Open Effective Field Theories and Universality

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EMMI

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Outline

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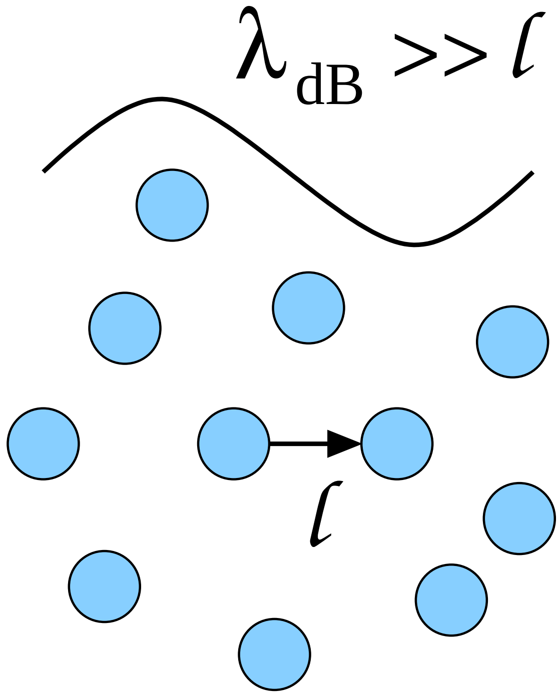
- Introduction
  - Low-Energy Universality
  - Resonant interactions and the unitary limit
- Universality in Few-Body Systems
- Inelastic Processes and Open EFT

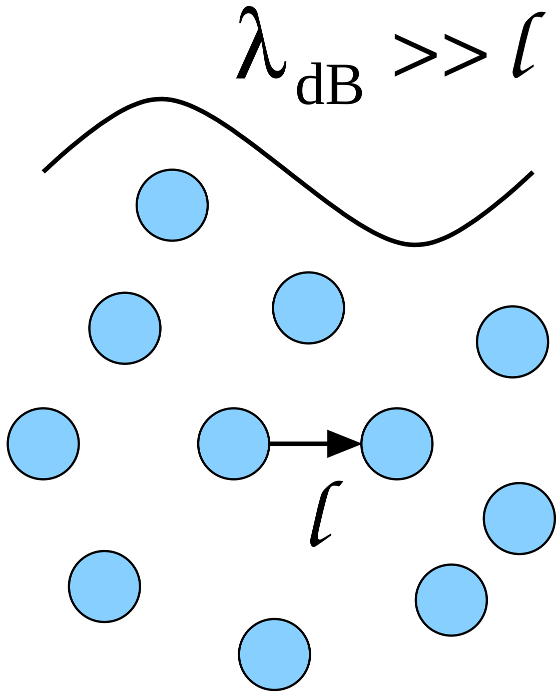
  (with E. Braaten, G.P. Lepage)
- Summary and Outlook

Low-Energy Universality

- Separation of scales:
  $1/k = \lambda_{dB} \gg \ell$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of $k\ell$
- Generic/natural case: $|a| \sim \ell$
- Resonant case: $|a| \gg \ell$



Low-Energy Universality



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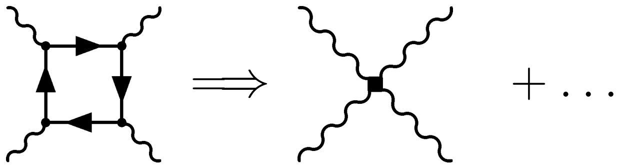
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)
  Contact interactions for $\omega \ll m_e$:
  $\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \to \mathcal{L}_{eff}[A_\mu]$



- Change of resolution: Renormalization group (Kadanoff, Wilson)

Physics Near the Unitary Limit

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- Consider system with short-ranged, resonant interactions
- Unitary limit: $a \to \infty$, $\ell \to 0$ (cf. Bertsch problem, 2000)

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \implies i/k$$

  - Scattering amplitude scale invariant, saturates unitarity bound

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- Use as starting point for description of few-body properties
  - Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
  - Natural expansion parameter: $\ell/|a|$, $k\ell$,...
  - Universal dimer with energy $E_d = -\hbar^2/(ma^2)$ $(a > 0)$

    size $\langle r^2 \rangle^{1/2} = a/2$

Physics Near the Unitary Limit

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- Unitary limit important in many areas of physics
- Nuclear physics: dripline nuclei
  - $2N$-system: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - ^{11}Be $\Rightarrow$ ^{10}Be + n: n separation energy $\approx$ 0.5 MeV
- Particle physics: hadronic molecules
  - $X(3872)$ as a $D^0\bar{D}^{0*}$ molecule? $(J^{PC} = 1^{++})$

    $$B_X = (0.11 \pm 0.21) \text{ MeV}$$
- Atomic physics:
  - ^{4}He: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW} \longrightarrow B_d \approx 100$ neV
  - Feshbach resonances: a can be varied experimentally

