N coupled dipoles: from Anderson localization to Dicke subradiance

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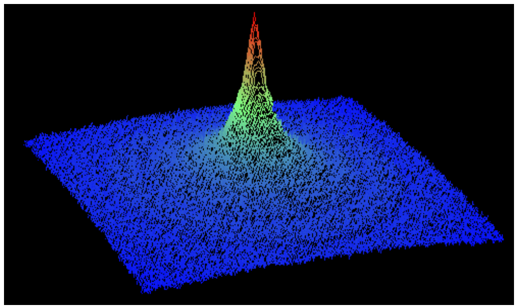
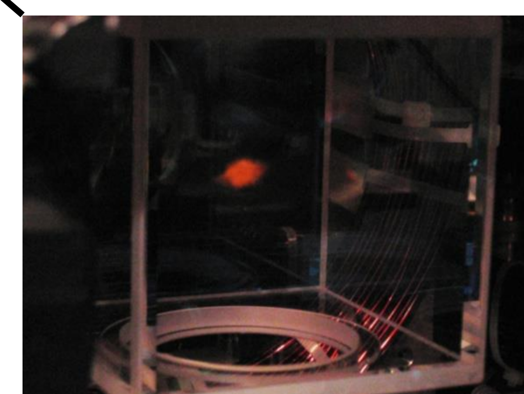
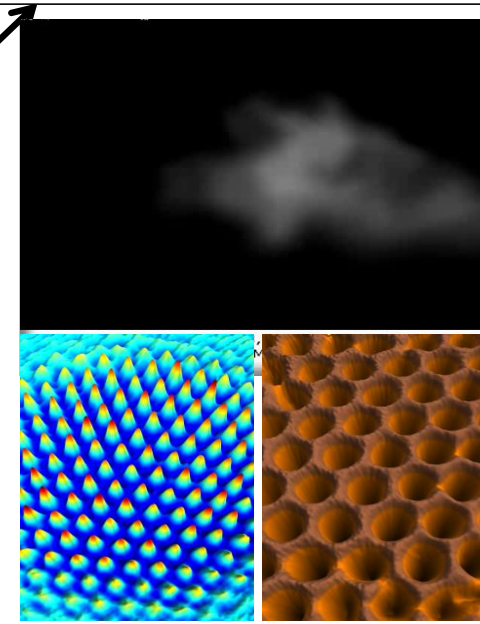
614. WE-Heraeus-Seminar on
"Few-body physics: Advances and prospects in Theory and Experiment"
17 – 20 April 2016 at the Physikzentrum Bad Honnef (Germany)

Dicke vs Anderson



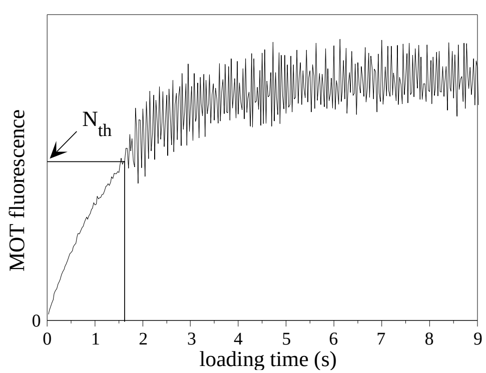
plasma physics / pattern formation

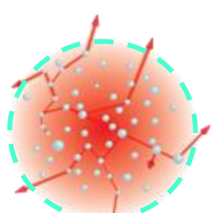


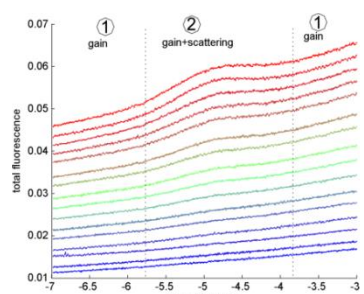


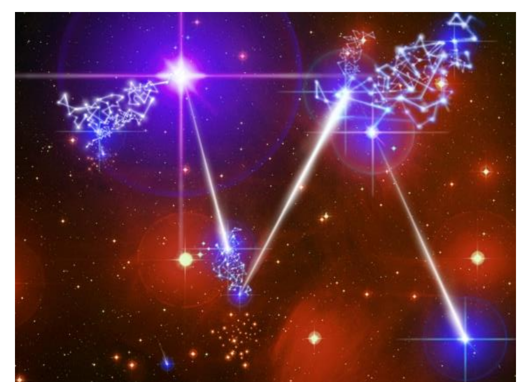
Nature Photonics 8, 321 (2014)

astrophysics
(self-oscillations, random lasing, Lévy flight of photons)



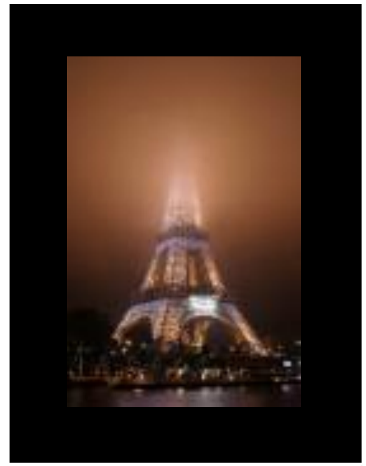






Nature Physics 9, 357 (2013)

Multiple Scattering of Light in Atomic samples : Disorder vs cooperative effects



Multiple Scattering

Anderson Localization

Dicke States

"Local"

"Global"

Interferences

The case for Anderson …

coherence of photons

Wave propagation in disordered media :

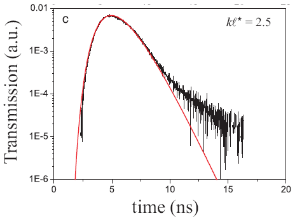
< 1958 : on average : interferences washed out : random walk / diffusion

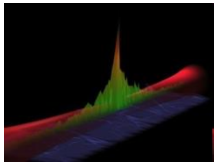
Light : radiation trapping in stars

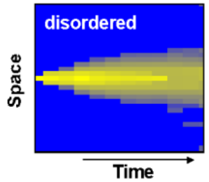
Electrons : metal (Drude model)

1958 : P.W. Anderson : vanishing diffusion for strong disorder !

- **Solid State Physics :**
  Metal-Insulator Transitions for electrons
- **Light Scattering :**
  Semiconductor powder, White Paint, **Atoms**
- **Matter Waves :**
  BEC in Disordered Potential, Kicked Rotator
- **Accoustics :**
  Aluminium Beads
- **NMR :**
  Nuclear Spins



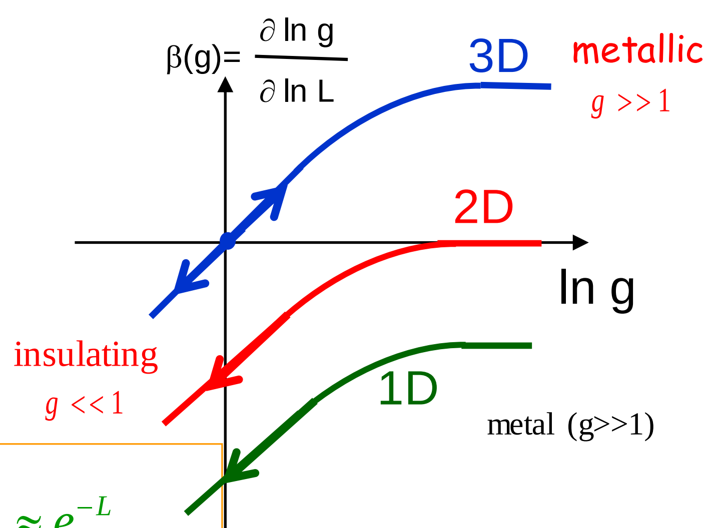




Anderson Localization of non interacting waves in 1,2 and 3D

- **Scaling theory of localization :** Abrahams et al., PRL **42**, 673 (1979)

g : dimensionless conductance



$$g \approx e^{-L}$$

$$\frac{\partial \ln g}{\partial \ln L} \approx \ln g$$

In 3D : threshold for disorder

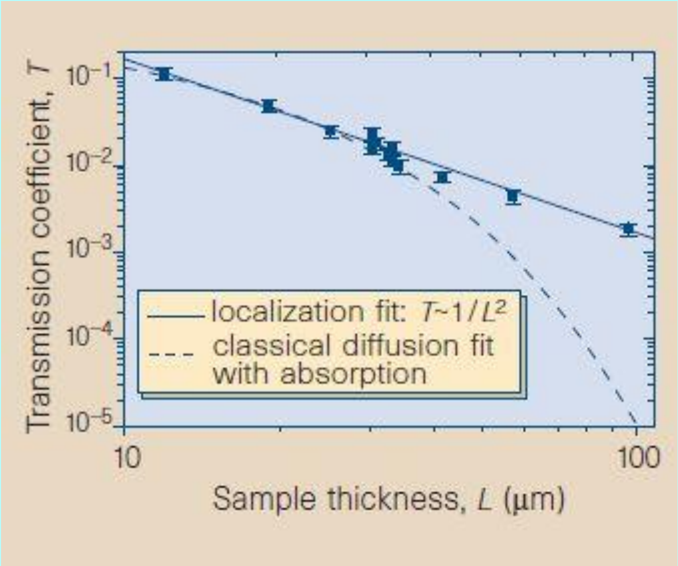
Ioffe-Regel criterion : $k\ell=1$

- **No microscopic theory**
  self consistent theory of localization,
  numerical simulations of toy systems

Anderson Localization of Light in 3D :

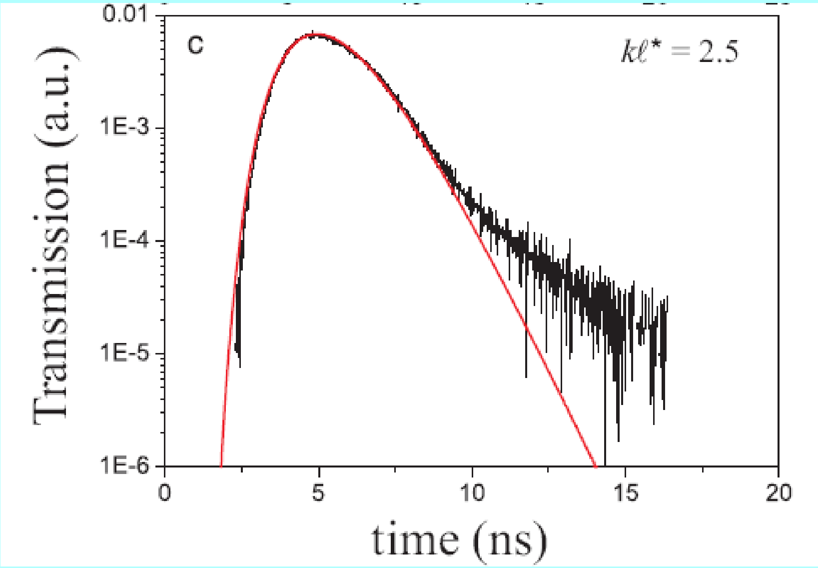
phase transition ⇒ strong scattering required

Semi-conductor powder



D.Wiersma et al., Nature 1997

White Paint



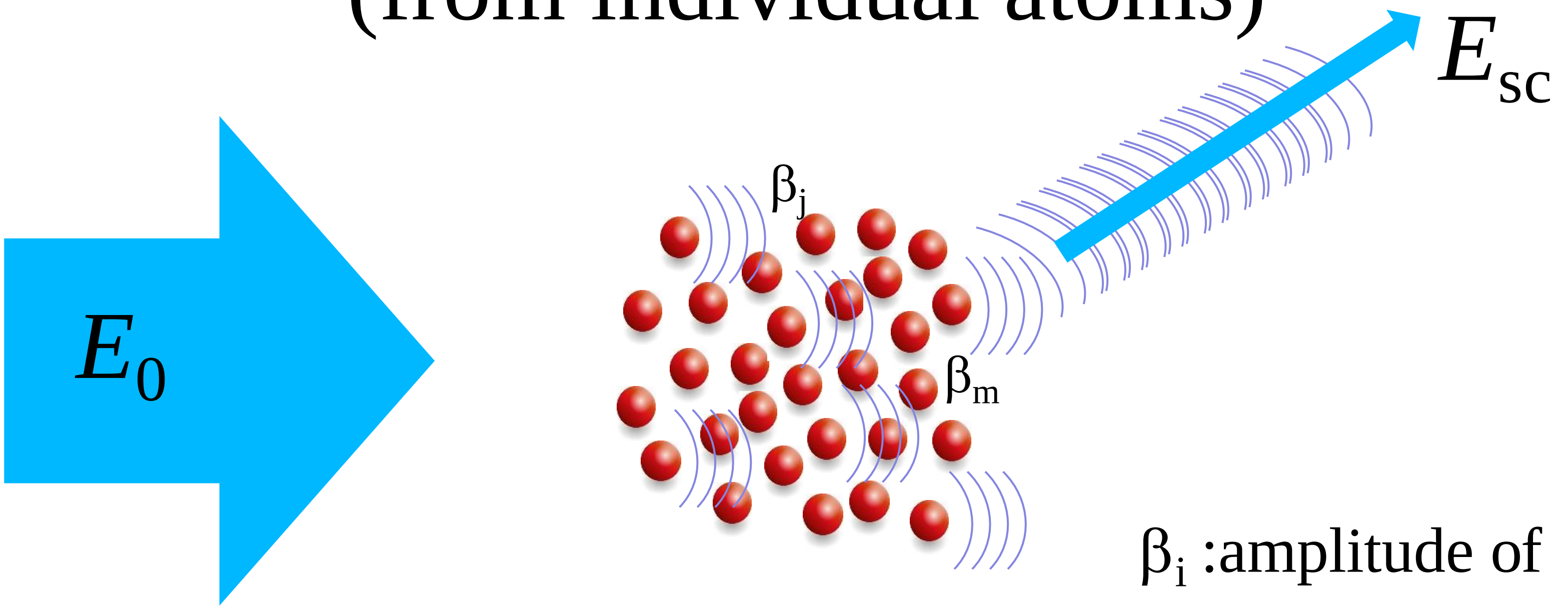
C.Aegerter et al., EPL 2006

F. Scheffold et al., Nature 398, 206(1999)
T. v. der Beek et al., PRB 85 115401 (2012)

F. Scheffold et al., Nat. Photon. 7, 934 (2013)
T Sperling et al., New J. Phys. 18, 013039 (2016)

=> Not observed so far

Building up a refractive index « ab inito » (from individual atoms)



β_i :amplitude of dipole i

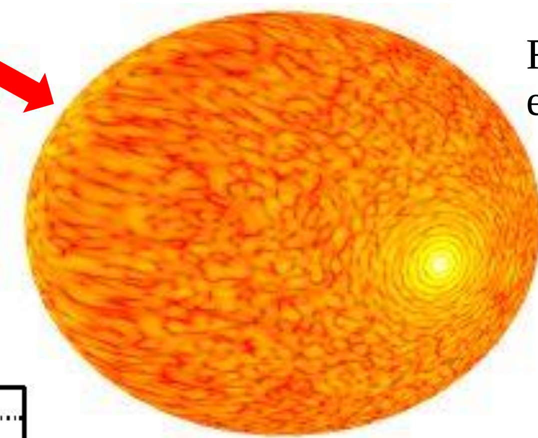
$$\dot{\beta}_j(t) = -\frac{i}{2}\Omega e^{i\mathbf{k}_0\cdot\mathbf{r}_j} + \left(i\Delta - \frac{\Gamma}{2}\right)\beta_j(t) - \frac{\Gamma}{2}\sum_{m\neq j}^{N}\beta_m \frac{\exp(ik_0 \mid \mathbf{r}_j - \mathbf{r}_m \mid)}{ik_0 \mid \mathbf{r}_j - \mathbf{r}_m \mid}$$

$$E_{\mathrm{sc}}(\mathbf{r}) = -\frac{\hbar\Gamma}{2d}\sum_{j=1}^{N}\beta_j \frac{e^{ik_0|\mathbf{r}-\mathbf{r}_j|}}{k_0|\mathbf{r}-\mathbf{r}_j|}$$

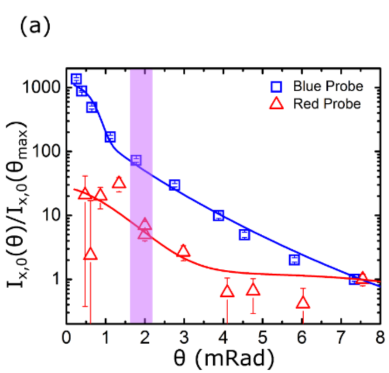
Spherical gaussian cloud : emission diagram

