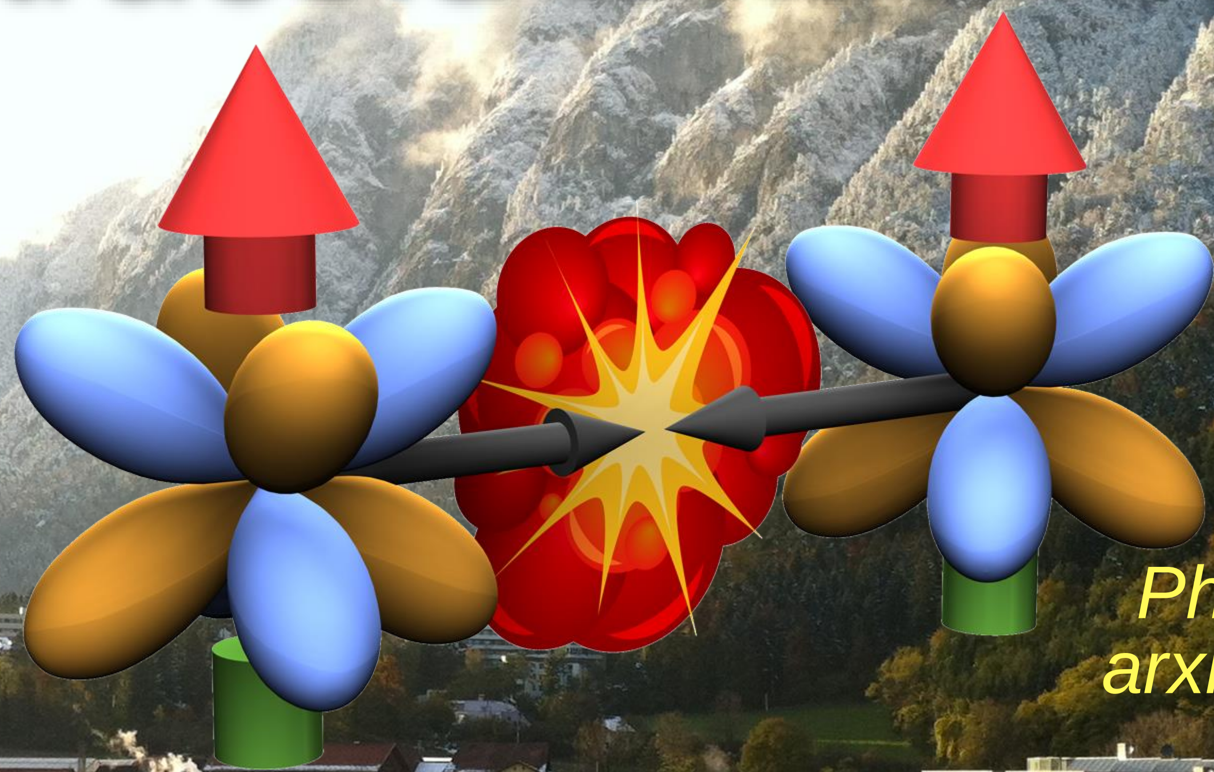
Emergence of chaotic scattering in ultracold lanthanides.



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Phys. Rev. X 5, 041029
arxiv preprint 1506.05221

A. Frisch, S. Baier, K. Aikawa, L. Chomaz, M. J. Mark, F. Ferlaino

in collaboration with :

Dy group in Stuttgart: T. Maier, H. Kadau, M. Schmitt, M. Wenzel, I. Ferrier-Barbut, T. Pfau

Theory group in Temple/Maryland: C. Makrides, A. Petrov, and S. Kotochigova, E. Tiesinga.

SFB FoQuS

START Prize 2009 FWF Der Wissenschaftsfonds

IQI

erc

Outline of the talk

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- ✦ Toward non-alkali atoms, the quest for atomic richness and the case of magnetic atoms.

- ✦ Specificity of scattering of ultracold lanthanides

- ✦ Random Matrix Theory: Complex systems, dense spectra and chaos

- ✦ Chaos in Feshbach resonances of Lanthanides: First Erbium study and beyond, describing lanthanides chaoticity.

- ✦ Towards a theoretical understanding of the origin of chaos in lanthanides scattering

Experimental achievement of BEC

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H hydrogen 1998																	2 He 4.003 2001
3 Li [6.938; 6.997] 1995	4 Be beryllium 9.012											5 B boron [10.80; 10.83]	6 C carbon [12.00; 12.02]	7 N nitrogen [14.00; 14.01]	8 O oxygen [15.99; 16.00]	9 F fluorine 19.00	10 Ne neon 20.18
11 Na 22.99 1995	12 Mg magnesium 24.31											13 Al aluminium 26.98	14 Si silicon [28.08; 28.09]	15 P phosphorus 30.97	16 S sulfur [32.05; 32.08]	17 Cl chlorine [35.44; 35.46]	18 Ar argon 39.95
19 K 2001	20 Ca 2009	21 Sc scandium 44.96	22 Ti titanium 47.87	23 V vanadium 50.94	24 Cr 2004	25 Mn manganese 54.94	26 Fe iron 55.85	27 Co cobalt 58.93	28 Ni nickel 58.69	29 Cu copper 63.55	30 Zn zinc 65.38(2)	31 Ga gallium 69.72	32 Ge germanium 72.63	33 As arsenic 74.92	34 Se selenium 78.96(3)	35 Br bromine 79.90	36 Kr krypton 83.80
37 Rb 85.47 1995	38 Sr 87.62 2009	39 Y yttrium 88.91	40 Zr zirconium 91.22	41 Nb niobium 92.91	42 Mo molybdenum 95.96(2)	43 Tc technetium	44 Ru ruthenium 101.1	45 Rh rhodium 102.9	46 Pd palladium 106.4	47 Ag silver 107.9	48 Cd cadmium 112.4	49 In indium 114.8	50 Sn tin 118.7	51 Sb antimony 121.8	52 Te tellurium 127.6	53 I iodine 126.9	54 Xe xenon 131.3
55 Cs 132.9 2002	56 Ba barium 137.3	57-71 lanthanoids	72 Hf hafnium 178.5	73 Ta tantalum 180.9	74 W tungsten 183.8	75 Re rhenium 186.2	76 Os osmium 190.2	77 Ir iridium 192.2	78 Pt platinum 195.1	79 Au gold 197.0	80 Hg mercury 200.6	81 Tl thallium [204.3; 204.4]	82 Pb lead 207.2	83 Bi bismuth 209.0	84 Po polonium	85 At astatine	86 Rn radon
87 Fr francium	88 Ra radium	89-103 actinoids	104 Rf rutherfordium	105 Db dubnium	106 Sg seaborgium	107 Bh bohrium	108 Hs hassium	109 Mt meitnerium	110 Ds darmstadtium	111 Rg roentgenium	112 Cn copernicium		114 Fl flerovium		116 Lv livermorium		

57 La lanthanum 138.9	58 Ce cerium 140.1	59 Pr praseodymium 140.9	60 Nd neodymium 144.2	61 Pm promethium	62 Sm samarium 150.4	63 Eu europium 152.0	64 Gd gadolinium 157.3	65 Tb terbium 158.9	66 Dy 162.5 2011	67 Ho holmium 164.9	68 Er 167.3 2012	69 Tm thulium 168.9	70 Yb 173.1 2003	71 Lu lutetium 175.0
89 Ac actinium	90 Th thorium 232.0	91 Pa protactinium 231.0	92 U uranium 238.0	93 Np neptunium	94 Pu plutonium	95 Am americium	96 Cm curium	97 Bk berkelium	98 Cf californium	99 Es einsteinium	100 Fm fermium	101 Md mendelevium	102 No nobelium	103 Lr lawrencium

The different condensed atoms

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Bose-Einstein condensates

cryogenic cooling

1 H hydrogen 1998

metastable state

2 He helium 2001

One-electron atom
“simple” electronic structure (L=0),
tunable contact interaction!

$$U_c = \frac{4\pi\hbar^2 a}{m}\delta(r)$$

3 Li lithium 1995 [6.938; 6.997]

11 Na sodium 1995 22.99

19 K potassium 2001 39.10

37 Rb rubidium 1995 85.47

55 Cs caesium 2002 132.9

20 Ca calcium 2009 40.08

38 Sr strontium 2009 87.62

24 Cr chromium 2004 52.00

$$V_{\mathrm{dd}} = \frac{\mu_0\mu^2}{4\pi}\frac{1-3\cos^2\theta}{r^3}$$

strong dipolar character

Multi-electron atom
“complex” electronic structure (L very large), tunable contact interaction and DDI!

