Experimental study of statistical structures and forces in granular matter

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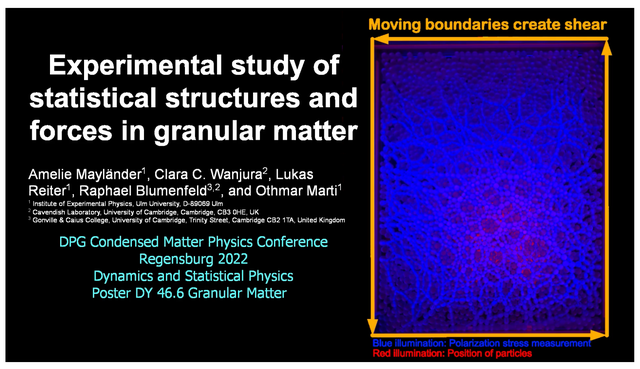
We investigated the structure-forces coevolution in rotational shear of a planar assembly of photo-elastic polyurethane-discs of four different sizes under constant confining stress [1,2]. Disc positions and contacts were determined using unpolarized red light. A dark-field polariscope, using circularly polarized blue light, detected mechanical deformations and the force network. The experiment ran through: a de-correlation step, initial state preparation, steady-state dynamics.
Repeated measurements of the structure and cell order distribution of the geometric contact network were carried out, validating theoretical predictions of detailed balance[3,4] and maximum entropy[5].
Simultaneously detected force chain networks had more cells of higher orders than the geometric network, providing less than maximum entropy. This is attributed to the sensitivity of force detection to low-force contacts. Characteristic differences also existed in the shapes of small cells.



Additional materials: Video showing the time evolution of forec chains
https://www.uni-ulm.de/fileadmin/website_uni_ulm/nawi.inst.100/DPG-Regensburg-DY46.6.mp4

Methods and Materials

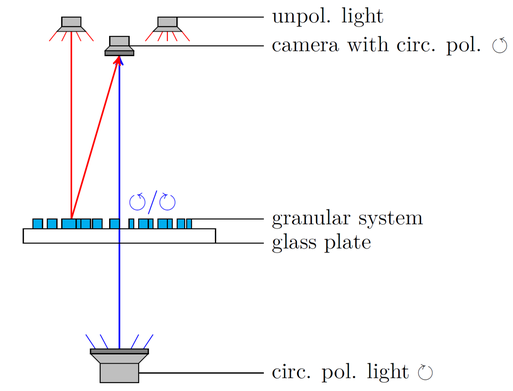


Fig. 1 The polariscope to measure the mechanical strain in the PU-cylinders [1]

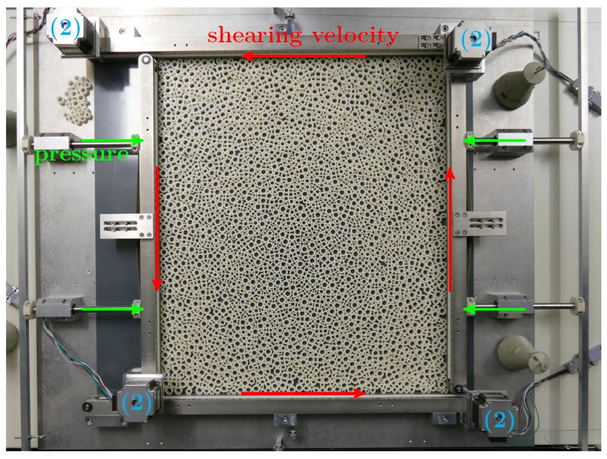


Fig. 2 The apparatus to excert a constant pressure while shearing with a constant (or variable) velocity [1]

- Cylinders:
  - Silicone mold: MoldStar™ 15 (Smooth-On Macungie, PA 18062, USA)
  - Particles: ClearFlex™ 50A (Smooth-On Macungie, PA 18062, USA)
  - Resulting friction coefficients [1]

	friction coefficient μ	force (mN)
base-particle (photoelastic, glass)	1.5±0.5	$F_R = 25\pm9$
base-particle (Teflon-coated photoelastic, glass)	0.270±0.029	$F_R = 4.5\pm0.5$
inter-particle (photoelastic)	1.46±0.14	$F_{ip} = 261.6$
base-particle (ABS cylinder, PVC)	0.170±0.024	$F_R = 1.718\pm0.025$
inter-particle (ABS cylinder)	0.43±0.07	$F_{ip} = 228.9$

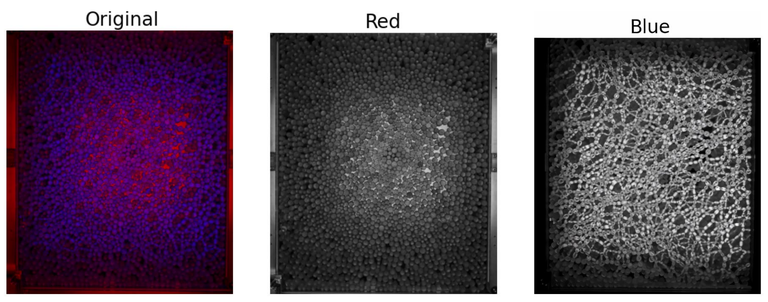


Fig. 4 Data extraction: Separation of geometry and force [2]

