Übungsblatt 2 - QIV I

2.1: (Linear dependence: example) Show that (1,-1, (1,2) and (2,1) are linearly dependent.

2.6: Show that any inner product (·,·) is conjugate-linear in the first argument,

$$\left(\sum_i \lambda_i |w\rangle, |v\rangle\right) = \sum_i \lambda_i^* (|w_i\rangle, |v\rangle).$$

2.7: Verify that $|w\rangle \equiv (1,1)$ and $|v\rangle \equiv (1,-1)$ are orthogonal. What are the normalized forms of these vectors?

2.9: (Pauli operators and the outer product) The Pauli matrices can be considered as operators with respect to an orthonormal basis $|0\rangle, |1\rangle$ for a two-dimensional Hilbert space. Express each of the Pauli operators in the outer product notation.

2.11: (Eigendecomposition of the Pauli matrices) Find the eigenvectors, eigenvalues, and diagonal representations of the Pauli matrices *X, Y*, and *Z.*

2.13: If $|w\rangle$ and $|v\rangle$ are any vectors, show that $(|w\rangle\langle v|)^+ = |v\rangle\langle w|$.

2.16: Show that any projector *P* satisfies the equation $P^2 = P$.

2.17: Show that a normal matrix is Hermitian if and only if it has real eigenvalues.

2.18: Show that all eigenvalues of a unitary matrix have modulus 1, that is, can be written in the form $e^{i\epsilon}$ for some real θ.

2.22: Prove that two eigenvectors of a Hermitian operator with different eigenvalues are necessarily orthogonal.

2.23: Show that the eigenvalues of a procector *P* are all either 0 or 1.

2.24: (Hermiticity of positive operators) Show that a positive operator is necessarily Hermitian . (Hint: Show that an arbitrary *A* can be written *A* = *B* + i*C* where *B* and *C* are Hermitian.)