66 Dy dysprosium 2011 162.5

68 Er erbium 2012 167.3

70 Yb ytterbium 2003 173.1

Two-electron atoms
“simple” electronic and nuclear structure (L=0, S=0 and I=0),
clock transitions, ...

More laser cooled species: Ho and Tm

Strongly dipolar systems

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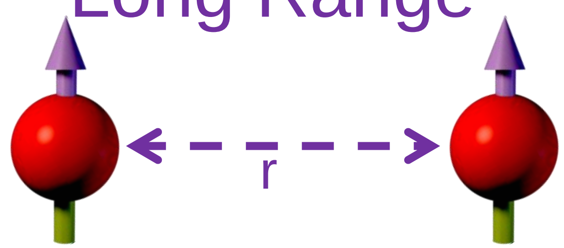
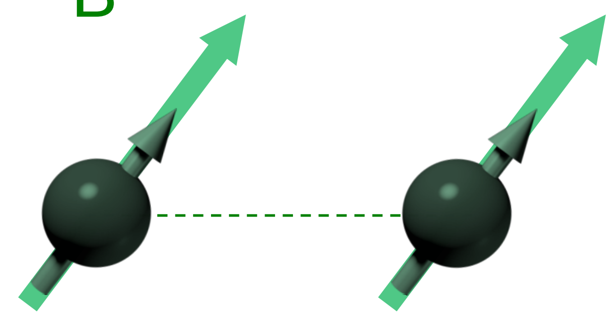
$$V_{\mathrm{dd}} = \frac{\mu_0 \mu^2}{4\pi} \frac{1 - 3\cos^2\theta}{r^3}$$

Long Range

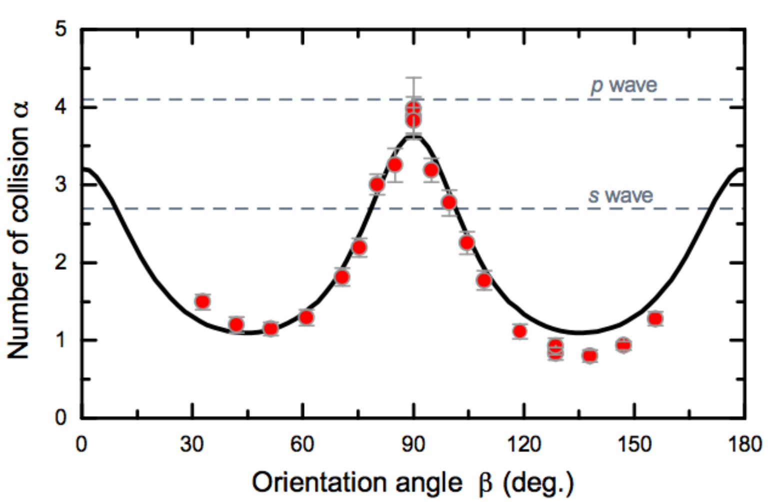


Anisotropy

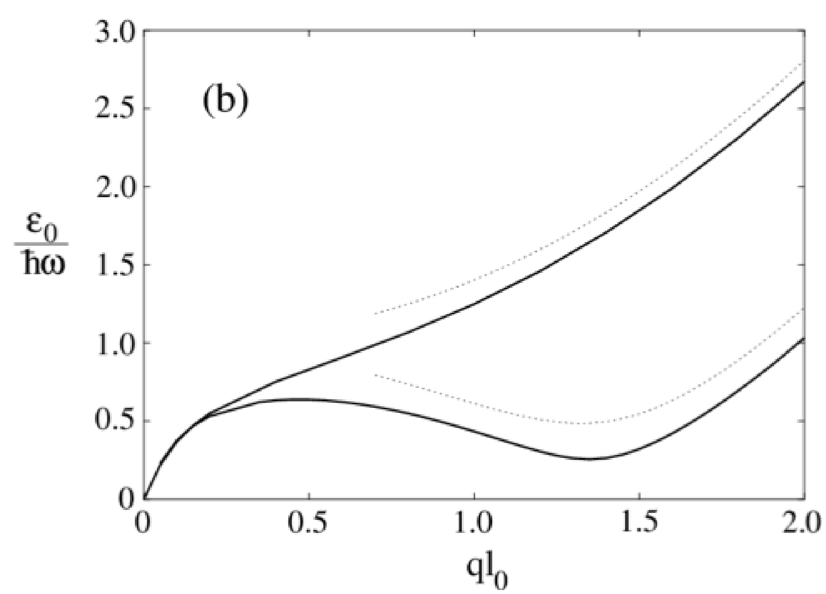
B

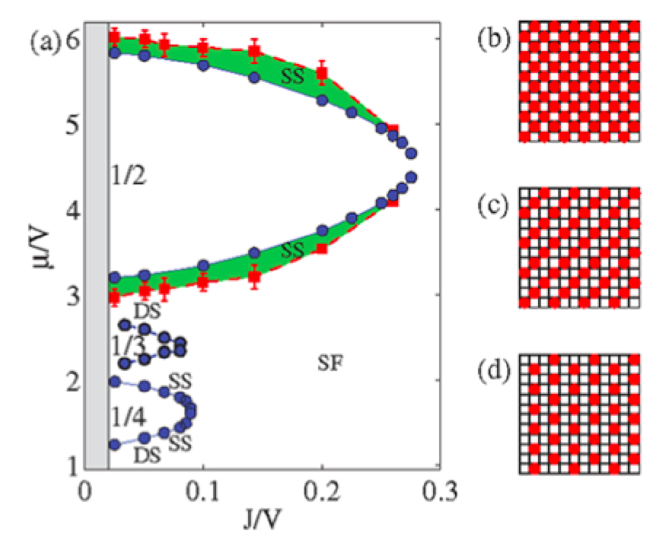


Few-body Physics
anisotropic scattering amplitude, different scattering behavior at low T, partial wave decomposition...



Many-body Physics
e.g. ground state properties, spectrum of excitations, novel quantum phases





Review articles: M.A. Baranov, Phys. Rep. 464, 71 (2008); T. Lahaye et al., Rep. Prog. Phys. 72, 126401 (2009); M.A. Baranov, & al. Chem. Rev., 2012, 112 (9) (2012)

Outline of the talk

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✦ Toward non-alkali atoms, the quest for atomic richness and the case of magnetic atoms.

✦ Specificity of scattering of ultracold lanthanides (Ln)

✦ Random Matrix Theory: Complex systems, dense spectra and chaos

✦ Chaos in Feshbach resonances of Lanthanides: First Erbium study and beyond, describing lanthanides chaoticity.