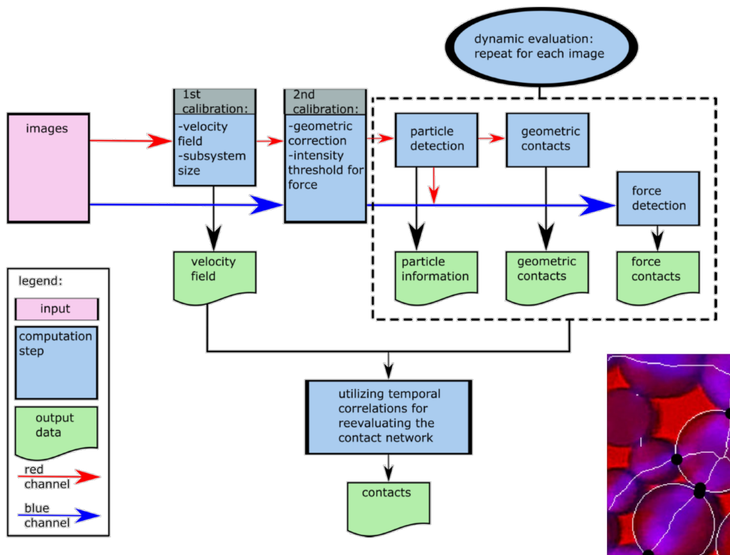


Fig. 6 Data analysis: Algorithm [1]

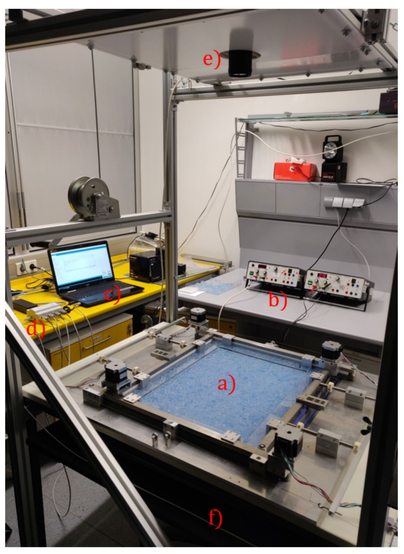


Fig. 3 The entire setup [2]

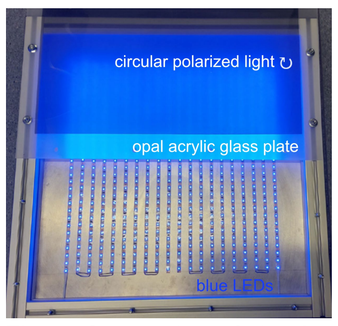


Fig. 5 Generation of circular polarized light [2]

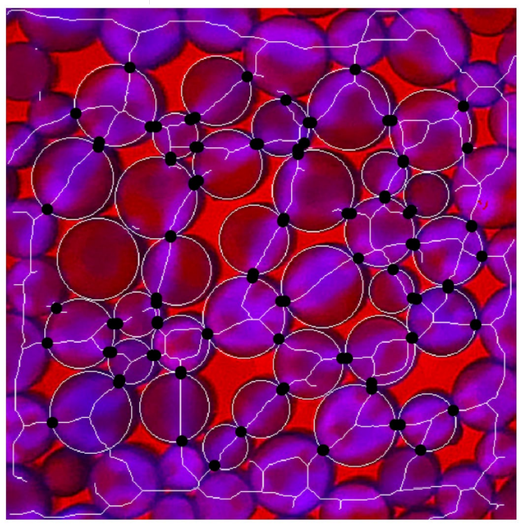
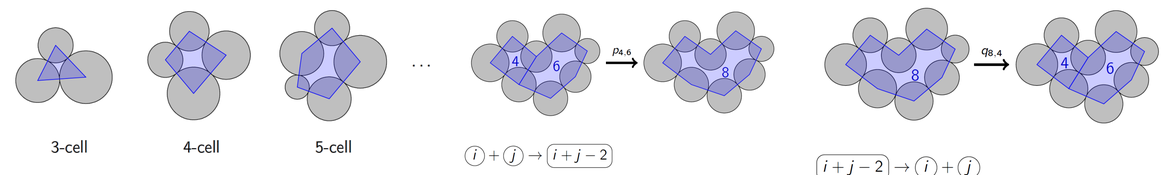


Fig. 7 Detected force networks an geometrical contacts [2]

Summary of the Theory (see [3-6])



- Number of k-cells: $\mathcal{N}_k$
- Total number of cells: $N_C = \sum_k \mathcal{N}_k$
- Cell order distribution: $Q_k \equiv \frac{\mathcal{N}_k}{N_C}$, normalised to 1

Fig. 9 Cell merging [3]

Fig. 10 Cell breaking [3]

Fig. 8 Cell order [3]