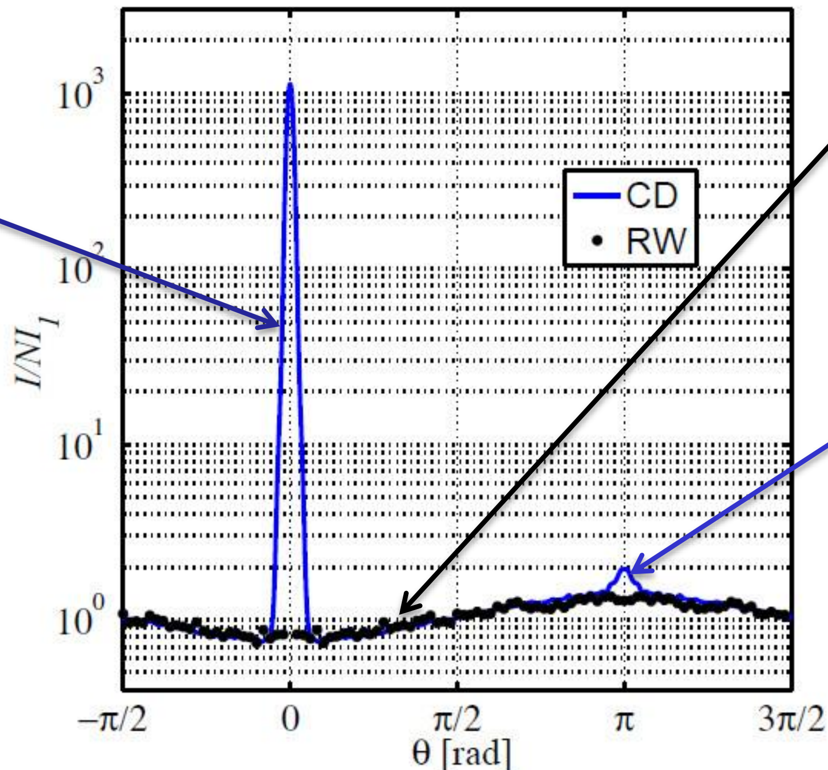
Cloud of atoms

Far field emission diagram



refractive index (mean field)



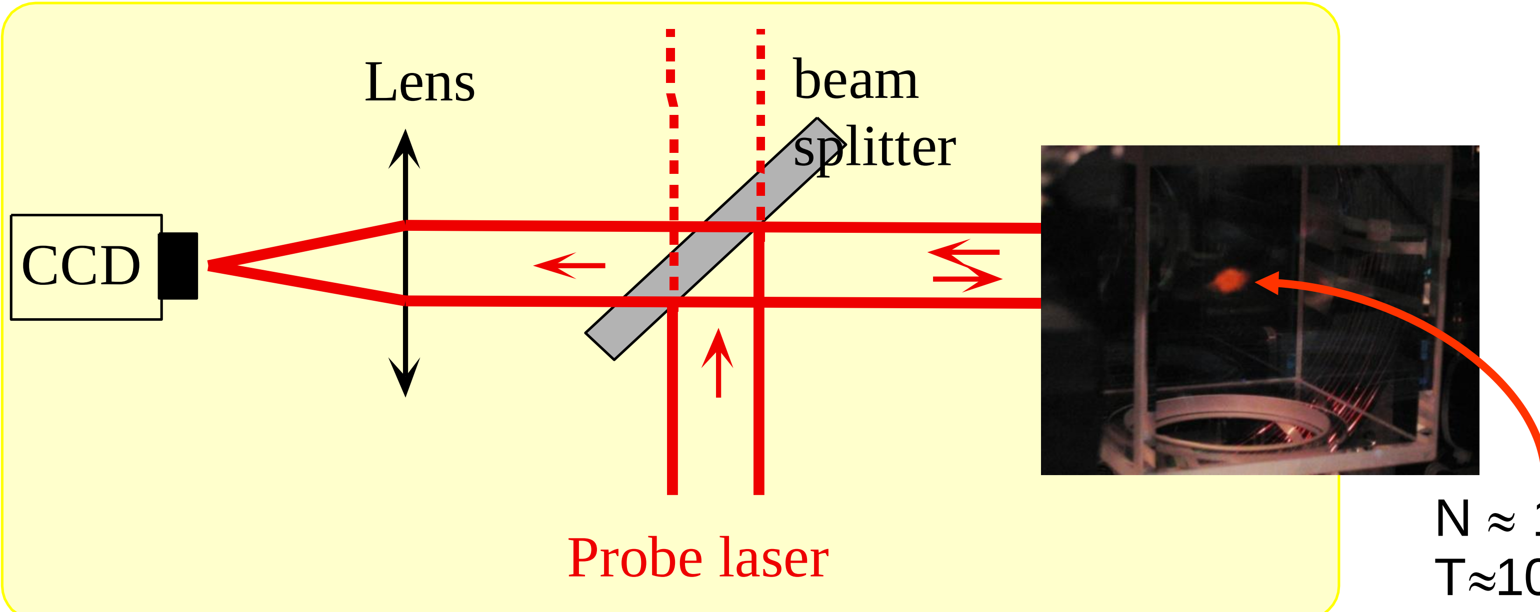


Incoherent model (particles trajectories, scattering in ‘empty modes’)

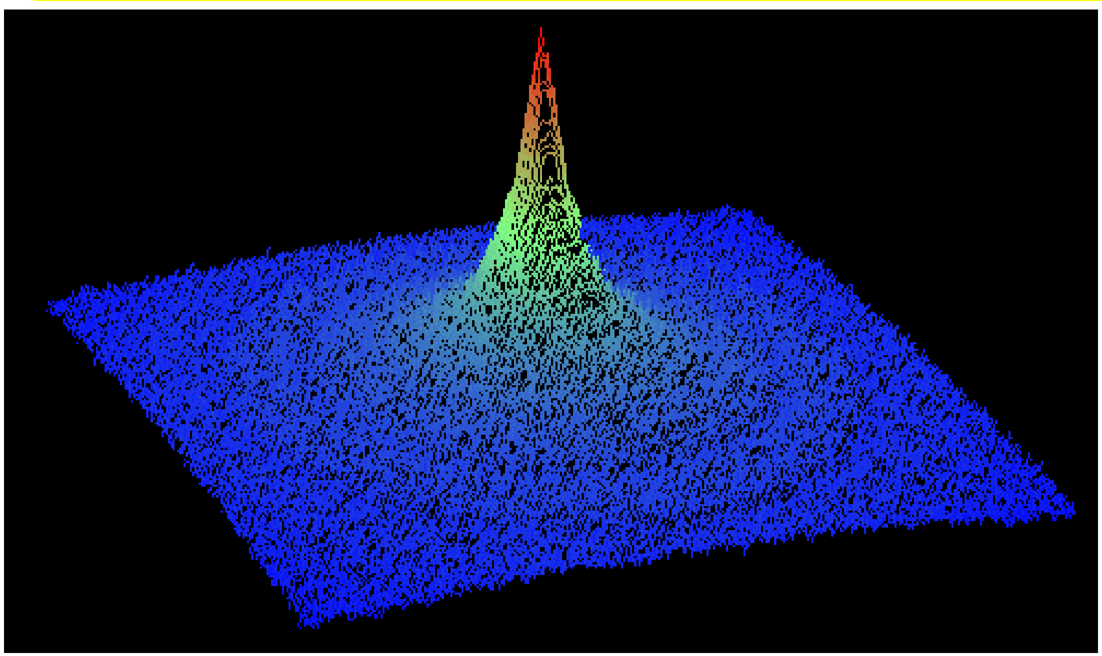
Mesoscopic physics: Weak localization (waves beyond mean field)

S. Bromley et al., Nat. Comm. 7, 11039 (2016)

Weak Localisation = precursor of strong Localisation?



$N \approx 10^{10}$
$T \approx 100 \mu K$
$k\ell \approx 1000$



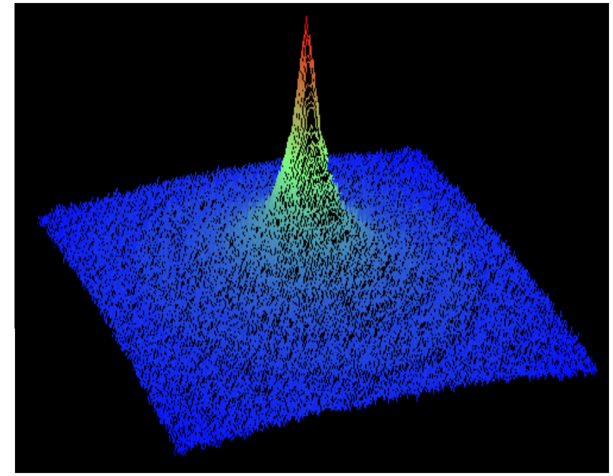
Coherence after resonant scattering with atoms !

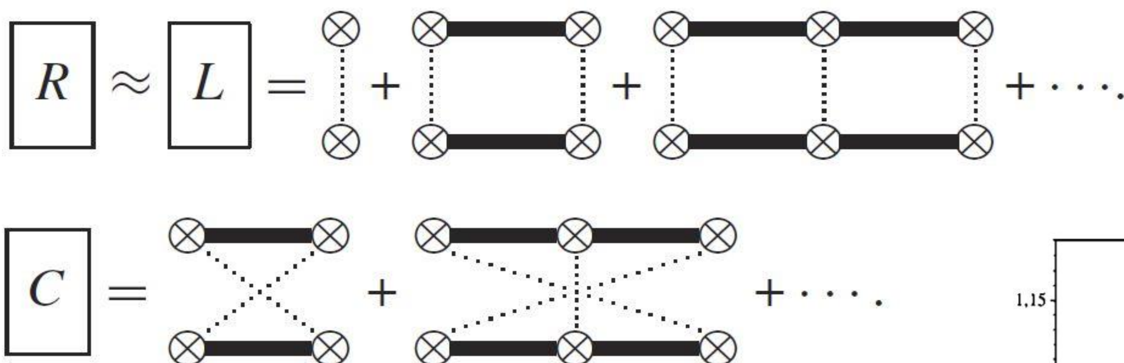
See also :
M. Havey's group

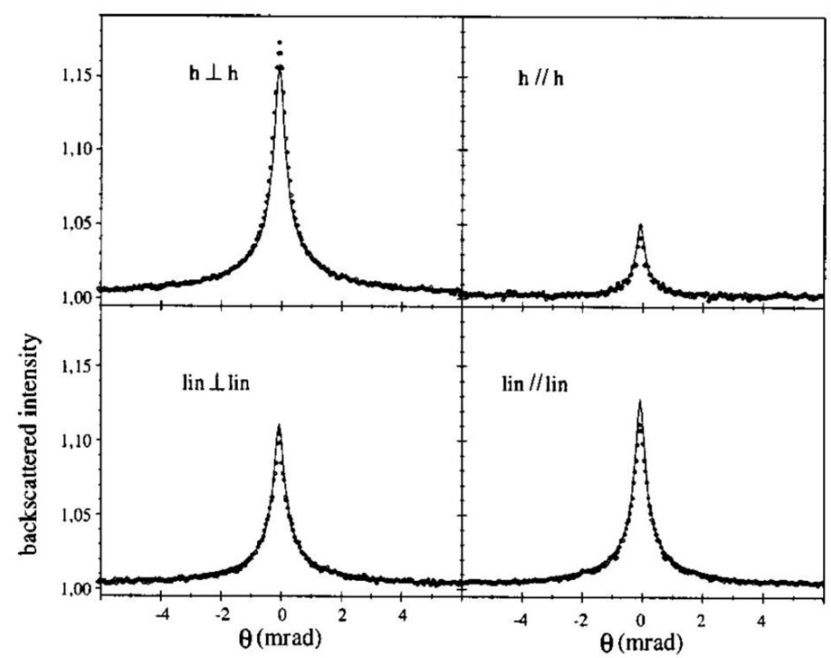
G. Labeyrie et al., Phys. Rev. Lett., **83**, 5266 (1999)

Theory :

- no “exact” solution
- diagrammatic approach



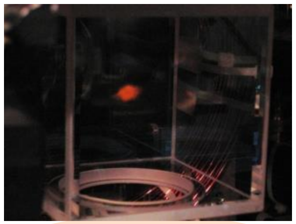




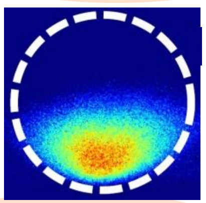
Excellent agreement
(no free parameter)

T. Jonckheere et al., Phys. Rev. Lett., **85**, 4269 (2000)

Towards strong localization of light : dense atomic clouds

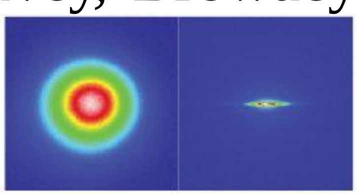


$k\ell \approx 1000$



$k\ell \approx 3$

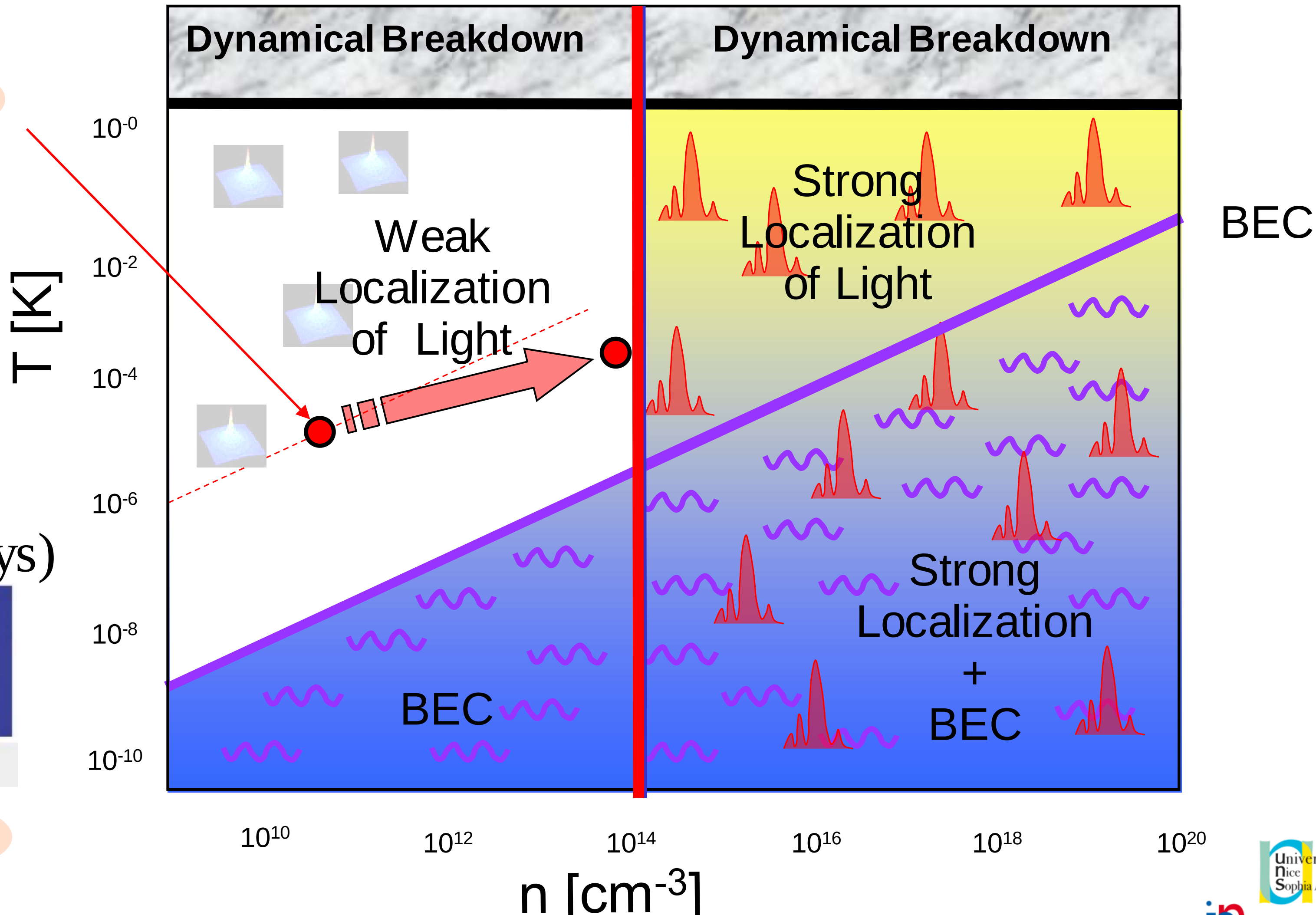
Dipole Trap
(Havey, Browaeys)



$N = 10^8$ $N = 10^7$

$k\ell < 1$

Ioffe-Regel : $k\ell \approx 1$



Theory : Effective Hamiltonian

$$H_{eff} = (\hbar\omega_0 - i\frac{\hbar\Gamma_0}{2}) \sum_i S_i^z + \frac{\hbar\Gamma_0}{2} \sum_{i \neq j} V_{ij} S_i^+ S_j^-$$

Diagonal :
On site energy

Off diagonal :
transport

$$V_{ij} = \beta_{ij} - i\gamma_{ij}$$

$$\beta_{ij} = \frac{3}{2}\left[-p\frac{\cos k_0 r_{ij}}{k_0 r_{ij}} + q\left(\frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^3} + \frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