    $\Longrightarrow$ tune system to the unitary limit

Broken Scale Invariance

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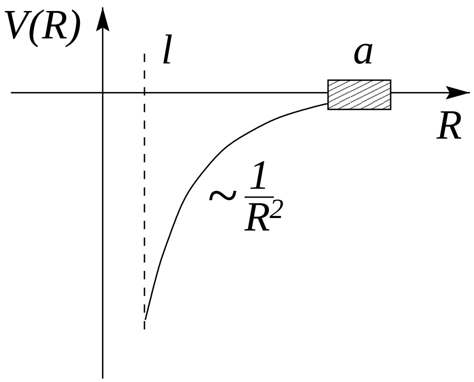
- Three-boson system near the unitary limit (Efimov, 1970)
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$



- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

  $\Longrightarrow$ scale invariance is anomalous
  $\Longrightarrow$ observables depend on boundary condition and a
- Universality concept must be extended $\Rightarrow$ 3-body parameter

Universal EFT

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- **Effective Lagrangian** (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4} (d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + \ldots$$

- 2-body amplitude:

- 2-body coupling g_2 near fixed point $(1/a = 0)$

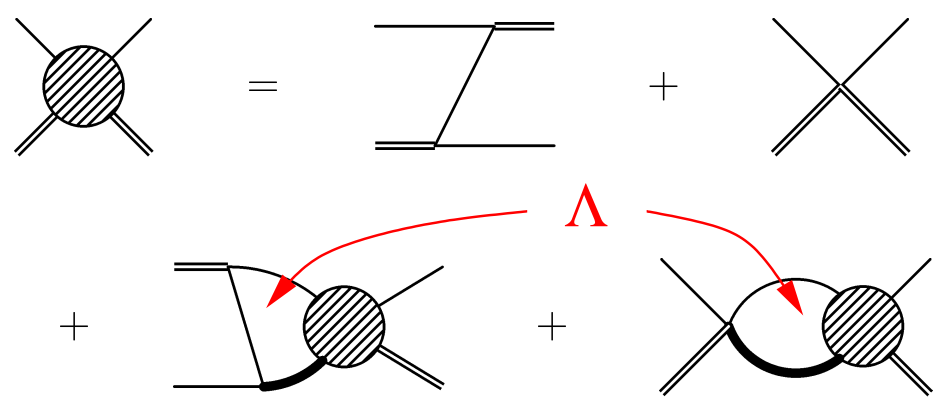
  $\Rightarrow$ scale and conformal invariance $\Longleftrightarrow$ unitary limit

  (Mehen, Stewart, Wise, 2000; Nishida, Son, 2007; ...)

- 3-body amplitude:

  $g_3(\Lambda)$ $\Rightarrow$ limit cycle

  $\Rightarrow$ discrete scale inv.

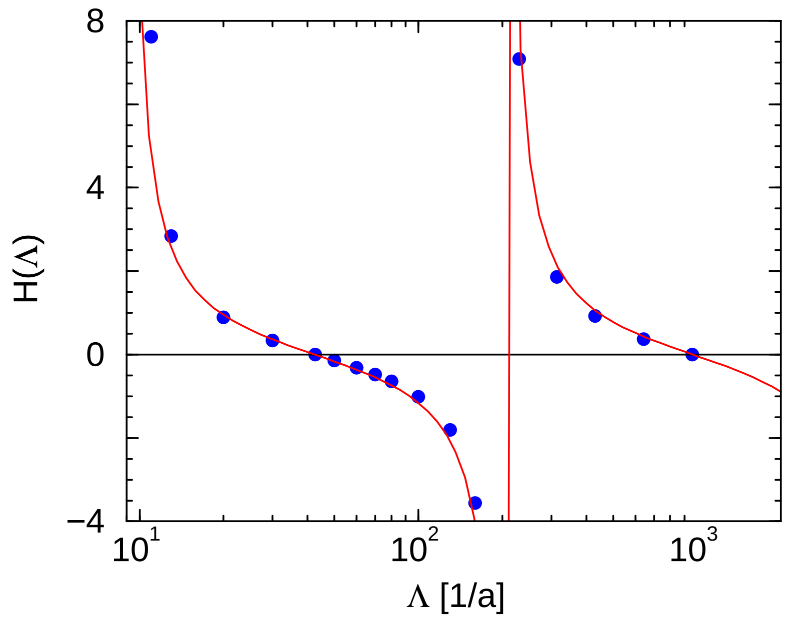


Limit Cycle

- RG invariance $\Longrightarrow$ running coupling $H(\Lambda) = g_3\Lambda^2/(9g_2^2)$
  - $H(\Lambda)$ periodic: limit cycle

