N coupled dipoles: from Anderson localization to Dicke subradiance

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Dicke vs Anderson

plasma physics / pattern formation

astrophysics
(self-oscillations, random lasing, Lévy flight of photons)

Nature Photonics 8, 321 (2014)

Nature Physics 9, 357 (2013)
Multiple Scattering of Light in Atomic samples: Disorder vs cooperative effects

Andersen Localization

Multiple Scattering

“Local”

Interferences

“Global”

Dicke States
The case for Anderson ...

coherence of photons
Wave propagation in disordered media:

< 1958: on average: interferences washed out: random walk / diffusion
   Light: radiation trapping in stars
   Electrons: metal (Drude model)

1958: P.W. Anderson: vanishing diffusion for strong disorder!

Solid State Physics:
   Metal-Insulator Transitions for electrons

Light Scattering:
   Semiconductor powder, White Paint, Atoms

Matter Waves:
   BEC in Disordered Potential, Kicked Rotator

Acoustics:
   Aluminium Beads

NMR:
   Nuclear Spins
Anderson Localization of non interacting waves in 1, 2 and 3D

Scaling theory of localization: Abrahams et al., PRL 42, 673 (1979)

\[ g : \text{dimensionless conductance} \]

\[ \beta(g) = \frac{\partial \ln g}{\partial \ln L} \]

In 3D: threshold for disorder

Ioffe-Regel criterion: \( k\ell = 1 \)

No microscopic theory

self consistent theory of localization,
numerical simulations of toy systems
Anderson Localization of Light in 3D: phase transition $\Rightarrow$ strong scattering required

Semi-conductor powder

- D. Wiersma et al., Nature 1997
- T. v. der Beek et al., PRB 85 115401 (2012)

White Paint

- C. Aegerter et al., EPL 2006
- F. Scheffold et al., Nat. Photon. 7, 934 (2013)

$\Rightarrow$ Not observed so far
Building up a refractive index « ab initio » 
(from individual atoms)

\[ E_0 \]

\[ \dot{\beta}_j(t) = -\frac{i}{2} \Omega e^{ik_0 \cdot r_j} + \left( i \Delta - \frac{\Gamma}{2} \right) \beta_j(t) - \frac{\Gamma}{2} \sum_{m \neq j}^N \beta_m \frac{\exp (ik_0 |r_j - r_m|)}{ik_0 |r_j - r_m|} \]

\[ E_{sc}(r) = -\frac{\hbar \Gamma}{2d} \sum_{j=1}^N \beta_j \frac{e^{ik_0 |r - r_j|}}{k_0 |r - r_j|} \]

\[ \beta_i : \text{amplitude of dipole } i \]
Spherical gaussian cloud: emission diagram

Cloud of atoms

Far field emission diagram

Refractive index (mean field)

Incoherent model (particles trajectories, scattering in ‘empty modes’)

Mesoscopic physics: Weak localization (waves beyond mean field)

S. Bromley et al., Nat. Comm. 7, 11039 (2016)
Weak Localisation = precursor of strong Localisation?

Coherence after resonant scattering with atoms!

See also:
M. Havey’s group

Theory:

- no “exact” solution
- diagrammatic approach

\[
\begin{align*}
R & \approx L = + + + \ldots \\
C & = + + \ldots 
\end{align*}
\]

Excellent agreement
(no free parameter)

Towards strong localization of light: dense atomic clouds

Dipole Trap (Havey, Browaeys)

\[ k \ell \approx 1000 \]

\[ k \ell \approx 3 \]

\[ k \ell < 1 \]

Ioffe-Regel: \[ k \ell \approx 1 \]

\[ N = 10^8 \quad N = 10^7 \]

Dynamical Breakdown

Weak Localization of Light

Strong Localization of Light

BEC

Strong Localization + BEC

\[ n \text{ [cm}^{-3}] \]

\[ 10^10 \quad 10^{12} \quad 10^{14} \quad 10^{16} \quad 10^{18} \quad 10^{20} \]
Theory: Effective Hamiltonian

\[ H_{\text{eff}} = \left( \hbar \omega_0 - i \frac{\hbar \Gamma_0}{2} \right) \sum_i S_i^z + \frac{\hbar \Gamma_0}{2} \sum_{i \neq j} V_{ij} S_i^+ S_j^- \]

