



Mathematisches Kolloquium

Ergodic Markov-Process as a generalized Interest-Rate-Model

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Nowadays there are two types of classical interest-rate-models used in practice: the Short-Rate- (Exponential Vasicek, Black-Karasinski, CIR++ ...) and Libor-Market-Models (LMM). Models of both types follow stochastic continuous-time differential equations and satisfy certain martingale properties. Both classes of models meet today a big problem because of their unstable converging property they have a too high ratio of ESG-trails with unrealistic too-high-yields and with unrealistic too-low-yields. The unstable converging property in all classical models is the consequence of only one matter: the high time dependence of the corresponding forward rate. Today we can fit the market swaption prices, if the forward rates have at least 98% annual time dependence. Remark that we find the same high dependence by historical governments bonds.

We have a lot of published models that are based on Markov functional valuations, e.g. Hughston's model with state-price density. But every tree-method based calibration model is in reality also a model with Markov-Process property, because we can always expand the states of the tree to the discrete Markov chain property. So the Chapman- Kolmogorov equation leads the risk-neutral property for every tree-method based calibration. But the realization of the stable convergence properties is essentially more complicated than the realization of the risk-neutral property. Furthermore, the Chapman-Kolmogorov equations alone do not guarantee the stable convergence property of the calibration process.

In this presentation, I will propose a new Markov-Process-Model (MPM), that is based on an ergodic Markov-Process. So the ergodic property automatically guarantees the optimal stable convergence property of the calibration process. We can then generalize the classical model and build the needed ergodic Markov-Process. This leads to a new interest rate model that is based on spherical-random-walk and has the optimal ergodic property.

The high time dependence of forward rates stochastically means, that every analytical calibration cannot be stable and cannot lead to good approximations. Because of high time dependence and asymmetrically distributed values, every analytical approximation is biased and the bias error increases rapidly with the number of time steps. I solve this numerical problem of high dependence by using the numerical step by step unbiased Monte Carlo valuations.

An example of the EURO market at Q4 2017 is presented. With this example of MPM calibration we can exclude all possible negative yields and at the same time MPM has no problem with unrealistic too-high-yields. I present also, that the model is able to match market option-prices and that the model has better risk-neutral and dynamic properties compared to other models. So the MPM is a model that solves the problem of high time dependence for the forward rate.