✦ Towards a theoretical understanding of the origin of chaos in lanthanides scattering

Specific structure of Ln atoms

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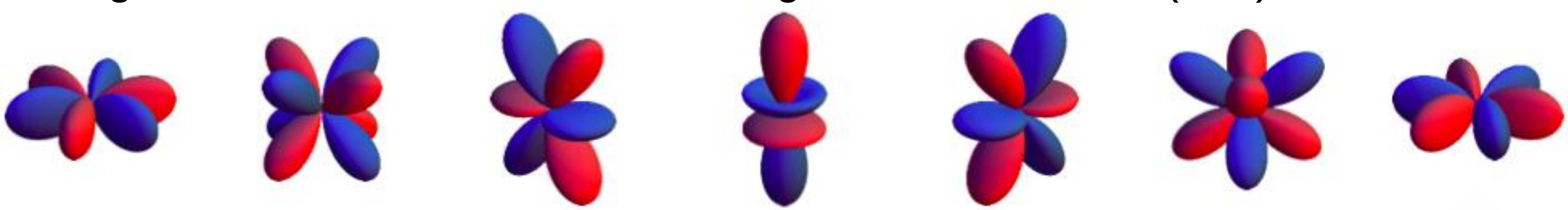
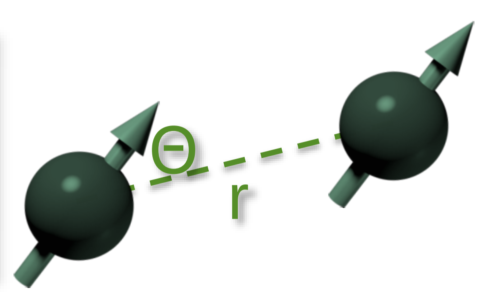
laser-cooled members
Lanthanide (Ln) family

gadolinium	terbium	dysprosium	holmium	erbium	thulium	ytterbium
64	65	66	67	68	69	70
Gd	Tb	Dy	Ho	Er	Tm	Yb
157.25	158.93	162.50	164.93	167.26	168.93	173.04
curium	berkelium	californium	einsteinium	fermium	mendelevium	nobelium
96	97	98	99	100	101	102
Cm	Bk	Cf	Es	Fm	Md	No
[247]	[247]	[251]	[252]	[257]	[258]	[259]

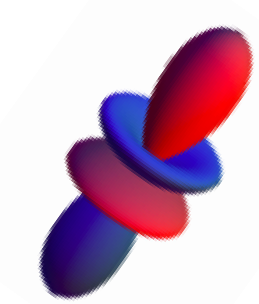
- partially filled 4*f* electron shell, submerged below a filled 6s shell.
- ground state with vacancies of large m_ℓ of the *f*-shell ($\ell=3$) *f* orbital basis



Large magnetic moment μ !



Large total angular momentum j !



Magnetic Anisotropy
(long range)
(Erbium $\mu \approx 7\mu_B$, Dysprosium $\mu \approx 10\mu_B$)

Large ground state degeneracy
+ Orbital Anisotropy *(short range)*
(Erbium j=6, Dysprosium j=8)

Interactions of lanthanides atoms

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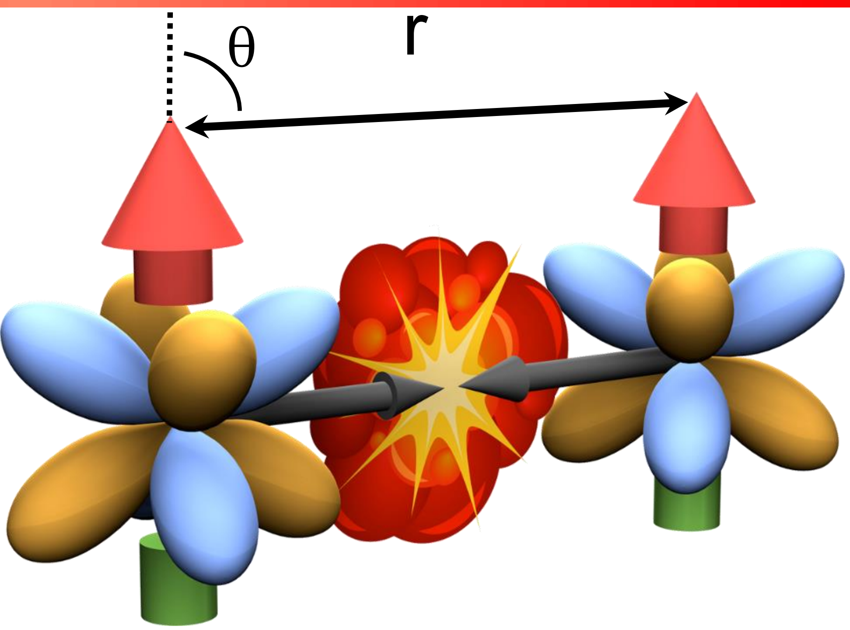


$$\hat{H}_{\mathrm{rel}} = -\frac{\hbar^2}{2\mu_r}\frac{\mathrm{d}^2}{d^2\vec{r}} + \hat{V}(\vec{r}) \left(+ \hat{H}_Z(B) \right)$$

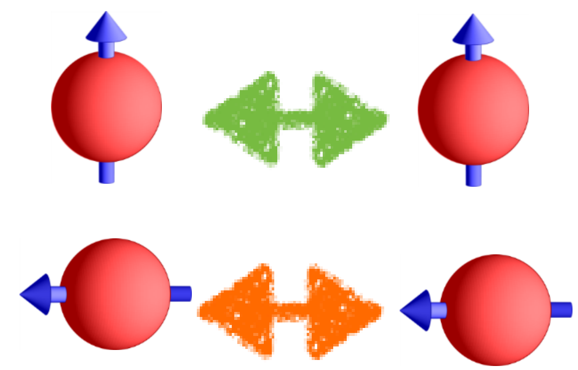
μ_r : reduced mass

Zeeman energy

Which specific interaction potential $\hat{V}(\vec{r})$ for lanthanide atoms?

Dipole-Dipole interaction (DDI):

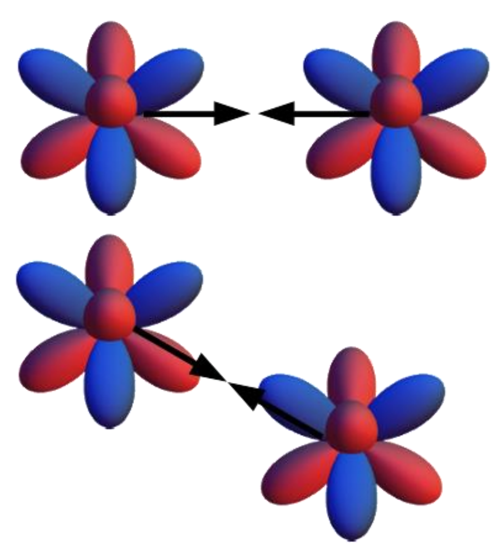
$$\boldsymbol{V_{\mathrm{dd}}(\vec{r}) = -\frac{C_3}{|r|^3}\left(3\cos^2\theta - 1\right)}$$



$$C_3 = \frac{\mu_0 \mu^2}{4\pi}$$

Anisotropic van der Waals dispersion:

$$\boldsymbol{V_{\mathrm{vdW}}(\vec{r}) = -\frac{C_6}{|r|^6} - \frac{C_6^{\mathrm{ani}}}{|r|^6}\left(3\cos^2\theta - 1\right)}$$



	C_6 (u.a)	ΔC_6 (u.a)
Er	1703	174
Dy	2003	188

Review: Svetlana Kotochigova, Report on progress in physics, 77, 093901 (2014)