Rate equations: [3]

$$\dot{\mathcal{N}}_3 = -2\,N_C\,Q_3^2\,p_{3,3} + 2\,N_C\,Q_4\,q_{4,3}$$
$$\dot{\mathcal{N}}_4 = N_C\,Q_3^2\,p_{3,3} - N_C\,Q_4\,q_{4,3}$$

$$\dot{\mathcal{N}}_k = N_C \left\{ \sum_{i=3}^{\lfloor(\frac{k+2}{2})\rfloor} (Q_i Q_{k+2-i} p_{i,k+2-i} - Q_k q_{k,i}) - \sum_{i=3}^{k-1} (Q_i Q_k p_{i,k} - Q_{i+k-2} q_{i+k-2,i}) - \sum_{i=k}^{N-k+2} (Q_k Q_i p_{k,i} - Q_{i+k-2} q_{i+k-2,k})(1+\delta_{k,i}) \right\}$$

$$\dot{Q}_k = \sum_{i=3}^{\lfloor(\frac{k+2}{2})\rfloor} (Q_i Q_{k+2-i} p_{i,k+2-i} - Q_k q_{k,i}) - \sum_{i=3}^{k-1} (Q_i Q_k p_{i,k} - Q_{i+k-2} q_{i+k-2,i}) - \sum_{i=k}^{N-k+2} (Q_k Q_i p_{k,i} - Q_{i+k-2} q_{i+k-2,k})(1+\delta_{k,i}) + Q_k \sum_{\substack{\text{all possible} \\ \text{processes } i,j}} (Q_i Q_j p_{i,j} - Q_{i+j-2} q_{i+j-2,i})$$

$$\dot{Q}_k = \frac{\dot{\mathcal{N}}_k}{N_C} + Q_k \sum_{\substack{\text{all possible} \\ \text{processes } i,j}} (Q_i Q_j p_{i,j} - Q_{i+j-2} q_{i+j-2,i})$$

Some recent Results

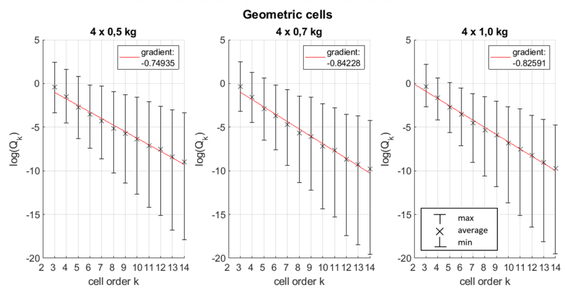


Fig. 11 Cell order distribution for geometric cells [2]

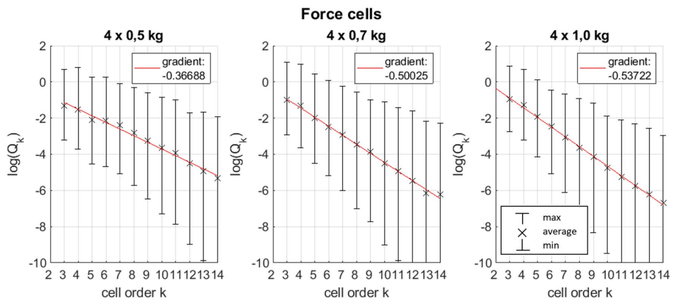


Fig. 12 Cell order distribution for force cells [2]

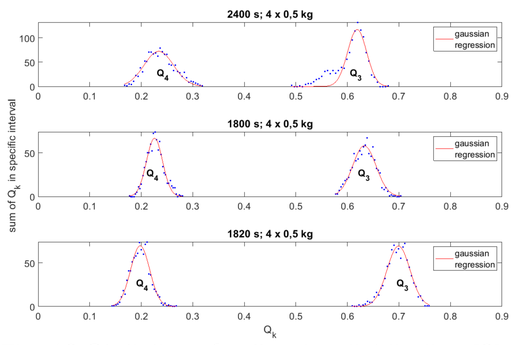


Fig. 13 Statistics of cell order distribution [2]

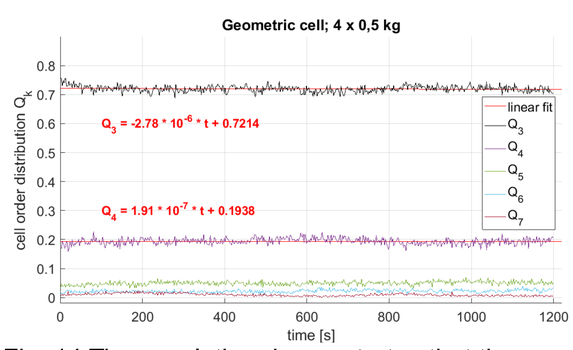


Fig. 14 Time evolution demonstrates that the equilibrium was reached [2]

Summary and Outlook

- We have built an apparatus which measures simultaneously geometry and forces
- Fewer force cells than geometrical cells: A threshold problem of optical detection
- Measured rates are in agreement with theoretical predictions
- Particle-particle friction controls rates

- Larger systems would be desirable
- Demonstrate that findings are independent of material pairing
- To be done:
  - Quantitative analysis of stress in particles
  - Larger range of pressures
  - 3D experiments?

References

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Acknowledgements and Contact Information

The authors thank Martin Müller for building and debugging the apparatus.

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