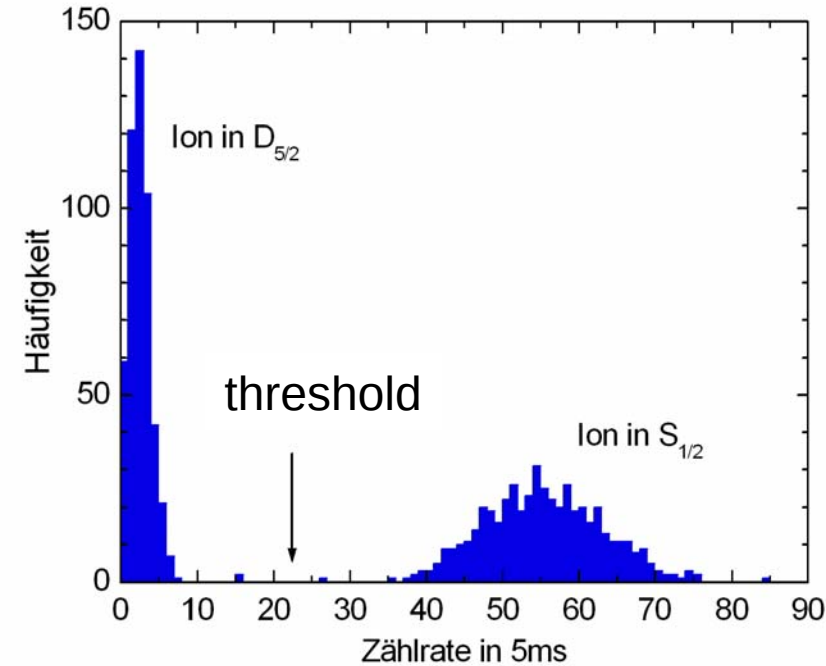
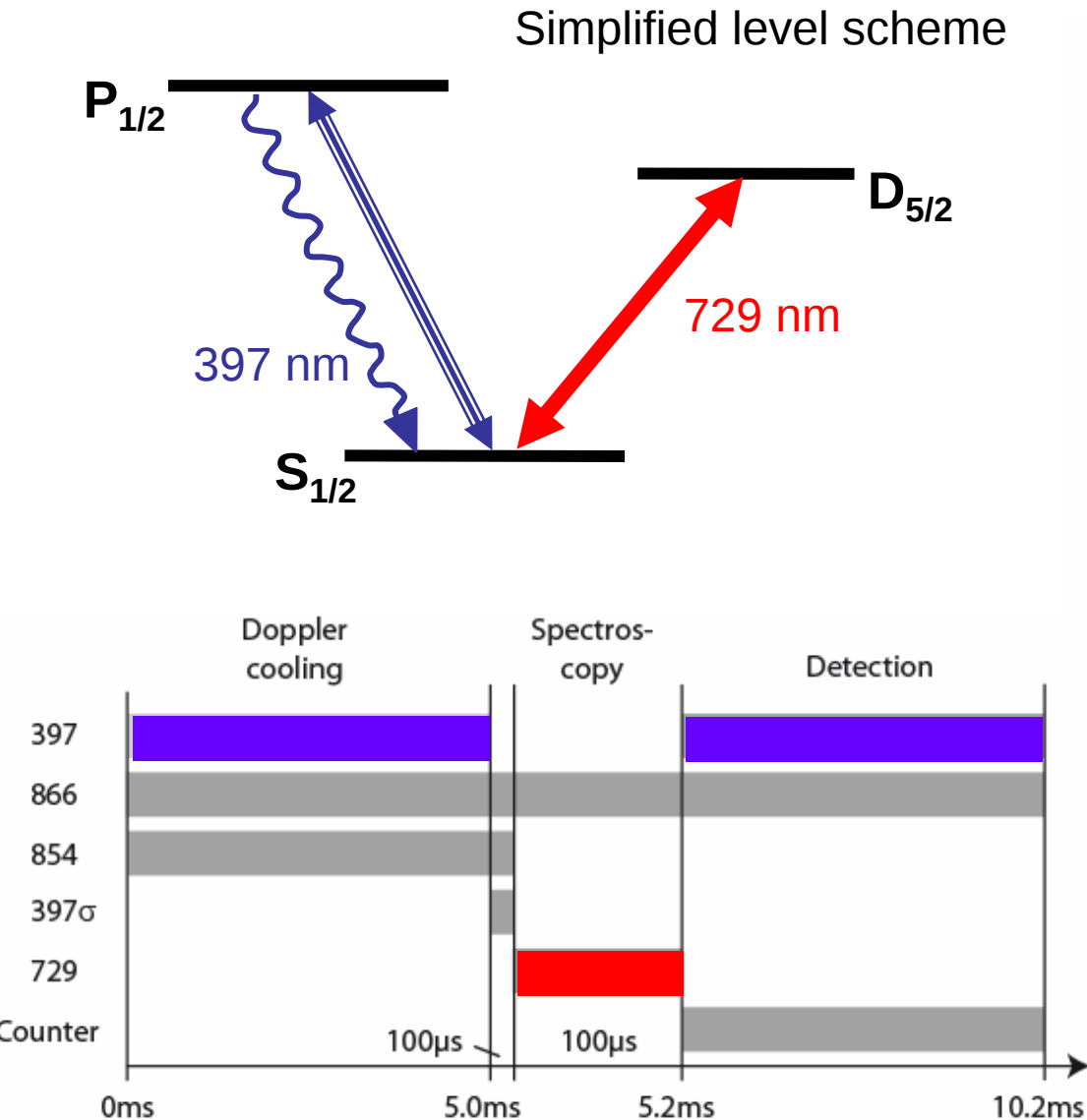


Zustandsnachweis

Quantum jump spectroscopy: Sequence

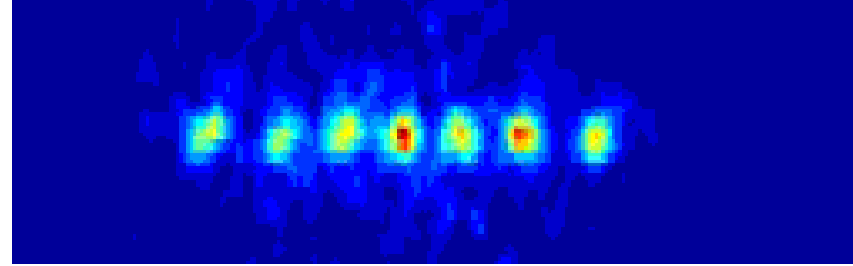


Quantum state detection

Pulse-Sequence

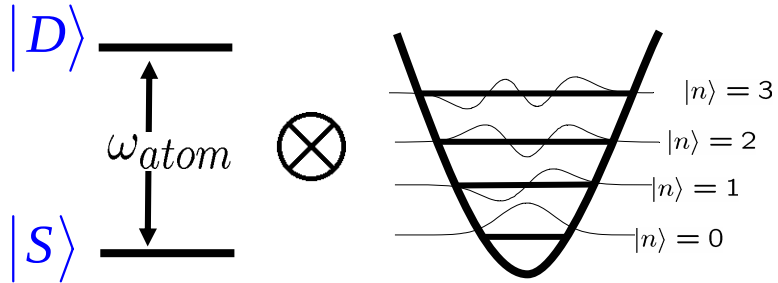
Rabioszillationen

Laser coupling

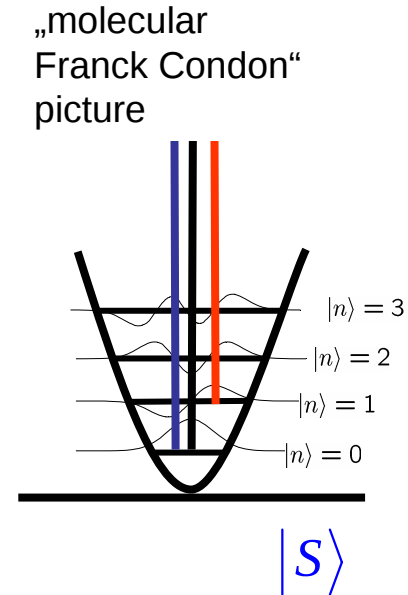
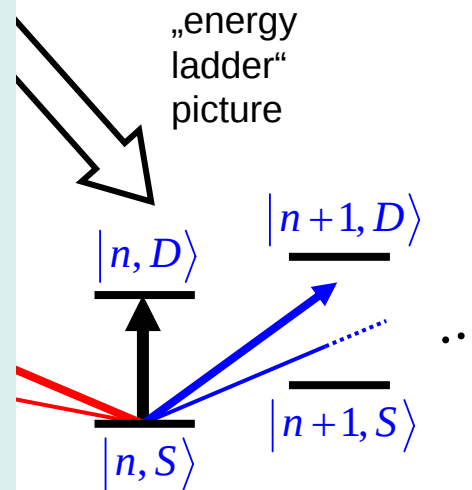
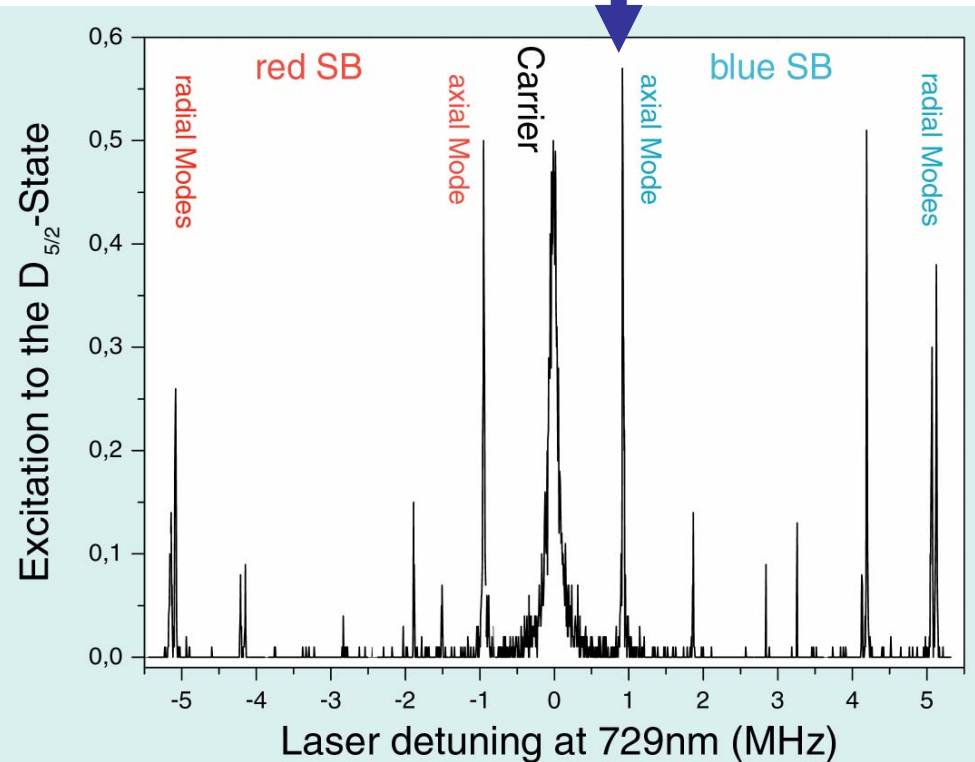
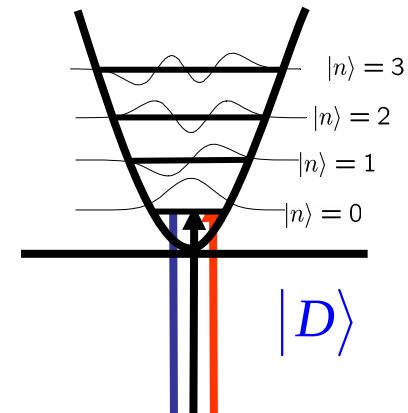


2-level-atom

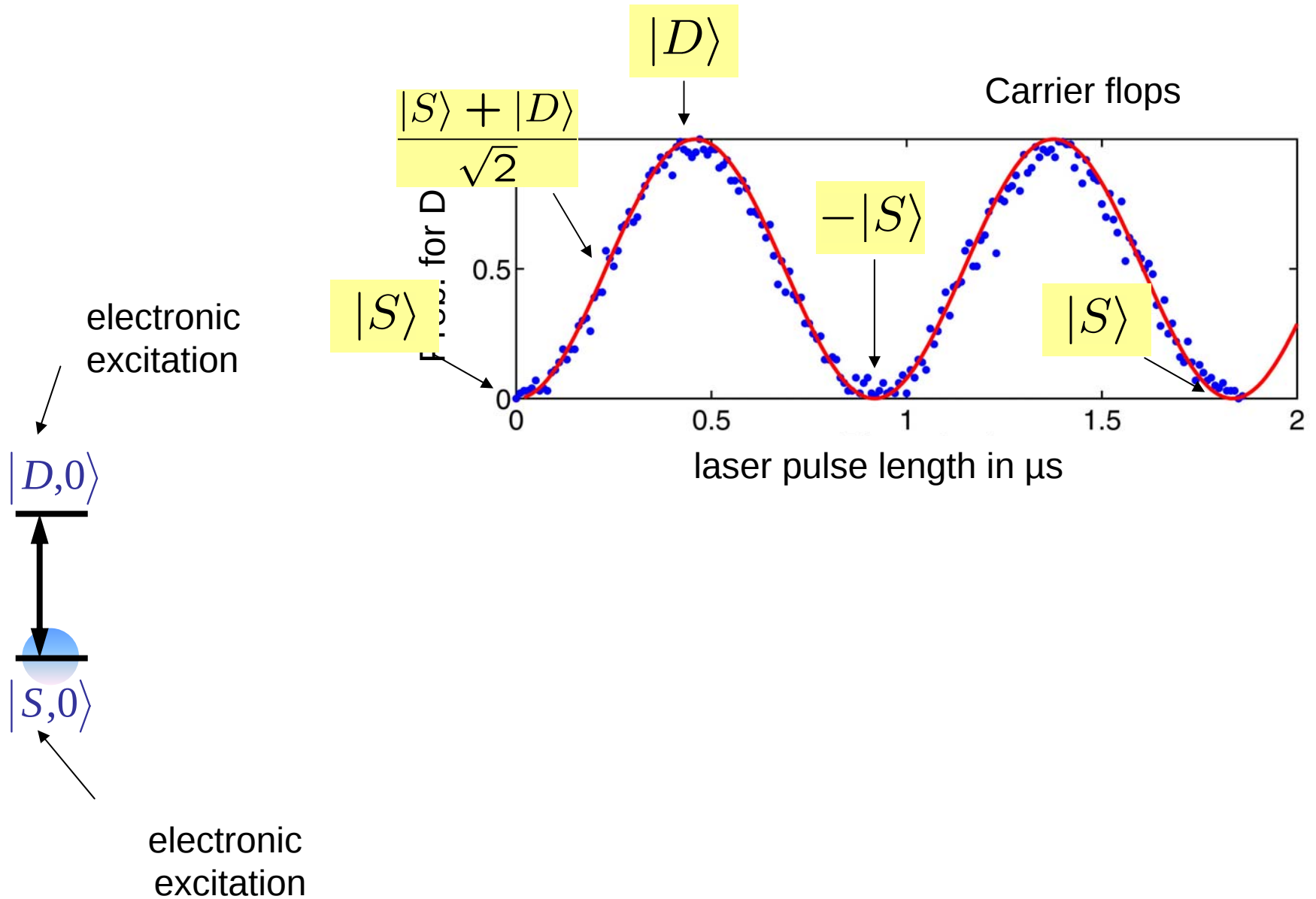
harmonic trap



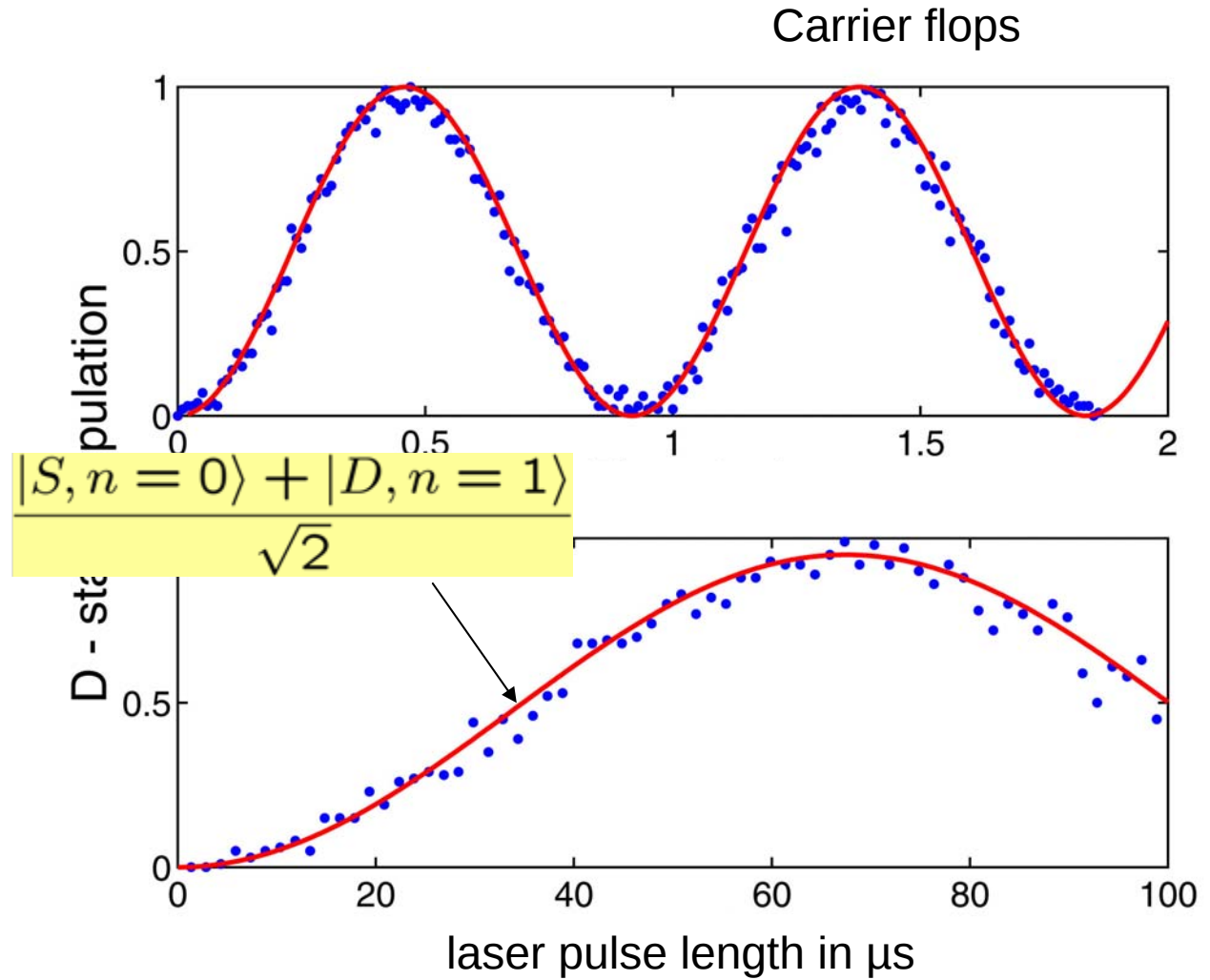
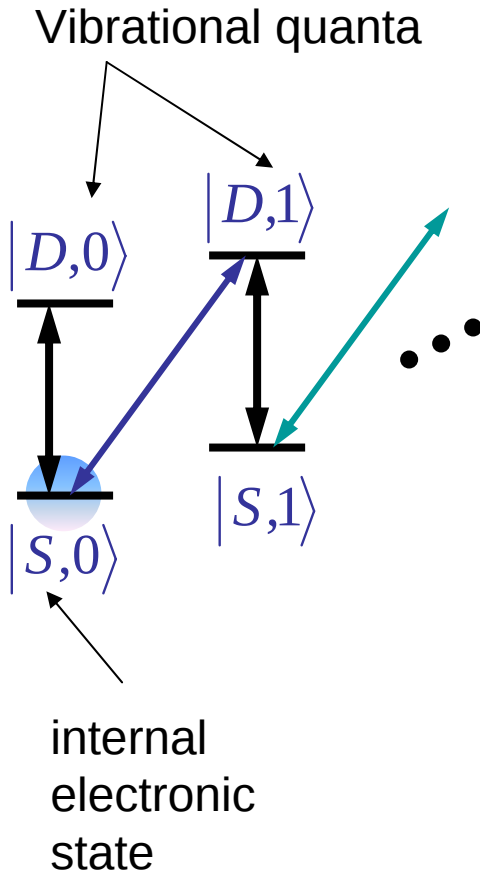
dressed system



Coherent qubit rotation

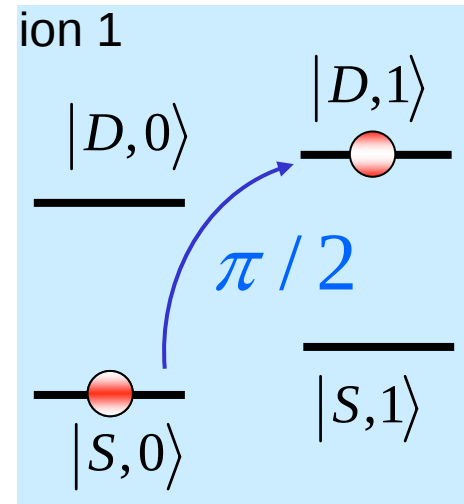
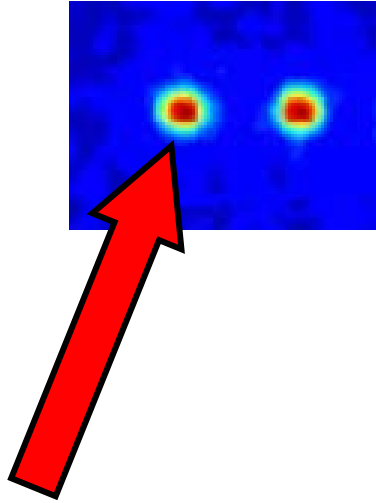