Feshbach resonances in Ln

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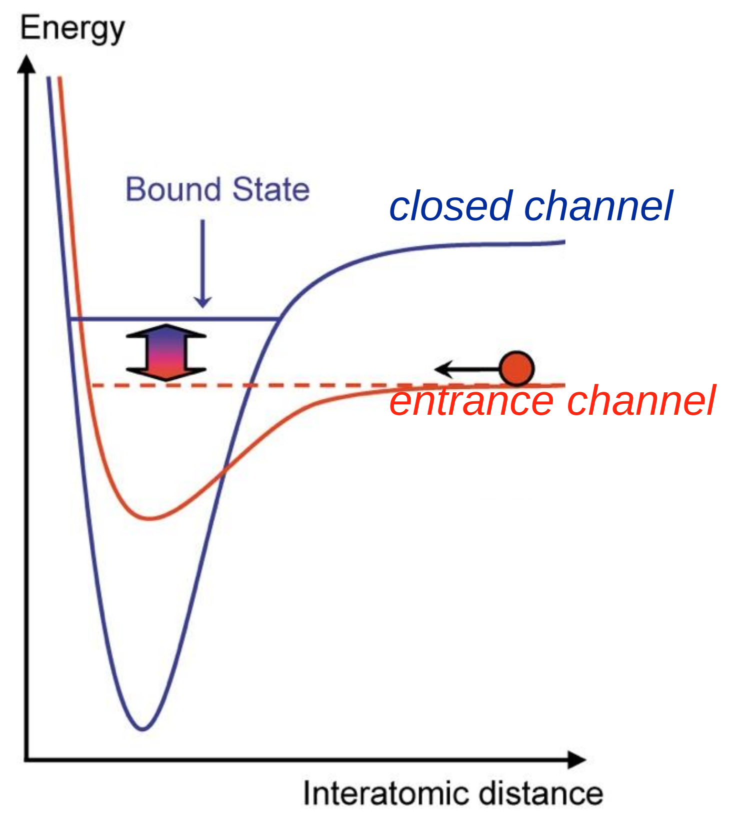

$$\hat{H}_{\mathrm{rel}} = -\frac{\hbar^2}{2\mu_r}\frac{\mathrm{d}^2}{d^2\vec{r}} + \hat{V}(\vec{r}) + \hat{H}_Z(B)$$

≠ Scattering scheme!

Alkali atoms

→ coupling btw Zeeman states induced by hyperfine coupling btw electron and nuclear spins.

★ perturbative, weak coupling

★ isotropic

★ few channels are coupled

Lanthanides atoms

$$\hat{V}(\vec{r}) = \hat{V}_{\mathrm{iso}}(r) + \hat{V}_{\mathrm{ani}}(\vec{r})$$

→ coupling btw Zeeman states induced by interaction potential itself (V_{ani}).

★ typically stronger coupling.

★ anisotropic: bound-states can have $\ell \neq 0$.

★ a large number of channels are coupled (large j).

★ [Partial wave are not a natural basis]

Feshbach Spectroscopy in Ln

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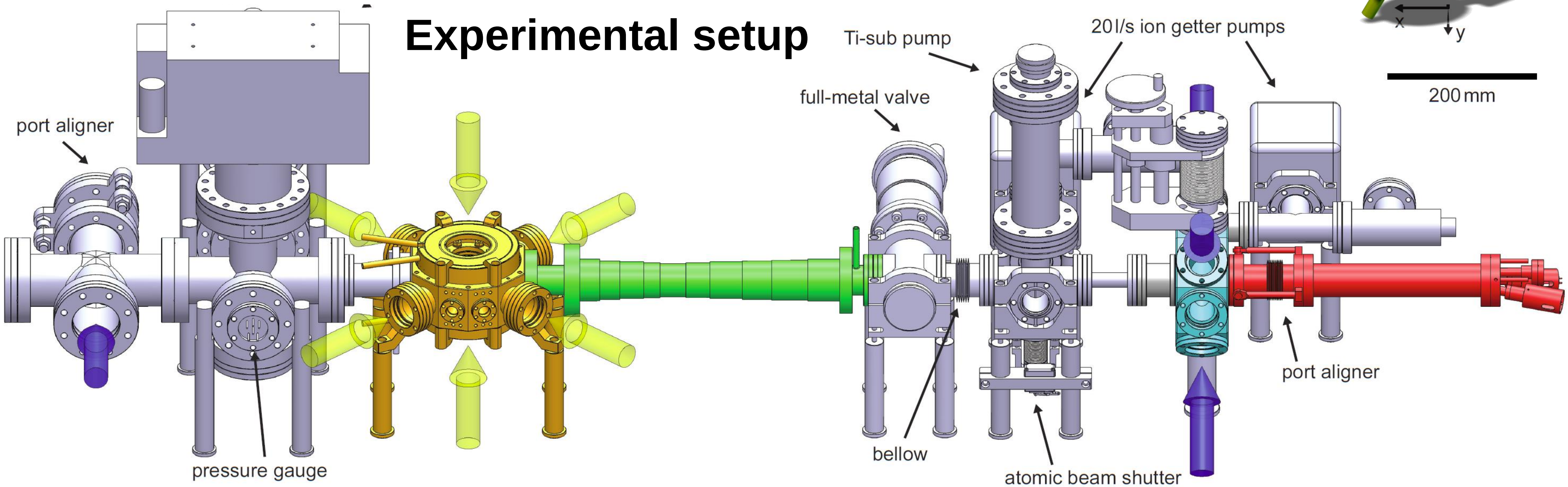
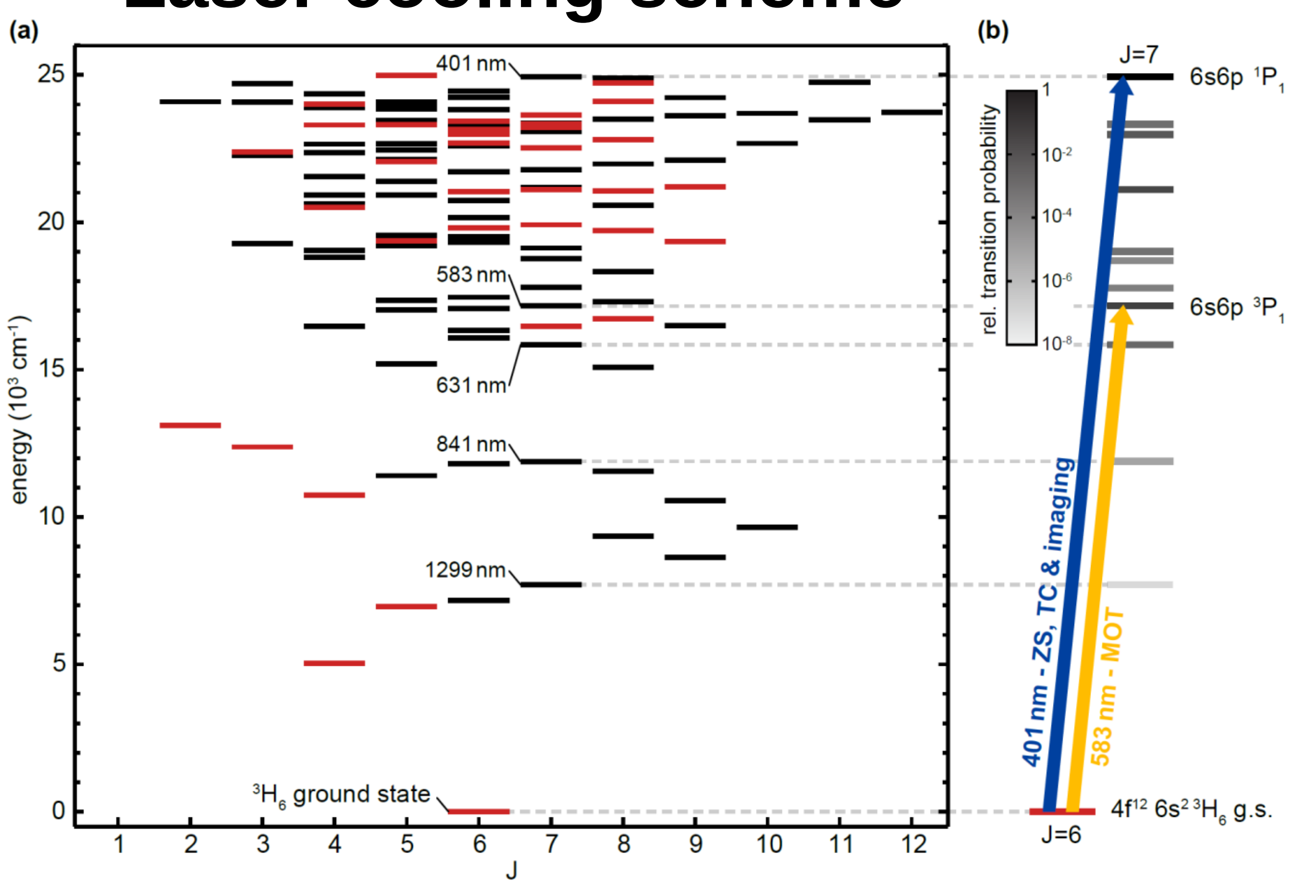
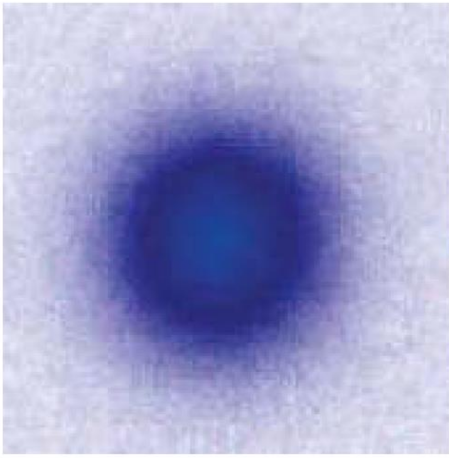
Experimental setup