Diagonal: On site energy

\[ V_{ij} = \beta_{ij} - i \gamma_{ij} \]
\[ \beta_{ij} = \frac{3}{2} \left[ -p \frac{\cos k_0 r_{ij}}{k_0 r_{ij}} + q \left( \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^3} + \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^2} \right) \right] \]
\[ \gamma_{ij} = \frac{3}{2} \left[ p \frac{\sin k_0 r_{ij}}{k_0 r_{ij}} - q \left( \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^3} - \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^2} \right) \right] \]

- Open System
- Reminiscent of Anderson Hamiltonian
- Heisenberg model with global coupling
- Long range hopping
- No decoherence (coupling to phonons, ...)
« Life time » of photons in the system (motivated by experimental approaches)

Photon Escape Rate = Spectrum \{ \text{Im} (H_{\text{eff}}) \}

size : a = L/\ell

disorder parameter W=1/k\ell

See also F. Pinheiro et al. 2004
Photon Escape Rates

\[ C(a, W) = 1 - 2 \int_{1}^{\infty} P(\Gamma) d\Gamma \]

Measure of long lived photons

Single parameter scaling
\[ \frac{N}{N_{\perp}} \]

cooperative effects dominate over disorder!
no phase transition observed with \( P(\Gamma) \)

Dicke > Anderson

E. Akkermans, A. Gero, RK, PRL, 101, 103602 (2008)
Eigenvalues

Beyond Photon escape times:

Cloud of Atoms = Large Molecule (with $10^{10}$ atoms)

‘dilute’ molecule

‘dense’ molecule

molecular spectrum?

SHENG LI AND ERIC J. HELLER.


FIG. 4. (a) Total cross section as a function of energy for a system of seven identical scatterers randomly placed on a plane. Each scatterer is the same as used in Fig. 1. The positions of the scatterers are shown in (b).

proximity resonances

doorway states

giant oscillator strength
Eigenvalues for N coupled dipoles

Important near field terms for high densities
Eigenvalues

\[ e^{i k r / k r} \]

\[ \Gamma_{at} \sim b_0 \Gamma_0 \]  
cooperative superradiance

\[ \Gamma_{at} = 2 \Gamma_0 \]  
superradiant pairs

\[ \Gamma_{at} \sim \Gamma_0 / b_0 \]  
cooperative subradiance

\[ \Gamma_{at} \sim E^{-2/3} \]  
subradiant pairs

- vectorial model
- scalar model

\[ e^{i k r} \left( 1/k r + 1/k r^2 + 1/k r^3 \right) \]
Eigenvalues: some statistics

No level repulsion

Phase rigidity
Resonance Overlap (« Thouless »)

\[ g = \left\langle \frac{1}{\left\langle 1/|\Gamma|\right\rangle \Delta E} \right\rangle \]

Scaling function \( \beta(g) \)

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**NO ANDERSON LOCALISATION FOR VECTORIAL LIGHT IN 3D**

Skipetrov & Sokolov, PRL 112, 023905 (2014)
LOCALISATION in 2D

\[ \frac{d\hat{\delta}_j^{(0)}}{dt} = -\frac{\Gamma_0}{2} \hat{\delta}_l^{(0)} - \frac{\Gamma_0}{2} \sum_{l=1}^{N} H_0(kr_{jl})\hat{\delta}_l^{(0)}, \]

Spatially extended mode (vectorial case)

\[ \frac{d\hat{\delta}_j^{(\pm 1)}}{dt} = -\frac{\Gamma_1}{2} \hat{\delta}_j^{(\pm 1)} - \frac{\Gamma_1}{2} \sum_{l \neq j} [H_0(kr_{jl})\hat{\delta}_l^{(\pm 1)} + e^{2i\phi_{jl}} H_2(kr_{jl})\hat{\delta}_l^{(\mp 1)}] \]

Spatially localized mode (scalar case)
Mode profiles

Spatially localized mode (scalar case)

Spatially extended mode (vectorial case)

Mode width NOT correlated to localisation length: temporal vs spatial localisation

Spatial and temporal localization of light in two dimensions

C. E. Máximo,¹ N. Piovella,² Ph. W. Courteille,¹ R. Kaiser,³ and R. Bachelard¹,⁸

PHYSICAL REVIEW A 92, 062702 (2015)
The case for Dicke …

coherence of atoms
1954 : Dicke super- and subradiant states

Fig. 1. Energy level diagram of an $n$-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.