Coherent qubit rotation



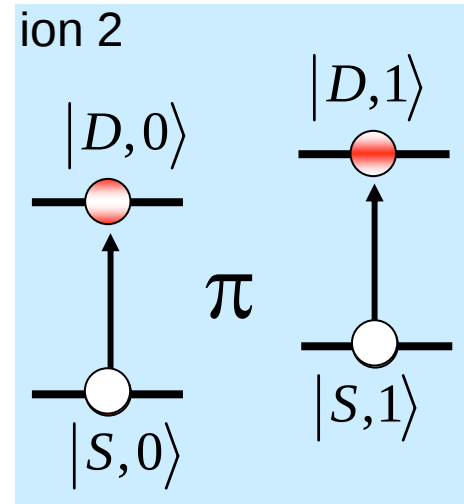
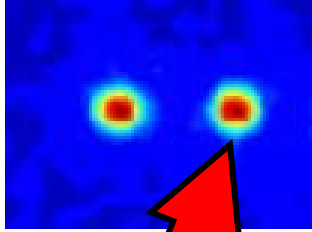
Bellzustände

Deterministic Bell state generation



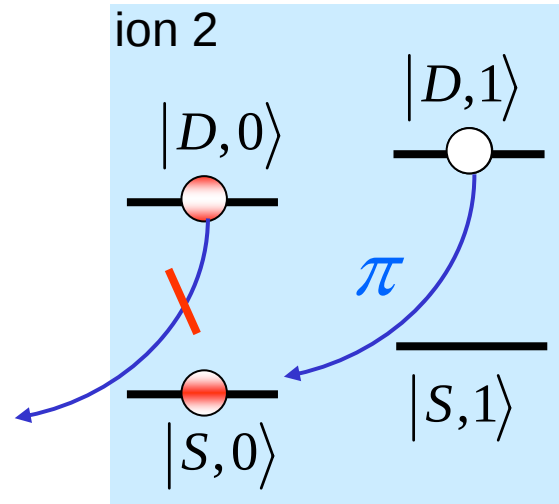
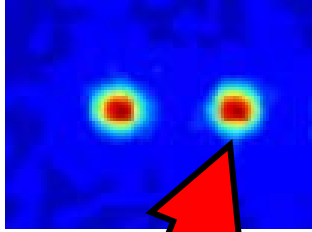
$$|SS\rangle|0\rangle \xrightarrow{\text{blue } \pi/2 \text{ pulse}} |SS, 0\rangle + |DS, 1\rangle$$

Deterministic Bell state generation



$$\begin{aligned}
 |SS\rangle|0\rangle &\xrightarrow{\text{blue } \pi/2 \text{ pulse}} |SS, 0\rangle + |DS, 1\rangle \\
 &\xrightarrow{\text{carrier } \pi \text{ pulse}} |SD, 0\rangle + |DD, 1\rangle
 \end{aligned}$$

Deterministic Bell state generation



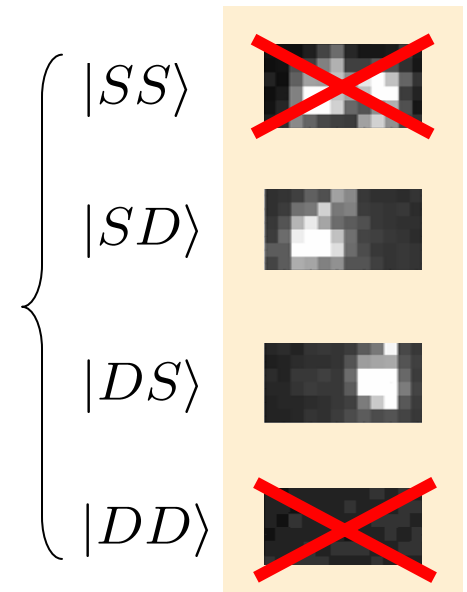
$$\begin{array}{lcl}
 |SS\rangle|0\rangle & \xrightarrow{\text{blue } \pi/2 \text{ pulse}} & |SS, 0\rangle + |DS, 1\rangle \\
 & \xrightarrow{\text{carrier } \pi \text{ pulse}} & |SD, 0\rangle + |DD, 1\rangle \\
 & \xrightarrow{\text{blue } \pi \text{ pulse}} & |SD\rangle|0\rangle + |DS\rangle|0\rangle
 \end{array}$$

Bell state analysis

$$\begin{aligned} &|SD\rangle|0\rangle + |DS\rangle|0\rangle \\ &= (|SD\rangle + |DS\rangle)|0\rangle \end{aligned}$$

- Coherent superposition of SD and DS states?
- Phase relation between both wave function components?

Fluorescence detection with CCD camera:



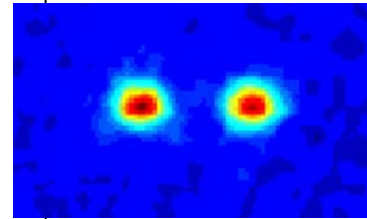
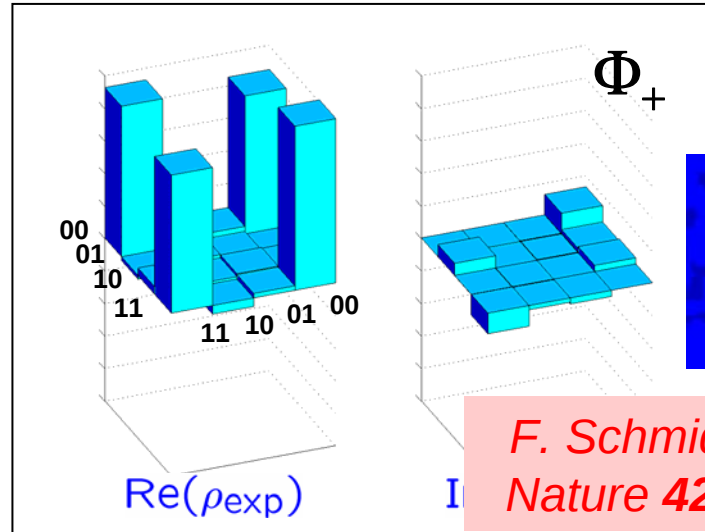
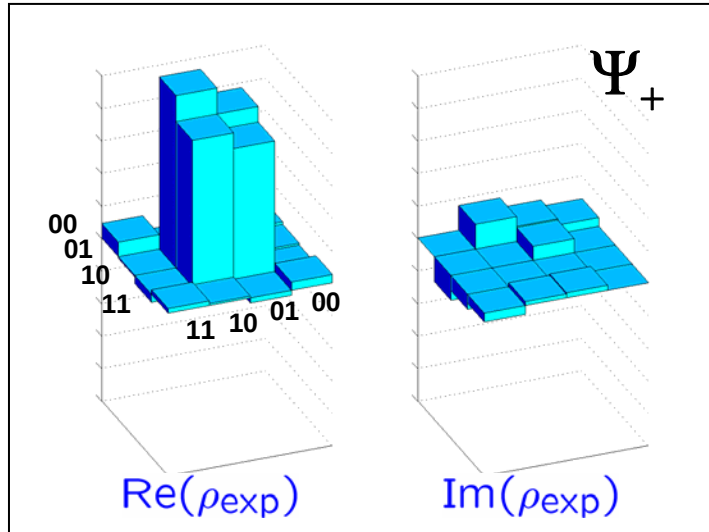
Entire information is contained in the density matrix

→ Measure the density matrix

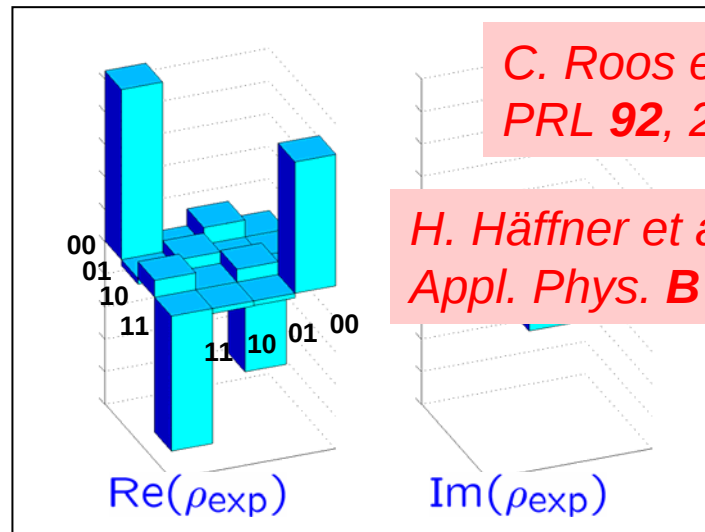
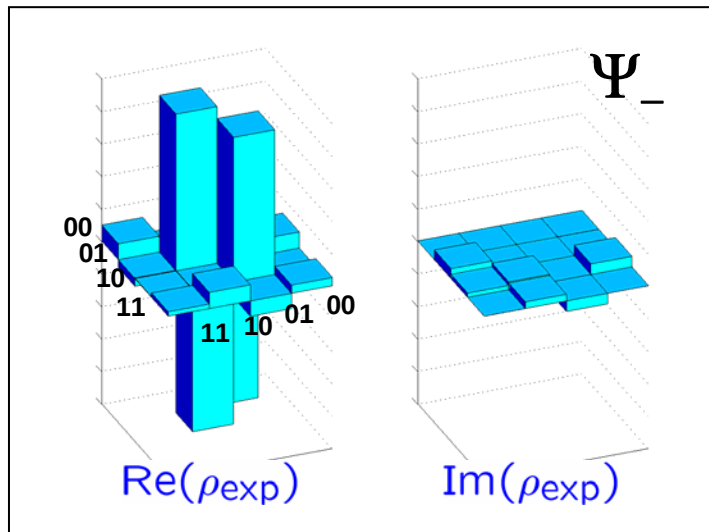
Density matrix for 2 ion entanglement

$$\psi = |0\rangle|1\rangle + e^{i\phi}|1\rangle|0\rangle$$

$$\phi = |1\rangle|1\rangle + e^{i\phi}|0\rangle|0\rangle$$



*F. Schmidt-Kaler et al.,
Nature **422**, 408 (2003)*



*C. Roos et al.,
PRL **92**, 220402 (2004)*

*H. Häffner et al.,
Appl. Phys. B **81**, 151 (2005)*

CNOT Gatter

Quantum gate proposal

74, NUMBER 20 4091

PHYSICAL REVIEW LETTERS

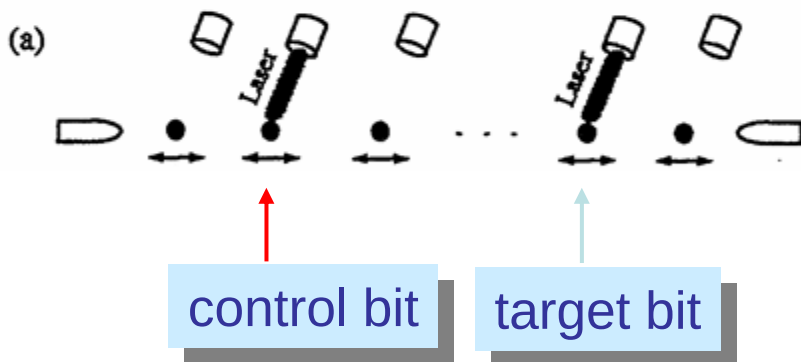
15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.



W. Paul



J. I. Cirac



P. Zoller

- single bit rotations and quantum gates
- small decoherence
- unity detection efficiency
- scalable

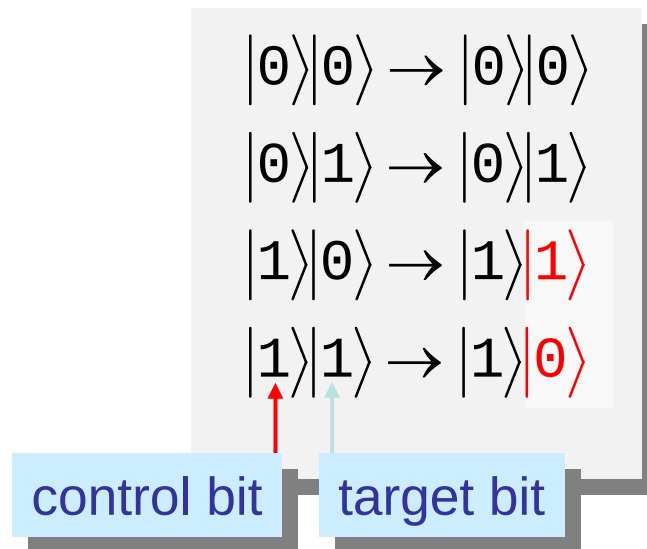
Quantum gate proposal



J. I. Cirac

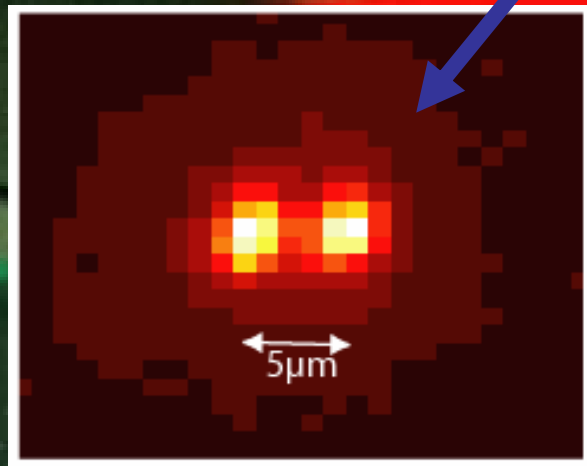
P. Zoller

$$\text{Controlled} - NOT : |\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$



- single bit rotations and quantum gates
- small decoherence
- unity detection efficiency
- scalable

Cirac & Zoller gate
with two ions



Controlled-NOT operation

$|\epsilon_1\rangle$ $|\epsilon_2\rangle$



$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$

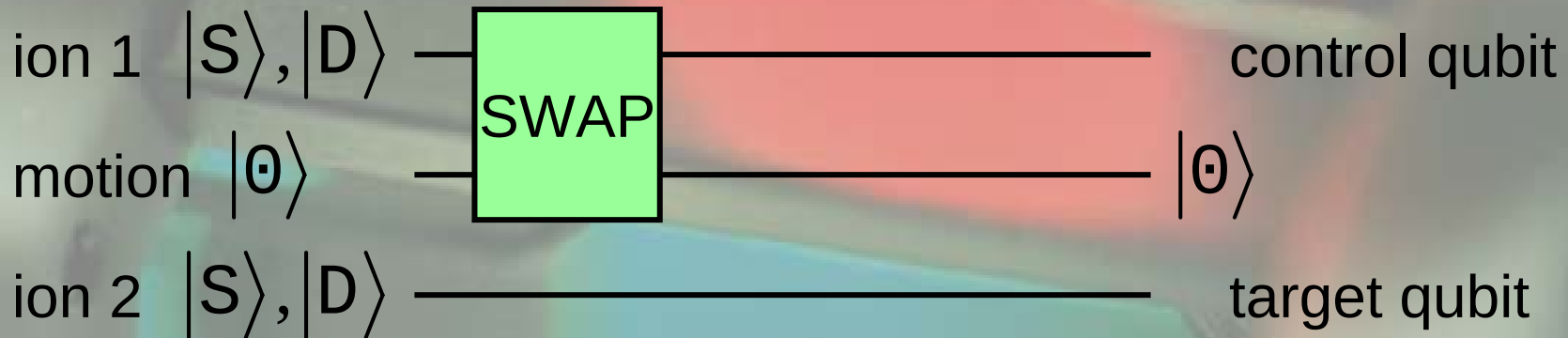
$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$

$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$

$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

control

target



Controlled-NOT operation

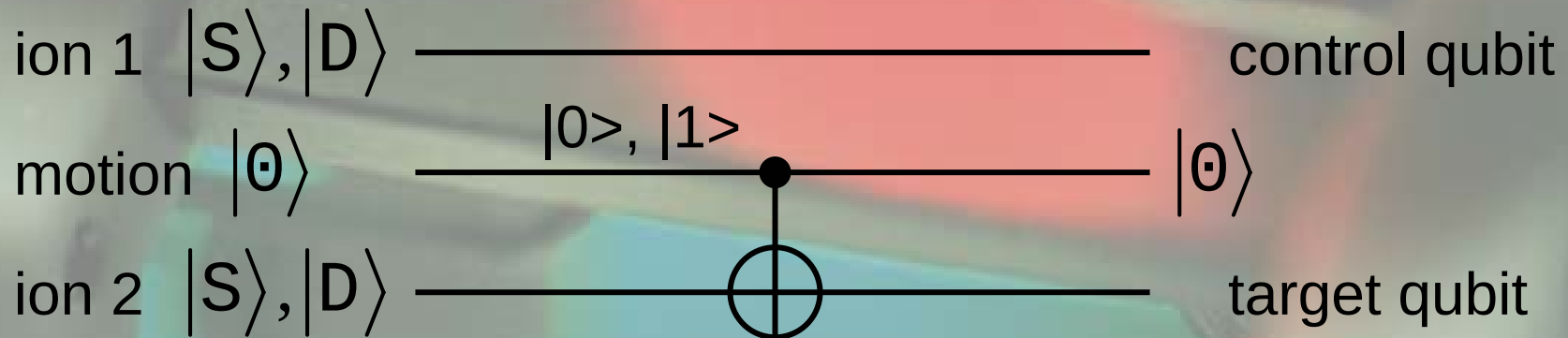


$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$

$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$

$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$

$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$



Controlled-NOT operation

$|\varepsilon_1\rangle$ $|\varepsilon_2\rangle$

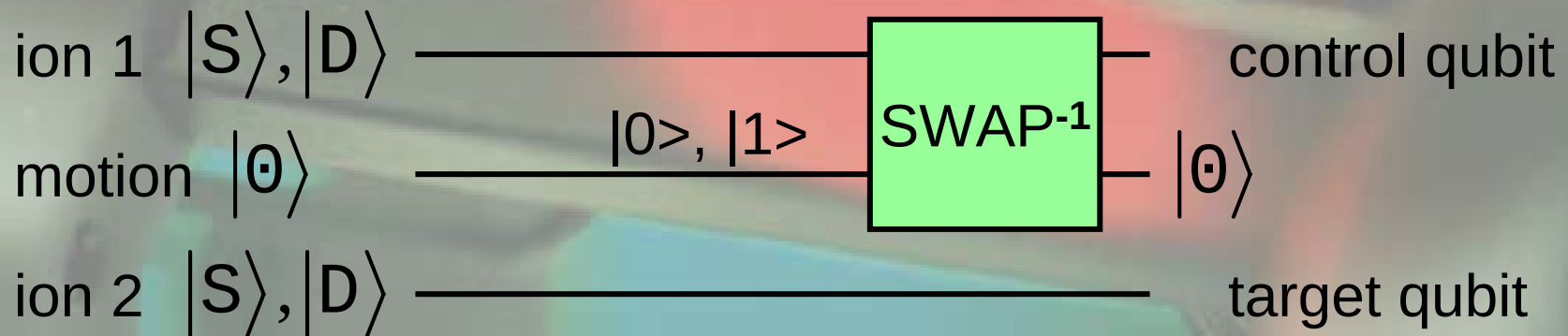


$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$

$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$

$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$

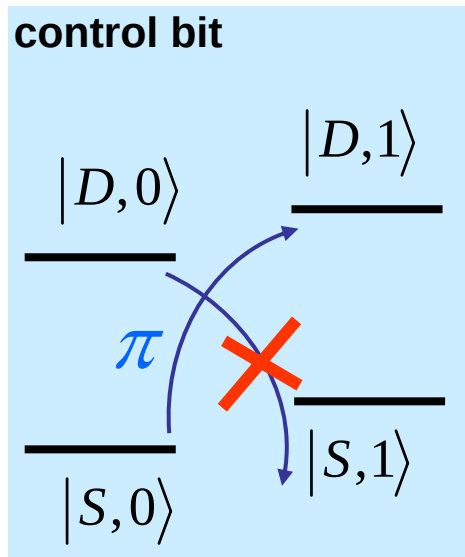
$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$



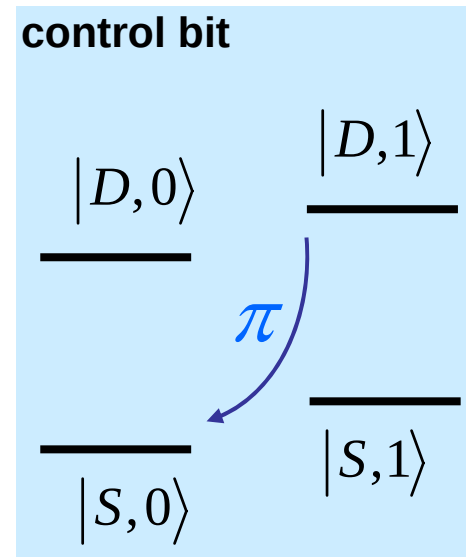
SWAP and SWAP⁻¹

starting with $|n=0\rangle$ phonons,
write into and read from the common vibrational mode

π -pulse on blue SB



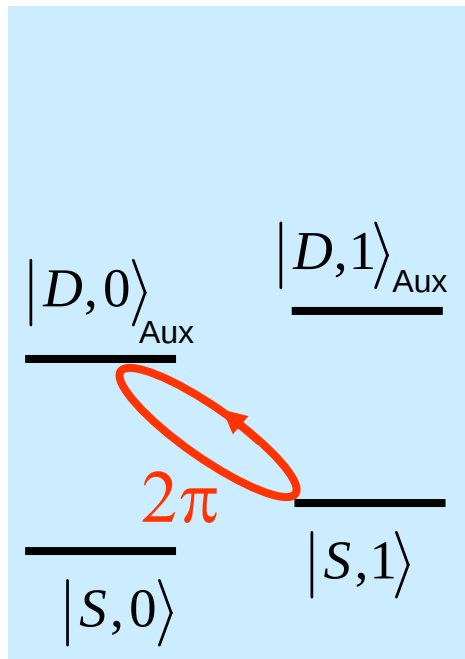
SWAP



SWAP⁻¹

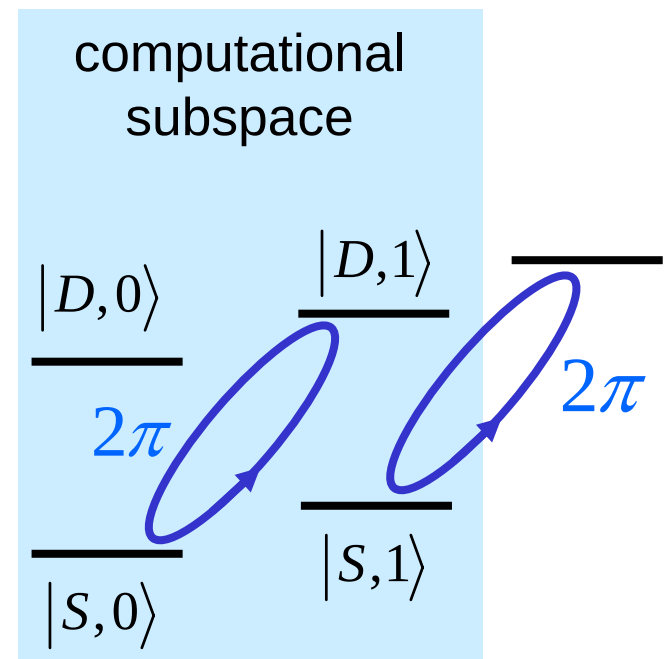
Conditional phase gates

Cirac & Zoller (1995)



