Exercise 15
Show that the sum
\[ a_n = \sum_{k=0}^{\infty} \frac{(-1)^k}{\ln(n+k)} \]
has the asymptotic behavior
\[ a_n = \frac{1}{2 \ln(n)} + \mathcal{O}\left(\frac{1}{n \ln^2(n)}\right) \]
as \( n \to \infty \).

(2 Points)

Exercise 16
Find the asymptotic behavior of the sum
\[ \sum_{k=1}^{\infty} \frac{1}{k(k+n)} \]
for large positive and integer \( n \).

(3 Points)

Exercise 17
By using Euler-Maclaurin formula, find the asymptotic expansion for the sum
\[ \sum_{n=1}^{\infty} \ln \left(1 - e^{-nx}\right) \]
as \( x \to 0 \) and \( x > 0 \), which is valid in the order of \( x \).

**Hint:** To find the correct result, use the definition of the Bernoulli numbers \( B_n \) via the generation function, that is
\[ \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n, \]
and study the dependence of all terms in the Euler-Maclaurin formula at small values of \( x \).

(4 Points)