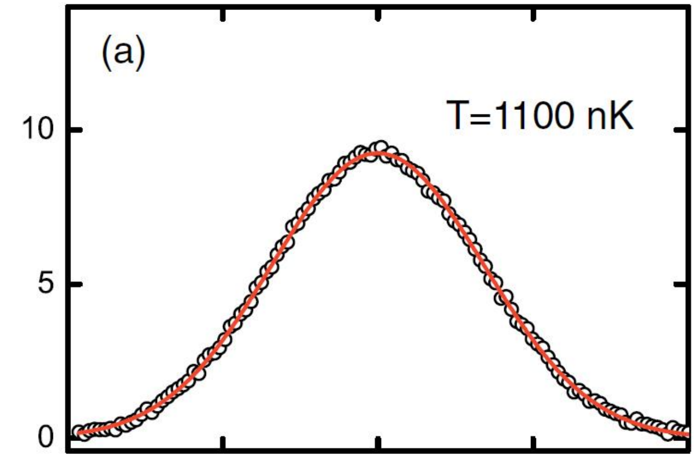


Laser cooling scheme



Detection





Feshbach Spectroscopy in Ln

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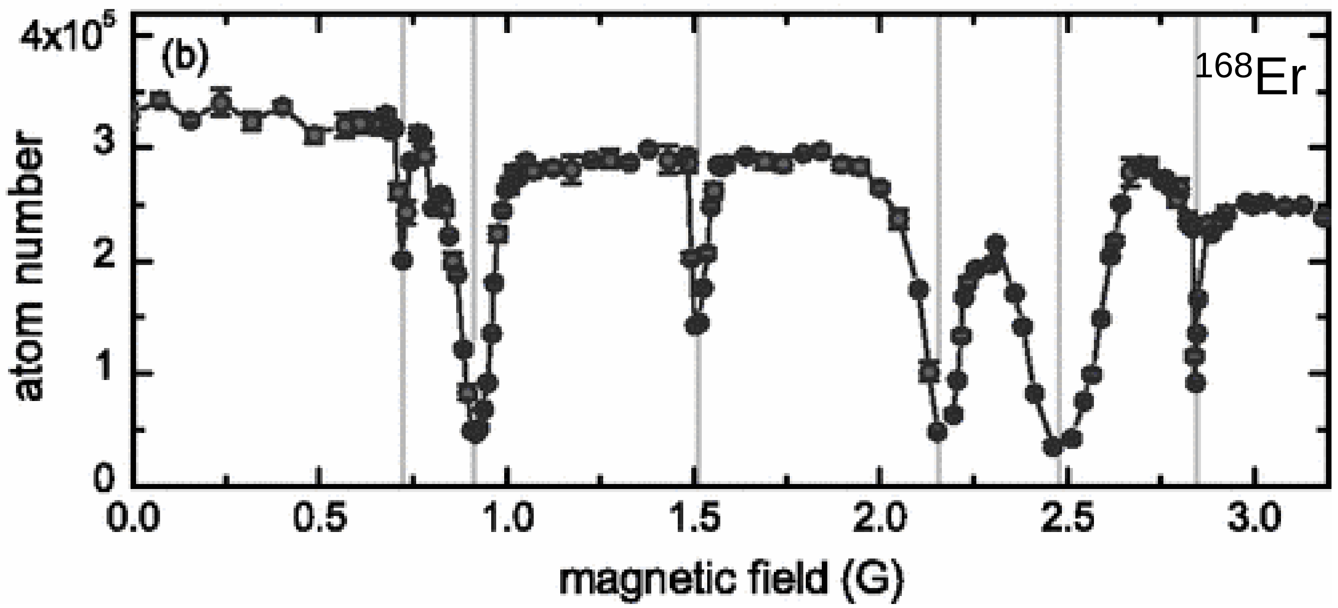
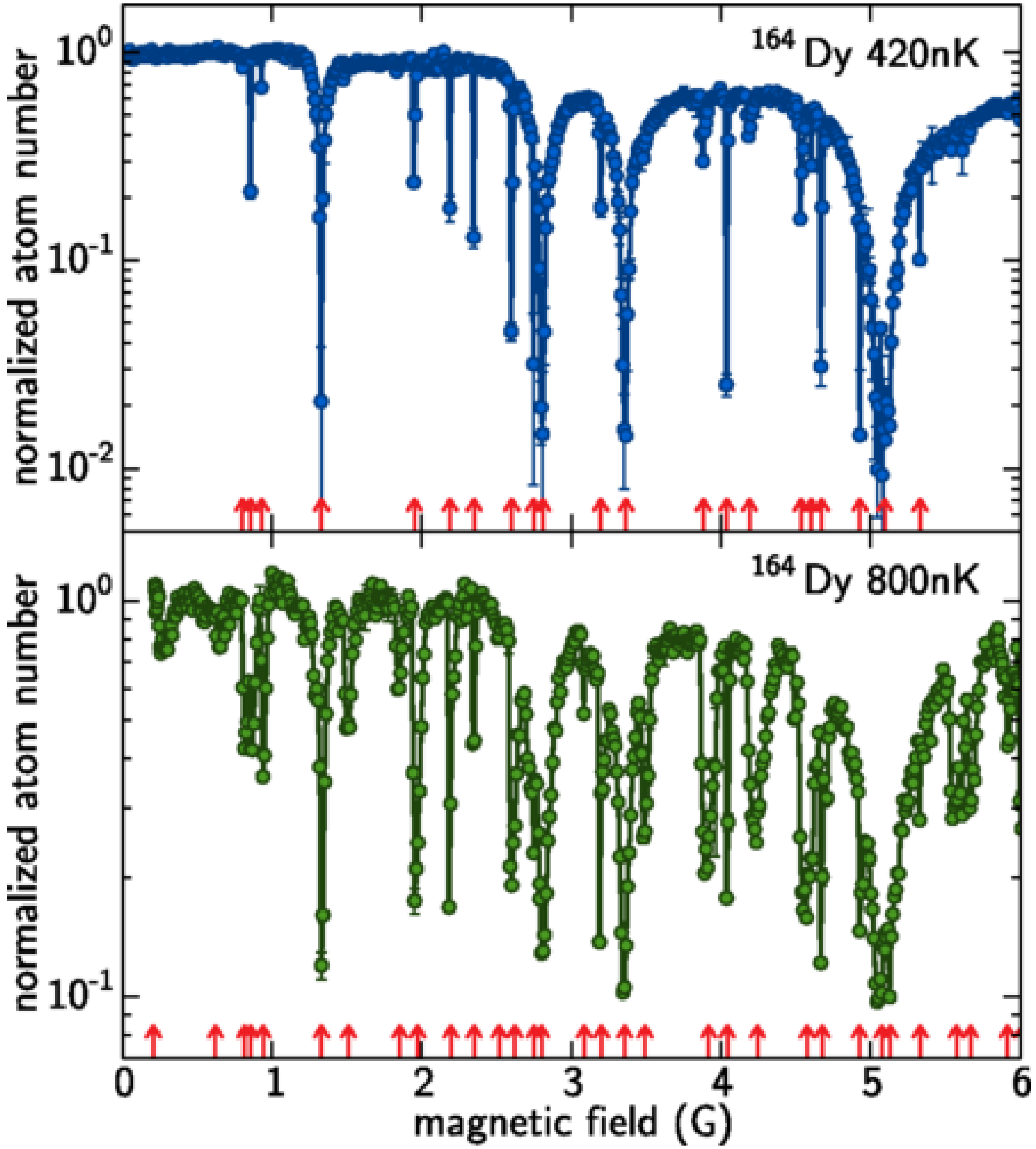