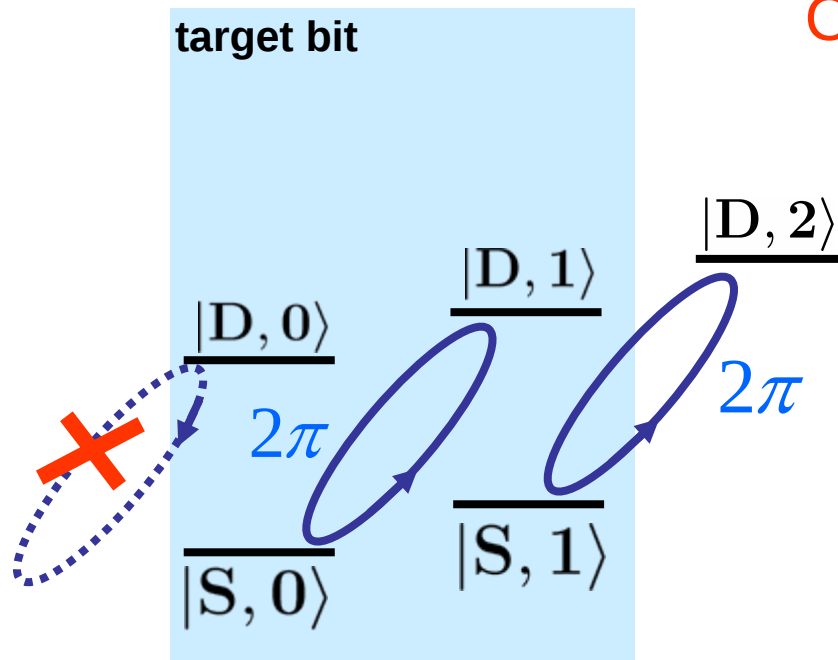
Effect:
phase factor of **-1**
for $|S,1\rangle$

Composite phase gate



Effect:
phase factor of **-1**
for all, except $|D,0\rangle$

Conditional phase gate



Composite pulse phase gate

*I. Chuang,
MIT Boston*

Rabi frequency:

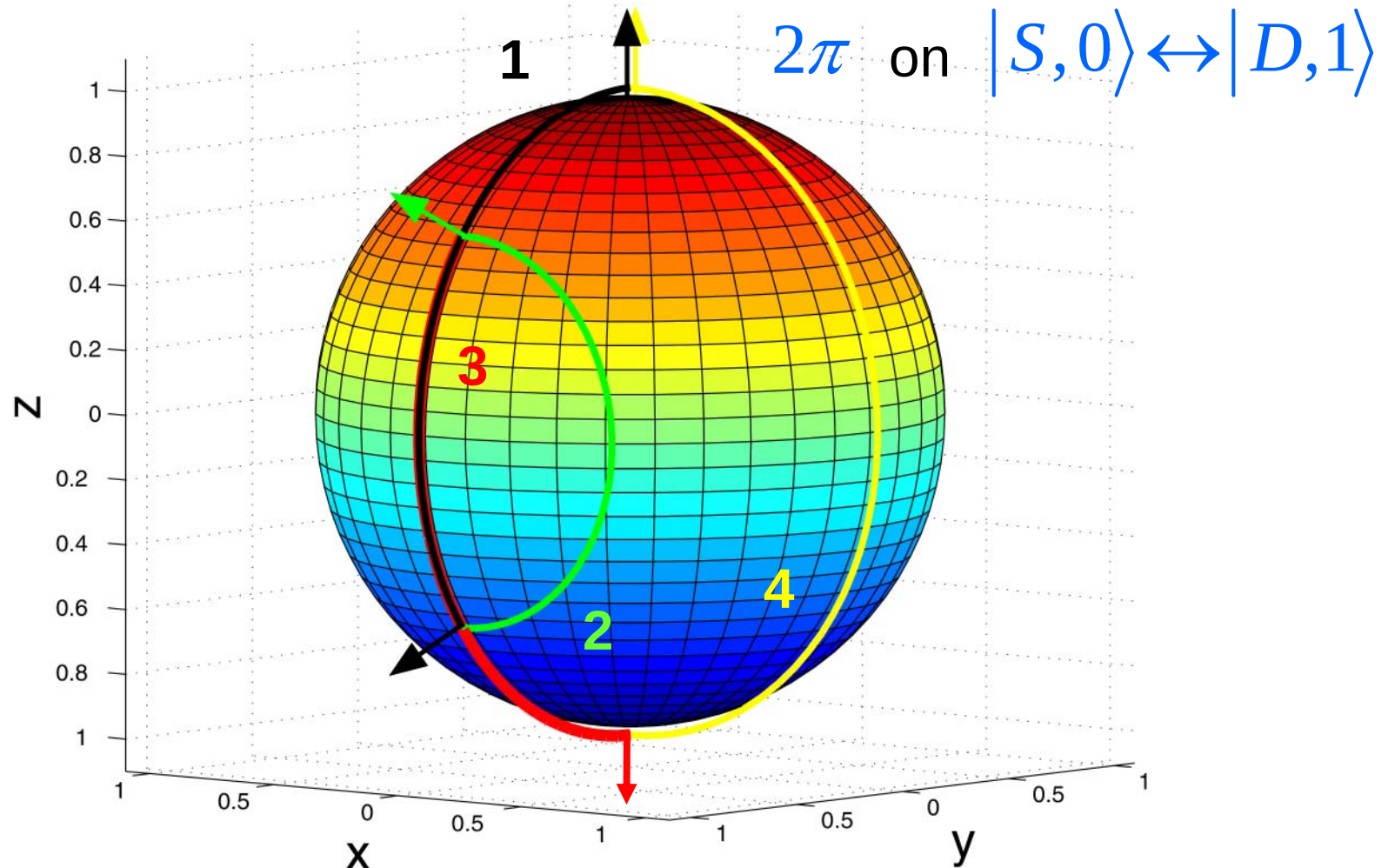
Blue SB: $\Omega \cdot \eta \cdot \sqrt{n+1}$

Effect:

phase factor of **-1**
for all, except $|D, 0\rangle$

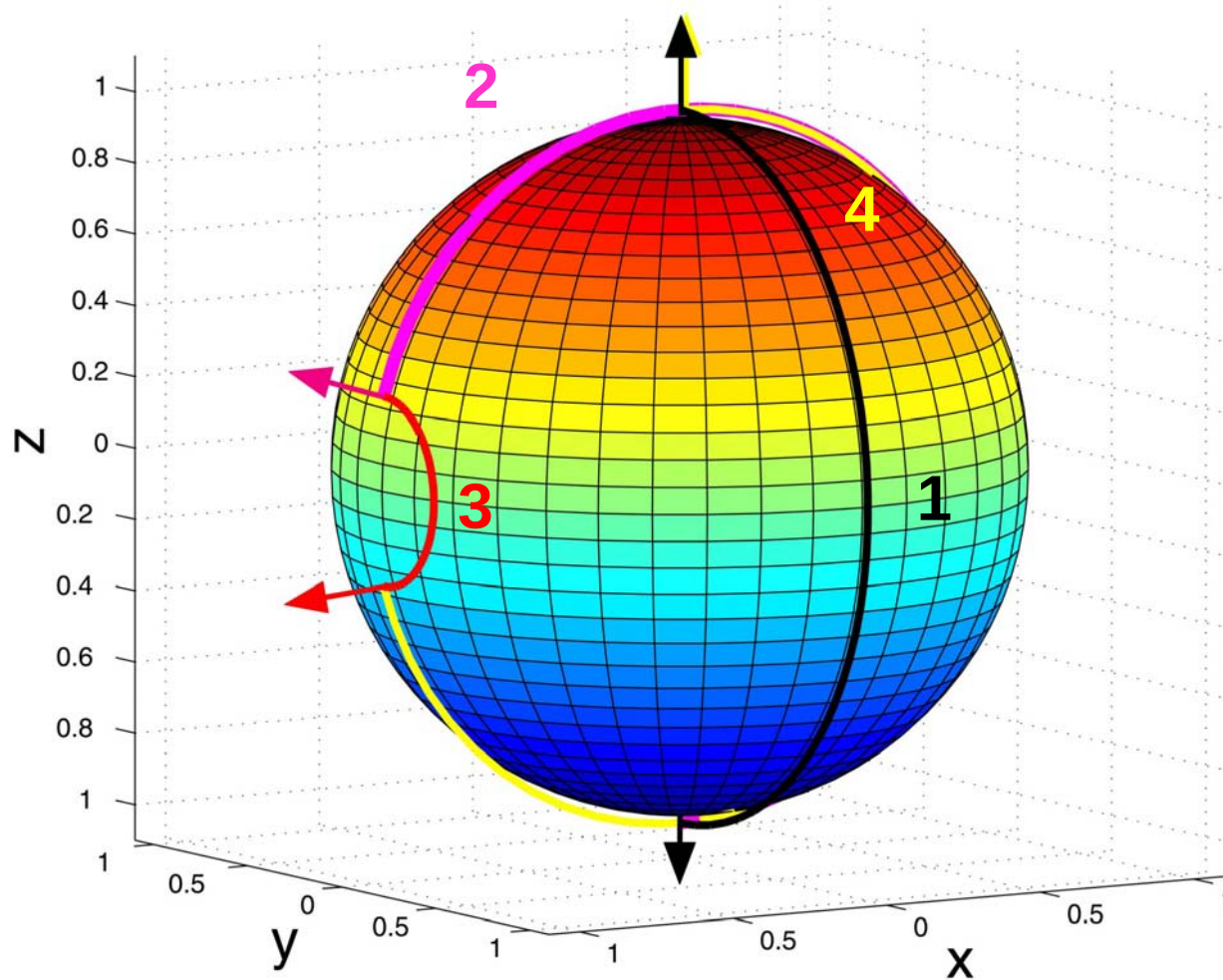
Composite phase gate (2π rotation)

$$R(\theta, \phi) = R_1^+ (\pi, \pi/2) R_1^+ (\pi/\sqrt{2}, 0) R_1^+ (\pi, \pi/2) R_1^+ (\pi/\sqrt{2}, 0)$$



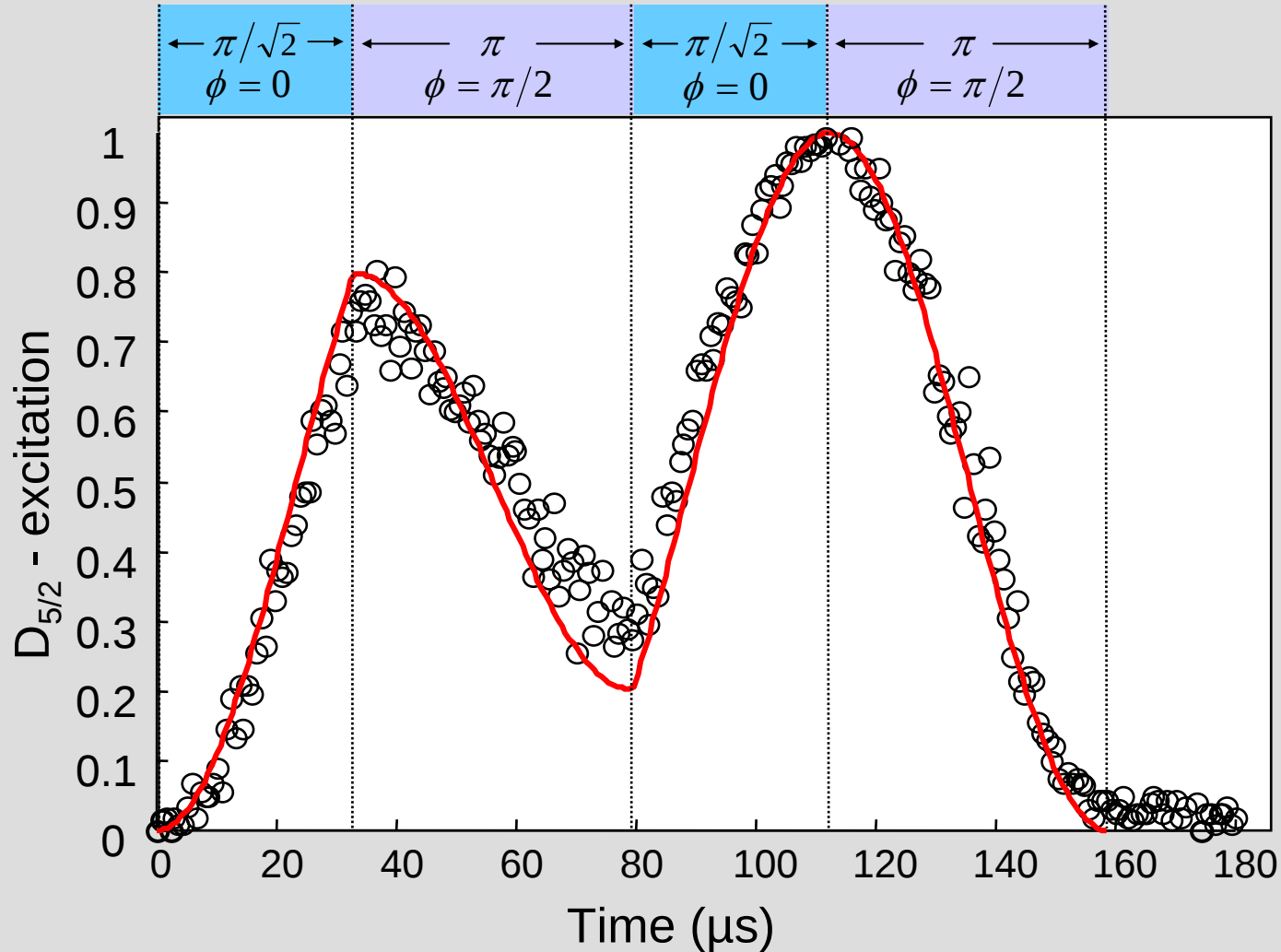
Population of $|S,1\rangle$ - $|D,2\rangle$ remains unaffected

$$R(\theta, \phi) = R_1^+ \left(\pi\sqrt{2}, \pi/2 \right) R_1^+ \left(\pi, 0 \right) R_1^+ \left(\pi\sqrt{2}, \pi/2 \right) R_1^+ \left(\pi, 0 \right)$$



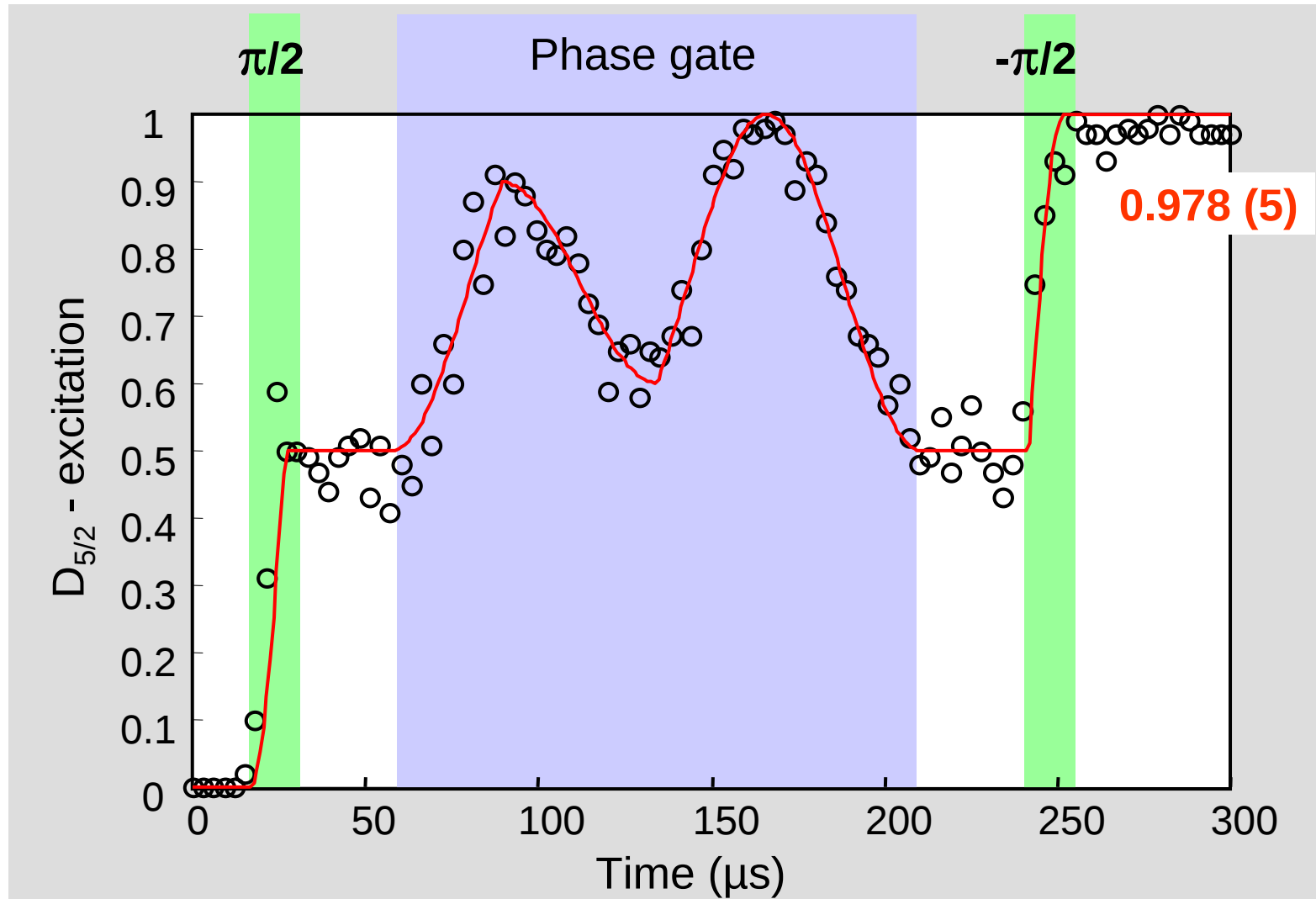
Single ion composite phase gate

state preparation $|S, 0\rangle$, then application of phase gate pulse sequence

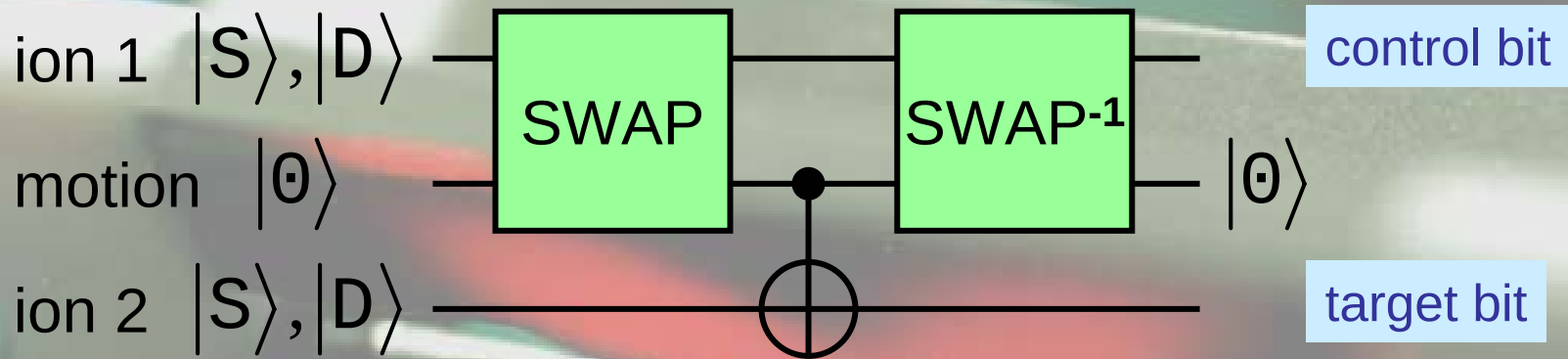


Single ion composite CNOT gate

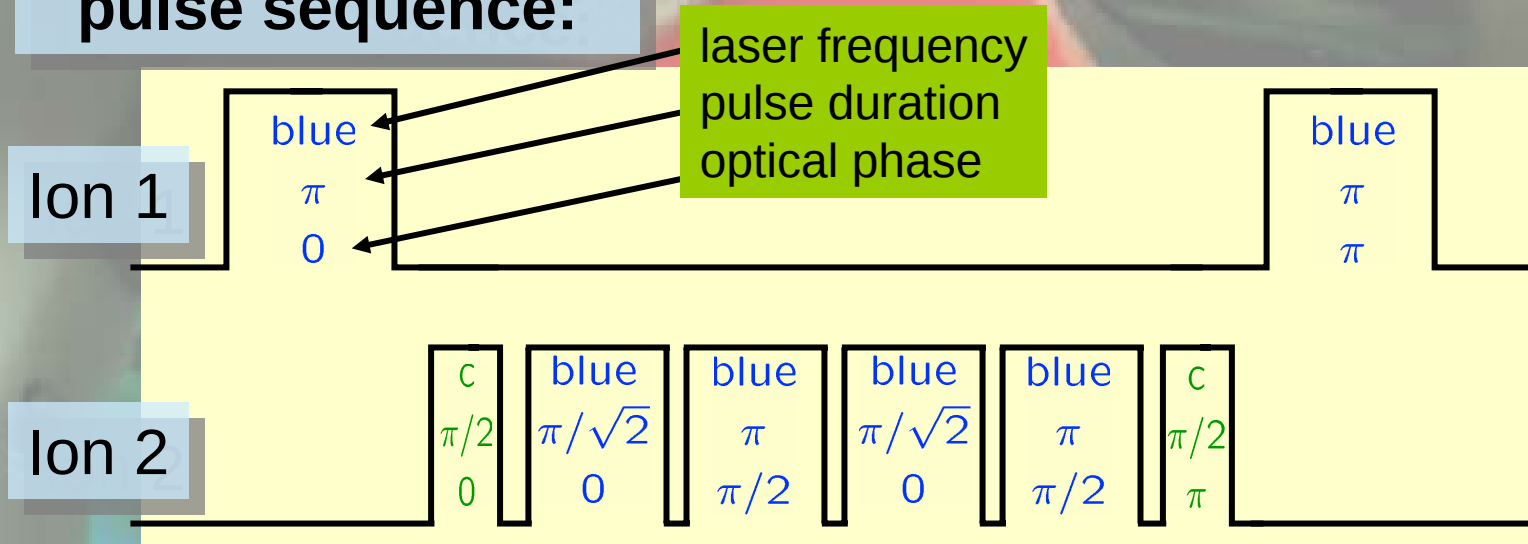
state preparation $|S, 0\rangle$, then application of CNOT gate pulse sequence



