Introduction to Asymptotic Methods

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Sheet 9

Exercise 28
Find the leading term of the asymptotic expansion for the integral

$$\int_0^\infty \exp\left(px - e^x\right) dx$$

as $p \to +\infty$.

(2 Points)

Exercise 29
Proof the following asymptotic behavior for the integral

$$\int_{-1}^1 \left(1 - x^2\right)^n dx \sim \sqrt{\frac{\pi}{n}} \quad \text{as } n \to +\infty.$$

(1 Point)

Exercise 30
Let us define the function

$$\mathfrak{F}(x, y) = \int_0^\infty \frac{d\xi}{\xi^2 + y^2} e^{-x\xi^2}$$

of two real positive variables x and y.
• Find the asymptotic expansion of $\mathfrak{F}(x, y)$ as $x \to \infty$ and fixed nonzero y. Define the regions for variables x and y, in which the expansions obtained are valid.

(1 Point)

• Find the asymptotic expansion of $\mathfrak{F}(x, y)$ as $y \to 0$ and large and finite x.

(1 Point)

• Performing the integration for the function $\mathfrak{F}(x, y)$ exactly, compare the exact behavior of $\mathfrak{F}(x, y)$ with the asymptotic expansions obtained above.

(2 Points)

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