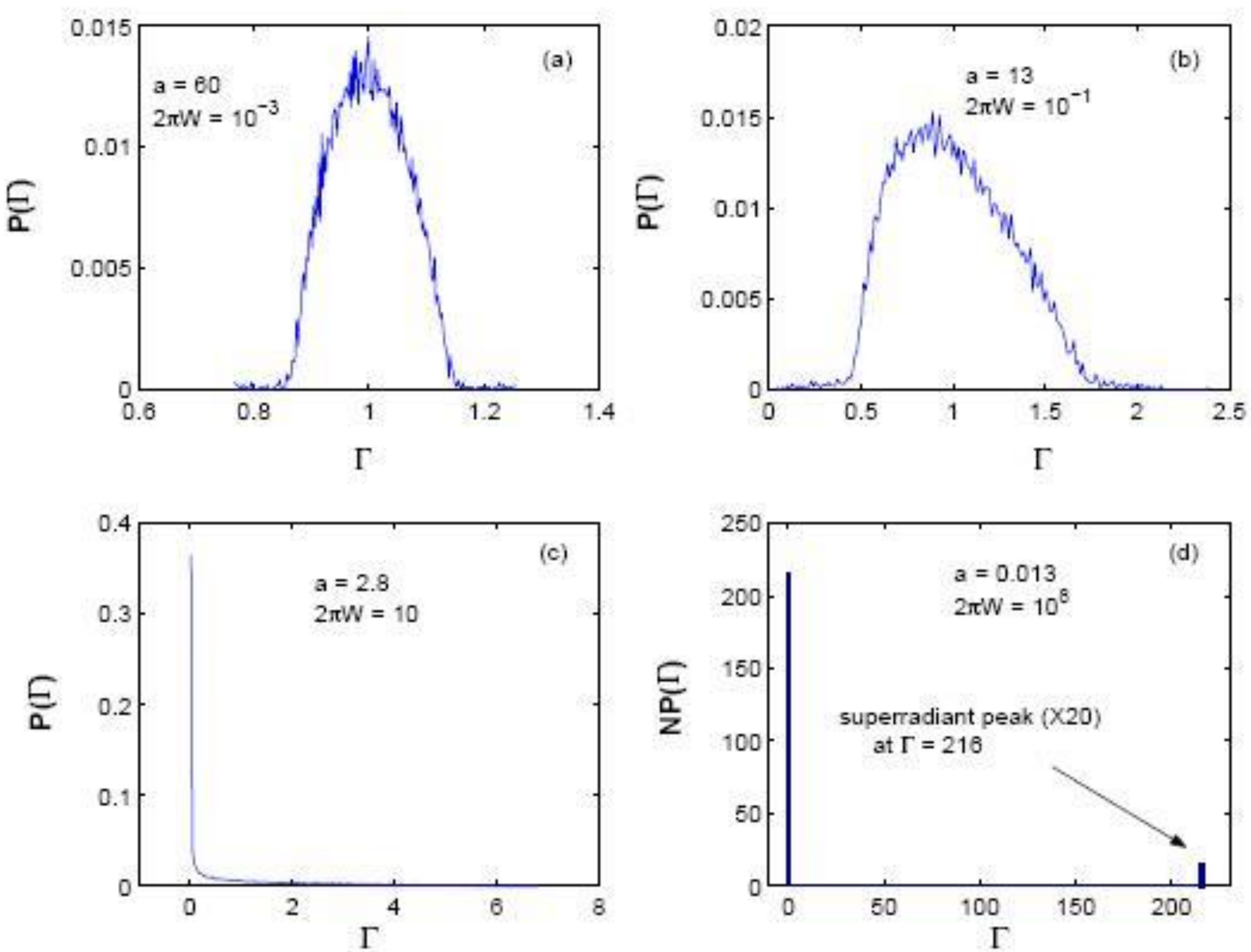
$$\gamma_{ij} = \frac{3}{2}\left[p\frac{\sin k_0 r_{ij}}{k_0 r_{ij}} - q\left(\frac{\sin k_0 r_{ij}}{(k_0 r_{ij})^3} - \frac{\cos k_0 r_{ij}}{(k_0 r_{ij})^2}\right)\right]$$

- Open System
- Reminiscent of Anderson Hamiltonian
- Heisenberg model with global coupling
- Long range hopping
- No decoherence (coupling to phonons, …)

« Life time » of photons in the system

(motivated by experimental approaches)

Photon Escape Rate = Spectrum {Im (H_{eff}) }



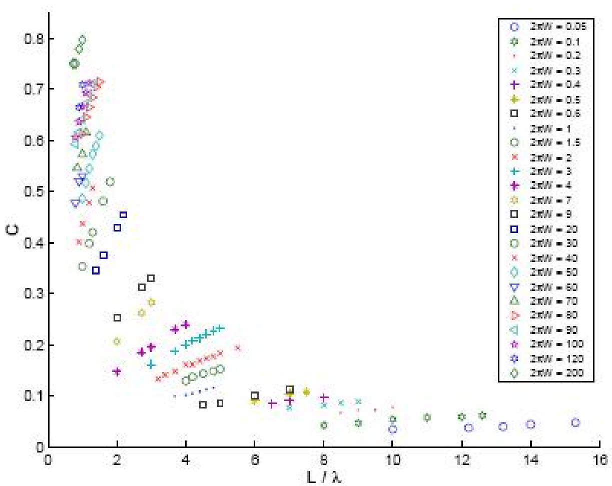
size : a = L/ℓ
disorder parameter W=1/kℓ

See also F. Pinheiro et al. 2004

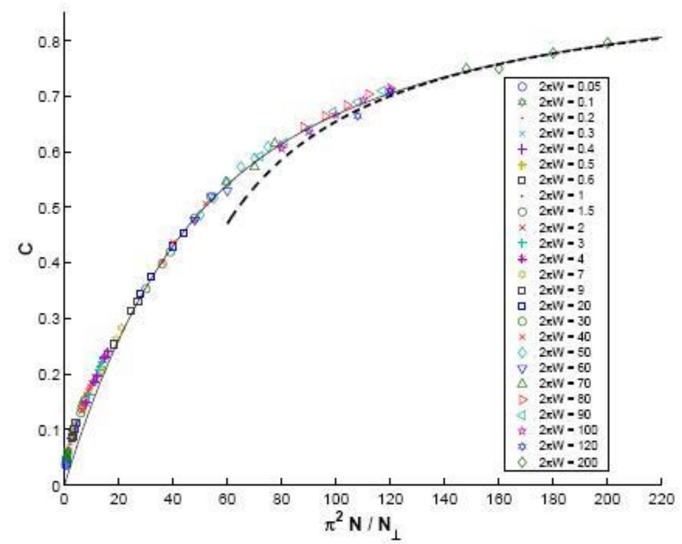
Photon Escape Rates

$$\mathbf{C(a, W) = 1 - 2\int_1^{\infty} P(\Gamma)d\Gamma}$$

Measure of long lived photons



Single parameter scaling
$N/N_{\perp}$



cooperative effects dominate over disorder !
no phase transition observed with P(Γ)

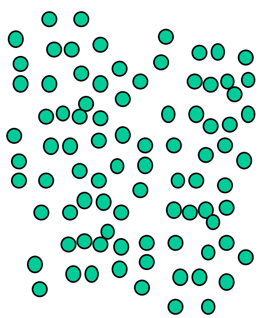
Dicke > Anderson

E. Akkermans, A. Gero, RK, PRL, **101**, 103602 (2008)

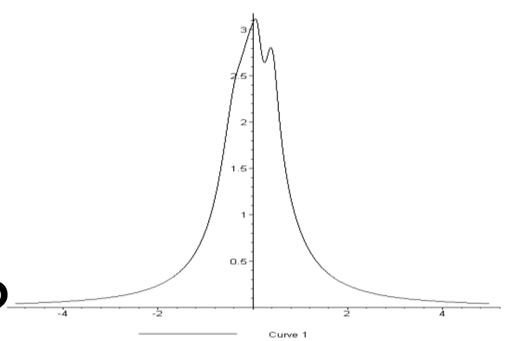
Eigenvalues

Beyond Photon escape times :

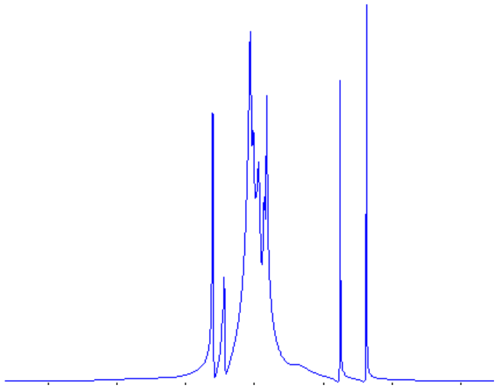
Cloud of Atoms = Large Molecule (with 10^{10} atoms)



'dilute' molecule



'dense' molecule



molecular spectrum ?

SHENG LI AND ERIC J. HELLER PHYSICAL REVIEW A **67**, 032712 (2003)

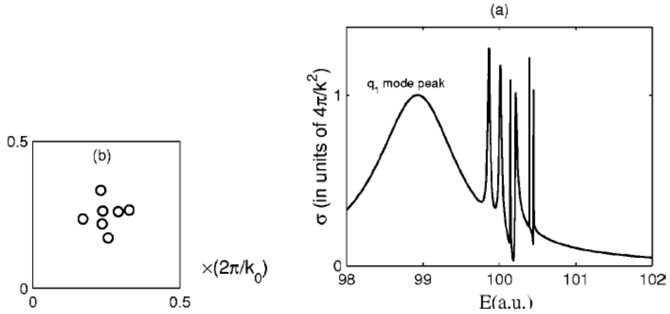
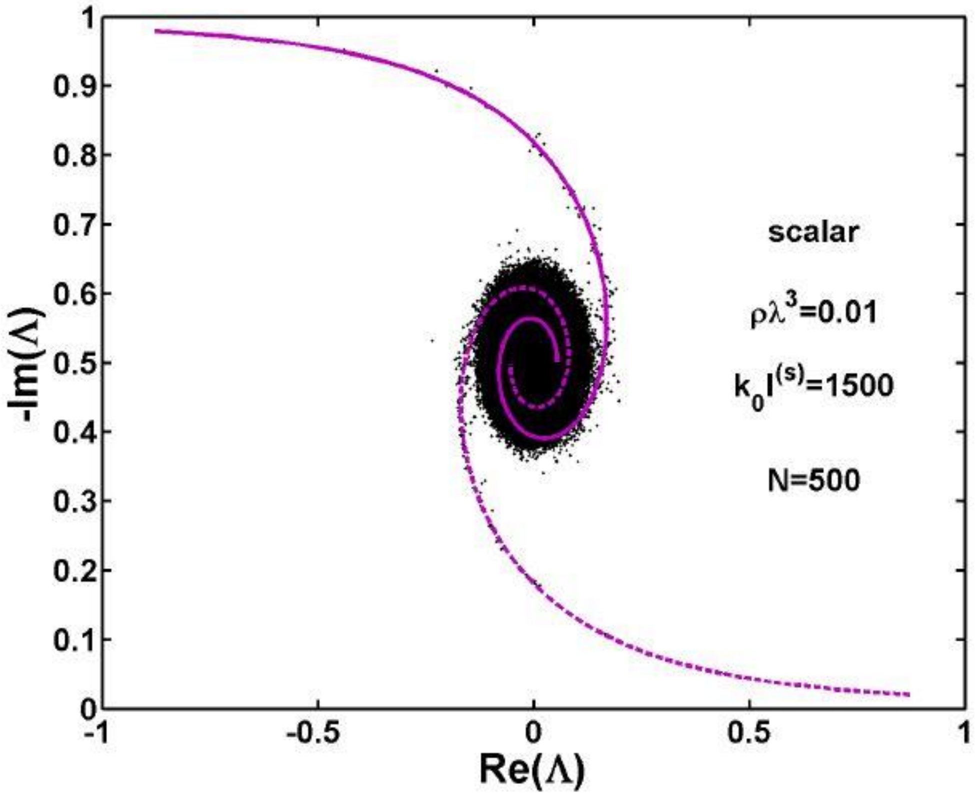


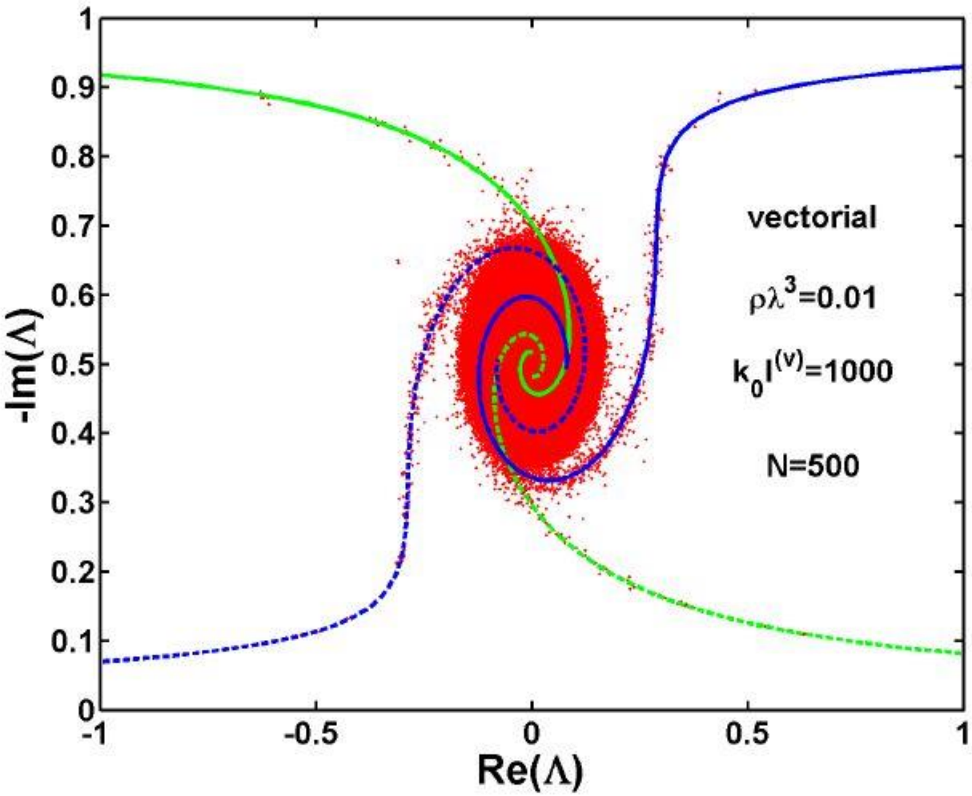
FIG. 4. (a) Total cross section as a function of energy for a system of seven identical scatterers randomly placed on a plane. Each scatterer is the same as used in Fig. 1. The positions of the scatterers are shown in (b).