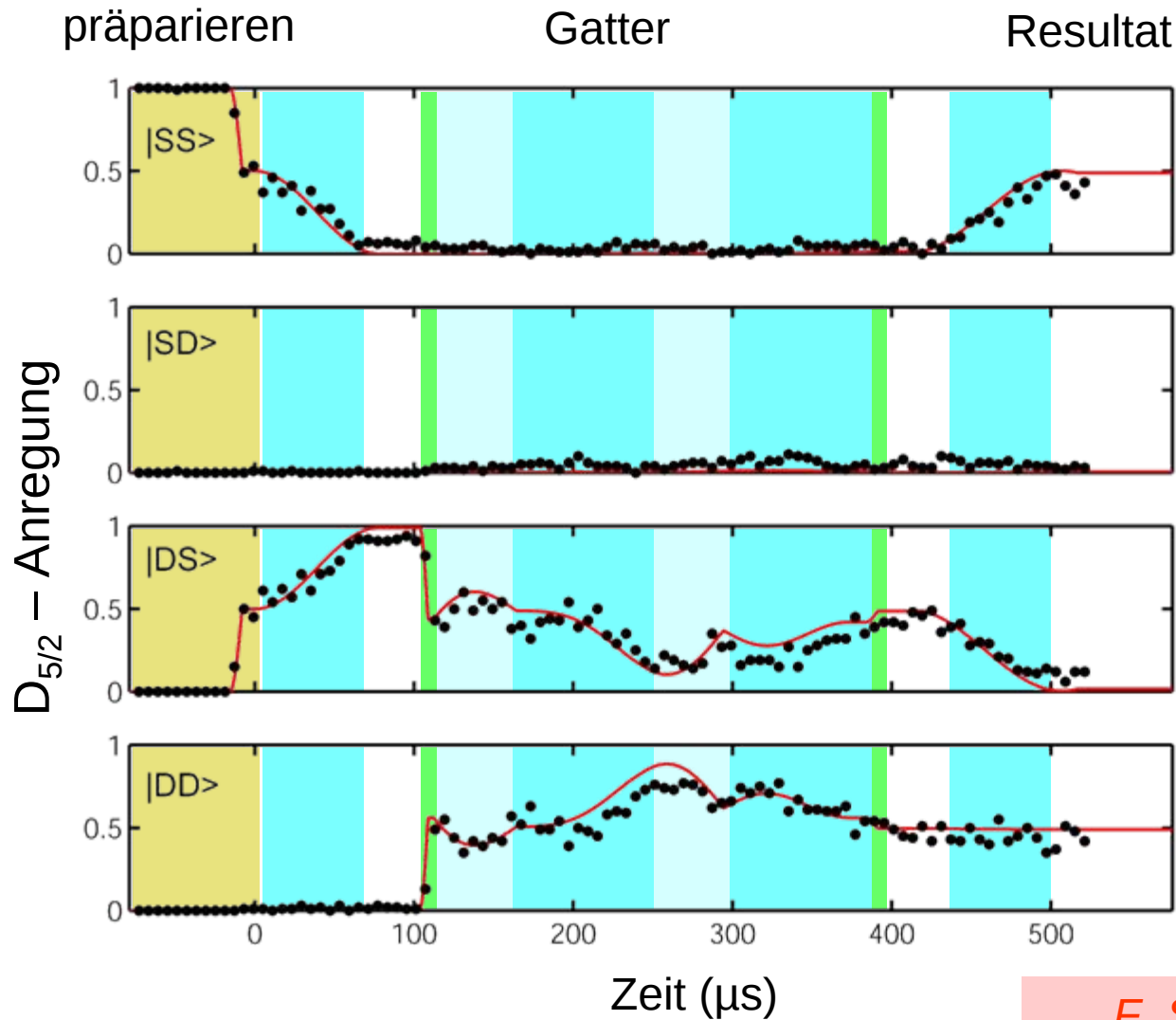
Controlled-NOT operation



pulse sequence:



$$|S+D, S\rangle \xrightarrow{\text{CNOT}} |SS\rangle + |DD\rangle$$

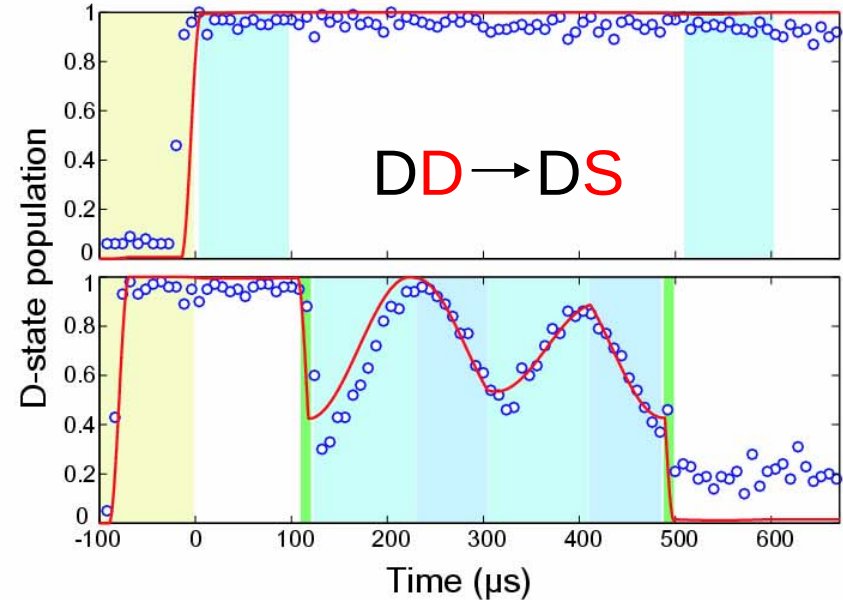
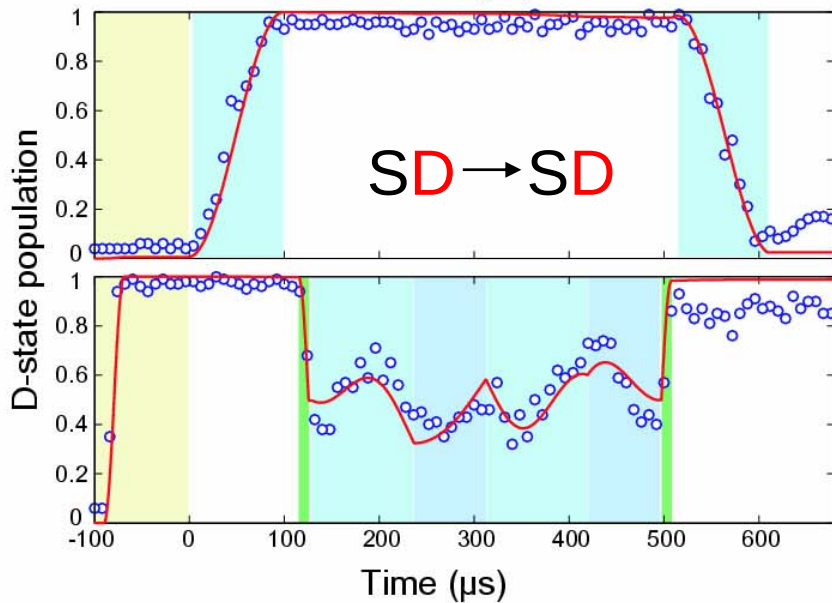
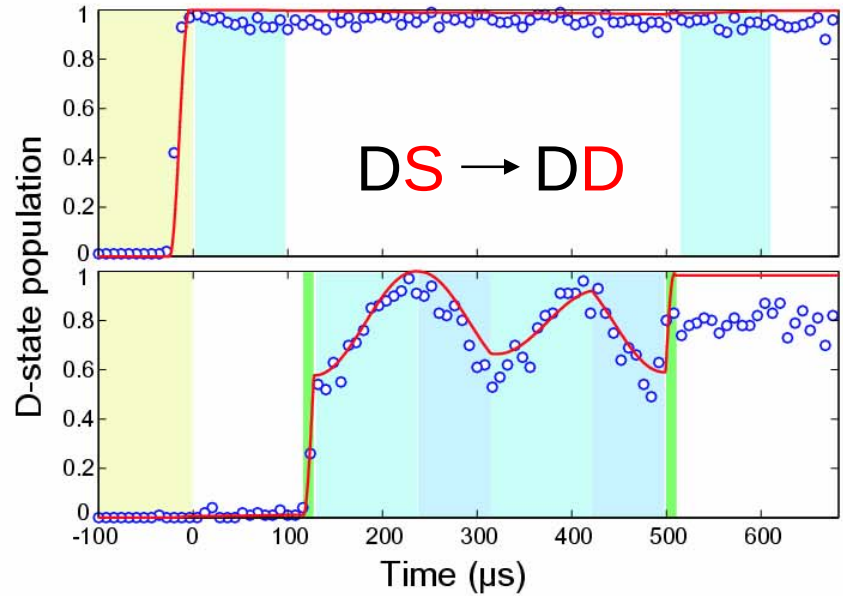
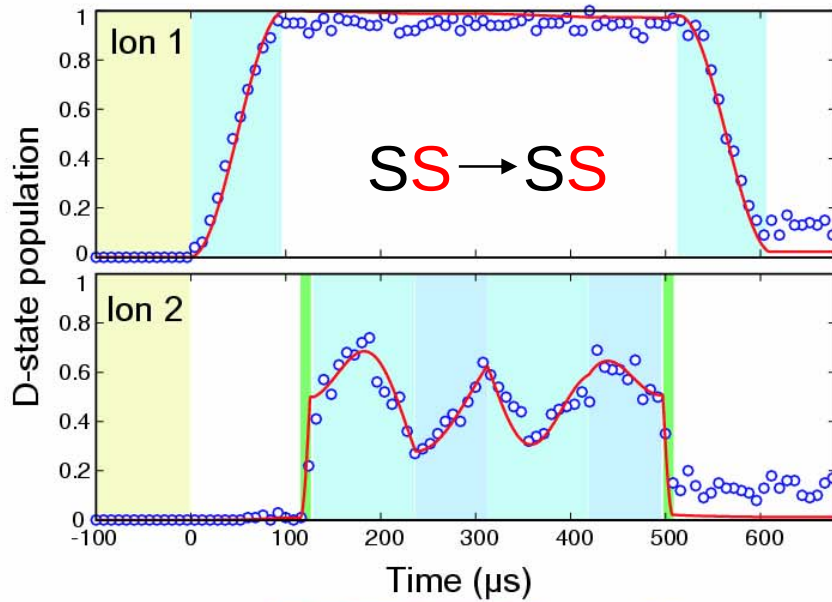


Nachweis

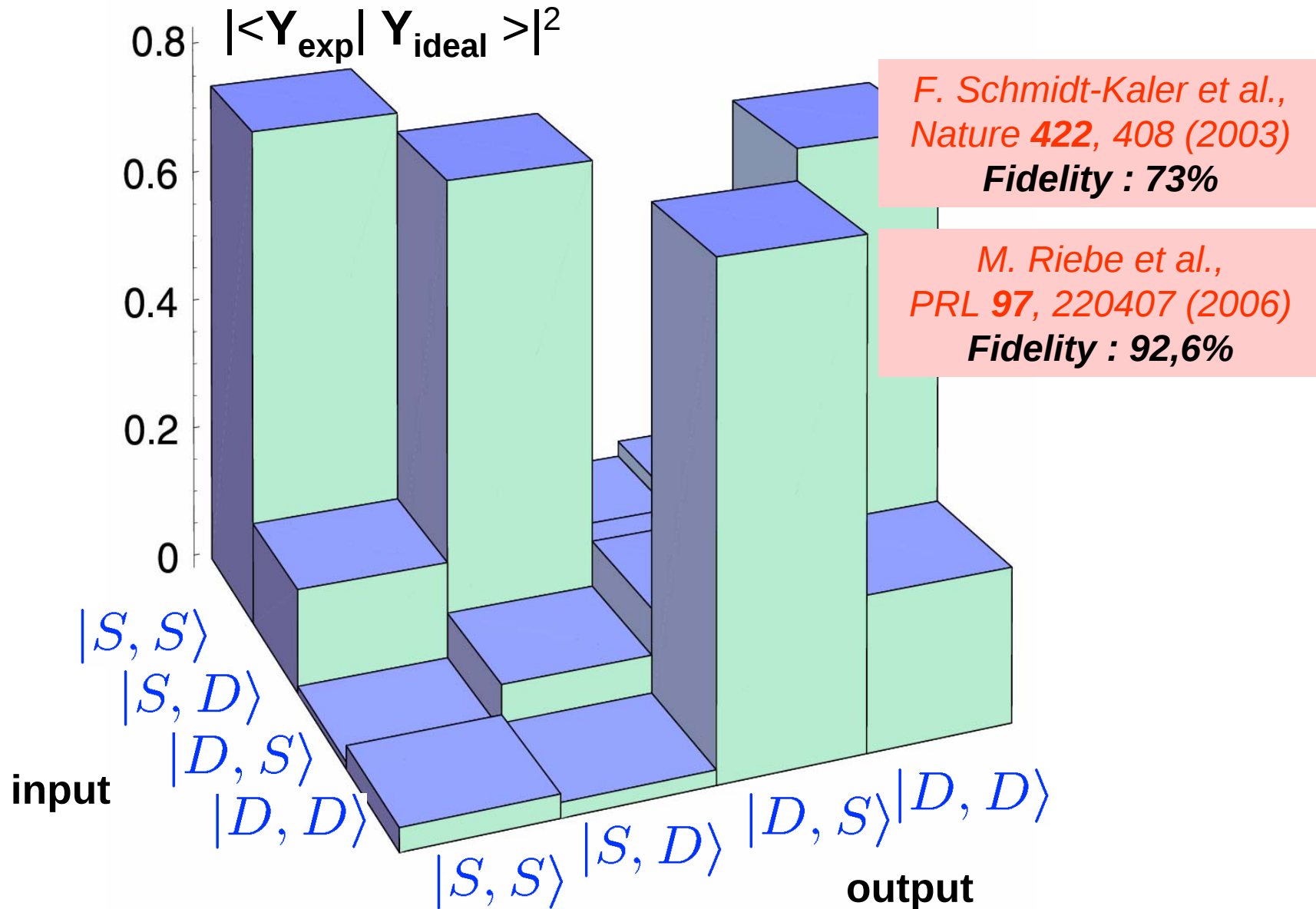


*F. Schmidt-Kaler et al.,
Appl.Phys.**B77**, 789 (2003)*

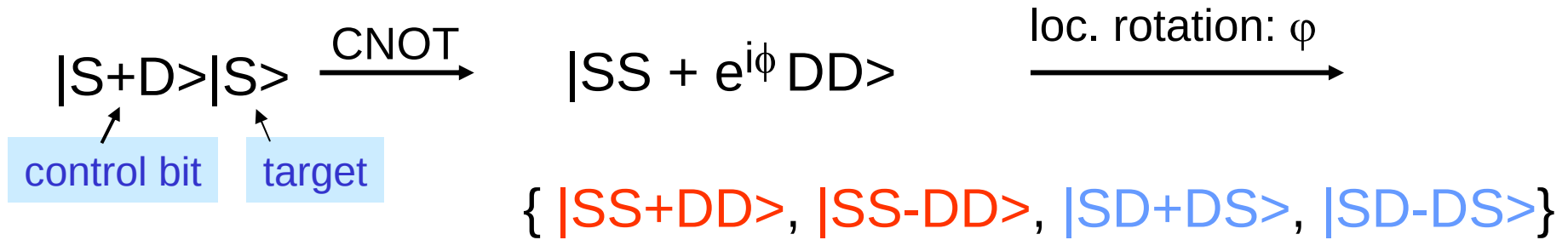
Cirac – Zoller CNOT gate operation



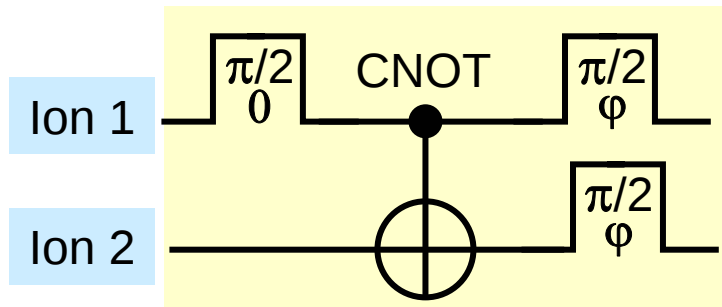
Fidelity of Cirac-Zoller CNOT



Entanglement

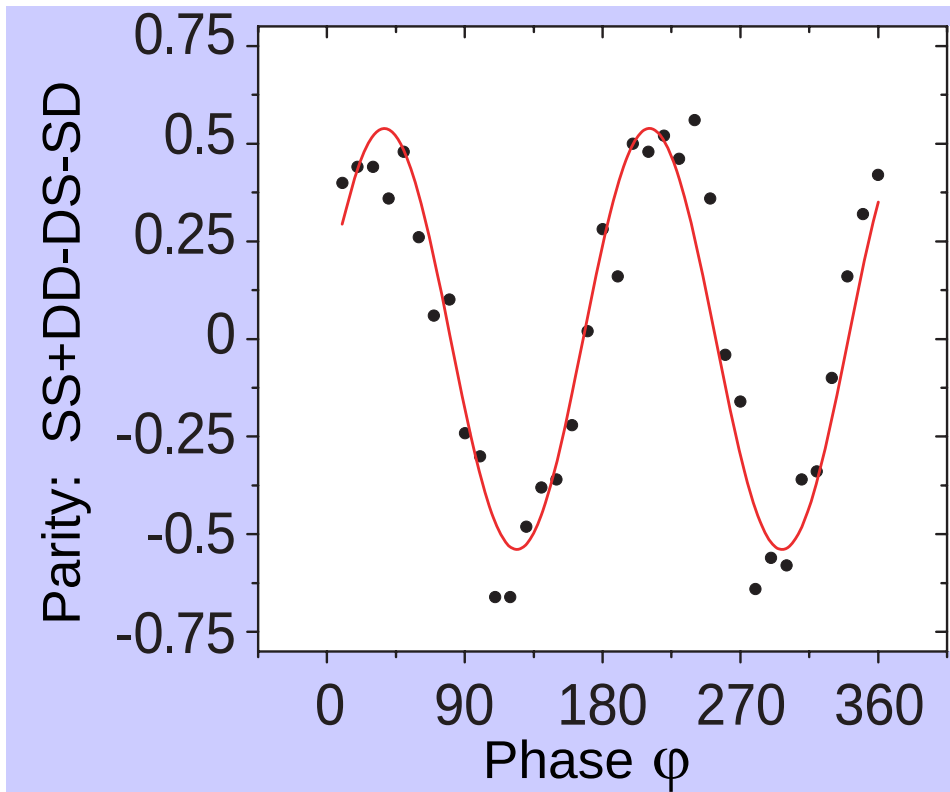
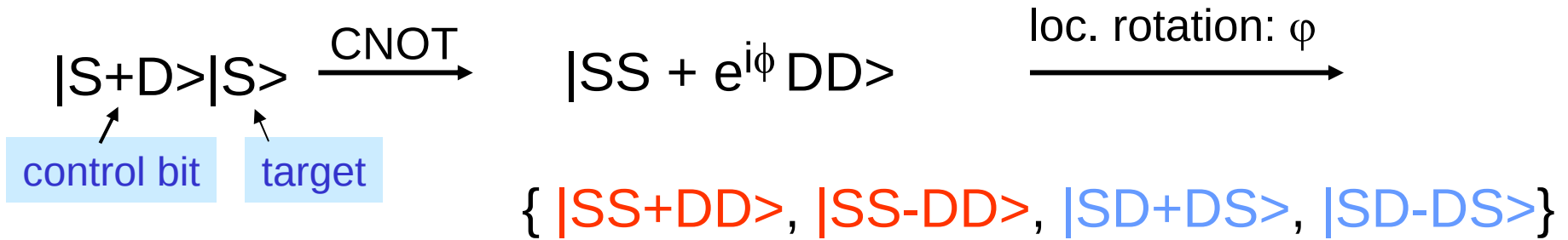


Sequence:



- Super-Ramsey experiment
- Parity check

Entanglement



oscillates with 2ϕ !
54% contrast



Fidelity =
 $0.5 (P_{SS} + P_{DD} + \text{contrast}) =$
71(3)%

Teleportation

Principle of the ion trap quantum processor

Universal two-qubit gate

Entangled states of two and three ions

Teleportation

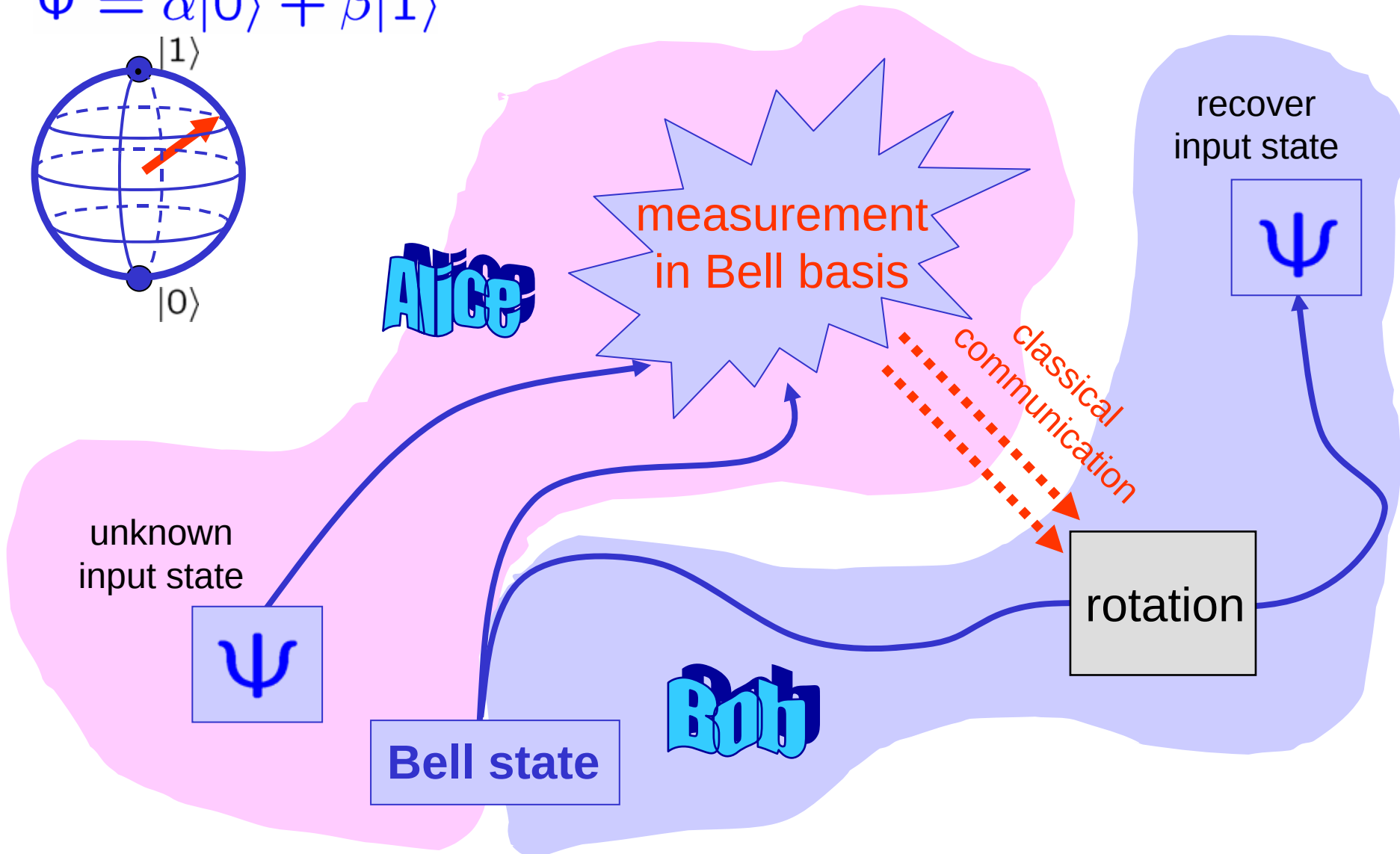
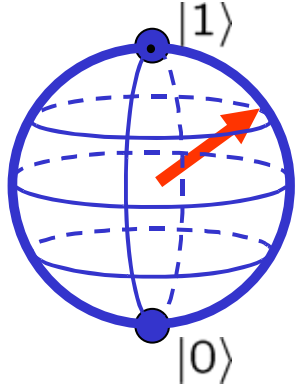
Theorie: D. James, Los Alamos



Teleportation

Bennett et al, Phys. Rev. Lett. 70, 1895 (1993)

$$\psi = \alpha|0\rangle + \beta|1\rangle$$



Quantum teleportation: No black magic

Source qubit(#1): pure state $|\chi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$

Target qubit(#3) and ancilla (#2): maximally entangled state

$$|\Psi^+\rangle_{23} = \frac{1}{\sqrt{2}} (|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3)$$

Combined state $|\varphi\rangle = |\chi\rangle_1 \frac{1}{\sqrt{2}} (|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3)$

Rearrange terms:

$$|\varphi\rangle = \frac{1}{2} (|\Phi^+\rangle_{12} \sigma_x |\chi\rangle_3 + |\Phi^-\rangle_{12} (-i\sigma_y) |\chi\rangle_3 + |\Psi^+\rangle_{12} |\chi\rangle_3 + |\Psi^-\rangle_{12} \sigma_z |\chi\rangle_3)$$

$$|\Psi^\pm\rangle_{12} = \frac{1}{\sqrt{2}} (|0\rangle_1|0\rangle_2 \pm |1\rangle_1|1\rangle_2)$$

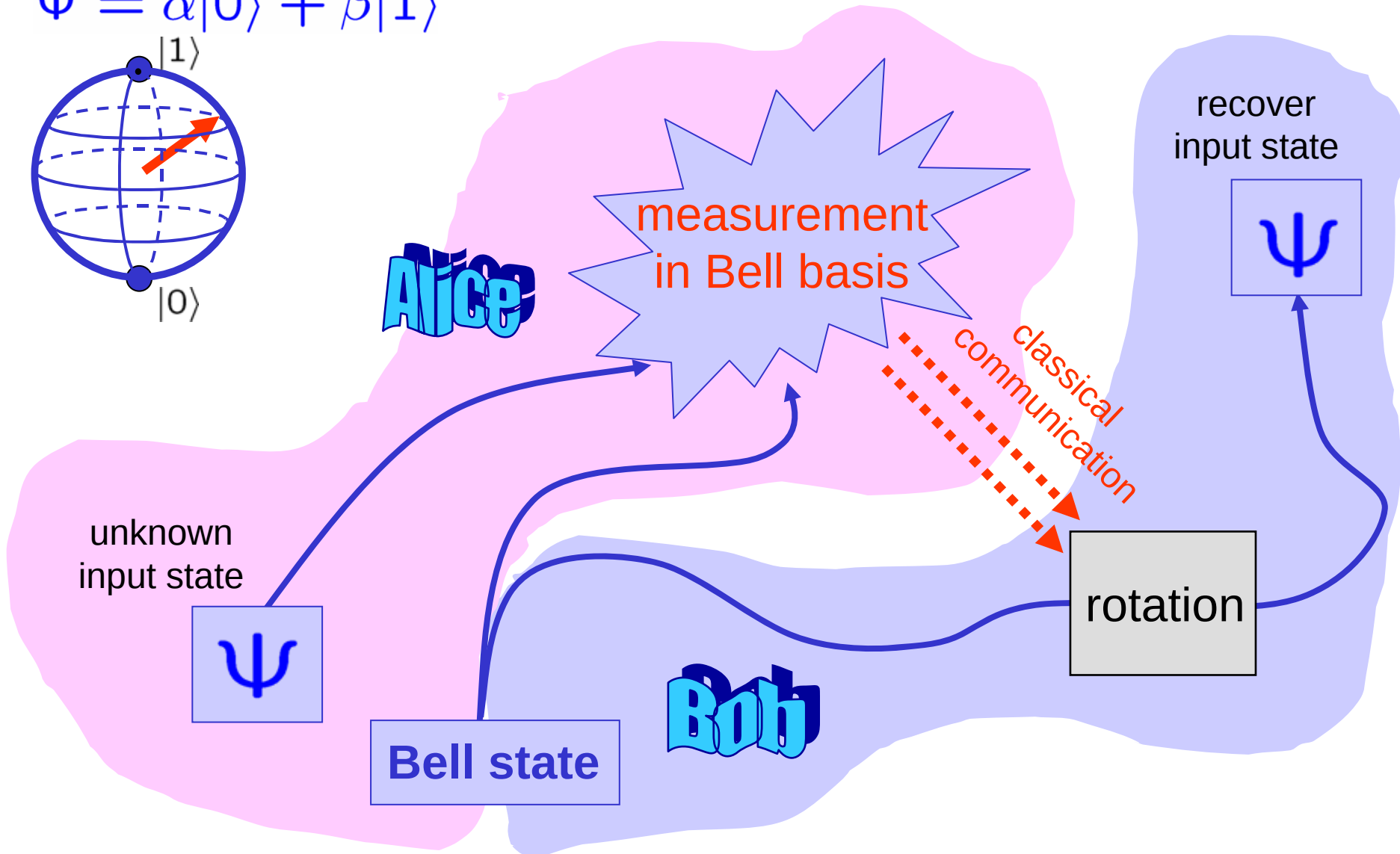
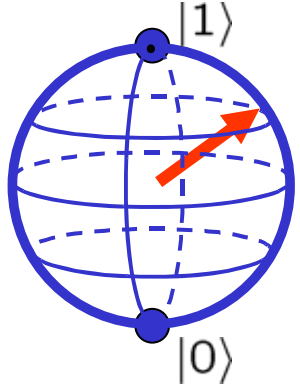
$$|\Phi^\pm\rangle_{12} = \frac{1}{\sqrt{2}} (|0\rangle_1|1\rangle_2 \pm |1\rangle_1|0\rangle_2)$$

measure #1 and #2 in Bell basis: $|\varphi\rangle$ is projected onto one of 4 pure states
e.g. measure $|\Psi^-\rangle_{12}$: perform $-\sigma_z$ operation on qubit #3 to yield input state back

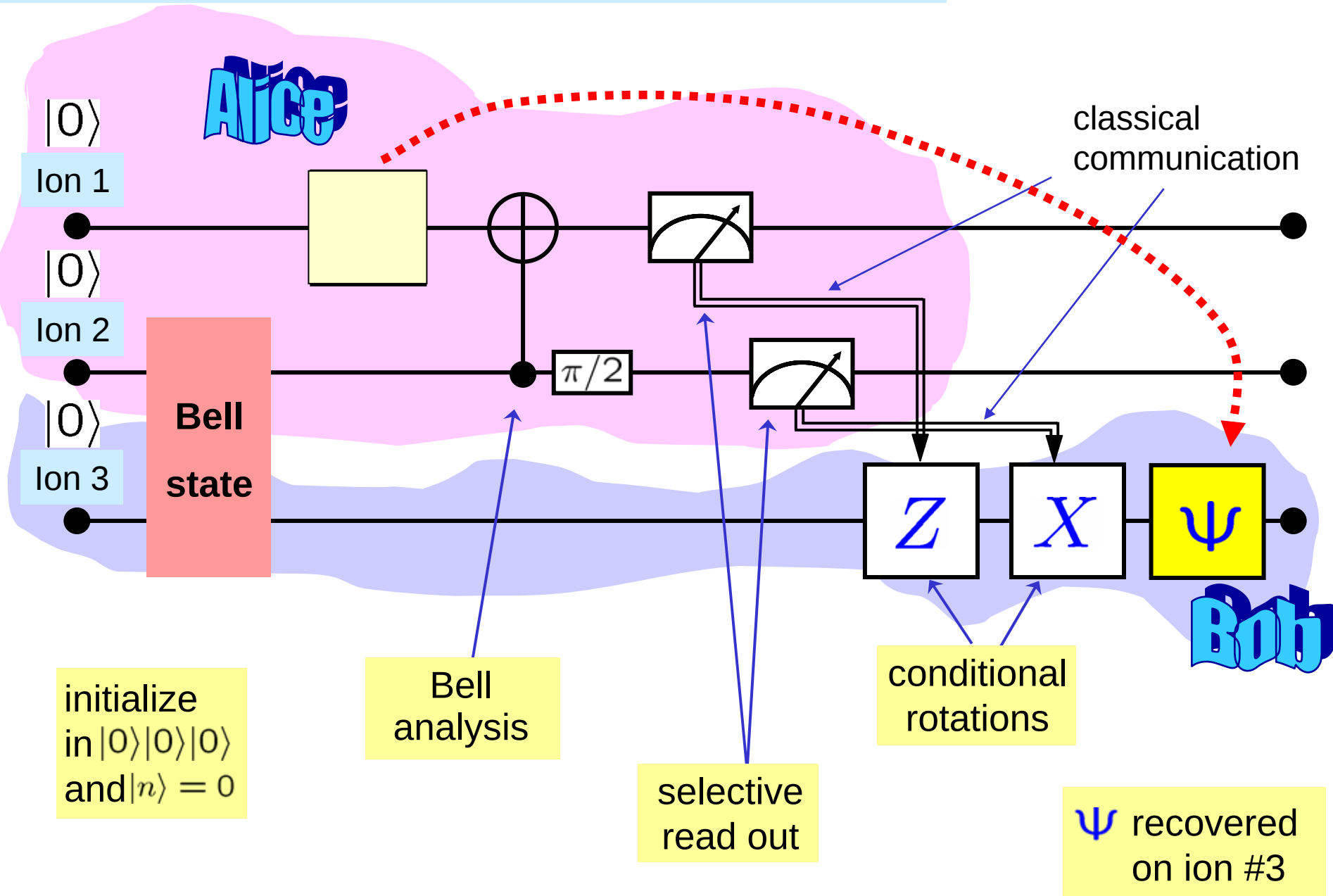
Teleportation

Bennett et al, Phys. Rev. Lett. 70, 1895 (1993)

$$\psi = \alpha|0\rangle + \beta|1\rangle$$

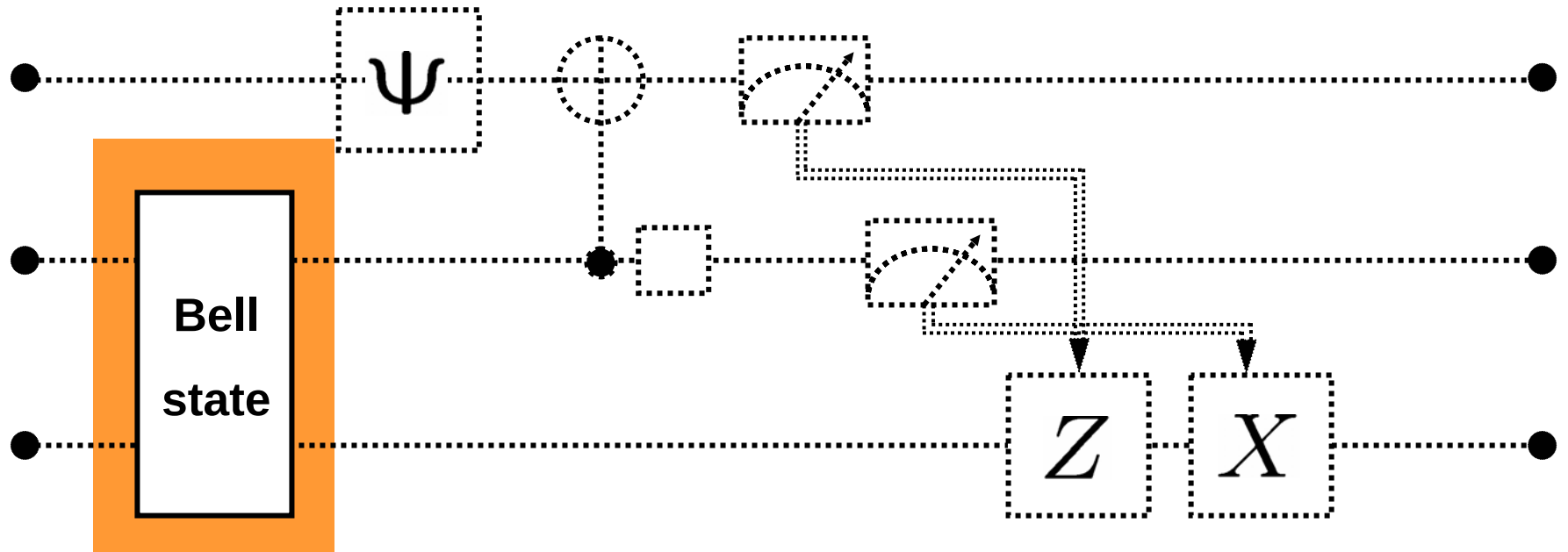
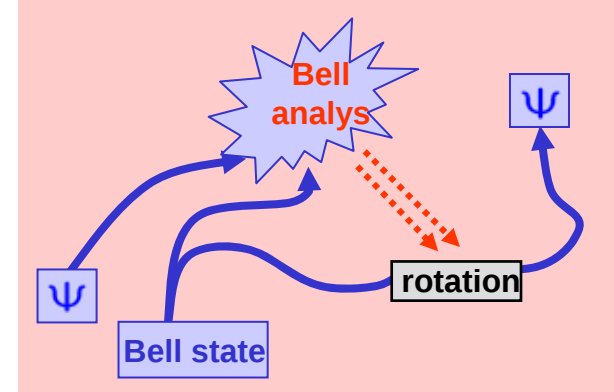


Quantum teleportation protocol



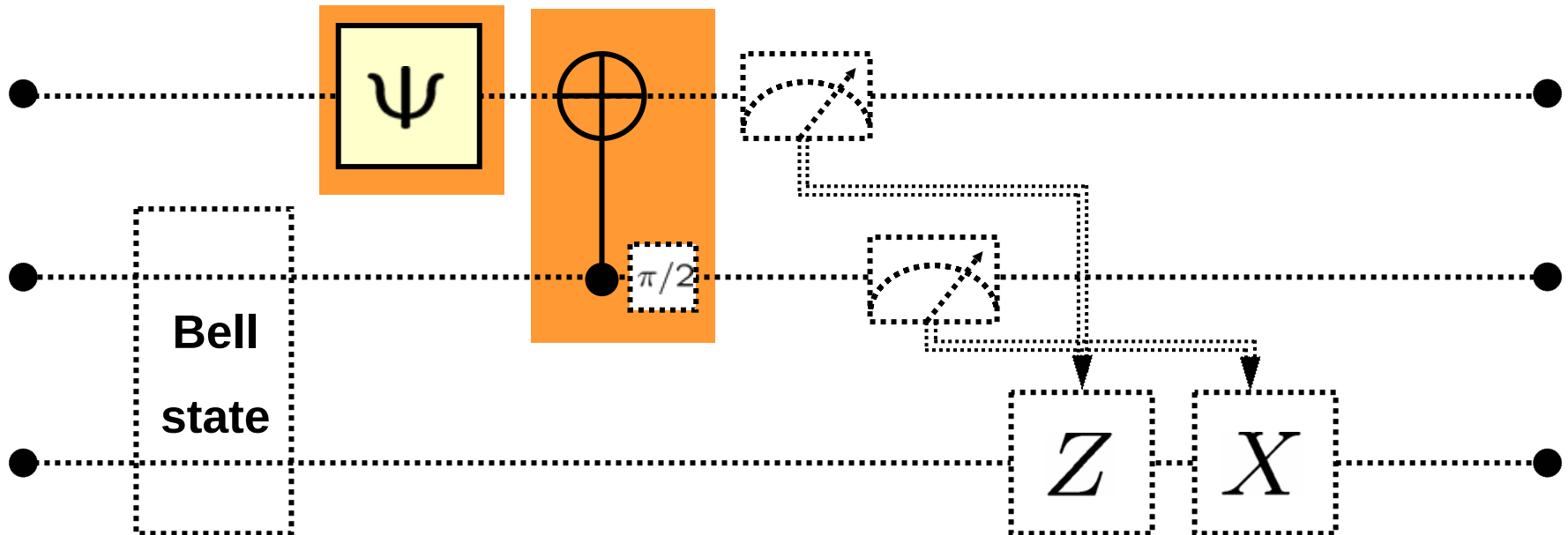
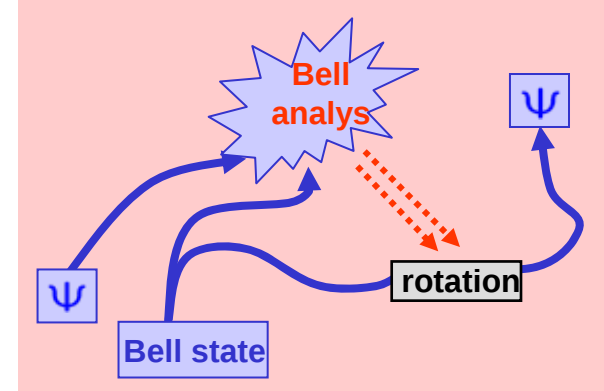
Step by step

