Introduction to Asymptotic Methods

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Sheet 2

Exercise 4
For $x \to +\infty$, find the first two terms in the asymptotic expansion of the function $f(x)$, defined as

$$f(x) = \int_0^x \sqrt{1+t^2}\, dt,$$

and compare the result obtained with the exact analytical formula.

(2 Points)

Exercise 5
For $x \to +\infty$, the function $f(x) \sim x^{\nu}$ with $\mathrm{Re}(\nu) = -1$ and $\mathrm{Im}(\nu) \neq 0$. By considering an example of such a function, $f(x) = x^{\nu} + 1/(x \log^{\mu}(x))$ with $\mu > 0$, find the condition on the parameter μ, under which the following estimation of the integral

$$\int_a^x f(t)dt = \mathcal{O}(1)$$

for $x \to +\infty$ and any finite $a > 0$ is correct.

(2 Points)

Exercise 6
Find the solution $w \equiv w(z, \alpha_1, \beta_1, \alpha_2, \beta_2)$ of the transcendental equation

$$z^{\alpha_1 w + \beta_1} = \alpha_2 w + \beta_2$$

in terms of the Lambert function. Here $z > 0$, $\alpha_{1,2}$ and $\beta_{1,2}$ are the constants.

(1 Point)

Exercise 7
Calculate the integrals

$$\int W_0(x)dx \text{ and } \int x^{-1} W_0(x)dx,$$

with $W_0(x)$ being the Lambert function, that is $W_0(x) \geq -1$ and $W_0(0) = 0$.

(2 Points)

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