proximity resonances
doorway states
giant oscillator strength

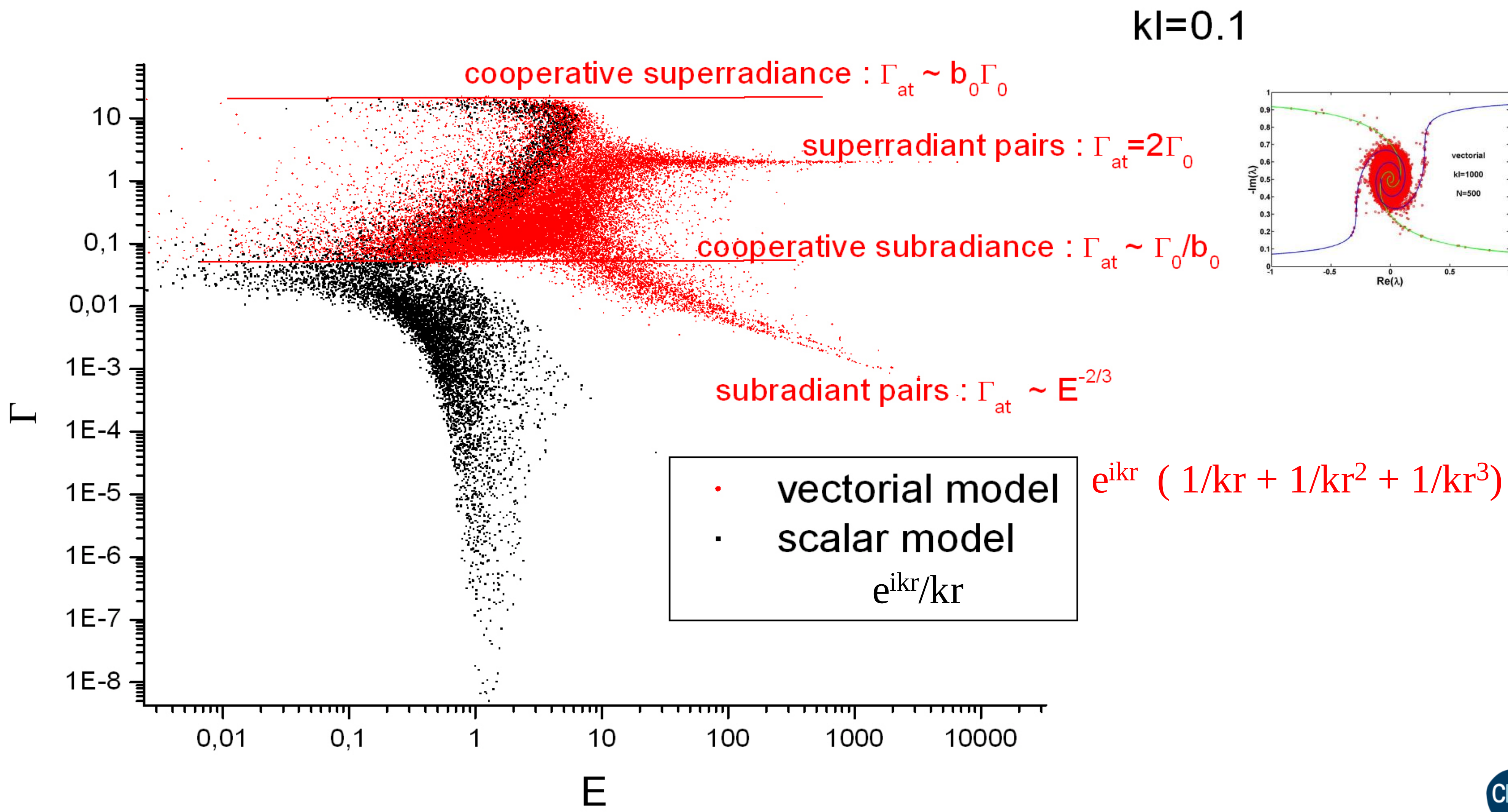
Eigenvalues for N coupled dipoles

Important near field terms for high densities

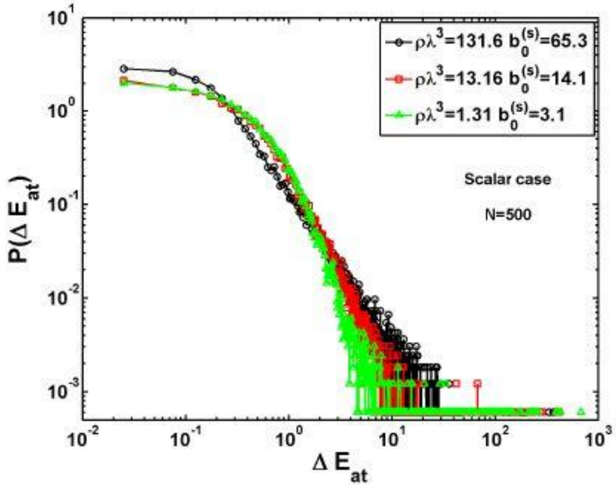


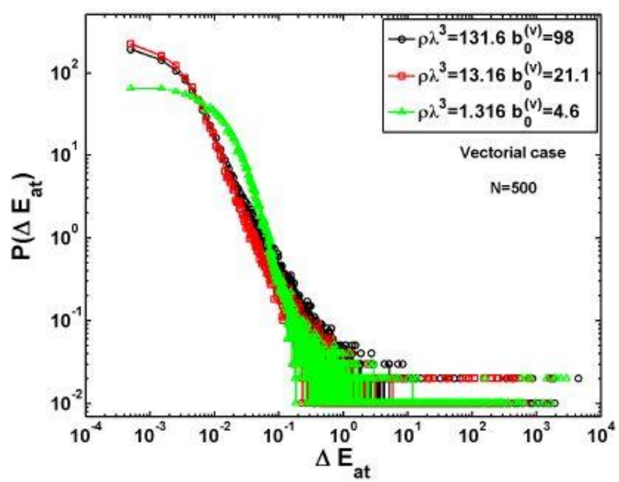


Eigenvalues

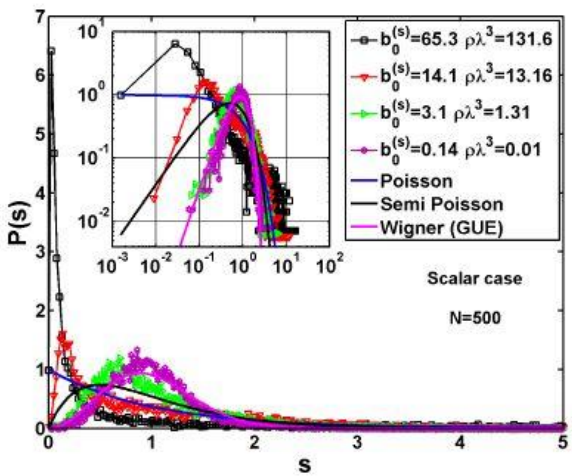


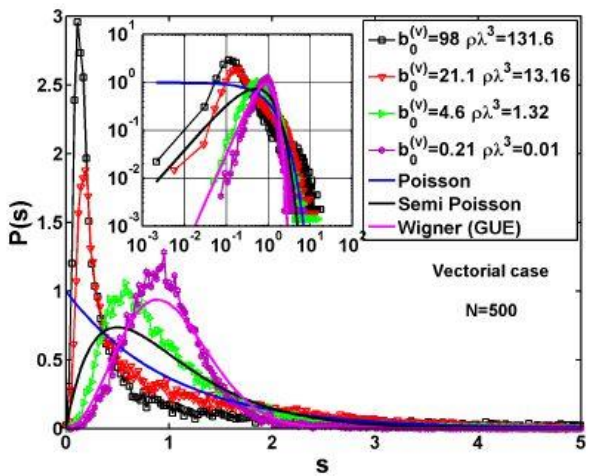
Eigenvalues: some statistics





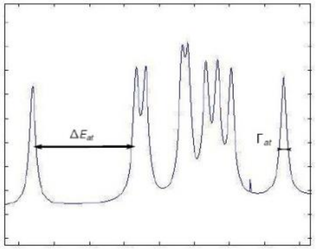
No level repulsion



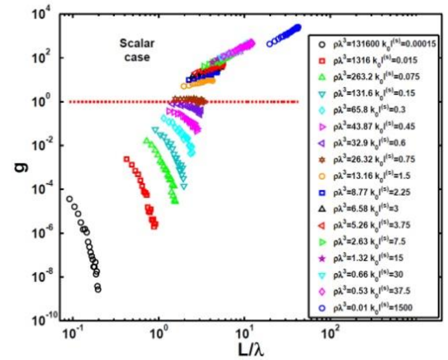


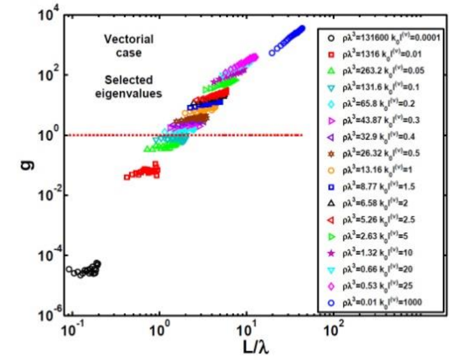
Phase rigidity

Resonance Overlap (« Thouless »)

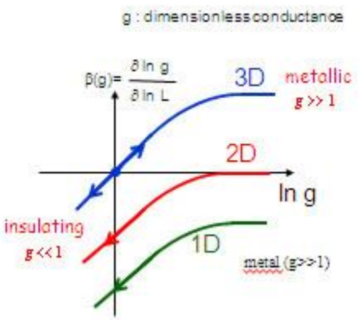


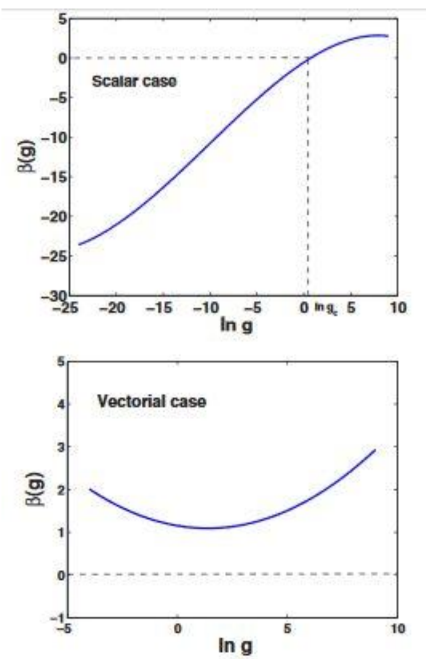
$$g = \left\langle \frac{1}{\langle 1/\Gamma \rangle \langle \Delta E \rangle} \right\rangle$$





Scaling function β(g)





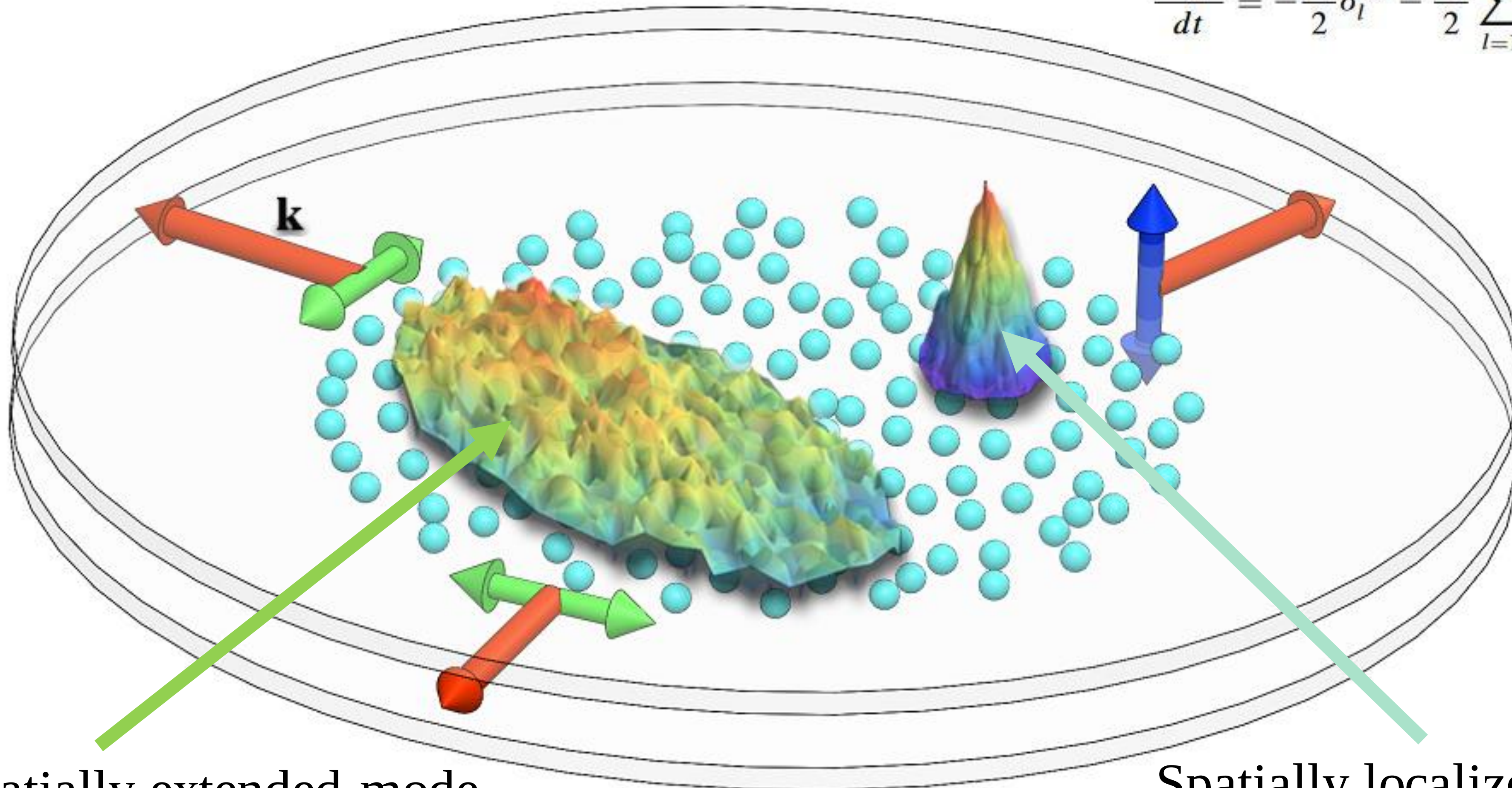
NO ANDERSON LOCALISATION FOR VECTORIAL LIGHT IN 3D

Skipetrov & Sokolov, PRL 112, 023905 (2014)

Bellando et al,. Phys. Rev. A 90, 063822 (2014)

LOCALISATION in 2D

$$\frac{d\hat{\sigma}_j^{(0)}}{dt} = -\frac{\Gamma_0}{2}\hat{\sigma}_l^{(0)} - \frac{\Gamma_0}{2}\sum_{l=1}^{N} H_0(kr_{jl})\hat{\sigma}_l^{(0)},$$



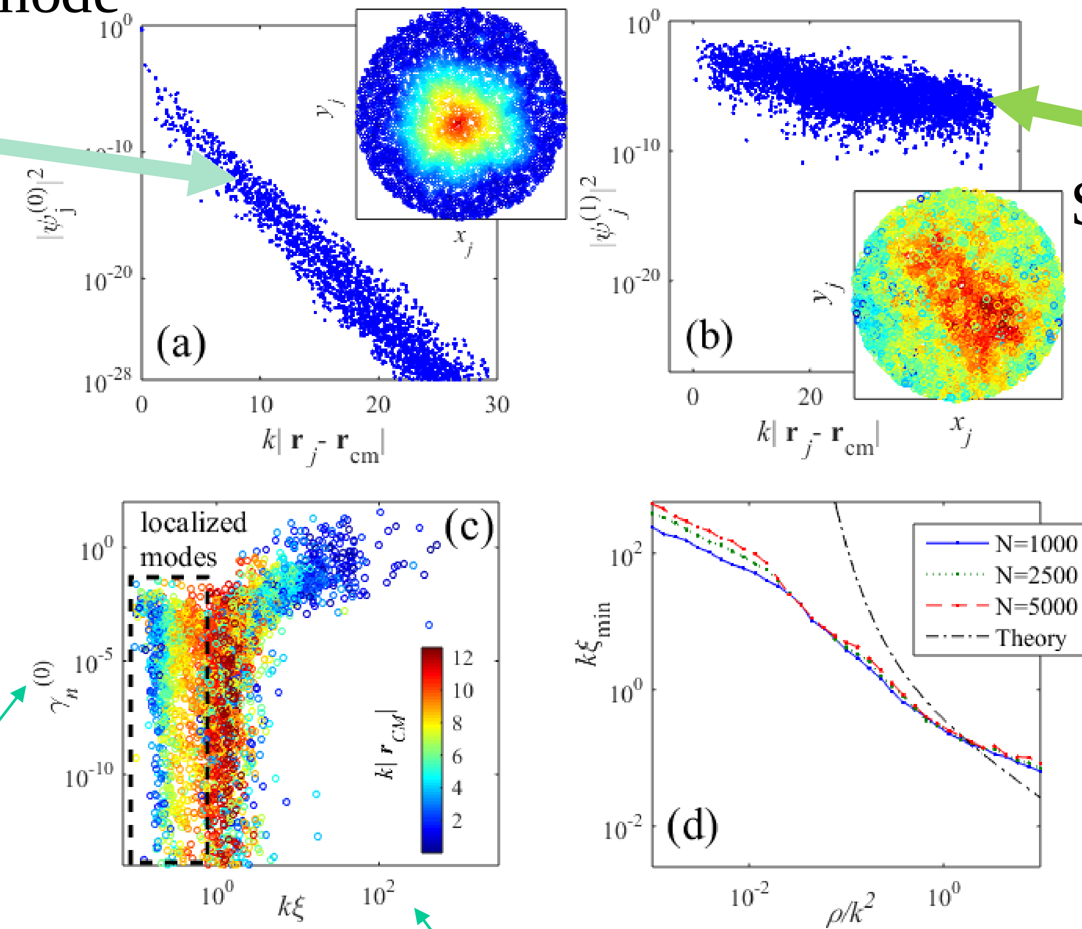
Spatially extended mode
(vectorial case)

Spatially localized mode
(scalar case)

$$\frac{d\hat{\sigma}_j^{(\pm1)}}{dt} = -\frac{\Gamma_1}{2}\hat{\sigma}_j^{(\pm1)} - \frac{\Gamma_1}{2}\sum_{l\neq j}\left[H_0(kr_{jl})\hat{\sigma}_l^{(\pm1)} + e^{2i\varphi_{jl}}H_2(kr_{jl})\hat{\sigma}_l^{(\mp1)}\right]$$

Mode profiles

Spatially localized mode (scalar case)



Spatially extended mode (vectorial case)

Mode width NOT correlated to localisation length : temporal vs spatial localisation

PHYSICAL REVIEW A **92**, 062702 (2015)

Spatial and temporal localization of light in two dimensions

C. E. Máximo,[1] N. Piovella,[2] Ph. W. Courteille,[1] R. Kaiser,[3] and R. Bachelard[1,*]

The case for Dicke …

coherence of atoms

1954 : Dicke super- and subradiant states

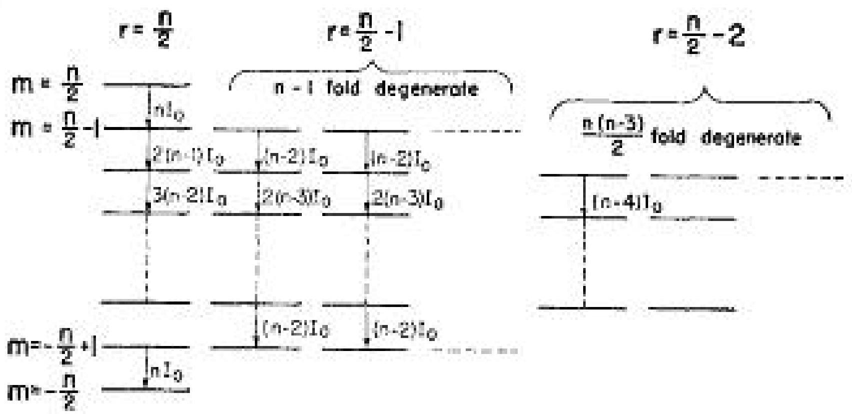
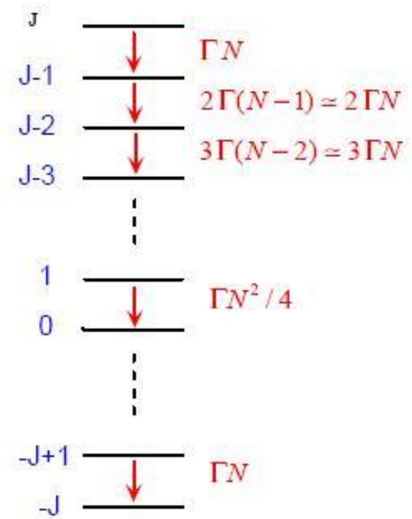
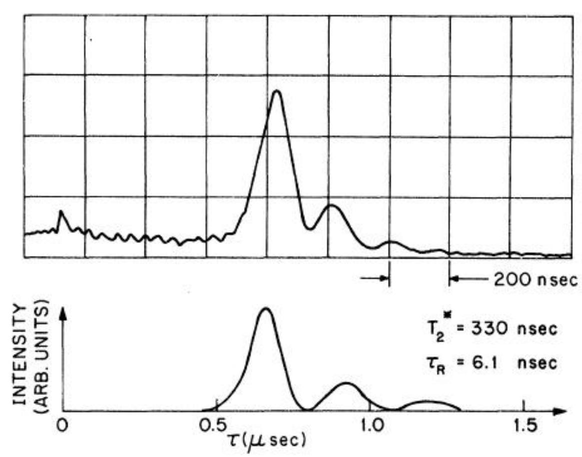


FIG. 1. Energy level diagram of an n-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.



