Exercise 24
Find the leading term of the asymptotic expansion for the modified Bessel function $I_n(x)$ as $x \to +\infty$, defined by the integral

$$I_n(x) = \frac{1}{\pi} \int_0^{\pi} e^{x \cos \varphi} \cos(n \varphi) d\varphi.$$  

(2 Points)

Exercise 25
Proof the following asymptotic behavior for the integral

$$\int_0^{\pi} x^n \sin(x) dx \sim \frac{\pi^{n+2}}{n^2} \quad \text{as} \quad n \to \infty.$$  

(1 Point)

Exercise 26
Find the first three terms of the asymptotic expansion for the integral

$$\int_{-\infty}^{\infty} d\eta \ e^{-x\eta^2} \ln(1 + \eta + \eta^2)$$

as $x \to \infty$.  

(2 Points)

Exercise 27
Find the asymptotic behavior for the integral

$$\int_0^1 \exp \left( -\frac{1}{x} - \lambda x \right) dx$$

for large and positive $\lambda$.  

(2 Points)

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