    $$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

    (cf. Wilson, 1971)
  - Anomaly: scale invariance broken to discrete subgroup



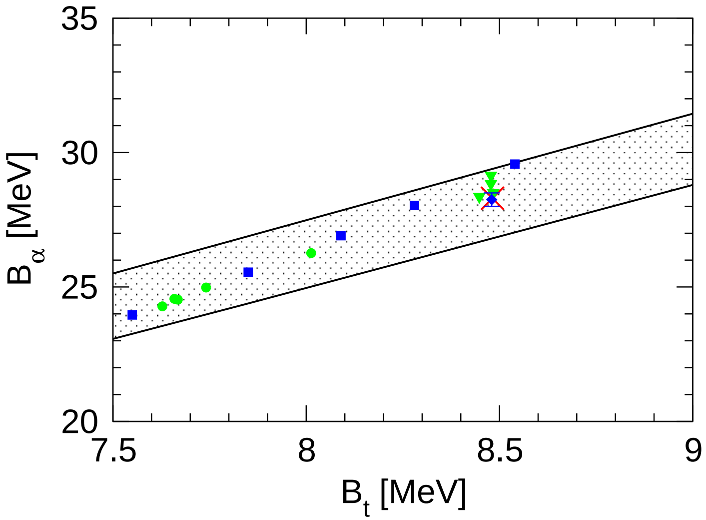
$$H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

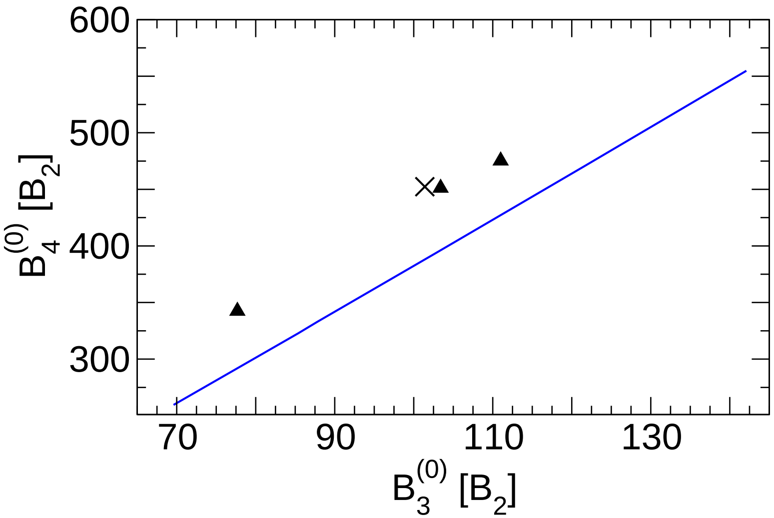
(Bedaque, HWH, van Kolck, 1999)

- Three-body parameter: $\Lambda_*, \ldots$
- Limit cycle $\Longleftrightarrow$ Discrete scale invariance

Universal Correlations

- 2 Parameters at LO $\Rightarrow$ 3-body observables are correlated
  $\Longrightarrow$ Phillips line (Phillips, 1968)
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)
  $\Rightarrow$ 4-body observables are correlated $\Longrightarrow$ Tjon line

