that a similar behavior concerning the solvency ratio can be observed when varying the RfB funds initially available. As long as more RfB funds are available than expected to be required for loss coverage, the solvency ratio of alt. 3 is mostly independent of the initial RfB. In contrast, the amount of SF, and therefore the impact on the solvency ratio, varies with the initial RfB. So again, concerning the initial RfB, the solvency ratios in alt. 2 and 4 are rather volatile compared to alt. 3. In particular, in times of decreasing RfB funds solvency ratios may decrease rapidly for alt. 2 and 4 whereas they are more stable for alt. 3.

Overall, although SF represent funds available at the valuation date, a general classification of SF as Tier 1 capital without any adjustment for the aspects discussed above appears questionable.

5.3.2 Impact of statutory requirements on the Risk Margin

The previous section showed the importance of local statutory requirements for an appropriate recognition of the risk reducing capacity of the RfB under Solvency II. This section goes one step further, discussing implications of statutory requirements on the calculation of the RM. However, note that an in-depth analysis of the RM methodology is beyond the scope of this paper.

The transfer value of Technical Provisions is based on a market-consistent valuation of expected cash flows. However, local accounting rules may not follow this approach. Hence, although adequate for a market-consistent valuation based environment, the transfer scenario underlying the calculation of the RM may lead to problems concerning the local accounting perspective which also applies to the reference undertaking.

According to the RM provisions, the reference undertaking's assets amount to the sum of SCR^{RU} and BEL. In general, technical provisions derived for the statutory accounting balance sheet differ from Solvency II, which may lead to an over- or underfunding in the initial statutory accounting balance sheet of the reference undertaking. To balance the underfunding, the company may need to raise additional money, which in consequence may cause additional costs of capital, increasing the RM.

Further note that the surplus participation process and in particular the RfB is also based on local statutory requirements. In Germany, MindZV and VAG have to be considered in this matter. FDB are derived from policyholders' participation in annual investment, cost and risk surplus. The reference undertaking does not receive any assets covering BOF items of the original company, which on the one hand implies that for the reference undertaking, policyholders' share in surplus equals 100 %. On the other hand, the reference undertaking has a smaller amount of initial assets. Due to that, especially the annual investment return will differ from the original company. Hence, for the reference undertaking not to require additional funds, it must be allowed to use risk/cost surplus to balance losses in the investment result of certain years. Only if this is the case, it may be appropriate to assume that FDB of the reference undertaking and FDB of the original undertaking coincide.

However, the question may arise whether a separate calculation of the reference undertaking's BEL and the relevant SCR based on adjusted input data and assumptions is required for the purpose of deriving the RM. For example, such a calculation could be based on initial assets which amount to the BEL of the original undertaking and assume higher expenses due to the run-off. In consequence, the RM can be decomposed into the cost of capital for the regulatory capital (based on the adjusted SCR) and additional capital required due to difference between BEL of the original and the reference undertaking.

6 Conclusion

Collective bonus reserves are primarily reserved for policyholders' future surplus participation, but may be used to cover future losses. The resulting risk sharing between policyholders and shareholders should be adequately reflected in a risk-based solvency framework. A natural approach would be the explicit modeling of such emergency withdrawals in a stochastic valuation model.

However, Solvency II adopts a different approach: a special balance sheet item called SF is introduced and derived from the expected benefit payments to policyholders resulting from the collective buffer reserves available at the valuation date. SF are meant to reflect the insurer's option to use the available bonus reserves in case of emergency. Hence, SF do not have to be considered as insurance liabilities but can be recognized as part of the BOF. On the one hand, this constitutes an exception from the market consistent valuation approach of Solvency II which is based on expected cash flows to policyholders. On the other hand, it requires a consistent inclusion of SF into the underlying valuation framework. In particular, any double counting of own funds has to be avoided.

Based on the implementation of SF in Germany and Austria, this paper shows in a first step that a careful implementation of SF in a stochastic valuation model can ensure overall consistency of quantitative solvency requirements without any double counting. Thereby, consistency is not limited to the derivation of additional BOF, but also affects further components of the capital requirements under Solvency II, in particular SCR, RM and DT.

In a second step, we show that the choice of the method used to reflect the risk sharing between policyholders and shareholders has a significant impact on the Solvency II results. Again, the analysis cannot be limited to BOF but a holistic evaluation has to be performed, also taking into account the components mentioned above. In particular, we apply three different valuation approaches using the same stochastic valuation model.

- Alt. 2 just allows for the loss absorbency of the initial RfB via SF.
- Alt. 3 considers explicit management rules for the application of § 140 VAG, which implies withdrawals from initial and future RfB funds for loss coverage.
- Alt. 4 combines alt. 2 and 3, allowing for both, SF and withdrawals to cover losses. It is the method currently used in most valuation models in Germany and Austria.