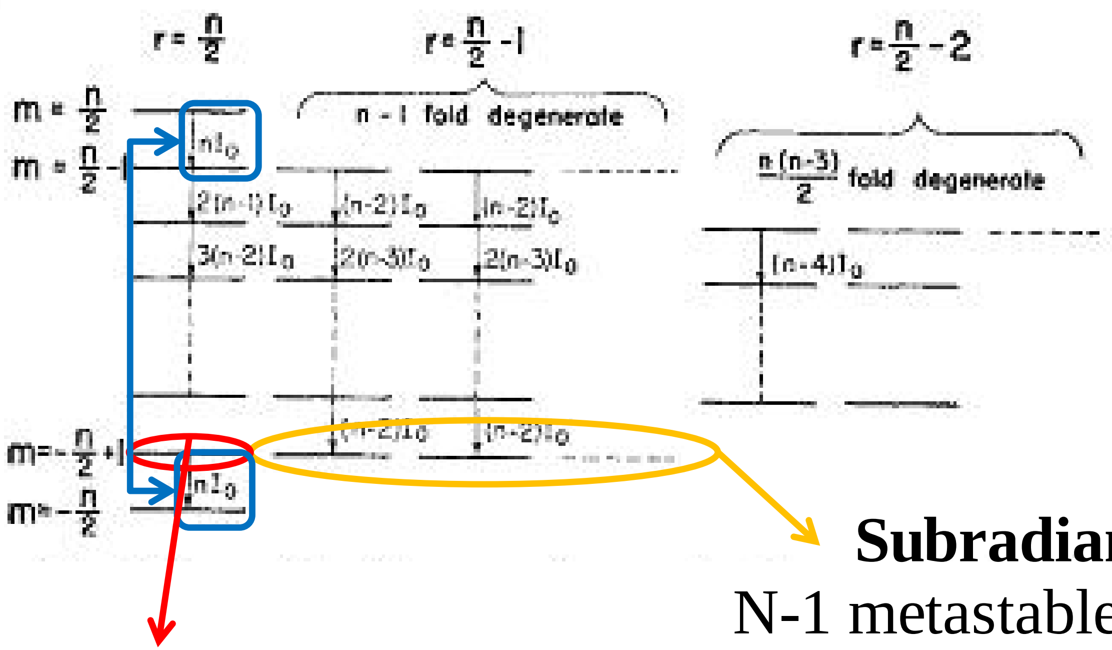
R. Dicke 1954

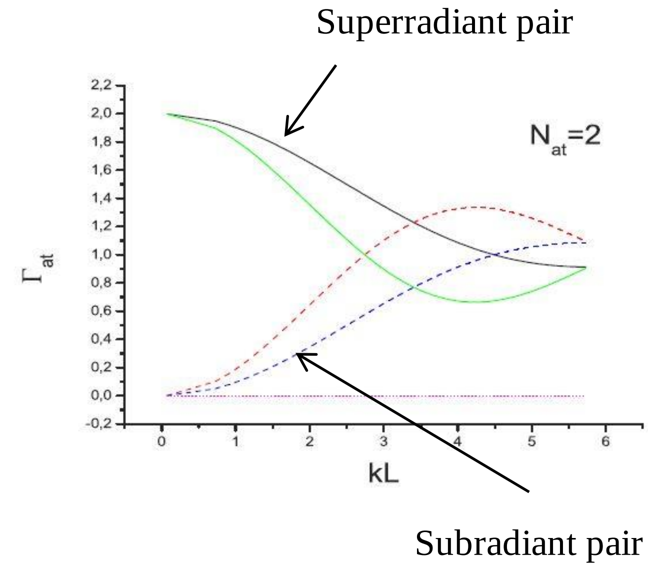


First experimental observation of superradiance

Feld et al. 1973

Single photon excitation / low intensity limit

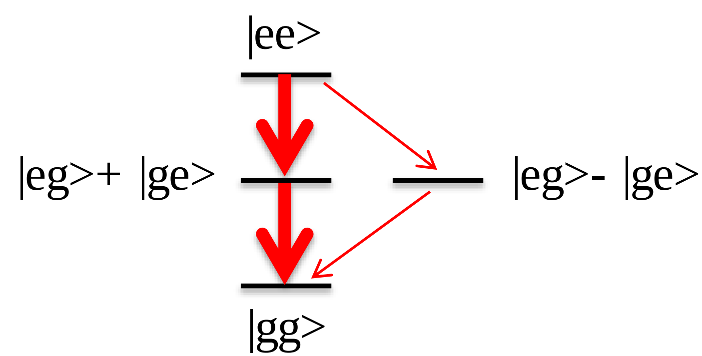




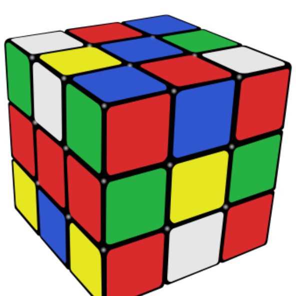
Subradiant

N-1 metastable states

$\Gamma_{max} \sim N\,\Gamma_0$

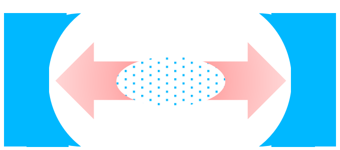


Extended Volume : $b_0 = N_{at}/N_{modes}$



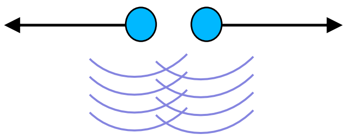
Cooperativity without cavity

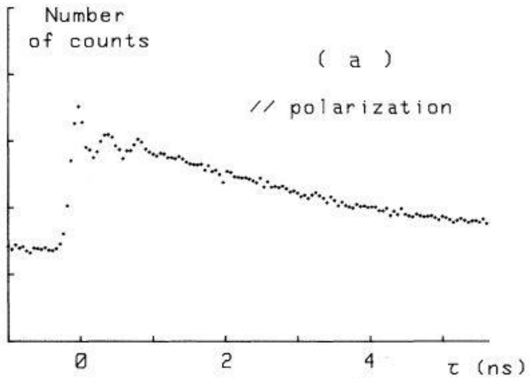
(also Random lasing)



The quest for Dicke subradiance

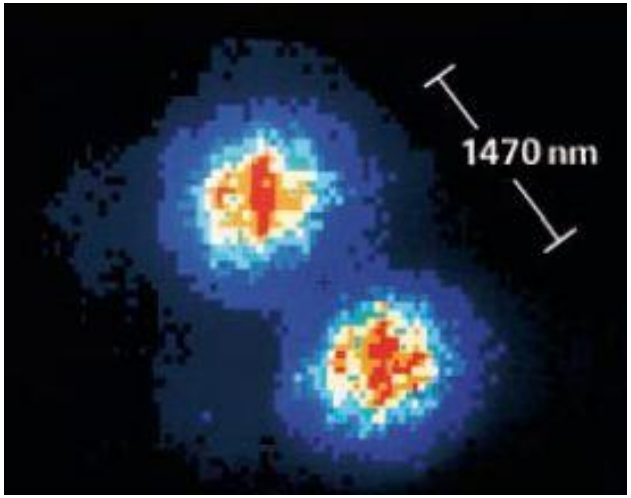
Single Photon interference from N=2 atoms

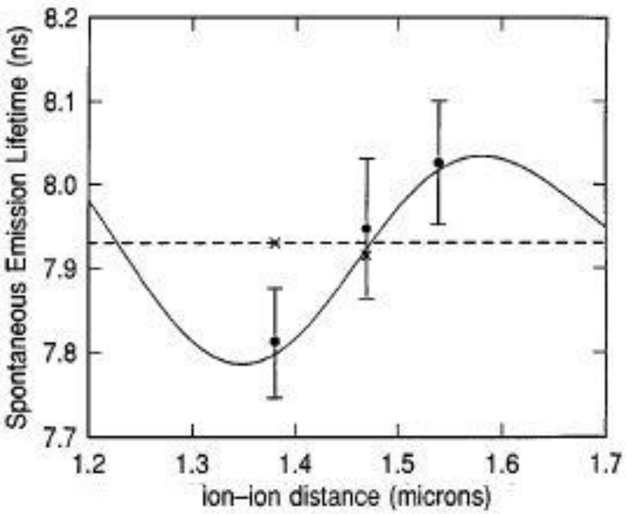




P. Grangier, A. Aspect, J. Vigue, PRL 54, 418 (1985).

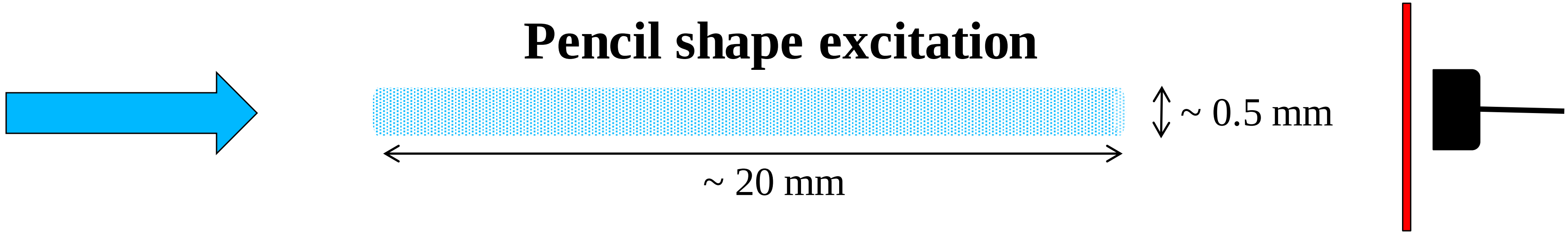
Subradiant pairs : N=2





R. G. DeVoe, R. G. Brewer, PRL 76, 2049 (1996).

Elusive Subradiance (for N>2)



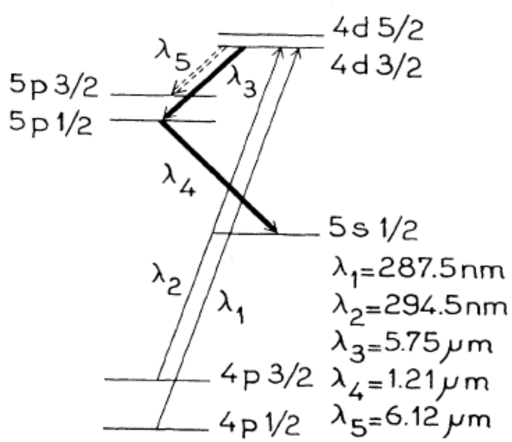


FIG. 1. Relevant level diagram of gallium.

τ_{nat}=7ns

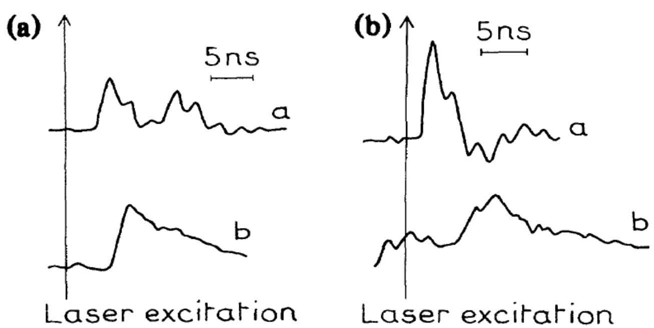


FIG. 2. Typical superradiant signals. (a) corresponds to the excitation with saturated linearly polarized light from the $4p_{3/2}$ level; (b), the $4p_{1/2}$ level. In both cases, traces *a* and *b* correspond respectively to $4d \rightarrow 5p$ and to $5p \rightarrow 5s$ transitions. The visible oscillations of the signals are most likely due to the hyperfine structure of the $5p_{1/2}$ level.

Forward ‘subradiance echo’

D. Pavolini et al. , Phys. Rev. Lett. 54, 1917 (1985)

Fragile subradiance

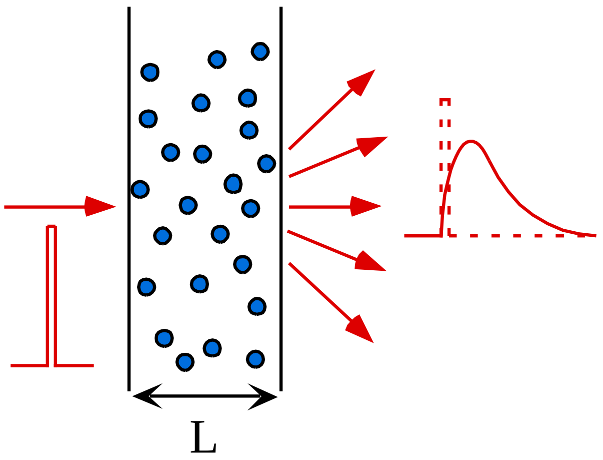
Dicke subradiance for N two level systems (in free space, N>>2) has **not yet been observed**

- Does **not** require large spatial densities
  (near field effect maybe even bad : Gross&Haroche 1982)

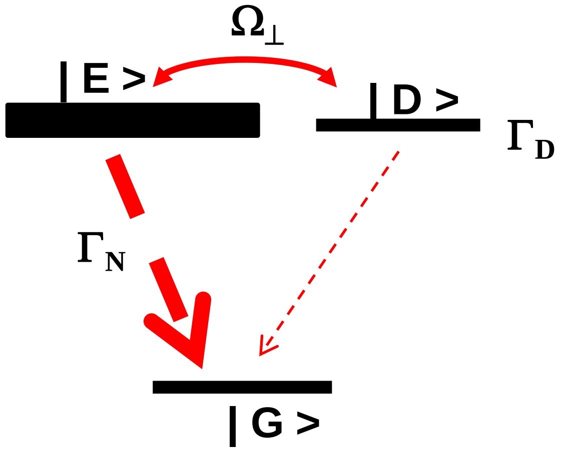
- Requires large optical densities in all directions (b_0>>1)

- Exploits the 1/r **long range** dipole-dipole interaction

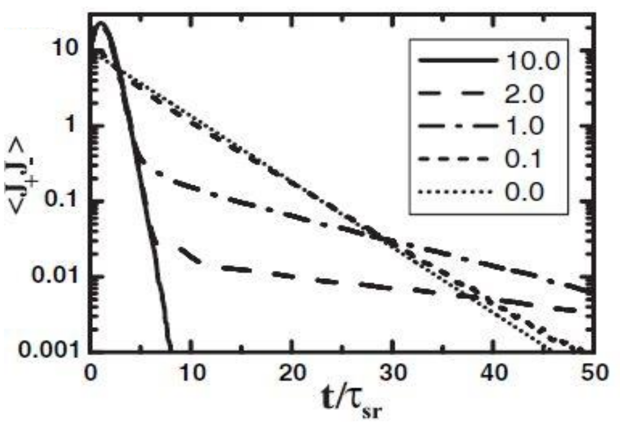
Time dependent experiments : coherent scattering



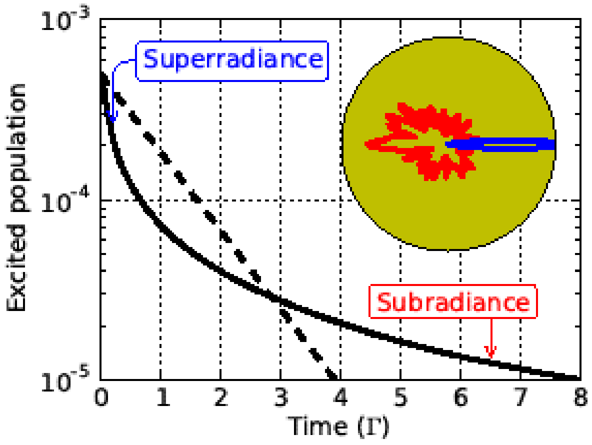
Superradiance = bright state
Subradiance = metastable 'dark' states