Er: K. Aikawa, A. Frisch, M. Mark, S. Baier, A. Rietzler, R. Grimm, and F. Ferlaino, Phys. Rev. Lett. 108, 210401 (2012).



Dy: K. Baumann, N. Q. Burdick, M. Lu, and B. L. Lev, Phys. Rev. A 89, 020701 (2014)

+ Theory works:

A. Petrov, E. Tiesinga, S. Kotochigova, PRL 109 103002 (2012)

for a review, see Svetlana Kotochigova, Report on progress in physics, 77, 093901 (2014)

- ✦ high # of resonances compared to alkali: **several per Gauss.**
- ✦ Not easy to encompass from the theory side: **traditional methods fail...**

Feshbach Spectroscopy in Ln

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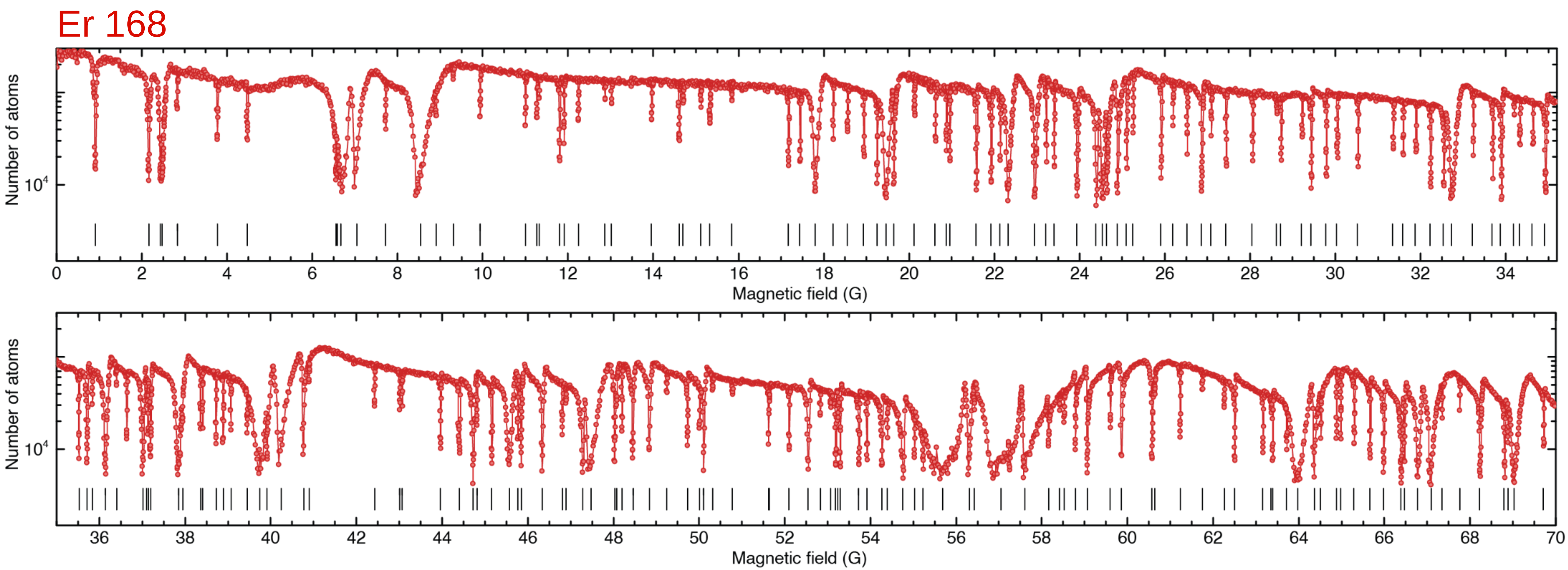
Er: A. Frisch, M. Mark, K. Aikawa, F. Ferlaino, J. L. Bohn, C. Makrides, A. Petrov, and S. Kotochigova, Nature 507, 475 (2014)



extensive high-resolution trap loss spectroscopy measurements: 13 200 points

- ✦ Very dense spectra (compare to alkali): **~200 resonances in [0,70]G!!**
- ✦ Not easy to encompass from the theory side: **traditional methods fail...**