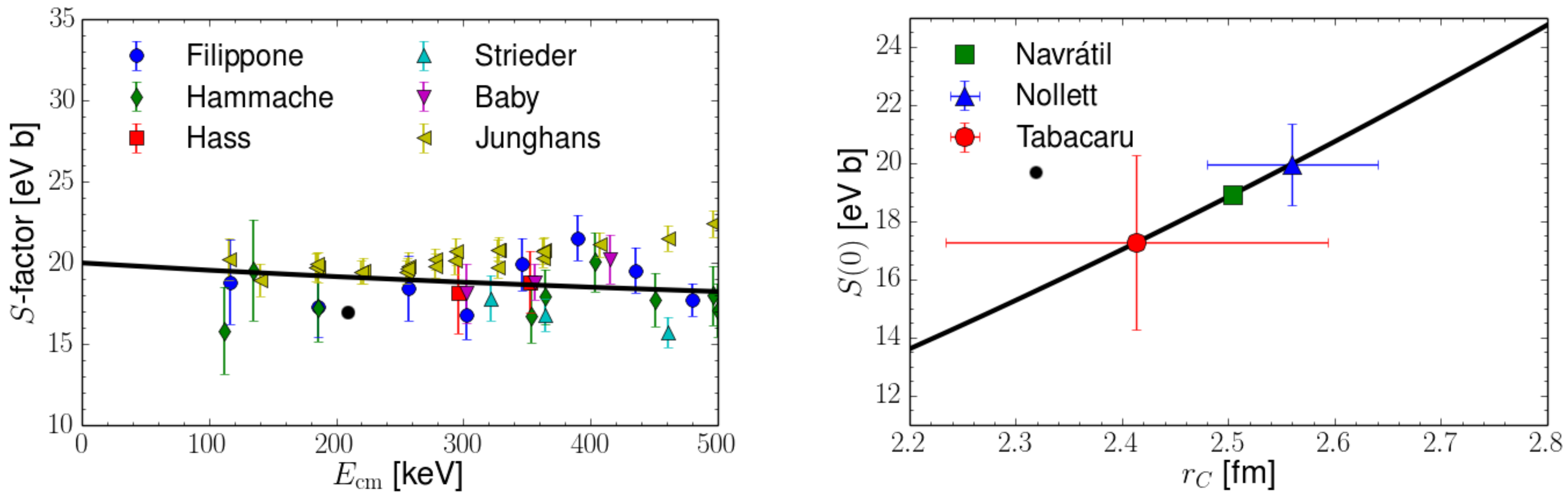




- Variation of 3-body parameter generates correlations
- Nuclear physics: RG evolution of 2-body forces

More Universal Correlations

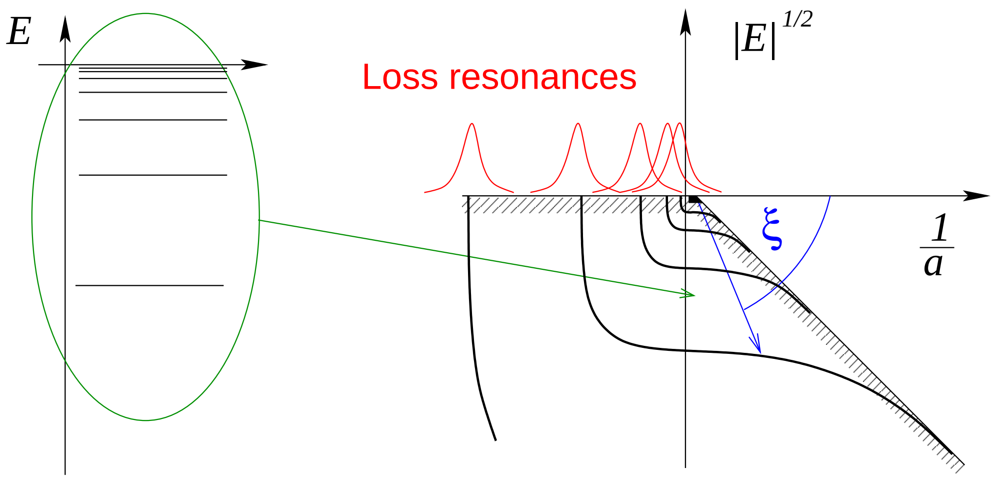
- Radiative proton capture on ^{7}Be determines high-energy part of solar neutrino spectrum
- EFT calculation of S-factor using ab initio ANCs
  (Zhang, Nollett, Phillips, 2014, 2015)
- S-factor for proton capture on ^{7}Be and charge radius of ^{8}B are correlated



Ryberg, Forssen, HWH, Platter, Eur. Phys. J. A **50** (2014) 170

Limit Cycle: Efimov Effect

- Universal spectrum of three-body states (Efimov, 1970)

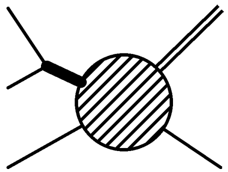


  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum for $1/a \to 0$

$$B_3^{(n)}/B_3^{(n+1)} \xrightarrow{1/a\to 0} \left(e^{\pi/s_0}\right)^2 = 515.035...$$

- Universal four- and higher-body states
- Ultracold atoms $\Longrightarrow$ variable scattering length $\Longrightarrow$ loss resonances