1. *Bell state generation*



Step by step

1. *Bell state generation*
2. **Generate ψ**
3. **Bell analysis**



complete Bell analysis

Two qubit
entangled state

CNOT

Superposition state

$\pi/2$

computational
basis state {S,D}

control bit

target bit

$$1/\sqrt{2}\{|S, S\rangle + |D, D\rangle\} \xrightarrow{\text{CNOT}}$$

$$1/\sqrt{2}\{|S + D\rangle|S\rangle\} \xrightarrow{\pi/2}$$

$$|S, S\rangle$$

complete Bell analysis

Two qubit
entangled state

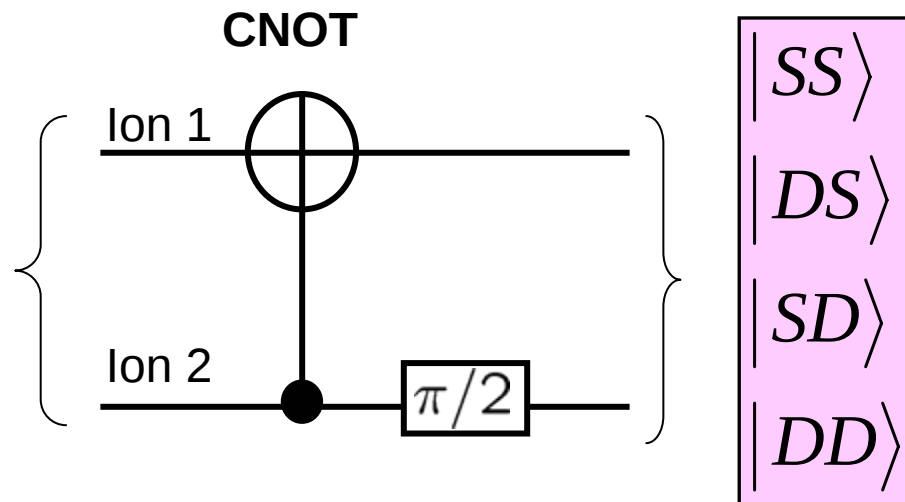
CNOT

Superposition state

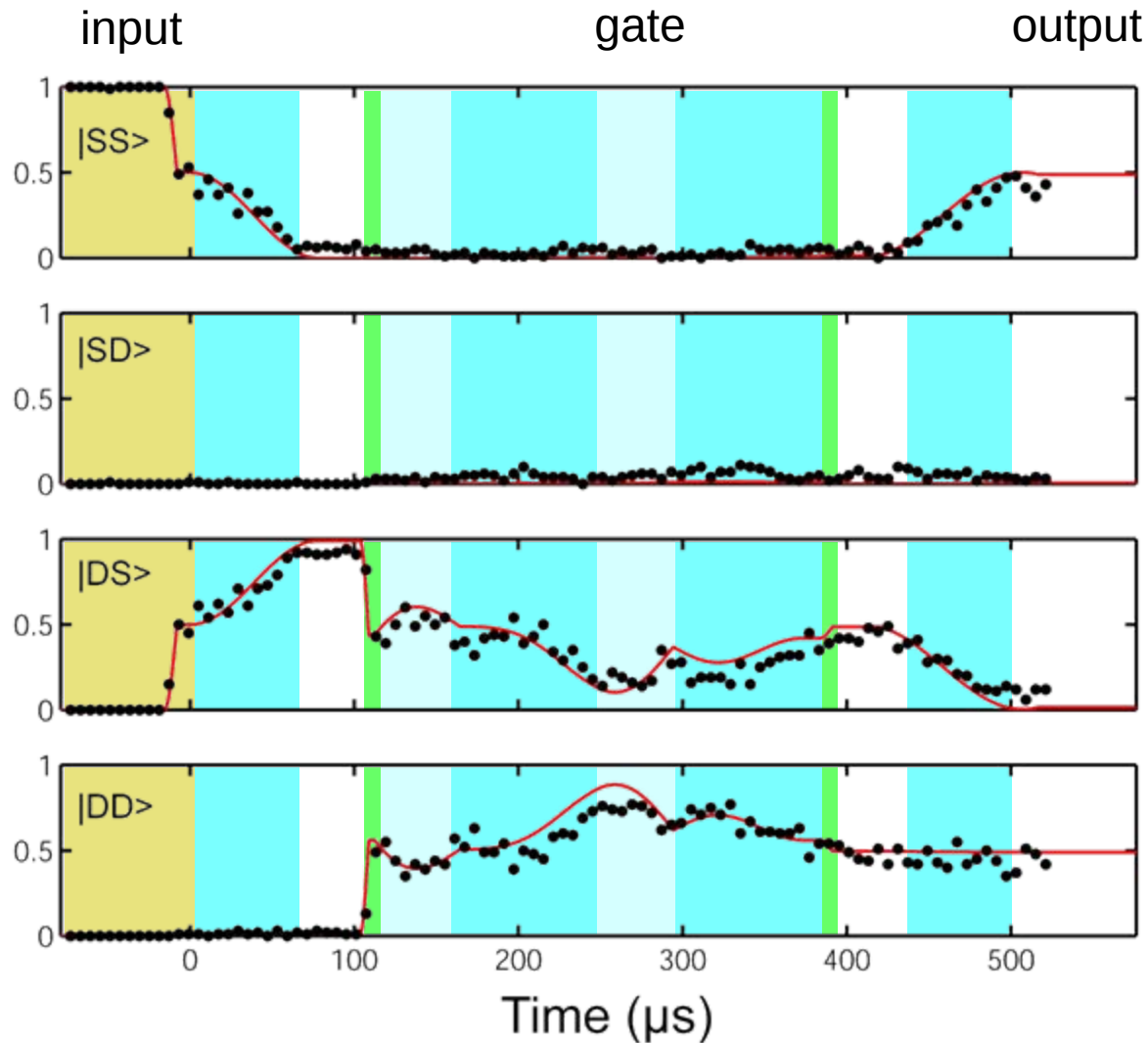
$\pi/2$

computational
basis state {S,D}

$$\begin{aligned}\beta_{00} &= \frac{1}{\sqrt{2}} (|SS\rangle + |DD\rangle) \\ \beta_{10} &= \frac{1}{\sqrt{2}} (|SD\rangle + |DS\rangle) \\ \beta_{01} &= \frac{1}{\sqrt{2}} (|SS\rangle - |DD\rangle) \\ \beta_{11} &= \frac{1}{\sqrt{2}} (|SD\rangle - |DS\rangle)\end{aligned}$$



$$|S+D, S\rangle \xrightarrow{\text{CNOT}} |SS\rangle + |DD\rangle$$

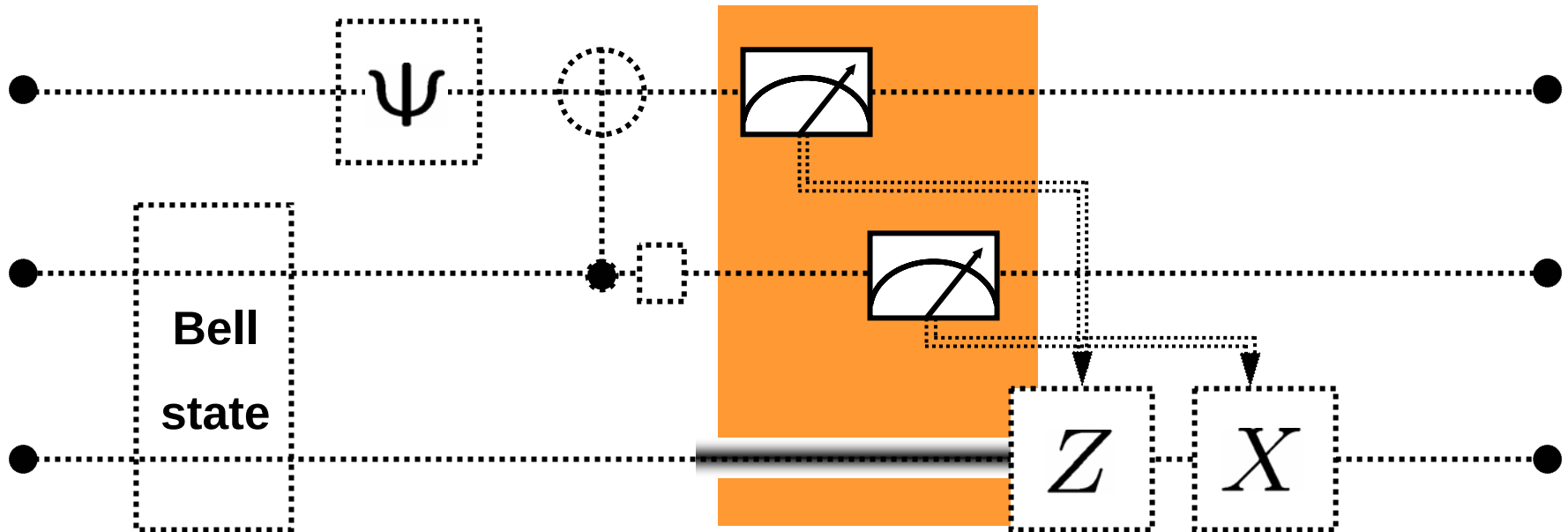
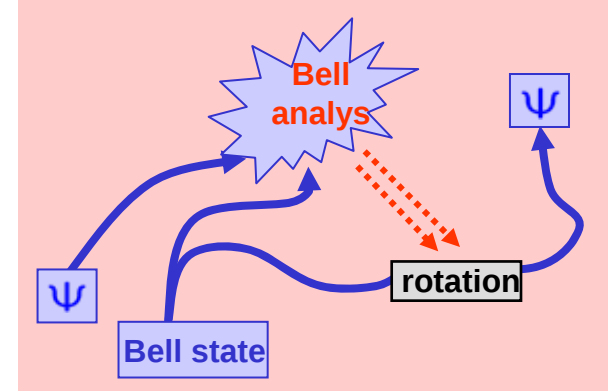


detect

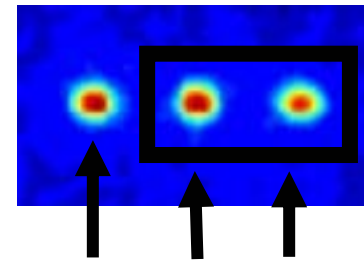


Step by step

1. Bell state generation
2. Generate Ψ
3. Bell analysis
4. **Selective read-out (and hiding)**



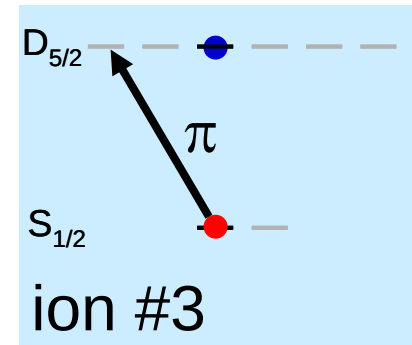
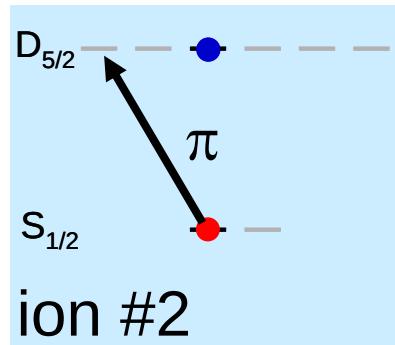
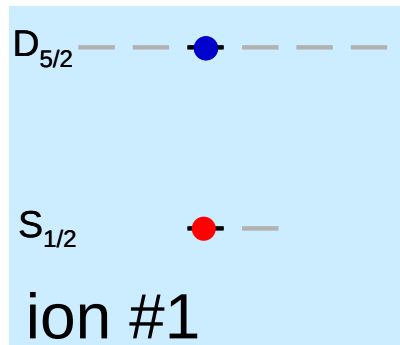
Hiding a qubit



protected !

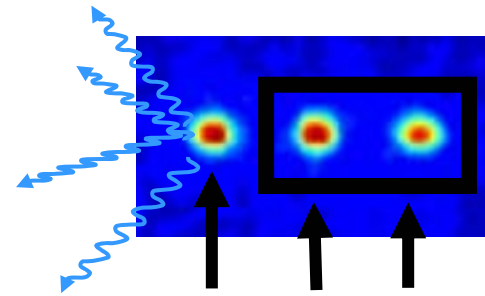
photon
scattering

Zeeman levels



detect quantum state of ion **#1** only

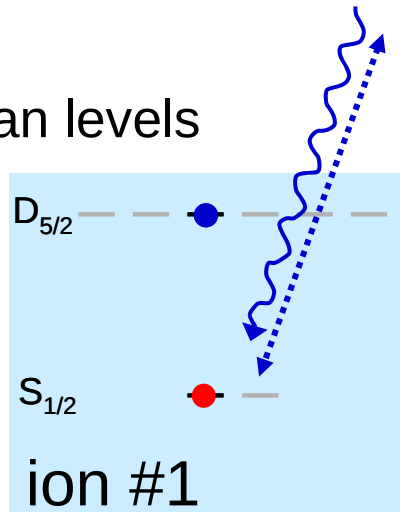
Hiding a qubit



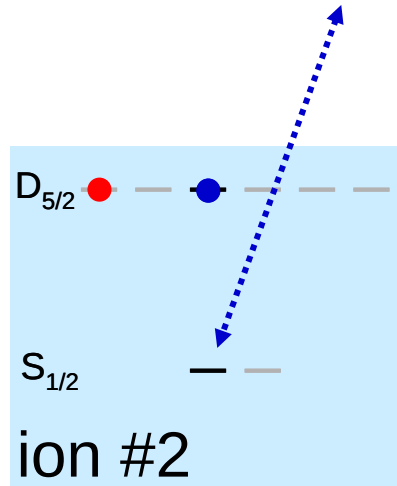
protected !

photon
scattering

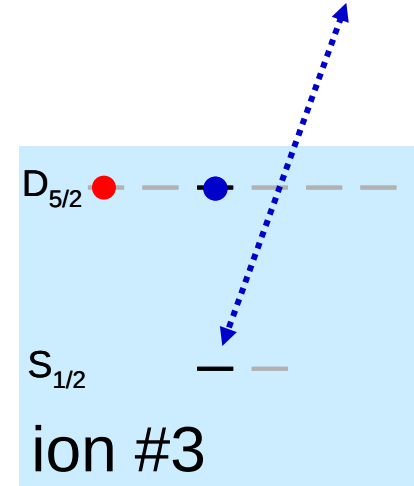
Zeeman levels



ion #1



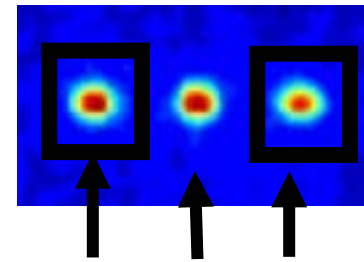
ion #2



ion #3

detect quantum state of ion **#1** only

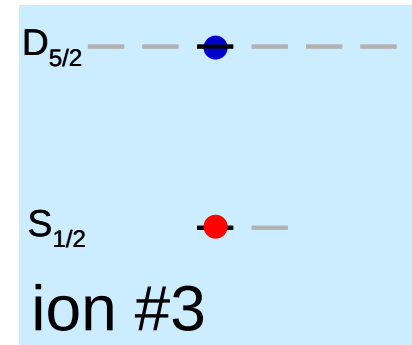
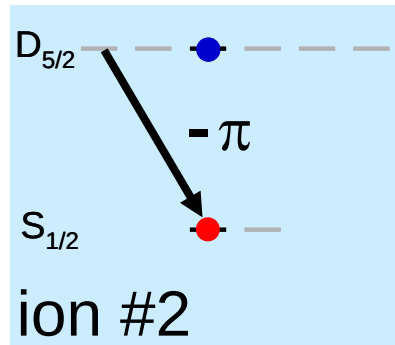
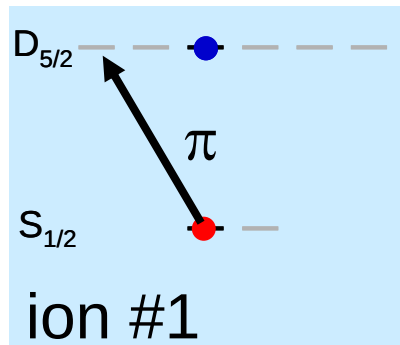
Hiding and unhiding



protected !

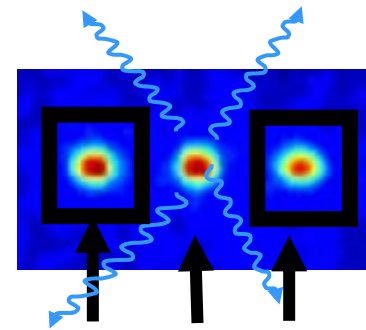
photon
scattering

Zeeman levels



detect quantum state of ion **#2** only

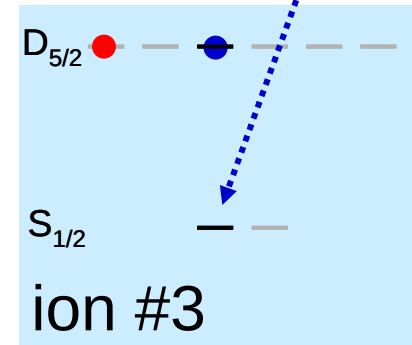
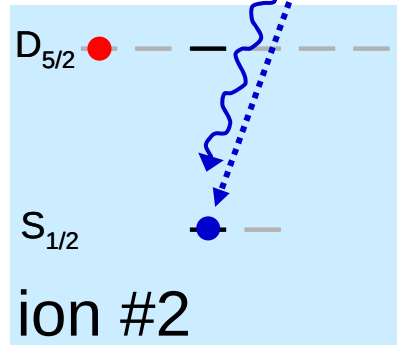
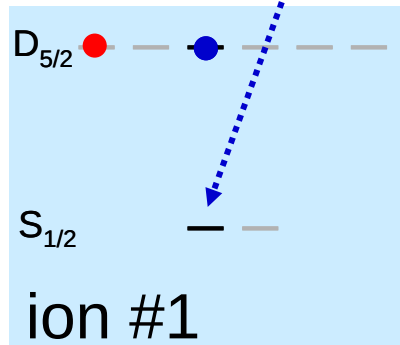
Hiding a qubit



protected !

photon
scattering

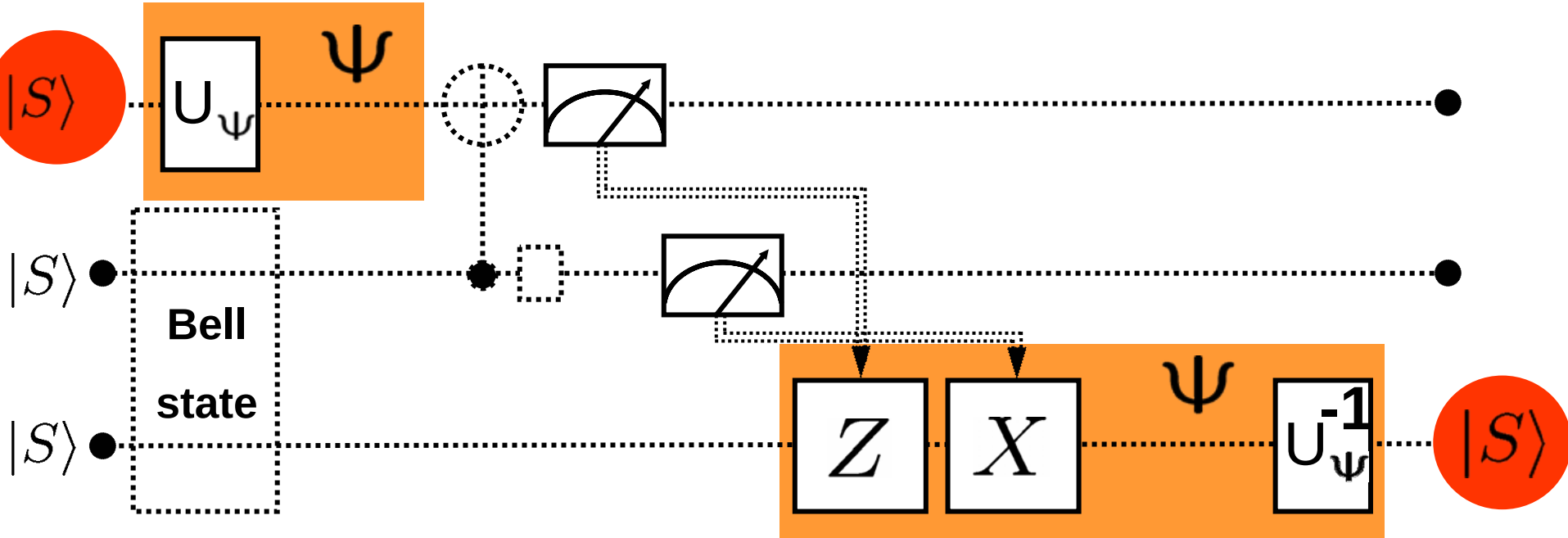
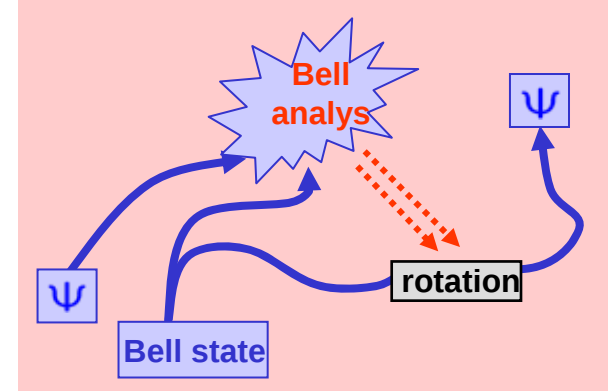
Zeeman levels



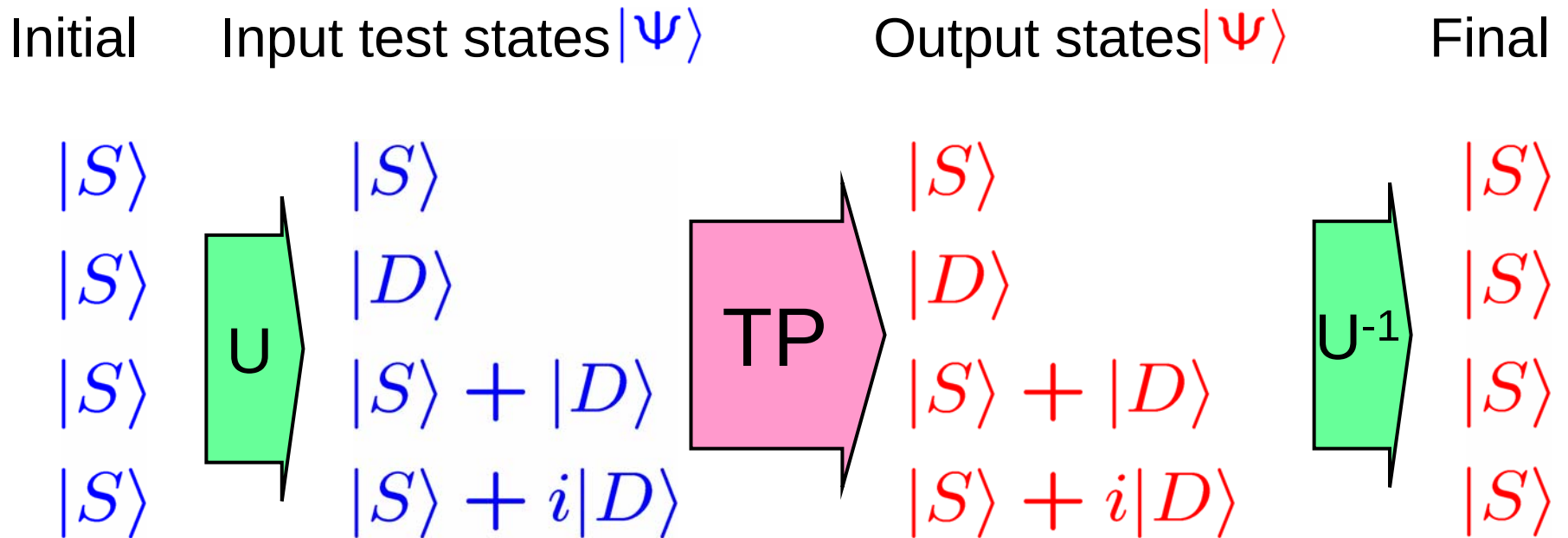
detect quantum state of ion **#2** only

Step by step

1. Bell state generation
2. Generate ψ
3. Bell analysis
4. Selective read-out
5. **Conditional rotations**
6. **Test performance !**



Analysis of teleportation I: Inverse preparation



BeamenNoPost13.seq - WordPad

Datei Bearbeiten Ansicht Einfügen Format ?

