Theoretical Quantum Optics

Sheet 9 **SS 2017**
Date of issue: 08-07-2017 Discussion: 21-07-2017

Exercise 14 *The Dark state in a Λ-system*

Let us consider three-level system consisting of the two low-energy states $|g_{1,2}\rangle$ and the high-energy state $|e\rangle$, see Fig. 1.

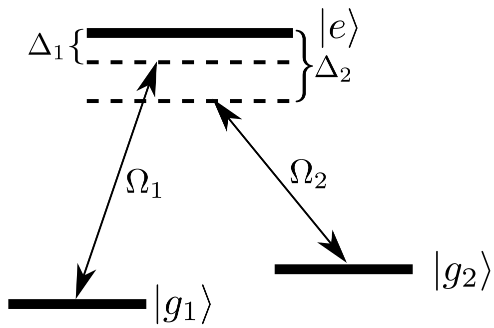

Figure 1: Scheme of atomic states in the Λ-type three-level system. Here $\Omega_{1,2} = \frac{1}{\hbar}\left|\vec{d}_{eg_{1,2}}\vec{\mathcal{E}}_{1,2}(t)\right|$ are the Rabi frequencies and $\Delta_{1,2} = \omega_{eg_{1,2}} - \omega_{1,2}$ are the corresponding detunings.

a) Within the rotating-wave approximation, derive the Hamiltonian, which describes the time dynamics of the Λ-system, Fig. 1, interacting with the two electric fields

$$\vec{E}_{1,2} = \vec{\mathcal{E}}_{1,2}(t)\cos(\omega_{1,2}t + \varphi_{1,2}) \ .$$

For simplicity, we can take $E_e = 0$ and the diagonal matrix elements $\vec{d}_{k,k} = 0$ with $k = e$, g_1, or g_2.

(2 points)

b) By making the linear transformation, reduce the Schrödinger equation for the coefficients c_e, c_{g_1}, c_{g_2} of the state vector

$$|\psi(t)\rangle = c_e(t)\,|e\rangle + c_{g_1}\,|g_1\rangle + c_{g_2}\,|g_2\rangle$$

to the form

$$i\frac{\mathrm{d}}{\mathrm{dt}}\begin{pmatrix}\tilde{c}_e\\ \tilde{c}_{g_1}\\ \tilde{c}_{g_2}\end{pmatrix} = \hat{H}_\Lambda\begin{pmatrix}\tilde{c}_e\\ \tilde{c}_{g_1}\\ \tilde{c}_{g_2}\end{pmatrix} \ ,$$

where

$$\hat{H}_\Lambda = \begin{pmatrix} 0 & -\frac{1}{2}\Omega_1 & -\frac{1}{2}\Omega_2 \\ -\frac{1}{2}\Omega_1 & -\tilde{\Delta}_1 & 0 \\ -\frac{1}{2}\Omega_2 & 0 & -\tilde{\Delta}_2 \end{pmatrix}$$

with $\tilde{\Delta}_{1,2} \equiv \Delta_{1,2} + \dot{\varphi}_{1,2}$.

Find the condition under which the Hamiltonian $\hat{H}_\Lambda$ has an eigenvalue independent of $\Omega_{1,2}$ and obtain the corresponding eigenvector in terms of $|e\rangle$, $|g_1\rangle$, and $|g_2\rangle$ state vectors.

(3 points)