Three-Body Recombination



- Three-body recombination:

  3 atoms $\rightarrow$ dimer + atom $\Rightarrow$ loss of atoms

- Recombination constant: $\dot{n}_A = -K_3\, n_A^3$
- K_3 has log-periodic dependence on scattering length

  (Nielsen, Macek, 1999; Esry, Greene, Burke, 1999; Bedaque, Braaten, HWH, 2000)

- Deep dimers: Efimov trimers aquire width $\Rightarrow$ resonances
- Loss term in short distance b.c.: $\Lambda_* \longrightarrow \Lambda_* \exp^{i\eta_*/s_0}$

  $\Longrightarrow$ non-hermitian Hamiltonian

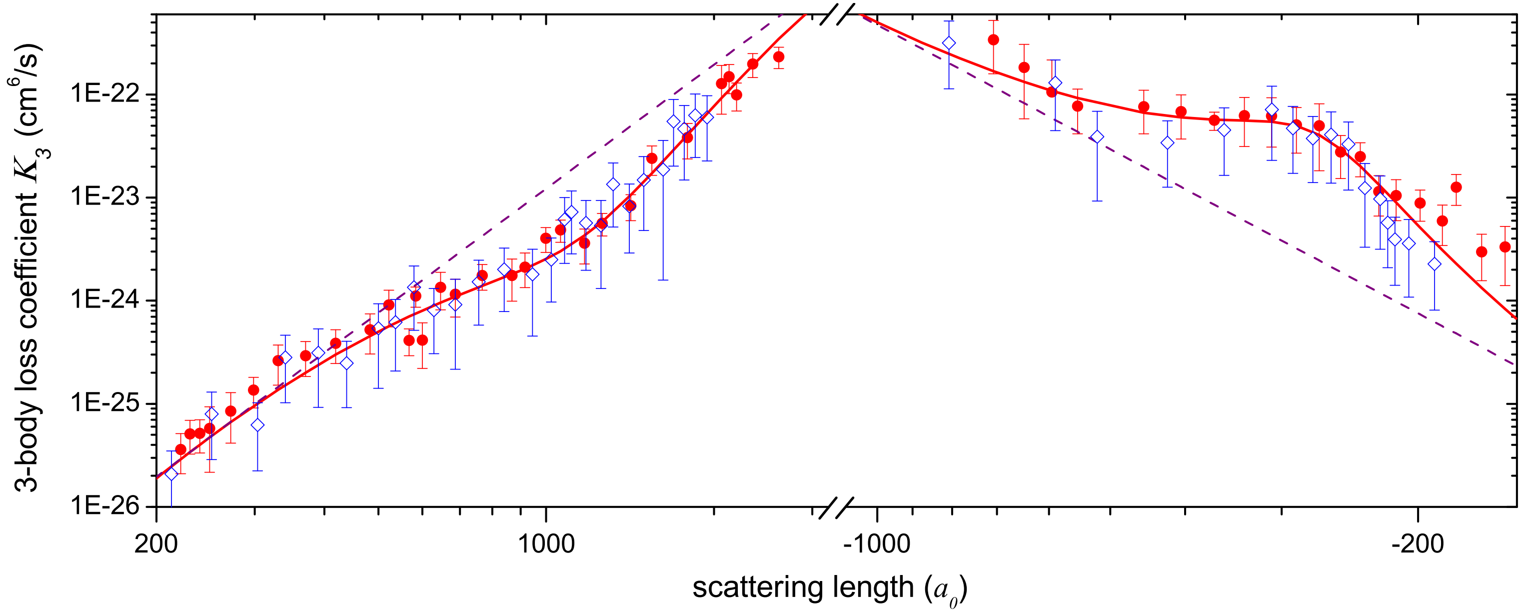
- Universal line shape of recombination resonance ($a < 0$)

$$K_3^{deep} = \frac{64\pi^2(4\pi - 3\sqrt{3})\ \coth(\pi s_0)\sinh(2\eta_*)}{\sin^2[s_0 \ln(a/a_-)] + \sinh^2 \eta_*}\frac{\hbar\, a^4}{m}\,, \qquad s_0 \approx 1.00624..$$

and other observables . . .

