Introduction to Asymptotic Methods

Maxim Efremov
Institut für Quantenphysik, Universität Ulm Summer Semester 2019

Sheet 5

Exercise 15
Show that the sum

$$a_n = \sum_{k=0}^{\infty} \frac{(-1)^k}{\ln(n+k)}$$

has the asymptotic behavior

$$a_n = \frac{1}{2\ln(n)} + \mathcal{O}\left(\frac{1}{n\ln^2(n)}\right)$$

as $n \to \infty$.

(2 Points)

Exercise 16
Find the asymptotic behavior of the sum

$$\sum_{k=1}^{\infty} \frac{1}{k(k+n)}$$

for large positive and integer n.

(3 Points)

Exercise 17
By using Euler-Maclaurin formula, find the asymptotic expansion for the sum

$$\sum_{n=1}^{\infty} \ln\left(1 - e^{-nx}\right)$$

as $x \to 0$ and $x > 0$, which is valid in the order of x.
Hint: To find the correct result, use the definition of the Bernoulli numbers B_n via the generation function, that is

$$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!}\, x^n,$$

and study the dependence of all terms in the Euler-Maclaurin formula at small values of x.

(4 Points)

Date: 29.05.2019
Discussion: 04.06.2019