Inverted system



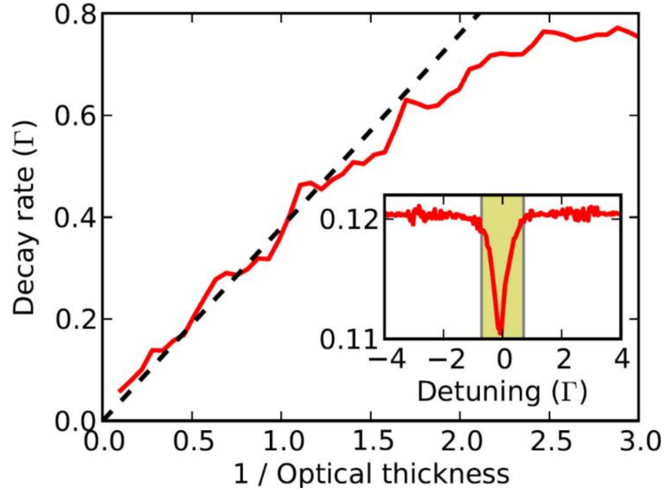
Temnov, Woggon, PRL 95, 243602 (2005)



T. Bienaimé, N. Piovella, R.K. PRL (2012)

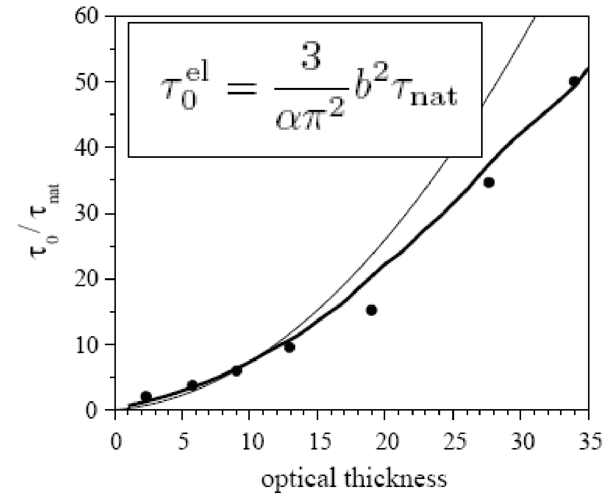
Subradiance vs incoherent scattering

$t_{sub} \propto b_0$



- Does not require large spatial densities
- Requires large optical densities

$t_{Rad.Trap.} \propto b(\delta)^2$

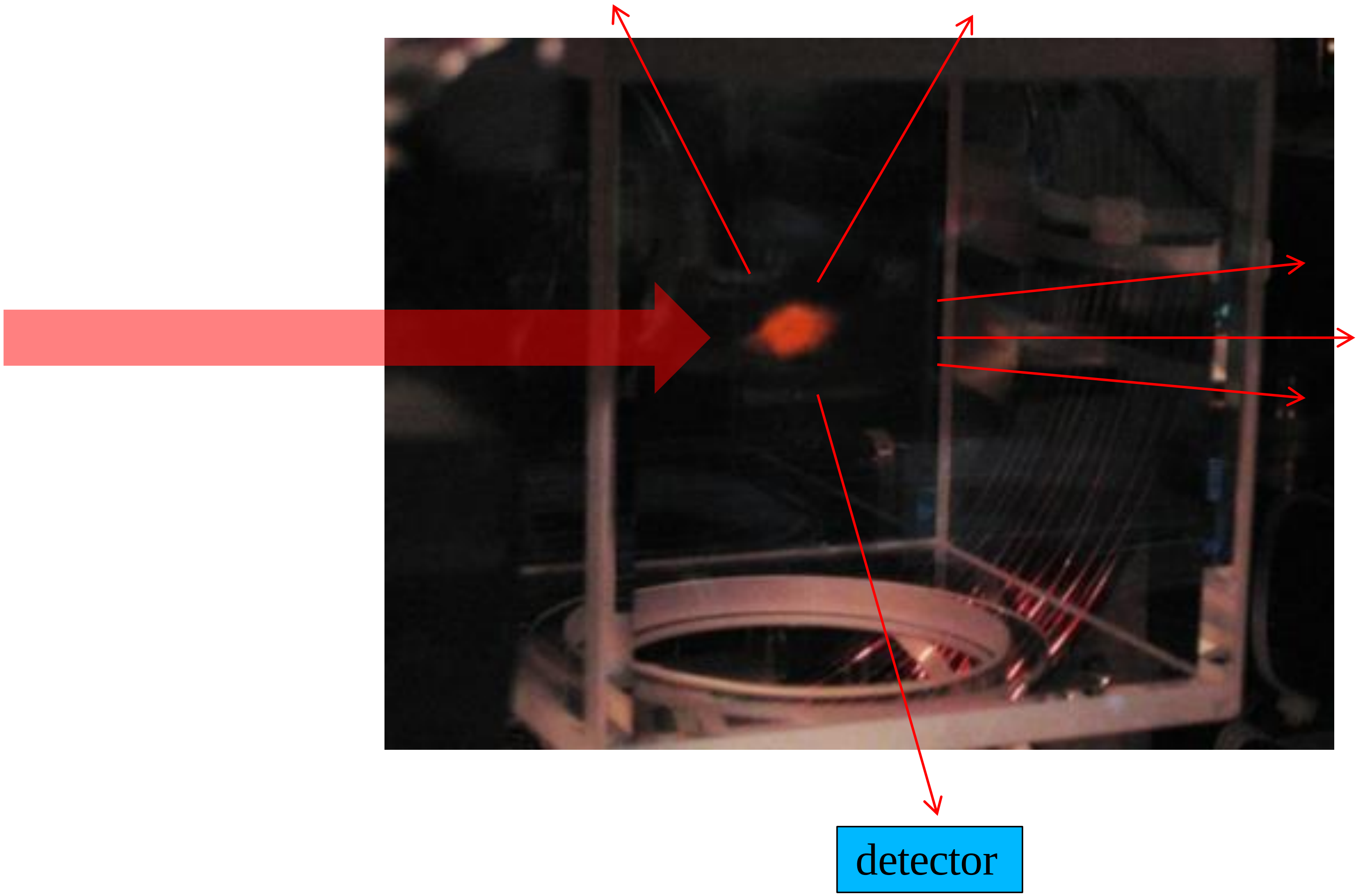


- Random walk of photons (without interference)
- Diffusion equation

$t_{Anderson} \propto \exp\{b(\delta)\}$

- Density Threshold ?

Experiment



$N=10^9$ ^{87}Rb

$T=50$ μK

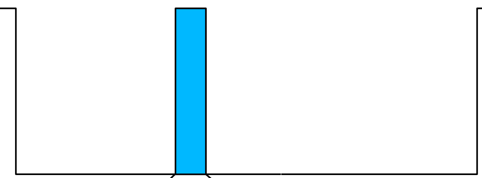
$R=1$ mm

$\rho=10^{11}$/cc

$b_0 = 20\dots100$

Average data (on multichannel scalar)

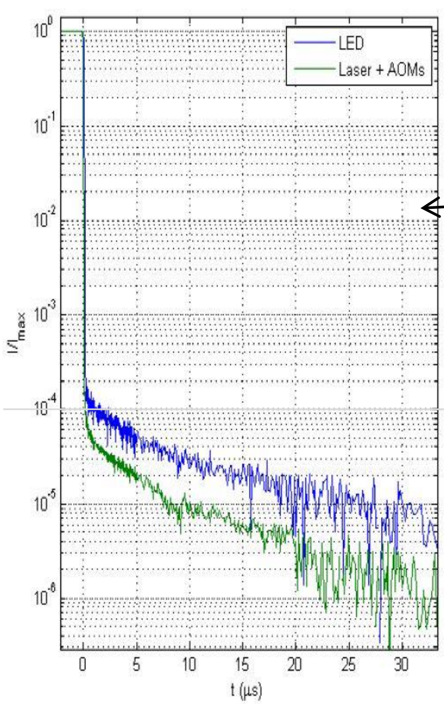
MOT + Dark MOT
(50+30 ms)

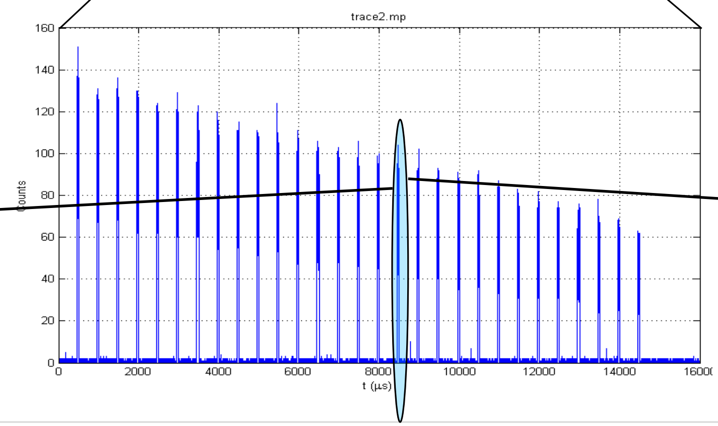


Subradiance Probe

Data average :
500 000 cycles
(1 curve/night)

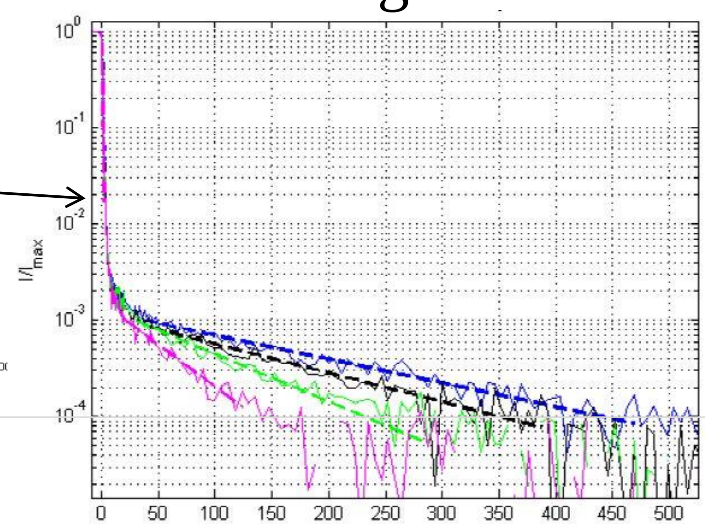
Calibration





12 pulses of 30µs

Signal

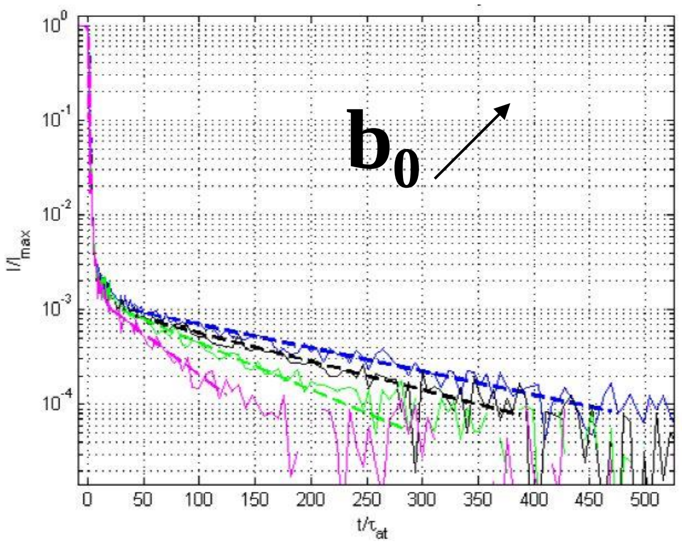


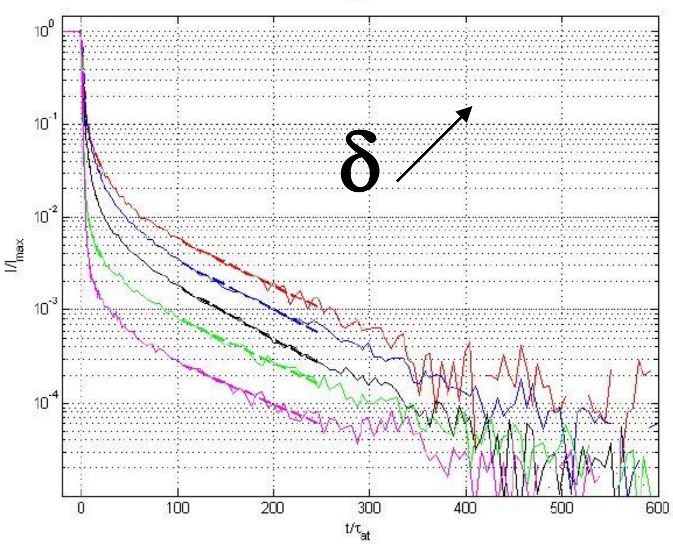
t/τ_{at}

Hybrid photomultiplier

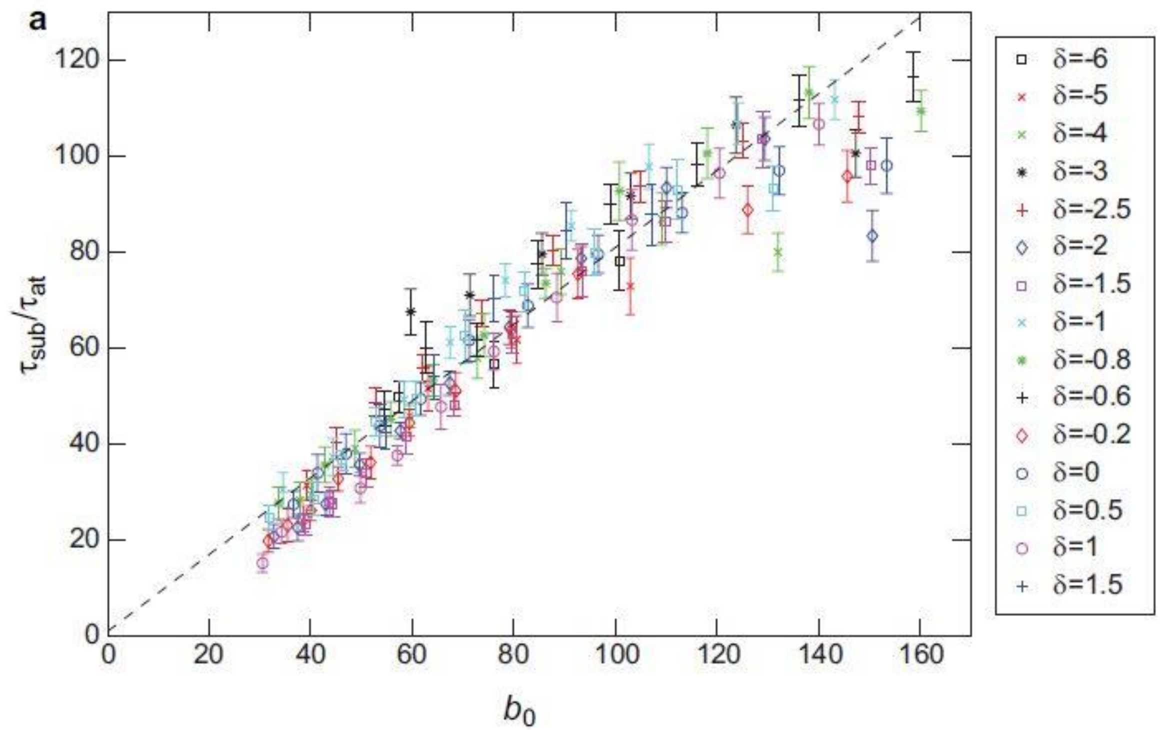
Experimental results

Long decay at b(δ)<1 ☺





$\tau(b_0)$



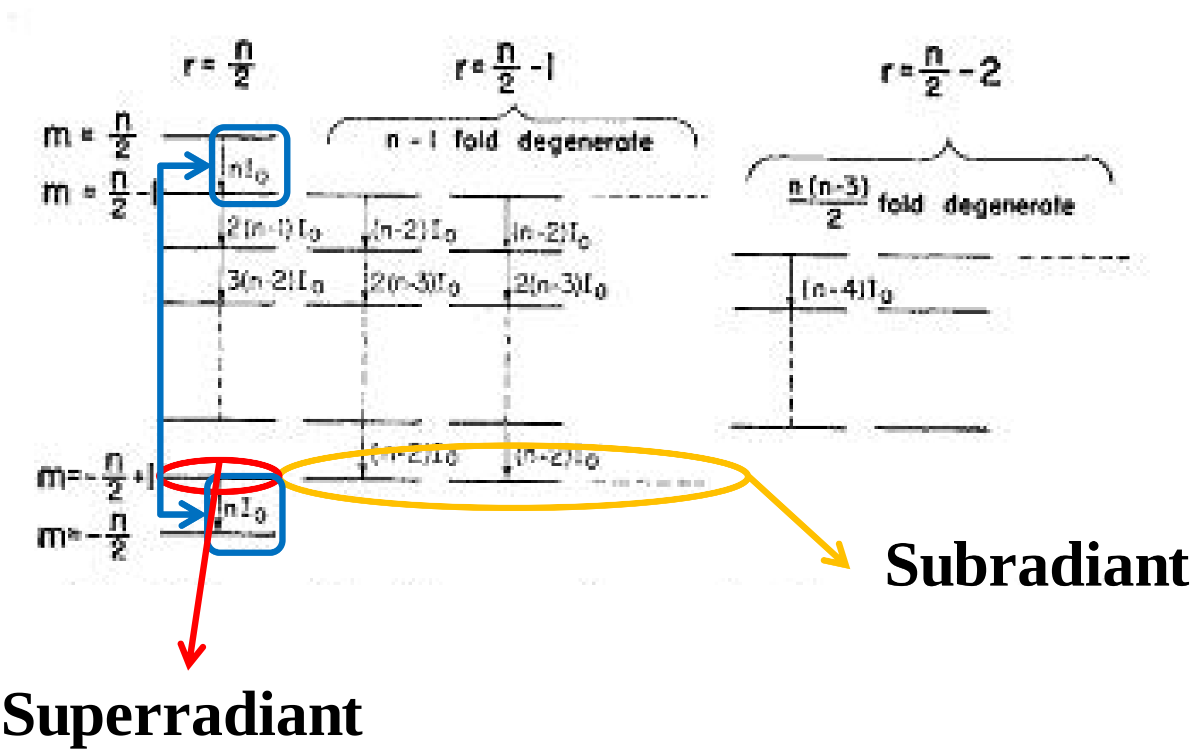
Increases as b_0 ☺

Selected for a Viewpoint in *Physics*
PRL 116, 083601 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