Efimov States in Ultracold Atoms

- First experimental evidence in ^{133}Cs (Krämer et al. (Innsbruck), 2006)
  now also ^{6}Li, ^{7}Li, ^{39}K, ^{41}K/^{87}Rb, ^{6}Li/^{133}Cs
- Example: Efimov spectrum in ^{6}Li/^{133}Cs mixture
  (Gross et al. (Bar-Ilan Univ.), Phys. Rev. Lett. **105** (2010) 103203)



- Van der Waals tail determines $a_-/l_{vdW} \approx -10\ (\pm 15\%)$
  (Wang et al., 2012; Naidon et al. 2012, 2014; ...)
  . . . but not η_* . . .

Inelastic Processes

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- Loss coefficients used in few-body rate equations
- Complete description requires density matrix
- Effective density matrix from tracing over high-energy states?
- Naive evolution equation for $H_{eff} = H - iK$

$$i\hbar\, \partial_t \rho = H_{\rm eff}\, \rho - \rho H_{\rm eff}^\dagger = [H, \rho] - i\{K, \rho\}$$

- Implies $\partial_t \mathrm{Tr}\,(\rho) = -\mathrm{Tr}(2K\rho)/\hbar$

  $\Longrightarrow$ probability not conserved
- Need evolution equation for open system

  $\Longrightarrow$ Lindblad equation (Lindblad; Gorini, Kossakowski, Sudarshan, 1976)
- Motivate from Quantum Field Theory

Inelastic Processes

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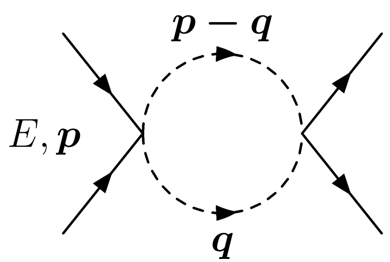
- Consider model with two fields ψ and ϕ: $H_{\text{full}} = H^{\psi} + H^{\phi} + H_{\text{int}}$

$$H_{\text{int}} = \frac{1}{4} g \int_{\boldsymbol{r}} \left(\psi^{\dagger 2}(\boldsymbol{r}) \phi^2(\boldsymbol{r}) + \psi^2(\boldsymbol{r}) \phi^{\dagger 2}(\boldsymbol{r}) \right)$$

- Reaction $\psi\psi \to \phi\phi$ has large energy release E_{deep}

$\Longrightarrow$ process is effectively local and instantaneous

- Leading contribution in g



$$T(E,\boldsymbol{p}) = \frac{1}{2} g^2 \int_{\boldsymbol{q}} \frac{1}{E - \omega_{\boldsymbol{q}} - \omega_{\boldsymbol{p}-\boldsymbol{q}} + i\epsilon}$$

$\Longrightarrow$ expand in powers of $\boldsymbol{p}^2/mE_{deep}$

- Effect of high-energy ϕ particles on low-energy ψ particles is local

Inelastic Processes

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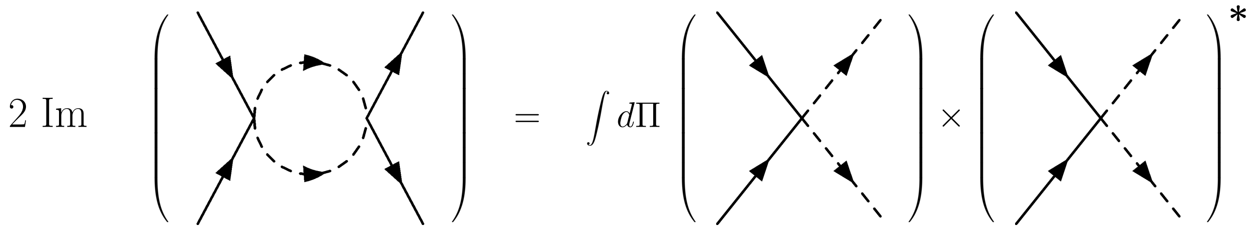
- Effective Field Theory without explicit ϕ dof

$$H - iK = H^{\psi} + T(2E_0^{\psi}, 0) \int_{\boldsymbol{r}} (\psi^{\dagger}(\boldsymbol{r})\psi(\boldsymbol{r}))^2$$

- Optical theorem



- Replacement for internal ϕ particles in correlation functions

$$g\,\phi^2(\boldsymbol{r}, t) \to T(2E_0^{\psi}, 0)\,\psi^2(\boldsymbol{r}, t), \qquad g\,\phi^{\dagger 2}(\boldsymbol{r}, t) \to T^*(2E_0^{\psi}, 0)\,\psi^{\dagger 2}(\boldsymbol{r}, t)$$