New Challenge in Scattering Physics

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Handling complex spectra

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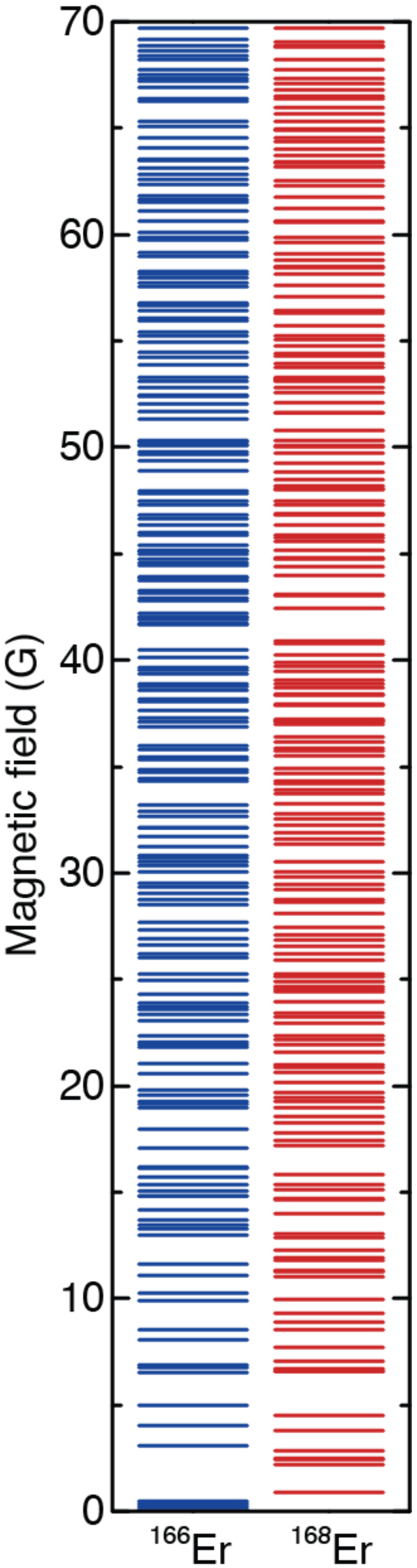
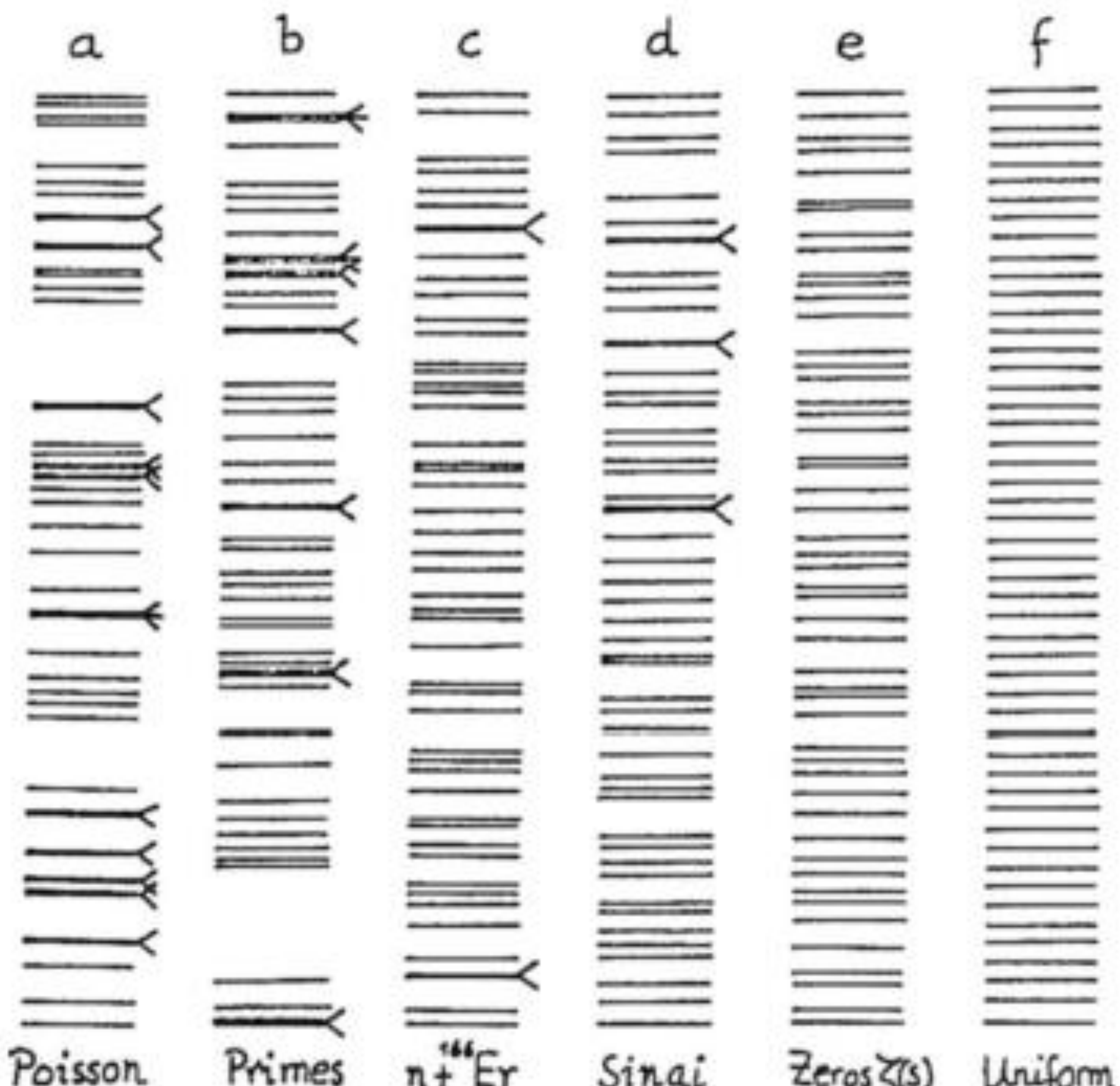


Wigner (1950):
*Can we identify / forecast **global** spectral properties of a system with complex interactions without integrating the microscopic Hamiltonian ? ⇒ statistical approach*



Bohigas and Giannoni (1984)

Random Matrix T

How to characterize some properties of systems with complex spectra?

Characterised by few **statistical properties of the energy spectrum**

=

correlations

Look at the global properties of the spectrum

1950's Wigner, 1960's Dyson, Mehta, and Gaudin. Originally designed for heavy nuclei

Random Matrix Theory

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Can predict global spectrum properties:

- **Without integrating the real Hamiltonian.**
- **Statistical analysis of a set of random Hamiltonians.**

$$\hat{H}_{\mathbf{real}} \longrightarrow \left\{ \hat{H}_i^{\mathrm{random}} = \begin{pmatrix} H_{1,1} & \cdots & H_{1,n} \\ \vdots & \ddots & \vdots \\ H_{n,1} & \cdots & H_{n,n} \end{pmatrix}_i \right\}_{i \in [1,N]}$$

random coupling terms

Gaussian distributed

Condition on the random matrix:
Respect the same symmetries as the real H !

Reproduce the correlations of the complex spectrum of a real system !!

1950's Wigner, 1960's Dyson, Mehta, and Gaudin. Originally designed for heavy nuclei

Level correlations & RMT

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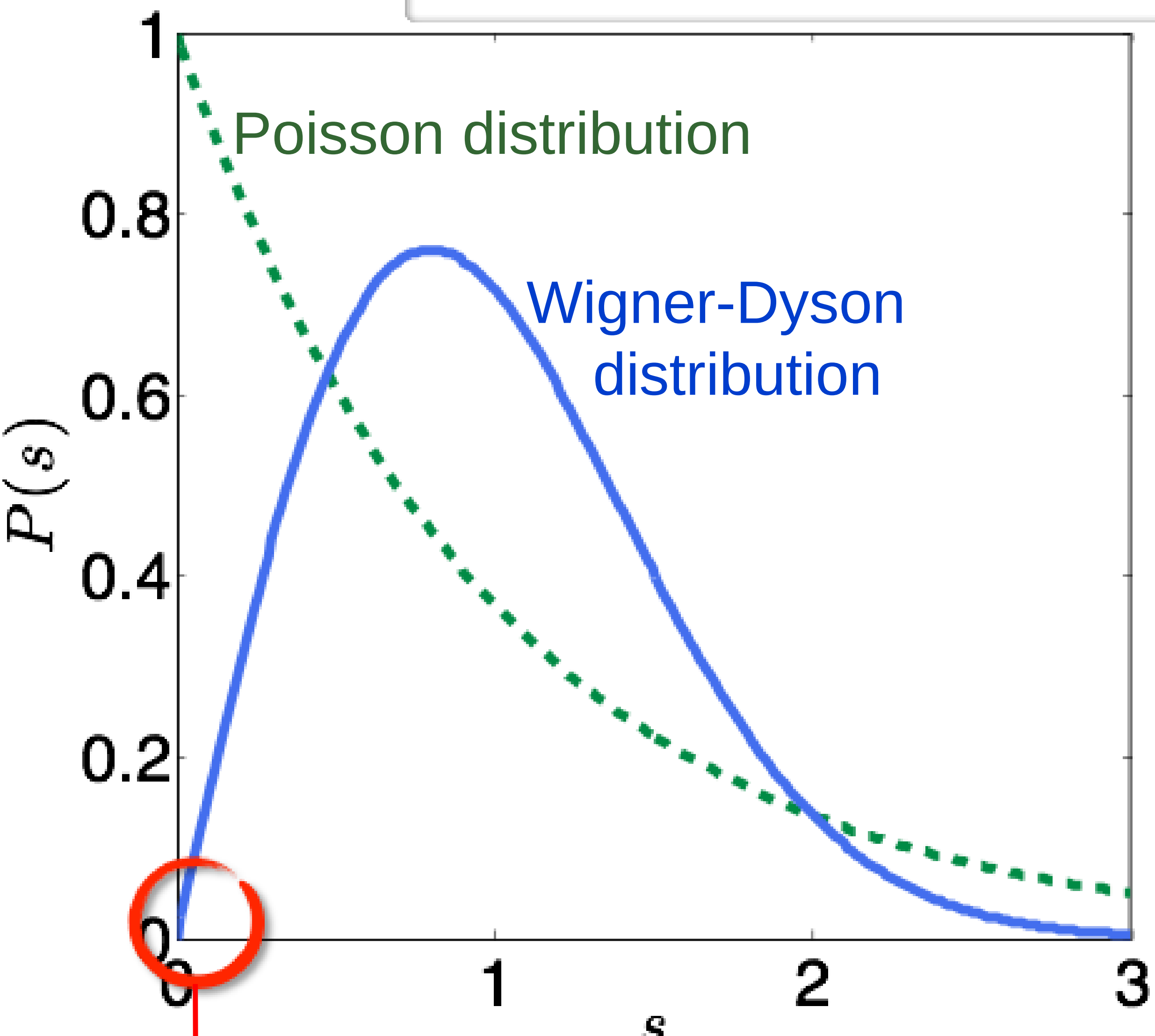
P(s): Nearest-neighbor distribution of energy levels



