Theoretical Quantum Optics

Sheet 7 **SS 2017**
Date of issue: 19-06-2017 Discussion: 23-06-2017

Exercise 11 *The Bloch vector and the 2π-pulse*

Within the rotating-wave approximation, we consider the dynamics of a two-level atom with the transition frequency ω_{eg} (corresponding to no change of the magnetic quantum number, that is $\Delta m = m_e - m_g = 0$), interacting with a near-resonant and linearly polarized electric field $\vec{E}(t) = \vec{E}_0(t)\cos(\omega_L t - \varphi)$ of the frequency ω_L. Here we focus on the case of the light phase $\varphi = 0$ and detuning $\Delta = \omega_{eg} - \omega_L \neq 0$. In this case, the pseudofield vector $\vec{\Omega}'(t) = \{-\Omega_0(t), 0, \Delta\}$ has components along the 1- and 3-axis. By solving the Bloch equations for the Bloch vector $\vec{r}'(t)$, find the form of the function $\Omega_0(t)$, such that $\vec{r}'(+\infty) = \vec{r}'(-\infty) = \{0, 0, -1\}$, valid for any values of the detuning Δ.

(4 points)

Hint: We can look for the solution of the Bloch equations in the form

$$r_3'(t) \;=\; -1 + 2B\sin^2\left[\frac{1}{2}\theta(t)\right] \quad \text{with} \quad \theta(t) = \int_{-\infty}^{t} \Omega_0(t')\,\mathrm{d}t' ,$$

where B is only a function of Δ, such that $|B(\Delta)| \leq 1$ and $B(\Delta = 0) = 1$. The aim of the exercise is to obtain and to solve the equation for $\theta(t)$ from the Bloch equations.