- Only imaginary part of T physically relevant, real part renormalized away

Inelastic Processes

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- Derive effective density matrix for low-energy particles

$$\rho(t) \equiv \mathrm{Tr}_\phi\left(\rho_{\text{full}}(t)\right) = \sum_{m=0}^{\infty} \int_{\boldsymbol{y}_1 \ldots \boldsymbol{y}_m} {}_\phi\langle \boldsymbol{y}_1 \ldots \boldsymbol{y}_m | \rho_{\text{full}}(t) | \boldsymbol{y}_1 \ldots \boldsymbol{y}_m \rangle_\phi$$

- Evolution of effective density matrix

$$i\hbar\partial_t \rho = \mathrm{Tr}_\phi\big(H_{\text{full}}\, \rho_{\text{full}} - \rho_{\text{full}}\, H_{\text{full}}\big)$$

- Four different contributions from interaction term

$$\mathrm{Tr}_\phi\left[\left(g\int_{\boldsymbol{r}} \psi^{\dagger 2}(\boldsymbol{r})\phi^2(\boldsymbol{r})\right)\rho_{\text{full}}\right] \longrightarrow T(E_0^\psi, 0)\int_{\boldsymbol{r}} (\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r}))^2 \rho$$

- Analog for other three contributions

$$\mathrm{Tr}_\phi[\rho_{\text{full}}(g\textstyle\int_{\boldsymbol{r}} \phi^{\dagger 2}(\boldsymbol{r})\psi^2(\boldsymbol{r}))],\ \mathrm{Tr}_\phi[(g\textstyle\int_{\boldsymbol{r}} \phi^{\dagger 2}(\boldsymbol{r})\psi^2(\boldsymbol{r}))\rho_{\text{full}}],\ \ldots$$

Open EFT

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- Evolution equation for effective density matrix

$$i\hbar\partial_t\rho = \left[H,\rho\right] - i\,\mathrm{Im}\,T\int_{\boldsymbol{r}}\left[\left(\psi^\dagger\psi(\boldsymbol{r})\right)^2\rho + \rho\left(\psi^\dagger\psi(\boldsymbol{r})\right)^2 - 2\,\psi(\boldsymbol{r})^2\,\rho\,\psi^{\dagger 2}(\boldsymbol{r})\right]$$

$\Longrightarrow$ Lindblad form

- General Hamiltonian with a loss term

$$H_{\text{eff}} = H - iK, \qquad K = \sum_i \gamma_i \int d^3r\, \Phi_i^\dagger \Phi_i$$

- Lindblad equation

$$i\hbar\partial_t\rho = [H,\rho] - i\sum_i \gamma_i \int d^3r \left(\Phi_i^\dagger\Phi_i\rho + \rho\Phi_i^\dagger\Phi_i - 2\Phi_i\rho\Phi_i^\dagger\right)$$

- Open EFT (Burgess et al., 2015)

Application: 2-body losses

- Fermionic atoms with a loss channel $\Rightarrow$ a complex

$$K = (4\pi\hbar^2/m)\mathrm{Im}(1/a)\int d^3r\,\Phi^\dagger\Phi, \qquad \Phi = 4\pi a\,\psi_2\psi_1$$

- Particle losses: $\langle N\rangle = \mathrm{Tr}(\rho N)$

$$\frac{d}{dt}\langle N_1\rangle = \frac{d}{dt}\langle N_2\rangle = -\frac{\hbar}{2\pi m}\mathrm{Im}(1/a)\int d^3r\,\left\langle \Phi^\dagger\Phi\right\rangle$$

where $\mathcal{C} = \left\langle \Phi^\dagger\Phi\right\rangle$ contact operator

- Note: regularization required

$\Rightarrow$ precise form of contact operator depends on regularization scheme

2-body Losses and the Contact

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- Universal relations involving the contact: $C = \int d^3r\, \mathcal{C}(\boldsymbol{r})$
  measures number of pairs at short distances (S. Tan, 2005-2008)

  e.g. adiabatic relation $$\frac{d}{da^{-1}} E = -\frac{\hbar^2}{4\pi m}\, C$$

  also RF spectroscopy, photoassociation, ...
- Here: inelastic loss rate for mixture of atom species $\sigma = 1, 2$
- Inelastic short-distance processes parameterized by complex scattering length

$$\frac{d}{dt} N_\sigma = -\frac{\hbar}{2\pi m}\, \mathrm{Im}(1/a)\, C$$

(Tan, 2008; Braaten, Platter, 2008)

- Incorrect field-theoretical derivation implied corrections

(Braaten, HWH, 2013)