Subradiance in a Large Cloud of Cold Atoms

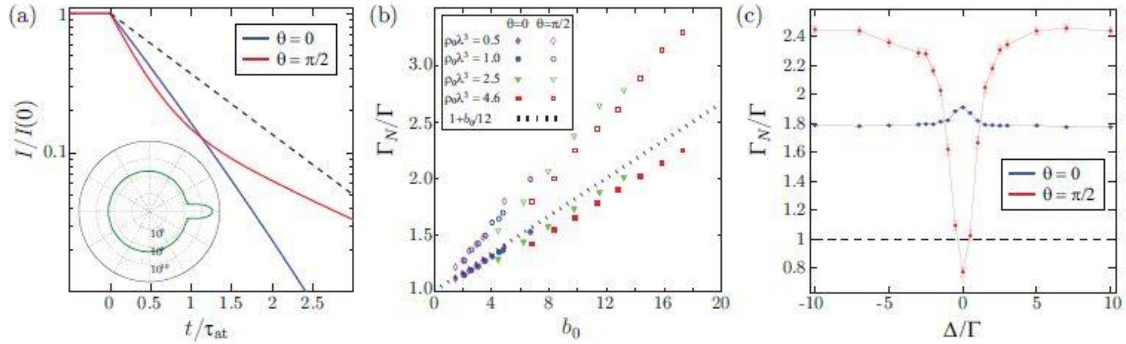
William Guerin,[1,*] Michelle O. Araújo,[1,2] and Robin Kaiser[1]

The ‘super’ of ‘single photon Dicke states’

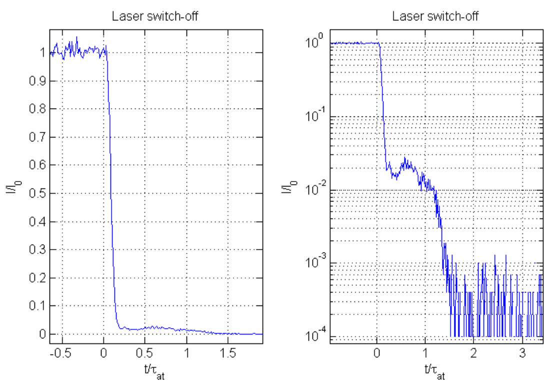


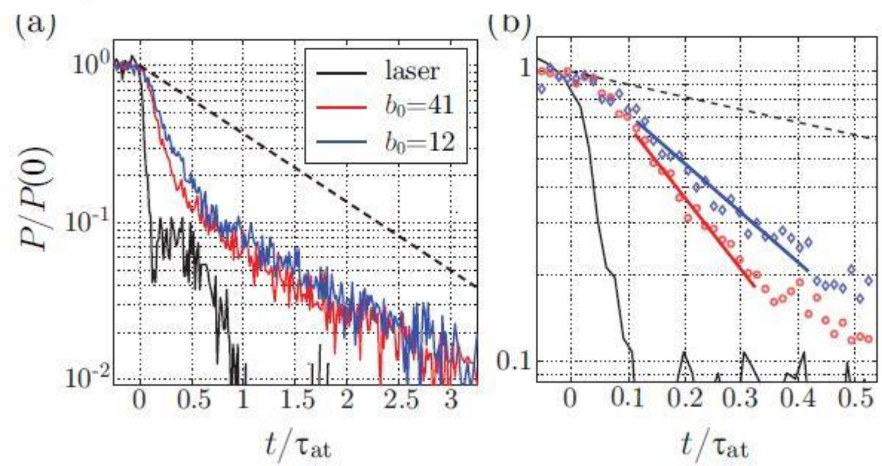
Off-axis Superradiance : physics/1603.07204

Simulations

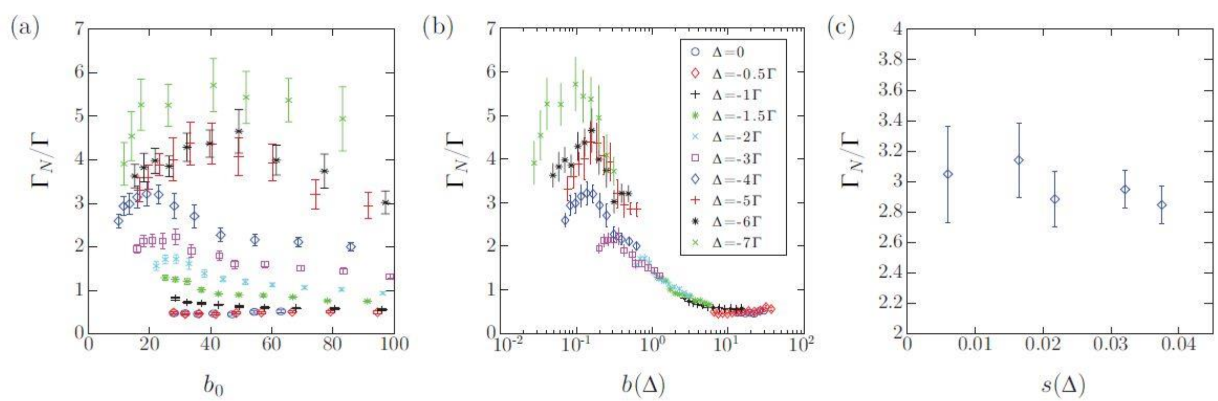


Exp. Data





Results



Université Nice Sophia Antipolis

Institut non linéaire de Nice SOPHIA ANTIPOLIS

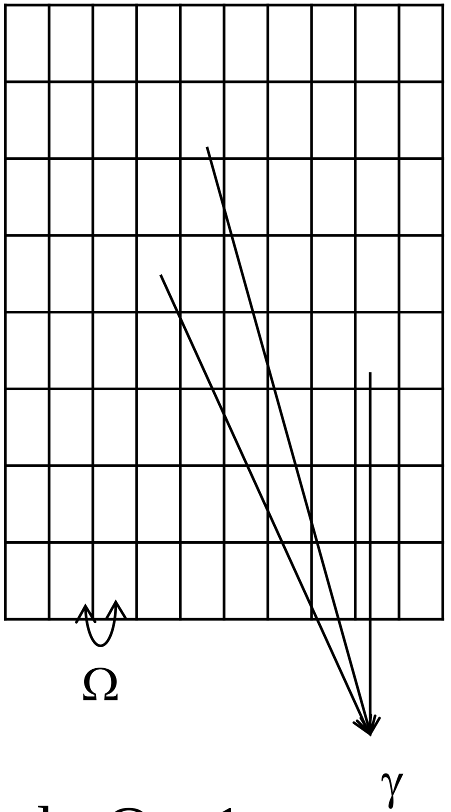
Combining Anderson and Dicke
Toy Model : Open Disordered System:

A. Biella et al., EPL, 103, 57009 (2013)

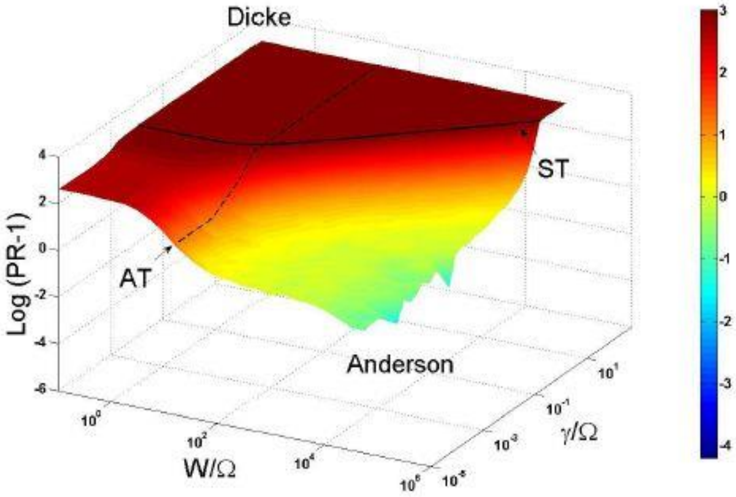
3D Anderson model on 10x10x10 lattice
hoping (Ω) + disorder (W) + opening (γ)

$$H_0 = \sum_{j=1}^{N} E_j |j\rangle\langle j| + \Omega \sum_{\langle i,j\rangle} \left(|j\rangle\langle i| + |i\rangle\langle j|\right)$$

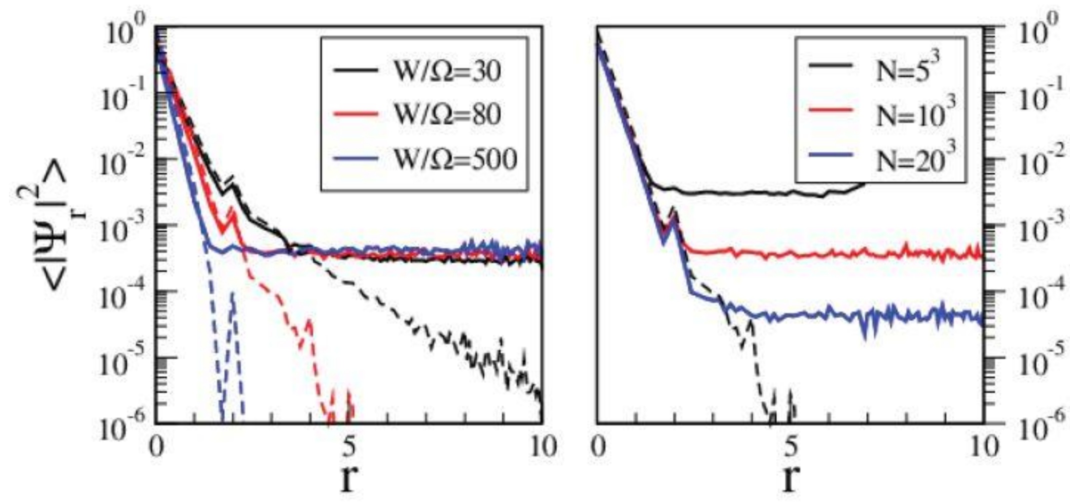
$$(H_{\mathrm{eff}})_{ij} = (H_0)_{ij} - \frac{i}{2}\sum_c A_i^c (A_j^c)^* = (H_0)_{ij} - i\frac{\gamma}{2} Q_{i,j}$$



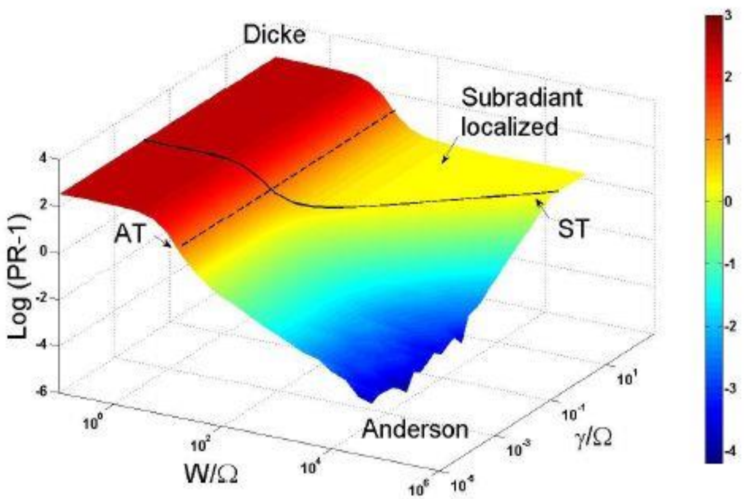
All sites coupled to one single decay channel : $Q_{ij}=1$



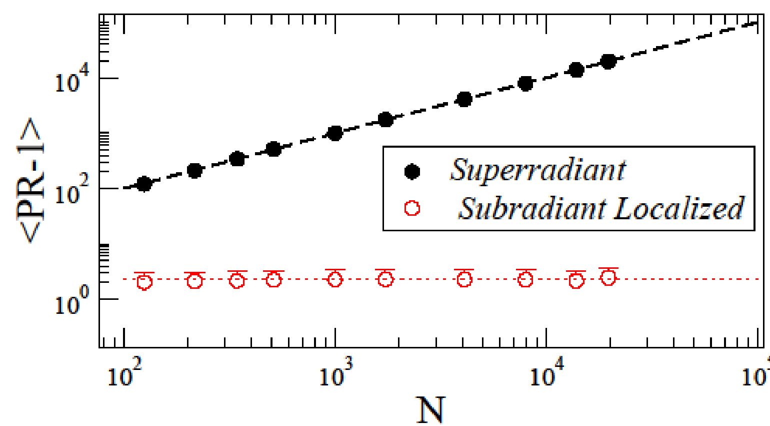
(a)



Hybrid Subradiant States « decoupled » from outside world



(b)

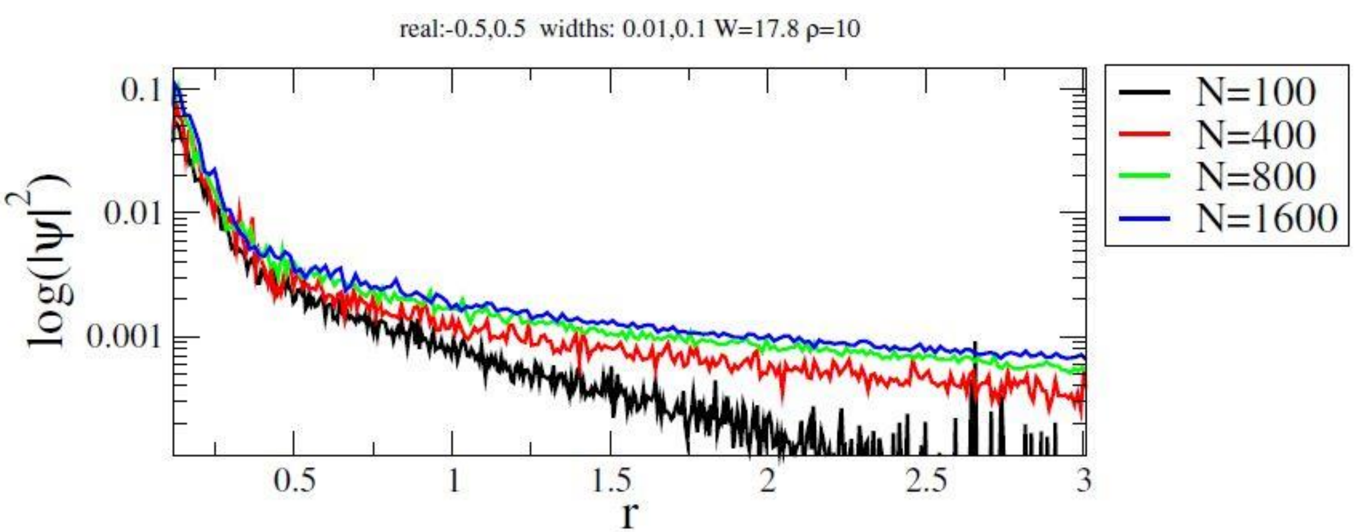


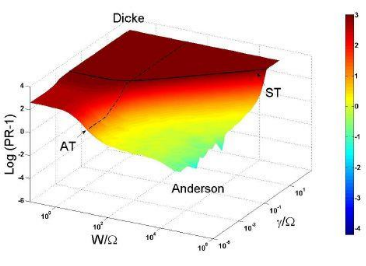
Outlook :

- **Subradiance vs Radiation trapping**

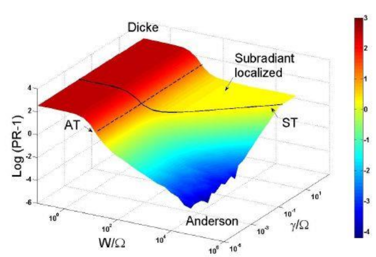
Radiation trapping for small beam and intermediate regimes: subradiance dominant at long times