The allowance for SF in alt. 2 and 4 creates a significant amount of additional BOF. At the same time, it also increases SCR and RM. In contrast, a purely economic valuation based on § 140 withdrawals in alt. 3 leads to a significant reduction of SCR and RM, but creates less additional BOF as the allowance for SF. Furthermore, the allowance for SF in alt. 4 reduces the positive impact of § 140 withdrawals on BOF, SCR and RM. In addition, the explicit modeling of § 140 withdrawals in alt. 3 and 4 increases DTL. In combination, alt. 4 leads to the highest amount of ExC.

Since the results presented are based on a simplified projection model they are of illustrative nature. However, our model covers the main characteristics of the German market regarding risk sharing between policyholders and shareholders via collective bonus reserves. In particular, we consider statutory requirements as well as typical management rules. Hence, although explicit numbers might change, we conclude that risk sharing between policyholders and shareholders via collective bonus reserves leads to a material improvement of the solvency ratio for life insurance companies in a risk-based solvency framework. However, the impact significantly depends on the approach used to reflect the risk sharing.

In a last step, we further analyze and also challenge the appropriateness of the SF methodology in the light of local statutory requirements. In this regard, a crucial point is the implicit assumption that in a SCR event the entire initial RfB is used to cover the loss and shareholder funds are only required if the loss exceeds the initial RfB. This is not in line with typical interpretations of statutory requirements which imply that shareholders always have to cover some portion of a loss. Furthermore, the usage of initial RfB funds for loss coverage is clearly linked to losses incurred under local accounting rules. In an economic stress scenario, it may take several years until losses occur under local accounting rules. Since there are time limits on the usage of RfB funds for surplus participation, all initial RfB funds may have been used for surplus participation already before a loss actually occurs under local accounting rules.

Overall, current Solvency II requirements regarding SF and RM do not reflect all relevant statutory requirements and the classification of SF as Tier 1 capital appears questionable. Hence, we conclude that the SF concept tends to overrate the loss absorbency of the initial RfB and the solvency ratio, respectively. Moreover, compared to the explicit modeling of emergency withdrawals, the allowance for SF leads to more volatile solvency ratios.

In conclusion, our analysis shows that the allowance for SF appears internally consistent but that the current valuation approach needs further refinements to better reflect the risk sharing between policyholders and shareholders. Given the material impact of SF on solvency ratios, a critical review of the corresponding valuation methodology is recommended.

A Appendix

Table 10 Historic data from the German life insurance market used for deriving the portfolio of insurance contracts

Year	Guaranteed interest rate[c]	Net investment return[c]	Administration cost rate[a,c]	Acquisition cost rate[b,c]
1990	3.50 %	6.78 %	5.40 %	-
1991	3.50 %	7.44 %	5.16 %	5.50 %
1992	3.50 %	7.39 %	4.92 %	5.50 %
1993	3.50 %	7.59 %	4.68 %	5.50 %
1994	3.50 %	7.15 %	4.44 %	5.50 %
1995	4.00 %	7.37 %	4.20 %	5.50 %
1996	4.00 %	7.37 %	4.06 %	5.52 %
1997	4.00 %	7.46 %	3.92 %	5.54 %
1998	4.00 %	7.57 %	3.78 %	5.56 %
1999	4.00 %	7.58 %	3.64 %	5.58 %
2000	4.00 %	7.51 %	3.50 %	5.60 %
2001	3.25 %	6.12 %	3.44 %	5.60 %
2002	3.25 %	4.68 %	3.38 %	5.60 %
2003	3.25 %	5.05 %	3.32 %	5.60 %
2004	2.75 %	4.90 %	3.26 %	5.60 %
2005	2.75 %	5.18 %	3.20 %	5.60 %
2006	2.75 %	4.82 %	3.00 %	4.90 %
2007	2.25 %	4.65 %	2.90 %	5.20 %
2008	2.25 %	3.54 %	2.80 %	4.90 %
2009	2.25 %	4.18 %	2.70 %	5.20 %
2010	2.25 %	4.27 %	2.40 %	5.10 %
2011	2.25 %	4.13 %	2.40 %	5.00 %
2012	1.75 %	4.59 %	2.40 %	5.00 %
2013	1.75 %	4.68 %	2.30 %	5.10 %
2014	1.75 %	4.63 %	2.20 %	5.00 %

[a] The administration cost rate is given as a percentage of gross written premium income. No data is given for years 1991 to 1994; the missing values are derived by interpolation.

[b] The acquisition cost rate is given as a percentage of new business premium sum. No data is given for 1994 and previous years; we use the acquisition cost rate for 1995 also for these years.

[c] Data are taken from GDV (2015).

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