File Edit View Insert Format ?

```
%DEFINE5 SpinEcho3 0
%DEFINE6 UseMotion 1

Include('DopplerPreparation.inc')
Include('SideBandCool.inc')

LineTrigger                % Turns line trigger on

Start729(0);
Trigger729(0);              % Also negative trigger to

%%COHERENT MANIPULATION

Rblue(0.5,1.5,3)            % entangle the target ion (
Rcar(1,1.5,2)
ifnot6 Rblue(1,0.5,2)       % write motional qubit to ion
Pause(#5)
if3 Rcar2(1,0,3)            % hide target ion

if(mod(round(#1),4)==0) Pause(10)      % id      In
if(mod(round(#1),4)==1) Rcar(1,0,1)      % not
if(mod(round(#1),4)==2) Rcar(0.5,0,1)    % x1
if(mod(round(#1),4)==3) Rcar(0.5,0.5,1)  % y1

ifnot6 Rblue(1,1.5,2)          % get motional qubit from ion

Rblue(1/sqrt(2),0.5,1)         % CNOT (only the phase
Rblue(1,0,1)                   % CNOT ;CNOT between mot
Rblue(1/sqrt(2),0.5,1)         % CNOT
Rblue(1,0,1)                   % CNOT

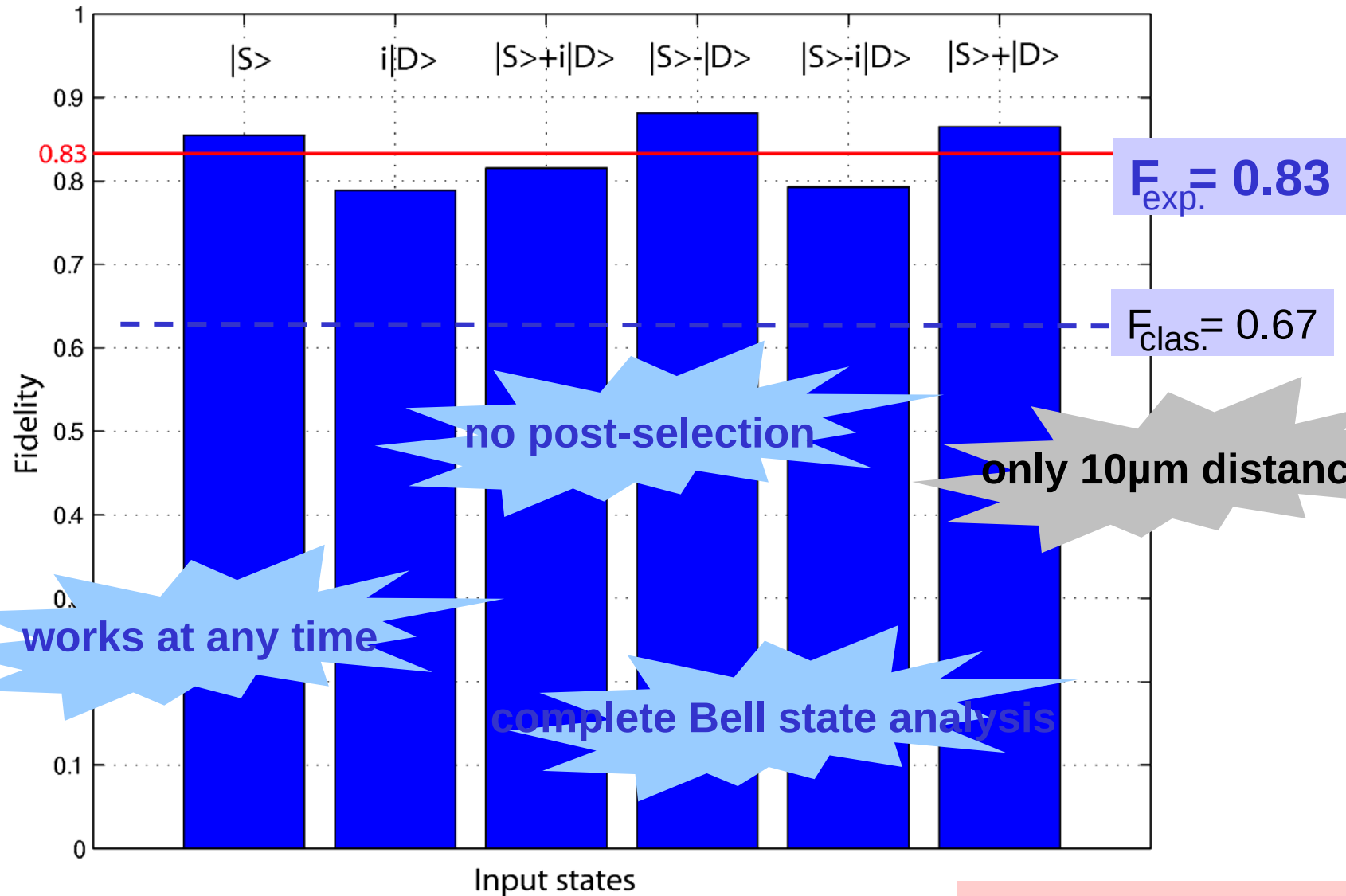
if4 Rcar(1,0.5,1)              %spinecho1

if5 if3 Rcar2(1,1,3)           %unhide for spinecho3
if5 Rcar(1,0.5,3) %spinecho3|
if5 if3 Rcar2(1,0,3)          %hide for spinecho3
```

Drücken Sie F1, um die Hilfe aufzurufen.

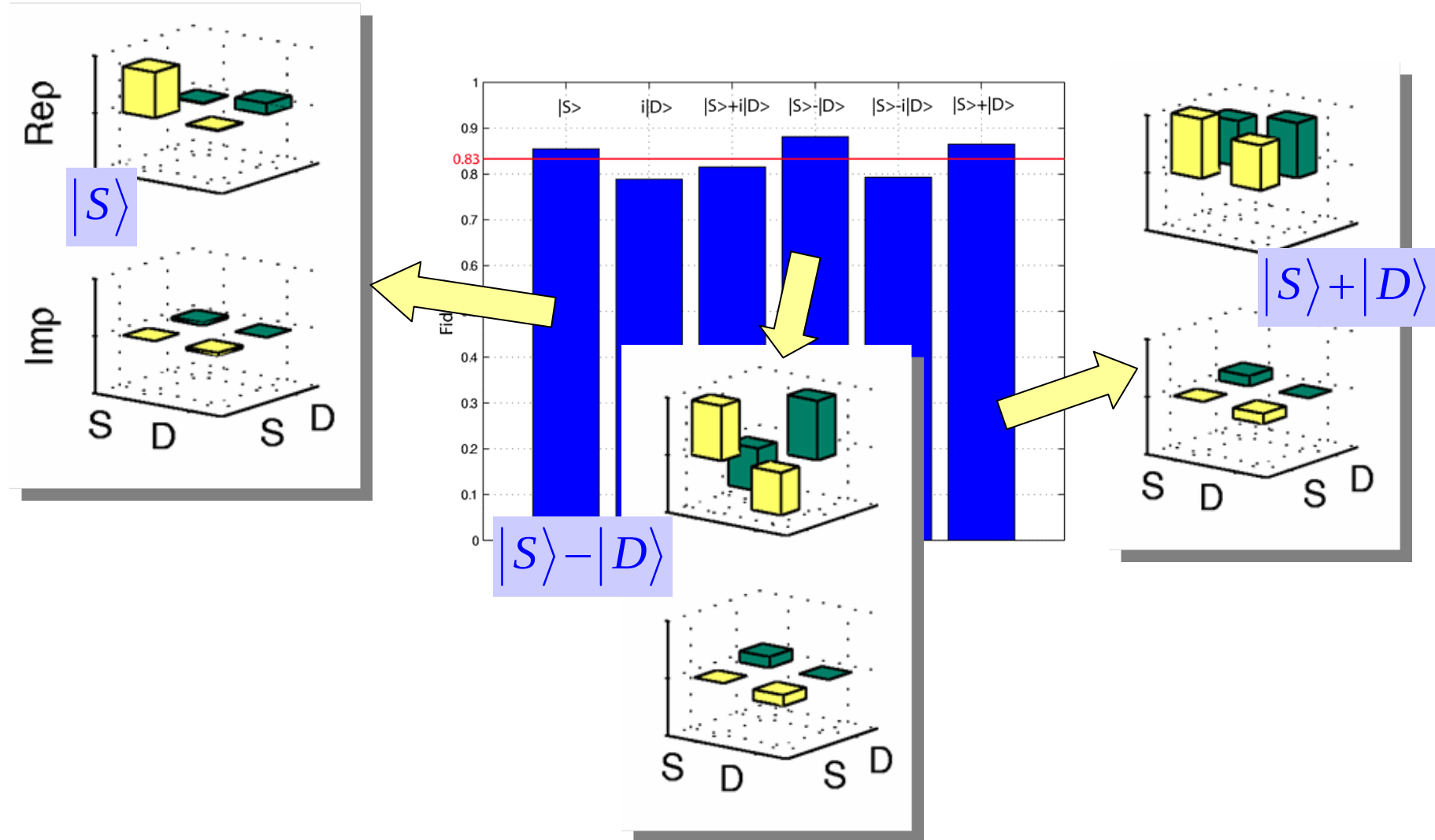


Teleportation „on demand“ : Results



*M. Riebe et al.,
Nature **429**, 734 (2004)*

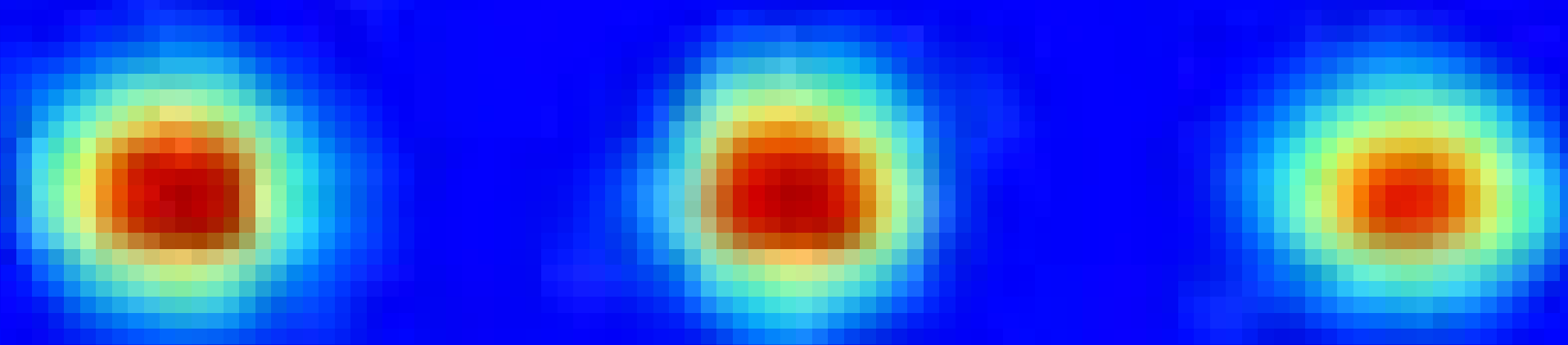
Analysis of teleportation II: Process tomography



GHZ und W Zustände

GHZ state:

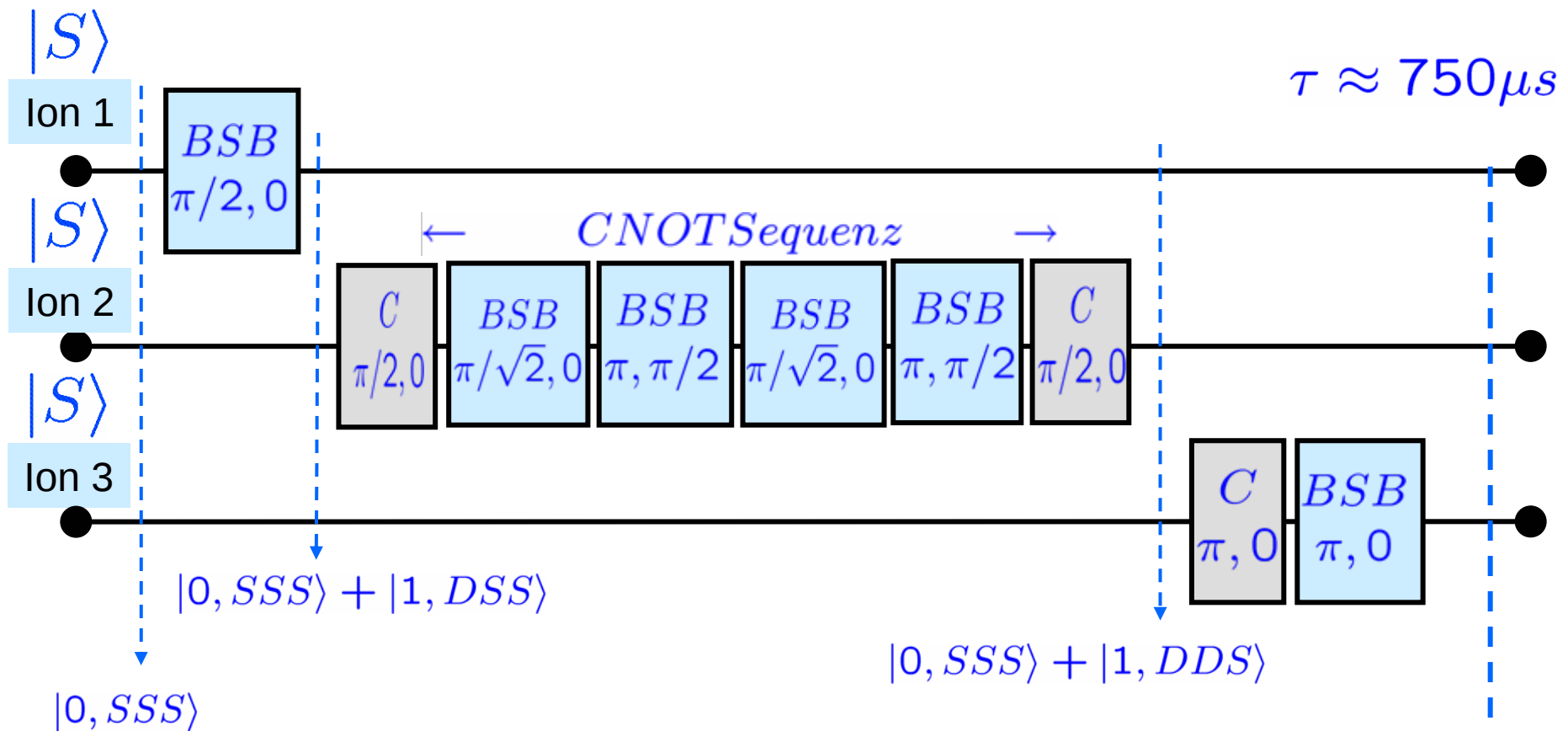
$$|SSS + DDD\rangle$$



W state:

$$|SSD + SDS + DSS\rangle$$

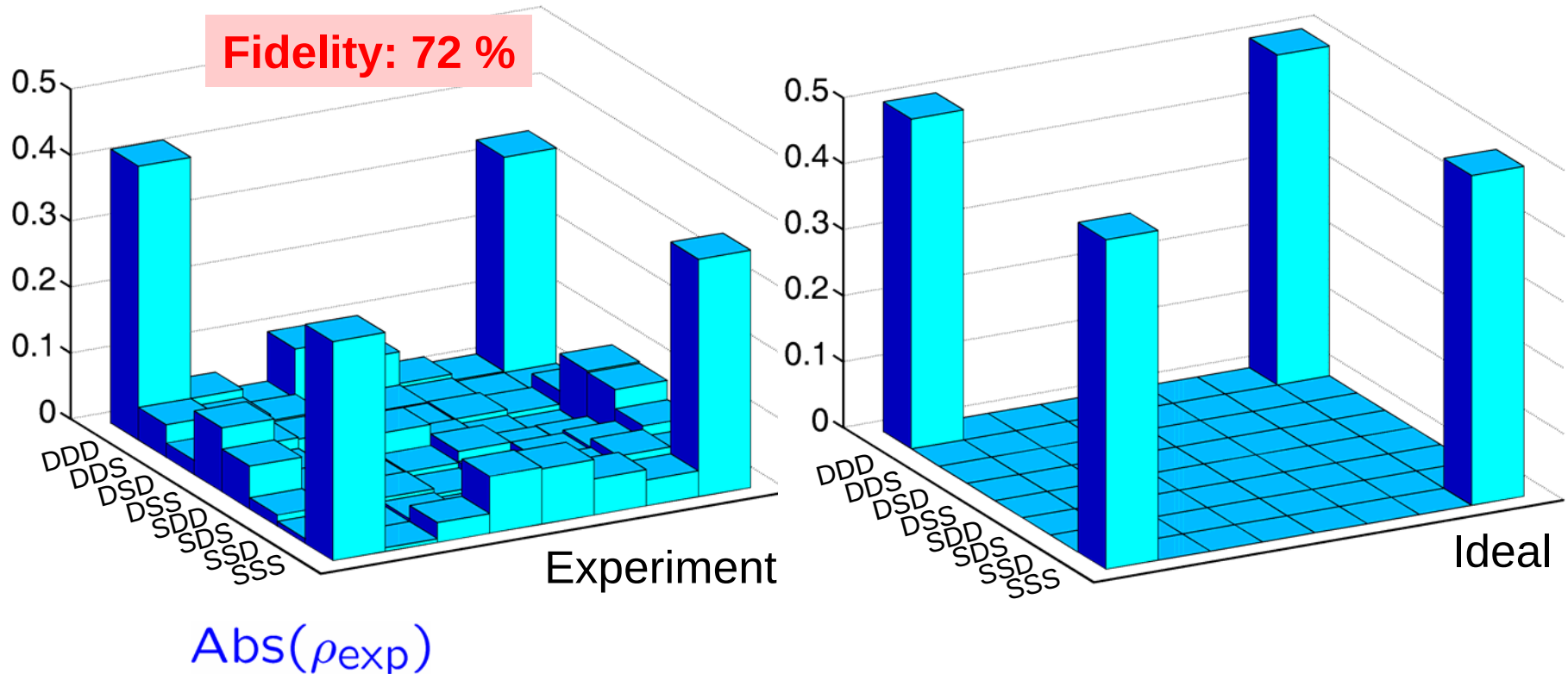
Deterministic generation of GHZ state



$$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}(|DDD\rangle + |SSS\rangle)|0\rangle$$

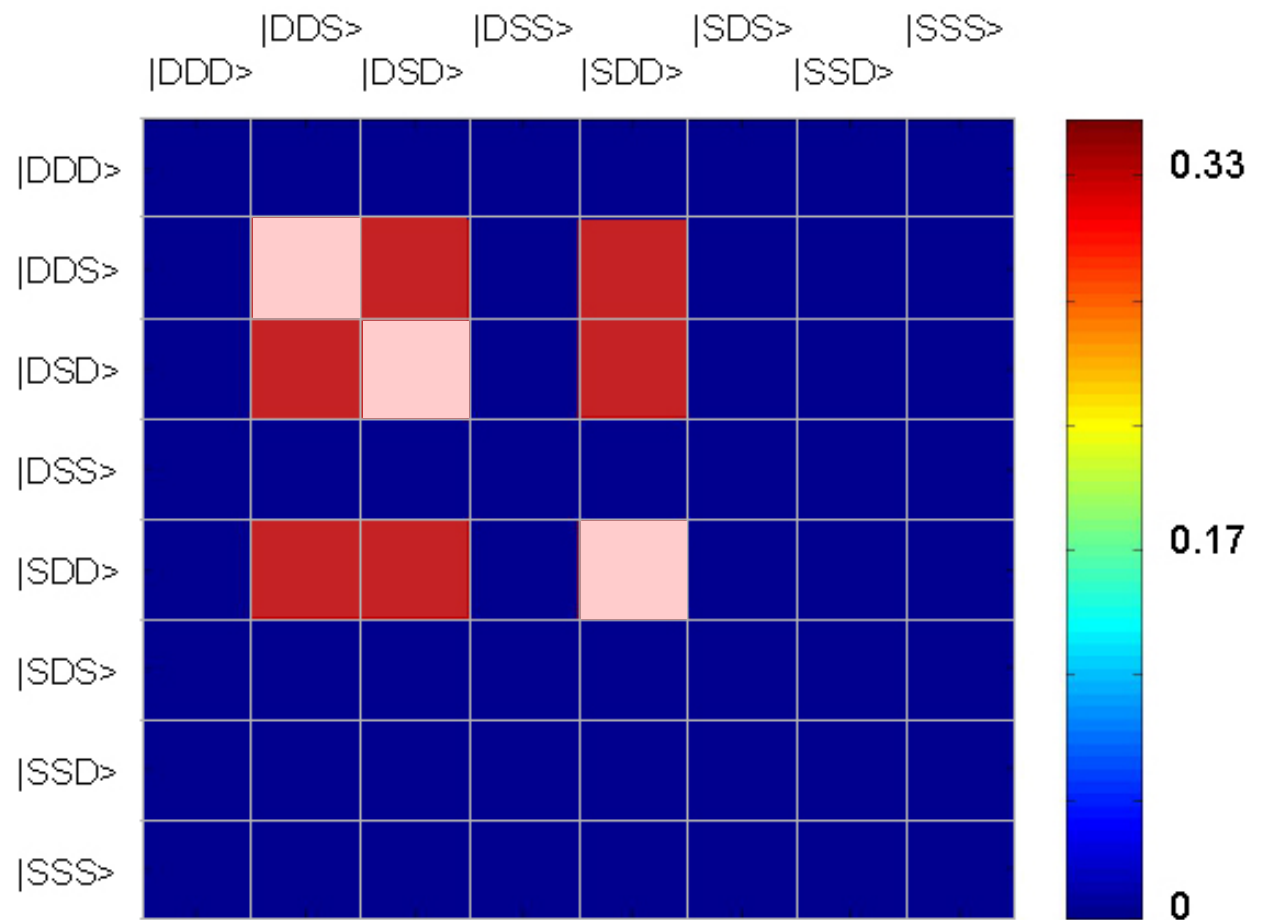
Tomography of the GHZ state

$$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}} (|SSS\rangle - |DDD\rangle)$$