- **Towards Anderson of subradiant Dicke states**





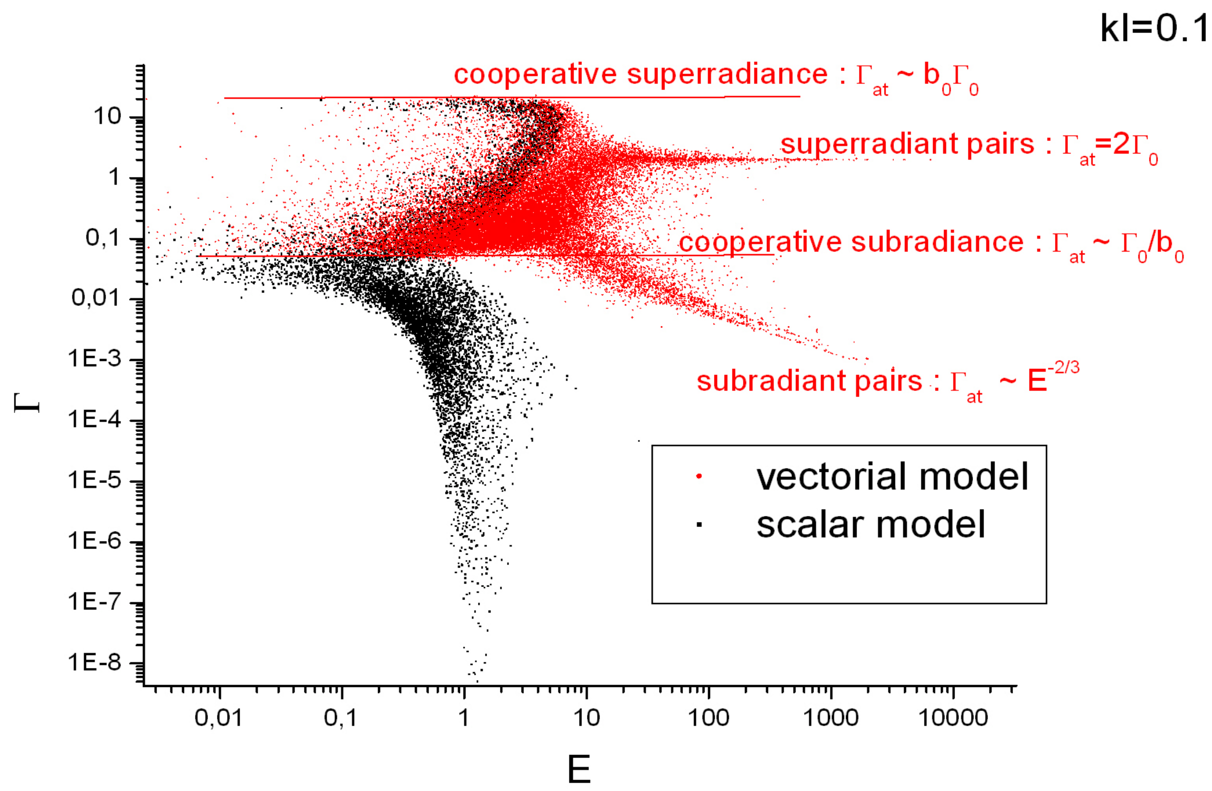
(a)



(b)

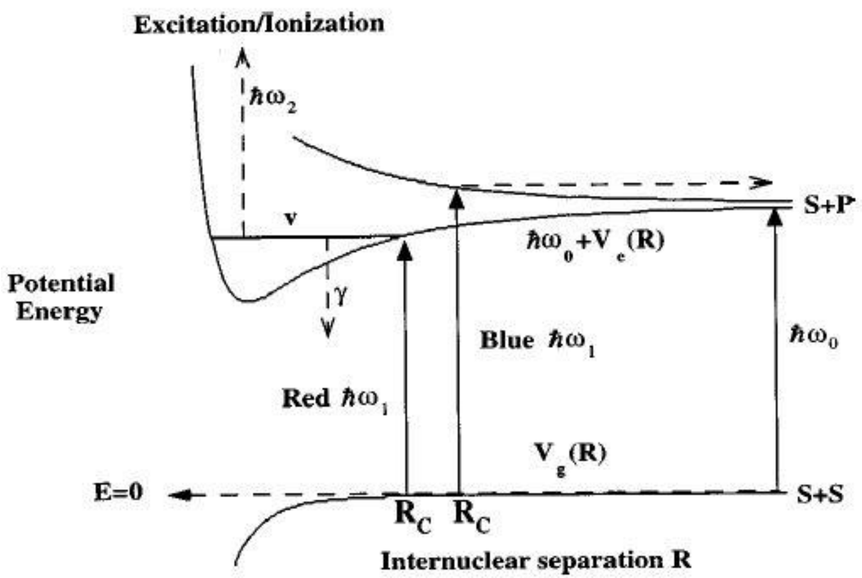
Now something new : not even yet in progress

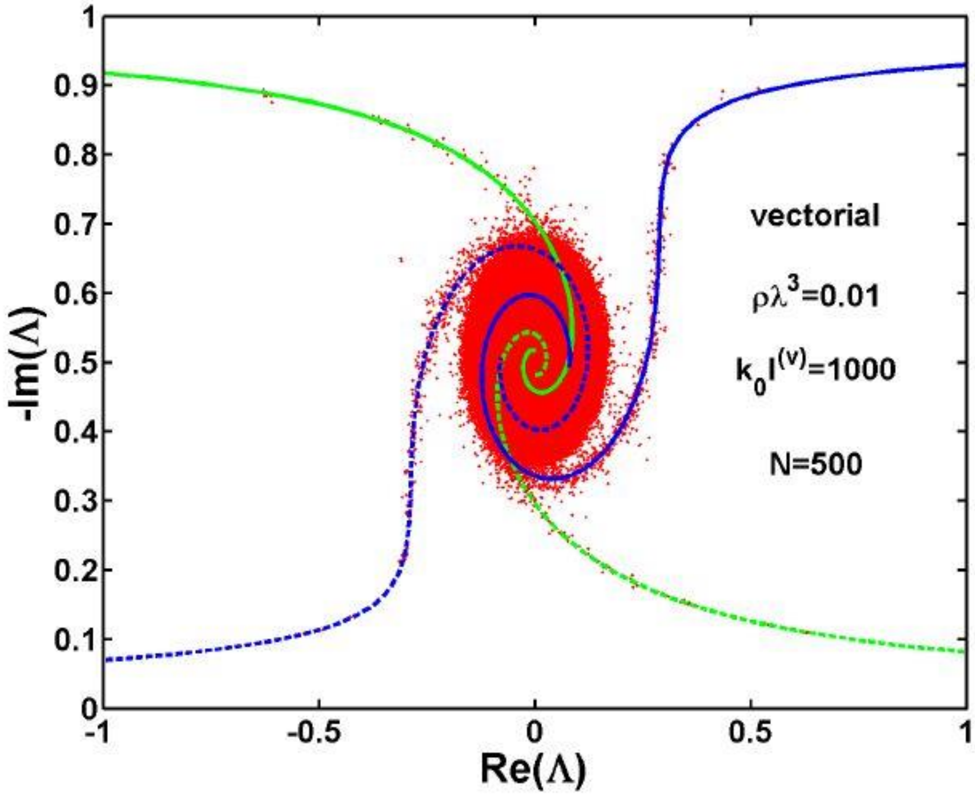
From few (N=2 to N=3) to many body (N>>1)



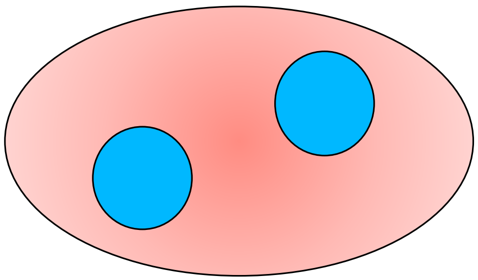
Few body physics with photons : N=2

Pair physics ($1/r^3$ for near field terms)

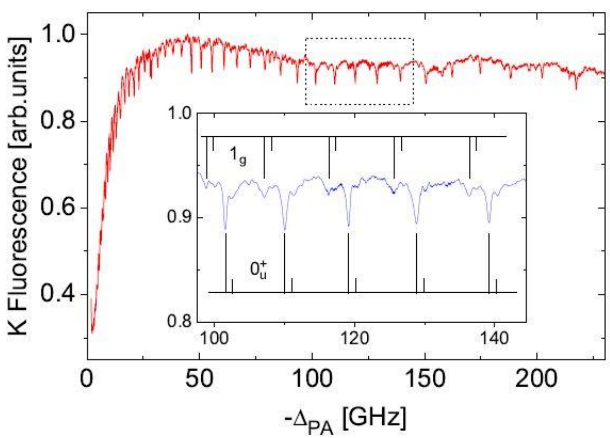


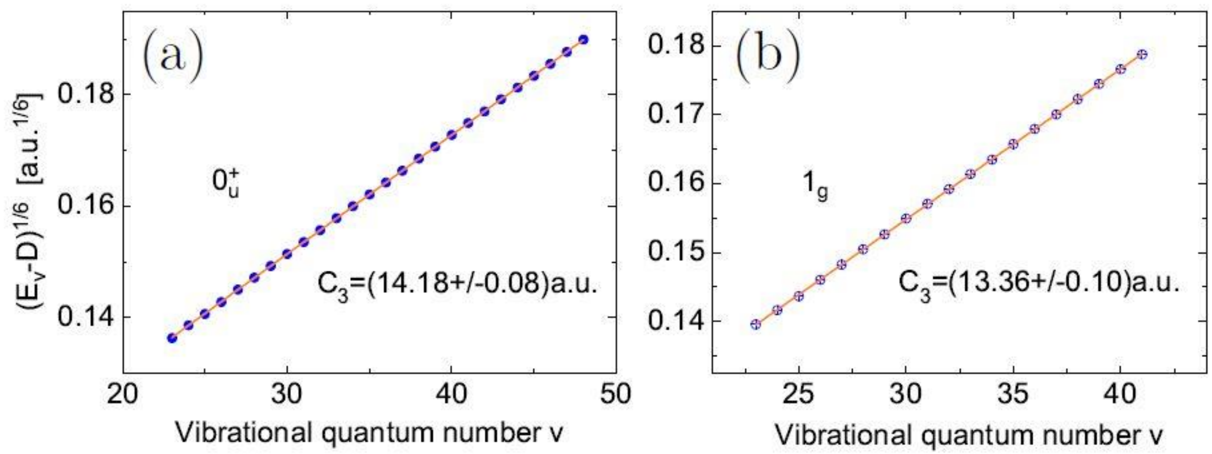


Leroy Bernstein (2 atoms)



$$E_n = -\frac{\hbar^2}{MR_*^2}\left(\frac{n_0 - n}{g}\right)^6$$

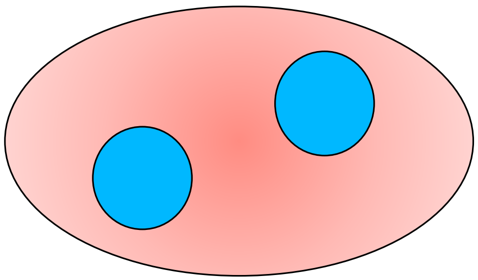




K trap loss
(Salomon group)

PhD A.RIDINGER

Photonic Efimov states



$$E_n = -\frac{\hbar^2}{MR_*^2}\left(\frac{n_0 - n}{g}\right)^6$$

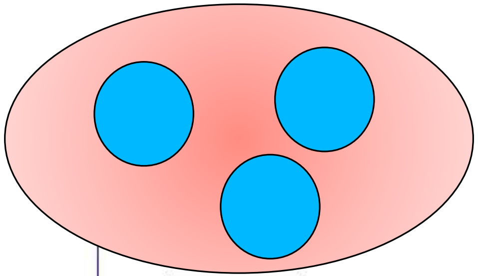
Three body (M+M=m)
vs Pair physics (M+M+photon)

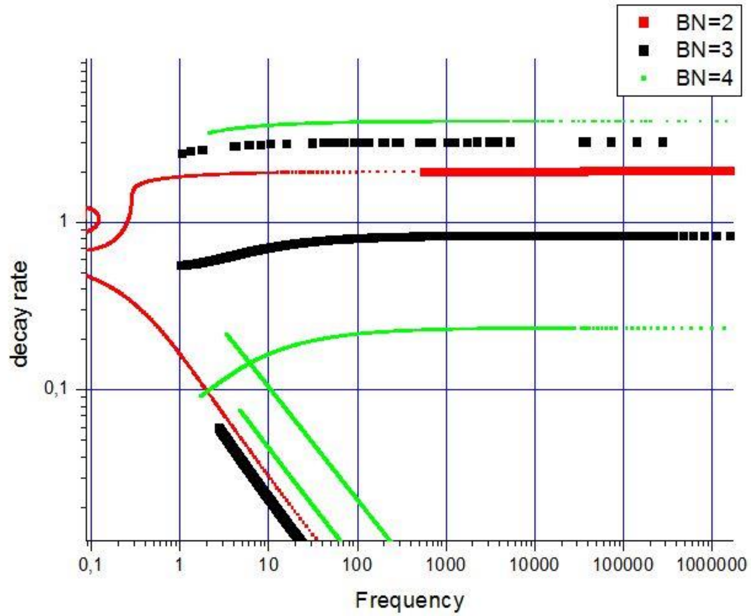
PRL **111,** 113201 (2013)   PHYSICAL REVIEW LETTERS   week ending 13 SEPTEMBER 2013

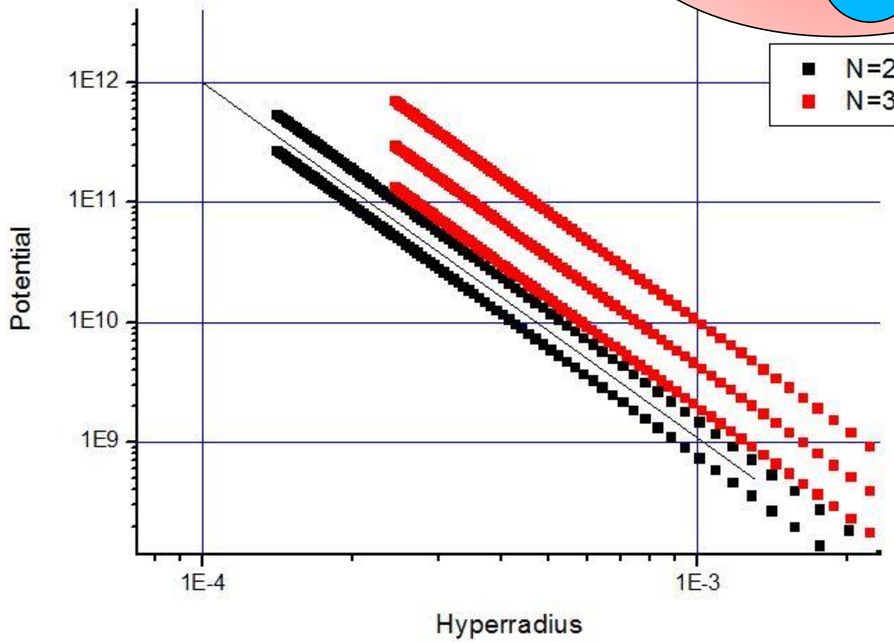
Three-Body Bound States in Atomic Mixtures With Resonant *p*-Wave Interaction

Maxim A. Efremov,[1,2,*] Lev Plimak,[1,3] Misha Yu. Ivanov,[3] and Wolfgang P. Schleich[1]

3 atoms + 1 photon







Shifted $1/r^3$ potential => shifted eigenstates
($1/r^2$ in 2D)

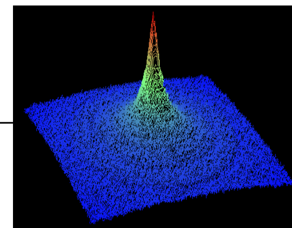
New lines ($\propto n^3$) to be looked for in experiments

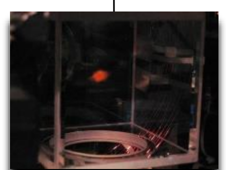
Princeton

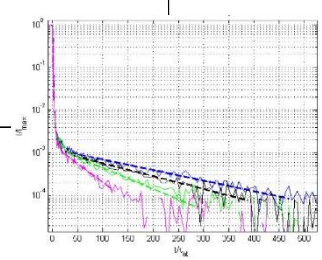
Anderson

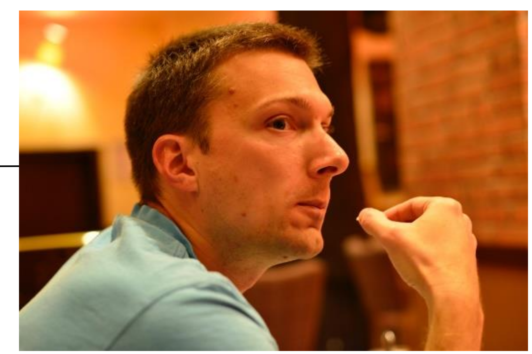
Nice

Labeyrie









Dicke

Guerin

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E. Perreira, M. Havey, T. Ackemann,, J. Tabosa, M. Chevrollier, T.Pohl, J. Ott,
T. Menconça, H. Tercas, G. Alvarez, G. Robb, A. Arnold, W. Firth, G.L. Oppo,
J. Ye, A. M. Rey, J. Schachenmayer, J. Stenflo, V. Bagnato,

Thank you for your attention