Introduction to Asymptotic Methods

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Sheet 8

Exercise 24
Find the leading term of the asymptotic expansion for the modified Bessel function $I_n(x)$ as $x \to +\infty$, defined by the integral

$$I_n(x) = \frac{1}{\pi} \int_0^\pi e^{x \cos \varphi} \cos(n\varphi) d\varphi.$$

(2 Points)

Exercise 25
Proof the following asymptotic behavior for the integral

$$\int_0^\pi x^n \sin(x) dx \sim \frac{\pi^{n+2}}{n^2} \quad \text{as } n \to \infty.$$

(1 Point)

Exercise 26
Find the first three terms of the asymptotic expansion for the integral

$$\int_{-\infty}^\infty d\eta \, e^{-x\eta^2} \ln(1 + \eta + \eta^2)$$

as $x \to \infty$.

(2 Points)

Exercise 27
Find the asymptotic behavior for the integral

$$\int_0^1 \exp\left(-\frac{1}{x} - \lambda x\right) dx$$

for large and positive λ.

(2 Points)

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