2-body Losses and the Contact

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- Application 1: low-density gas of atoms

$$\frac{d}{dt}n_1 = \frac{d}{dt}n_2 = -K_2\, n_1 n_2 = -\frac{\hbar}{2\pi m}\,\mathrm{Im}(1/a)\,\mathcal{C}_{AA}$$

where $$\mathcal{C}_{AA} = 8\lambda_T^3 \int_0^\infty dk\, \frac{k^2 e^{-\beta\hbar^2 k^2/(2\mu)}}{|-1/a - ik|^2}\, n_1 n_2$$

- Application 2: low-density gas of dimers

$$\frac{d}{dt}n_D = -\Gamma_D n_D = -\frac{\hbar}{2\pi m}\,\mathrm{Im}(1/a)\,\mathcal{C}_D$$

where $$\mathcal{C}_D = \frac{8\pi\,\mathrm{Re}(a)}{|a|^2} n_D$$

- Universal relation

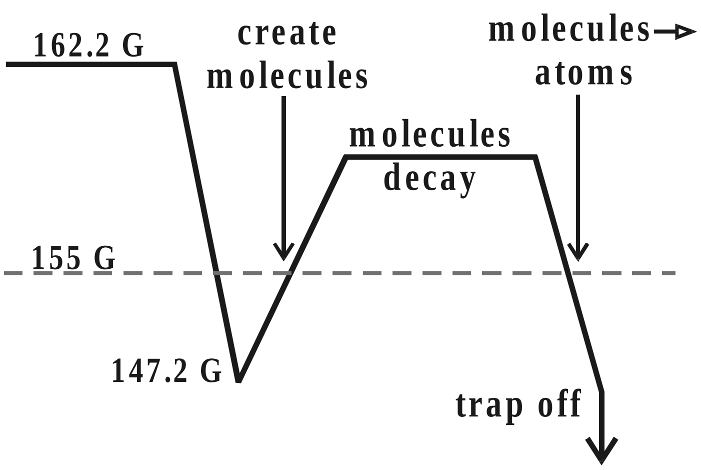
$$K_2(T=0) = \frac{2\pi|a|^4}{\mathrm{Re}(a)}\Gamma_D$$

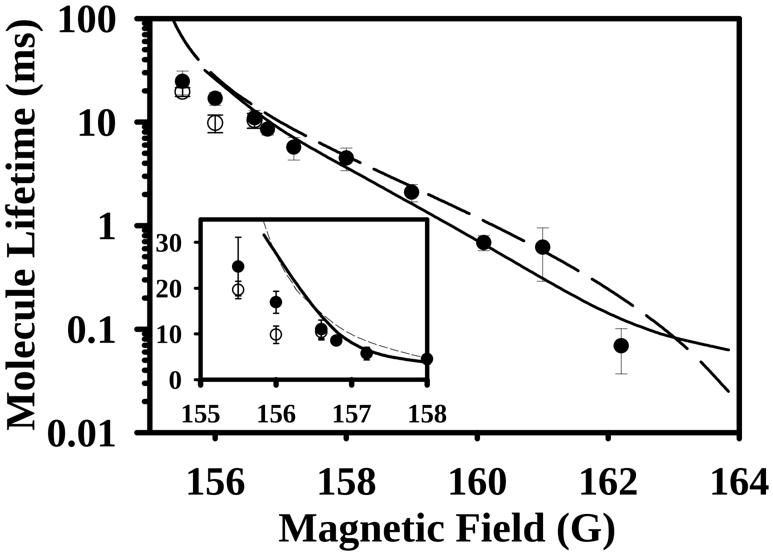
(for real a derived by Köhler, Tiesinga, Julienne, 2005)

2-body Losses and the Contact

- Experimental test using atoms with inelastic spin relaxation channel
- Previous experiment in ^{85}Rb: only molecular decay

(Thompson, Hodby, Wieman, PRL **94** (2005) 020401; Köhler, Tiesinga, Julienne, ibid. 020402)





- Problem: 3-body contact important?

$\longrightarrow$ use equal mass fermions

Summary and Outlook

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- Universality: Effective field theory for large scattering length
  - Discrete scale invariance, universal correlations,...
- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Few-body nuclei
  - Hadronic molecules: $X(3872)$, $\ldots$

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  - Hadronic molecules: $X(3872), \ldots$
- Open Effective Field Theory and inelastic processes
  - Universal relation for the inelastic loss rate
  - Losses proportional to $\mathrm{Im}(a)$ and Tan contact