First experimental observation of superradiance

Feld et al. 1973

R. Dicke 1954
Single photon excitation / low intensity limit

\[ \Gamma_{\text{max}} \sim N \Gamma_0 \]

Subradiant
N-1 metastable states

Extended Volume: \( b_0 = \frac{N_{\text{at}}}{N_{\text{modes}}} \)

Cooperativity without cavity
(also Random lasing)
The quest for Dicke subradiance
Single Photon interference from $N=2$ atoms


Subradiant pairs : $N=2$

Elusive Subradiance (for $N>2$)

Pencil shape excitation

$\tau_{nat}=7\text{ns}$

Forward ‘subradiance echo’

Fragile subradiance

Dicke subradiance for N two level systems (in free space, N>>2) has not yet been observed

- Does not require large spatial densities (near field effect maybe even bad: Gross&Haroche 1982)
- Requires large optical densities in all directions (b₀>>1)
- Exploits the 1/r long range dipole-dipole interaction
**Time dependent experiments : coherent scattering**

**Superradiance** = bright state  
**Subradiance** = metastable ‘dark’ states

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**Inverted system**

Temnov, Woggon, PRL 95, 243602 (2005)  
**Subradiance vs incoherent scattering**

- $t_{\text{sub}} \propto b_0$

- $t_{\text{Rad. Trap.}} \propto b(\delta)^2$

- Does not require large spatial densities
- Requires large optical densities

- Random walk of photons (without interference)
- Diffusion equation

- $t_{\text{Anderson}} \propto \exp\{b(\delta)\}$

- Density Threshold?
Experiment

\[ N = 10^9 \, ^{87}\text{Rb} \]
\[ T = 50 \, \mu\text{K} \]
\[ R = 1 \, \text{mm} \]
\[ \rho = 10^{11} / \text{cc} \]

\[ b_0 = 20 \ldots 100 \]
Average data (on multichannel scalar)

MOT + Dark MOT (50+30 ms)  
Data average: 500 000 cycles (1 curve/night)

Subradiance Probe

Calibration

Hybrid photomultiplier

12 pulses of 30μs
Experimental results

Long decay at $b(\delta)<1$ 😊

Increases as $b_0$ 😊
The ‘super’ of ‘single photon Dicke states’

Superradiant

Subradiant
Off-axis Superradiance: physics/1603.07204

Simulations

Exp. Data

Results
Combining Anderson and Dicke Toy Model: Open Disordered System:
A. Biella et al., EPL, 103, 57009 (2013)

3D Anderson model on 10x10x10 lattice hoping ($\Omega$) + disorder ($W$) + opening ($\gamma$)

$$H_0 = \sum_{j=1}^{N} E_j |j\rangle \langle j| + \Omega \sum_{\langle i,j \rangle} (|j\rangle \langle i| + |i\rangle \langle j|)$$

$$(H_{\text{eff}})_{ij} = (H_0)_{ij} - \frac{i}{2} \sum_{c} A_i^c (A_j^c)^* = (H_0)_{ij} - i\frac{\gamma}{2} Q_{i,j}$$

All sites coupled to one single decay channel: $Q_{ij}=1$
Hybrid Subradiant States « decoupled » from outside world
Outlook:

- **Subradiance vs Radiation trapping**

  Radiation trapping for small beam and intermediate regimes: subradiance dominant at long times

- **Towards Anderson of subradiant Dicke states**
Now something new : not even yet in progress

From few (N=2 to N=3) to many body (N>>1)
Few body physics with photons : N=2

Pair physics (1/r³ for near field terms)
Leroy Bernstein (2 atoms)

\[ E_n = -\frac{\hbar^2}{MR^2} \left( \frac{n_0 - n}{g} \right)^6 \]

K trap loss
(Salomon group)

PhD A. RIDINGER
Photonic Efimov states

\[ E_n = - \frac{\hbar^2}{MR^2_*} \left( \frac{n_0 - n}{g} \right)^6 \]

Three body (M+M=m) vs Pair physics (M+M+photon)

Three-Body Bound States in Atomic Mixtures With Resonant $p$-Wave Interaction

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3 atoms + 1 photon

Shifted $1/r^3$ potential $\Rightarrow$ shifted eigenstates
($1/r^2$ in 2D)

New lines ($\propto n^3$) to be looked for in experiments
Collaborators

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Thank you for your attention