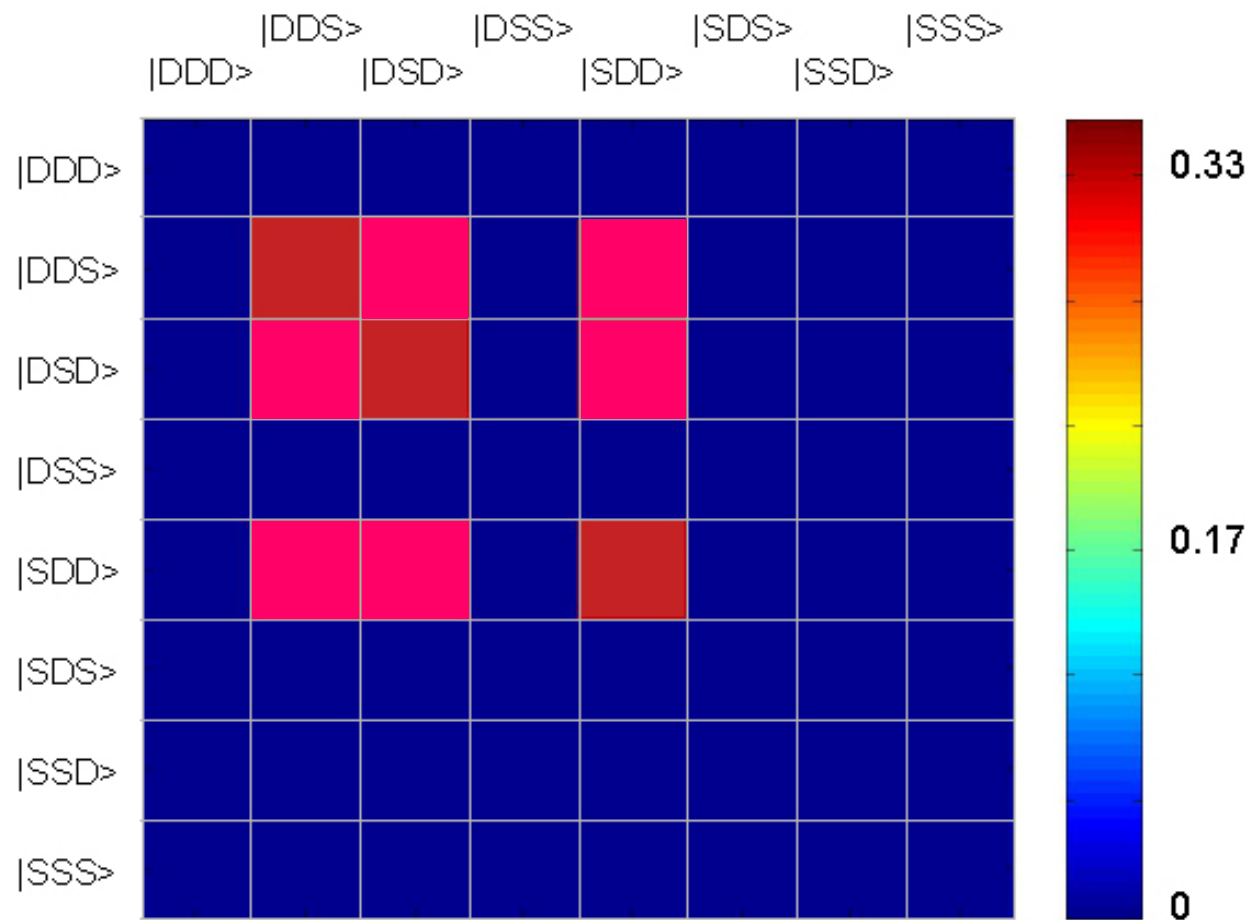
W state: $|SDD\rangle + |DSD\rangle + |DDS\rangle$

ideal 3-ion
density matrix

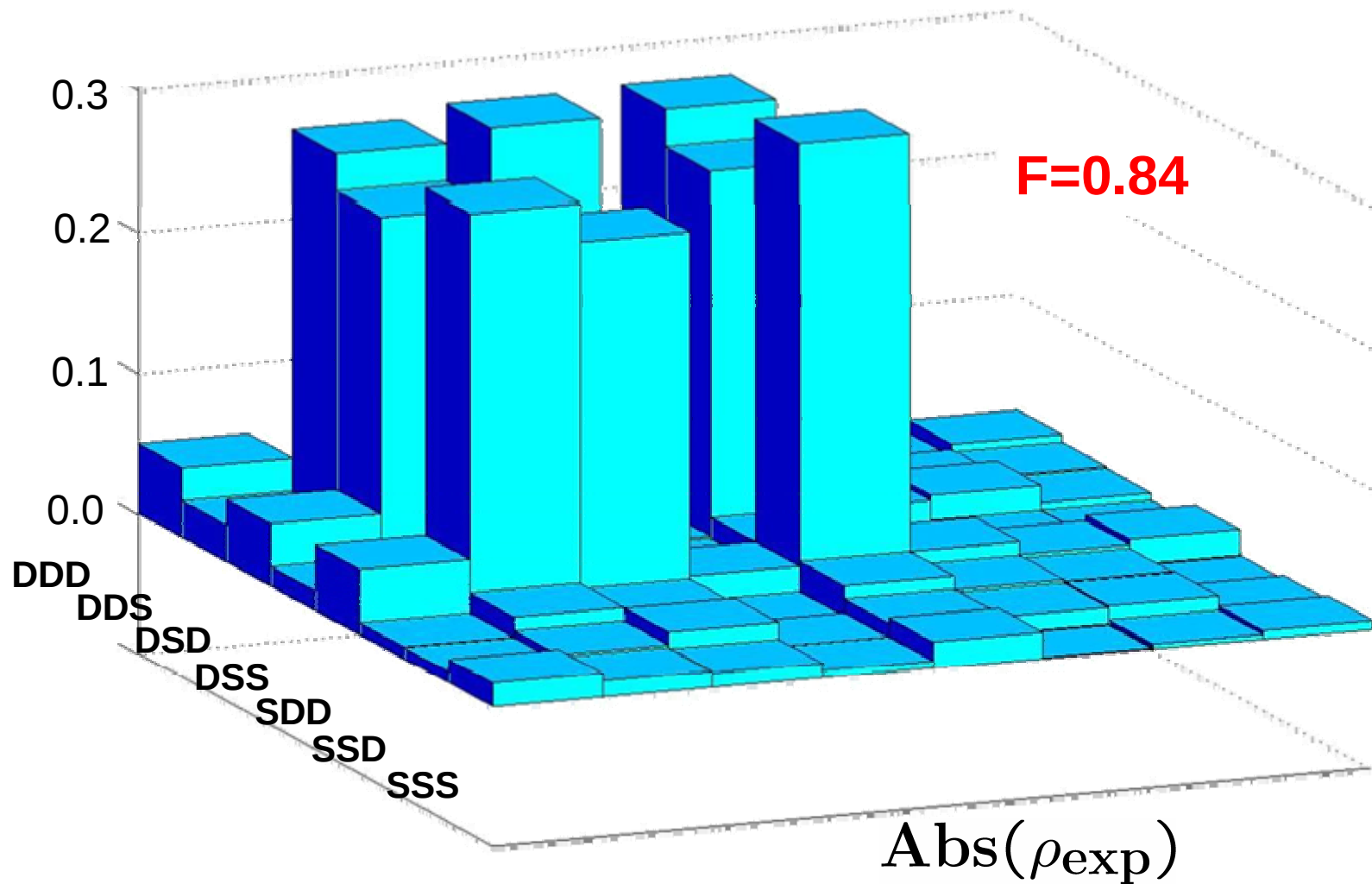


W state: $|SDD\rangle + |DSD\rangle + |DDS\rangle$

ideal 3-ion
density matrix



|SDD> - |DSD> - |DDS>



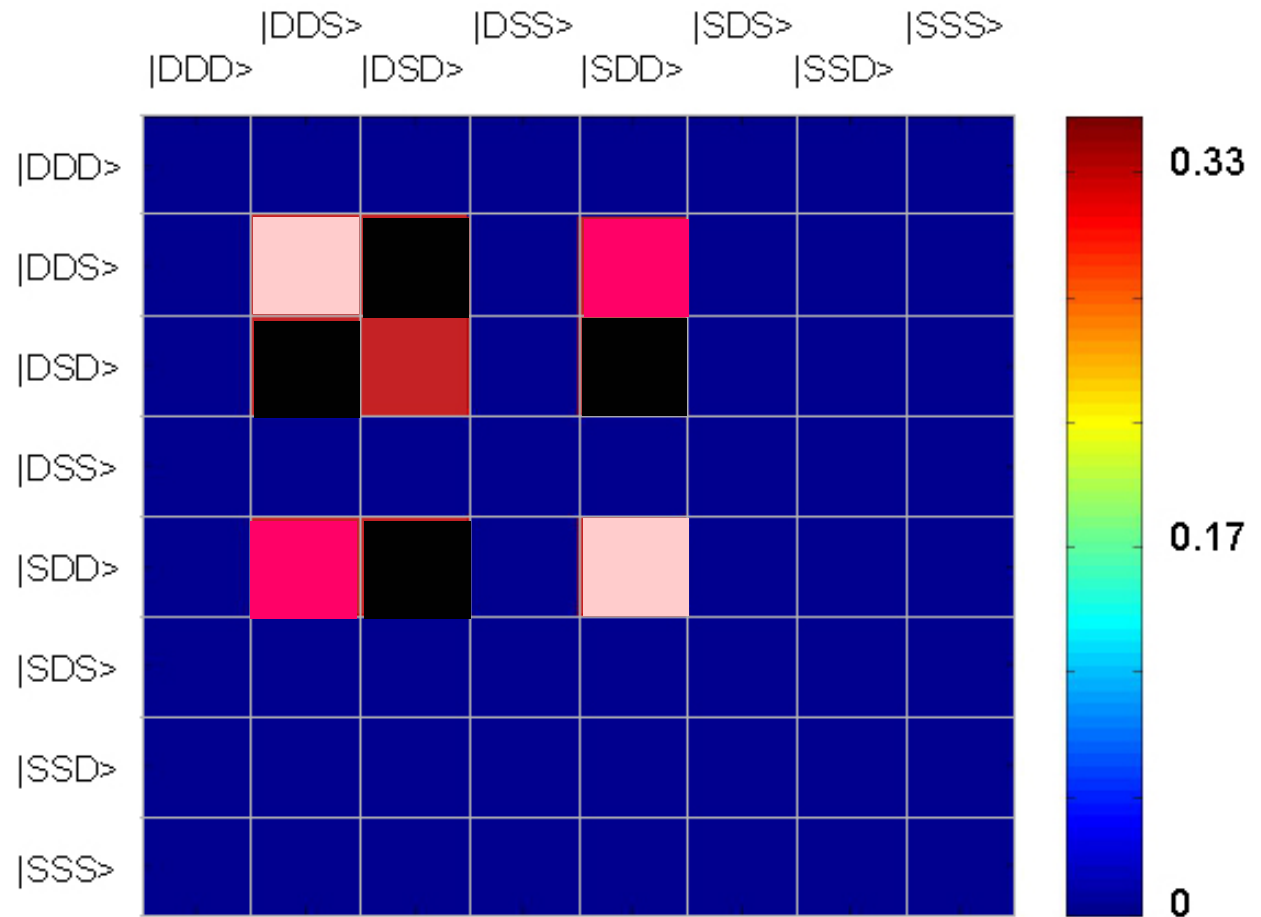
W state: $|SDD\rangle + |DSD\rangle + |DDS\rangle$

measure

measure only
the middle ion in D

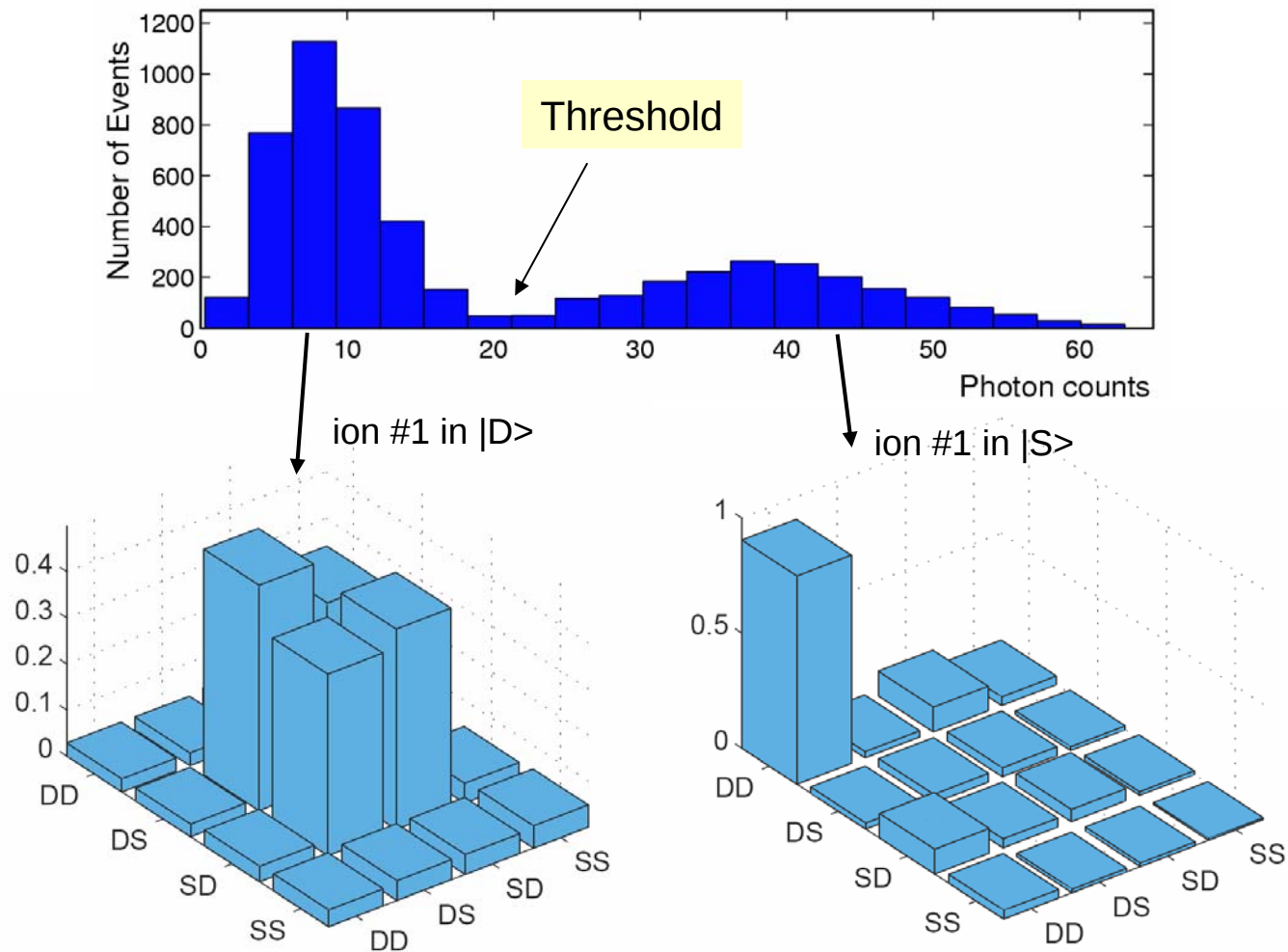


**Bell state
SD+DS**



Selective measurement

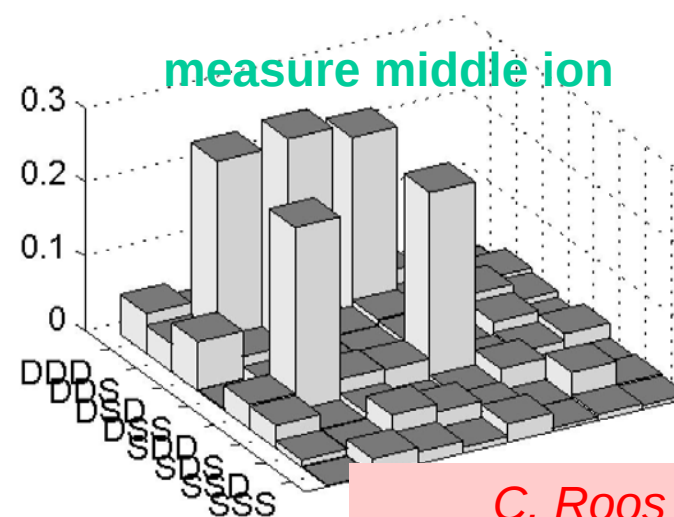
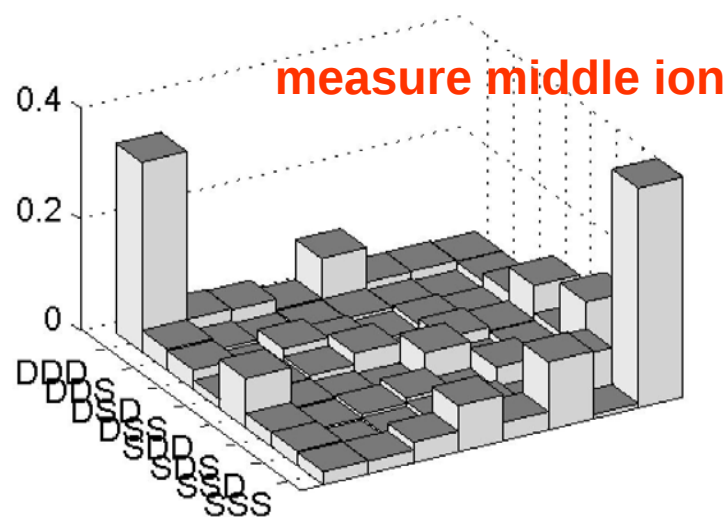
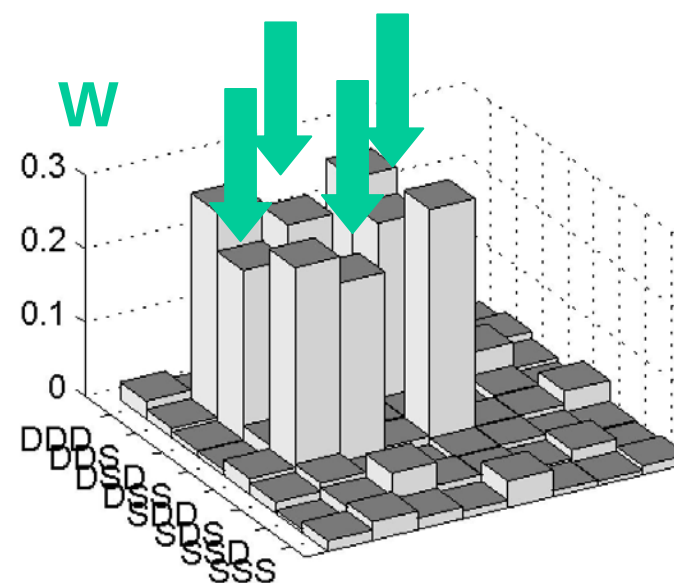
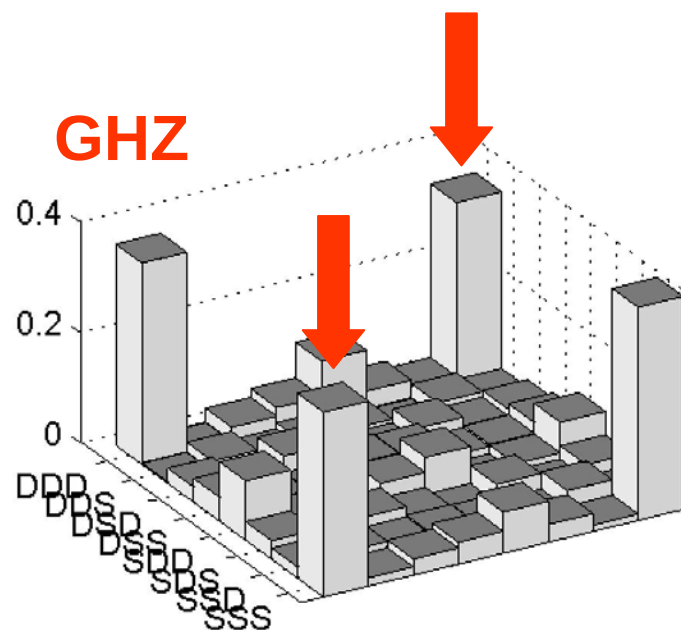
$$|\psi\rangle_W = \frac{1}{\sqrt{3}} (|SDD\rangle + |DSD\rangle + |DDS\rangle)$$



Tomography **after** the measurement result is available!

Selectively projected 3-ion entanglement

Greenberger Horne Zeilinger state



W - state

C. Roos et al.,
Science **304**, 1478 (2004)

selective read-out in rotated basis

$$|\Psi\rangle_{GHZ} = \frac{1}{\sqrt{2}} (|\textcolor{red}{S}DS\rangle - |\textcolor{red}{D}SD\rangle)$$

rotate qubit #1 by $\pi/2$

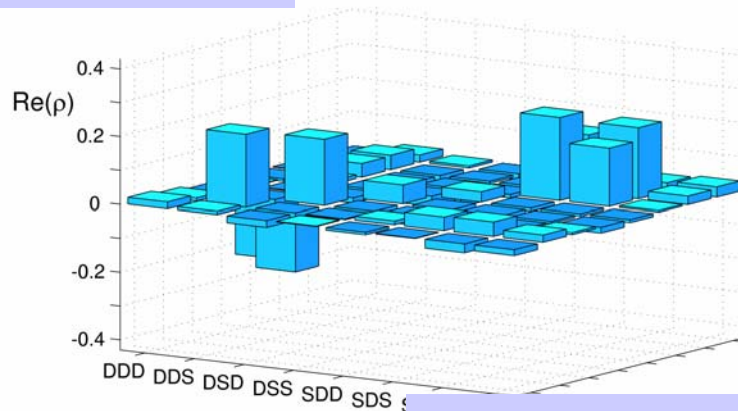
quantum algorithm
includes decisions

$$\rightarrow \frac{1}{\sqrt{2}} (|(S+D)DS\rangle - |(D-S)SD\rangle) = \frac{1}{\sqrt{2}} (|\textcolor{red}{S}DS\rangle + |\textcolor{blue}{D}DS\rangle - |\textcolor{blue}{D}SD\rangle + |\textcolor{red}{S}SD\rangle)$$

$$\rightarrow \{(|\textcolor{blue}{D}S\rangle - |\textcolor{red}{S}D\rangle), (|\textcolor{red}{D}S\rangle + |\textcolor{red}{S}D\rangle)\}$$

measure Ion #1

mixture of two Bell states
(Fidelity: 78 %)



finally: pure Bell state
(Fidelity: 77 %)

depending on the
outcome „ $|D\rangle_1$ “:

π -pulse on ion #1 and
Z-rotation on ion #3