s = normalized nearest-neighbor level spacing (in units of the mean level spacing)

Wigner surmise excludes degeneracies, $P^{(WD)}(0) = 0$

Poisson law ⇔ **uncorrelated** classical dynamics

$$P^{(P)}(s) = \exp(-s)$$

Wigner-Dyson distr. ⇔ Dynamics of a quantum system with time-reversal and rotational symmetry predicted within **Random Matrix Theory** (Gaussian Orthogonal ensemble)

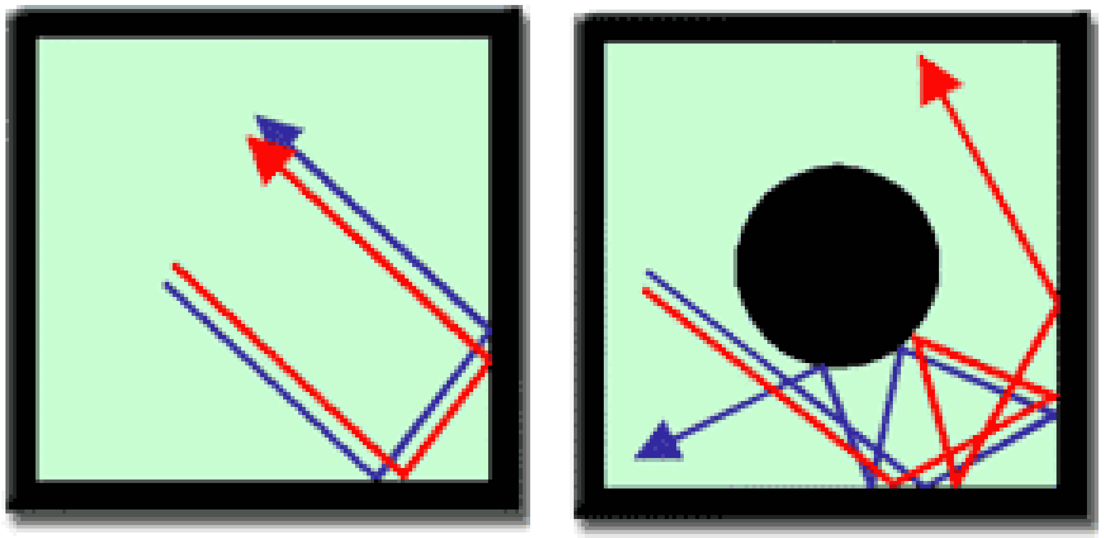
$$P^{(WD)}(s) = \frac{\pi s}{2} \exp\left(-\frac{\pi s^2}{4}\right)$$

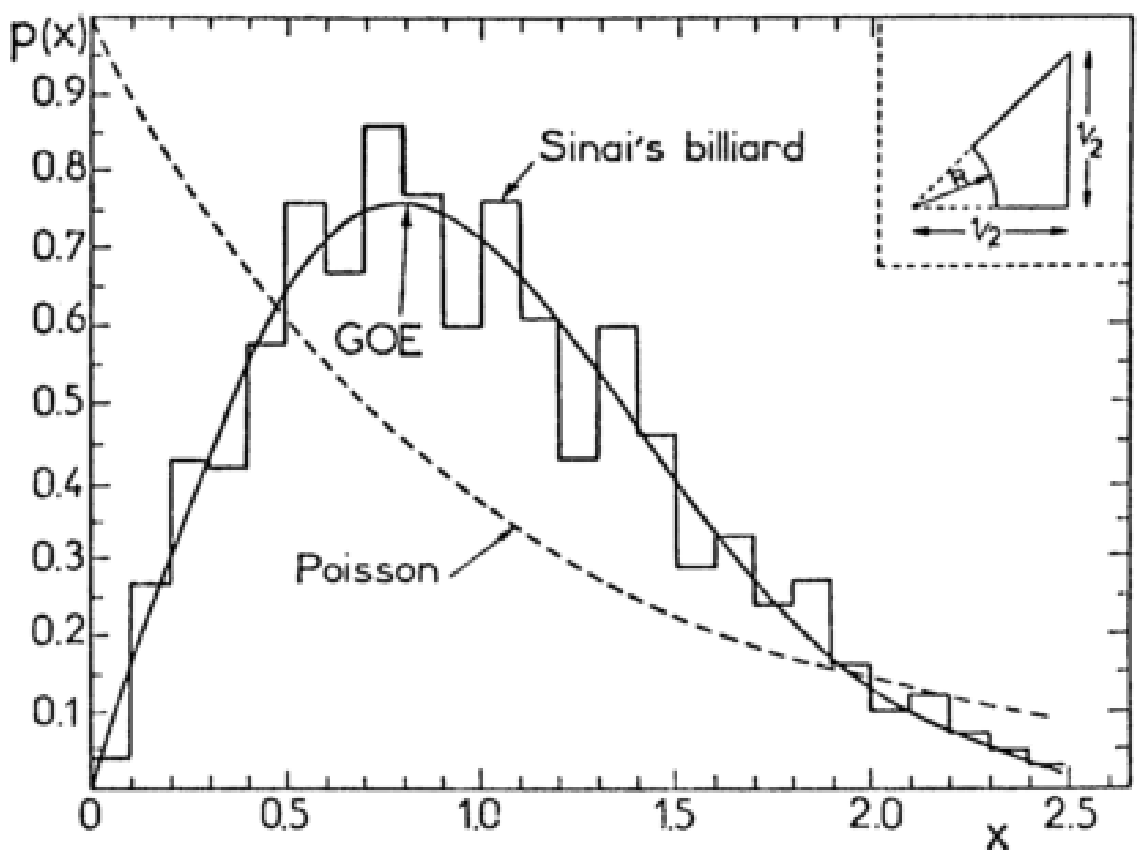
Same mean value, different variance.

Wigner-Dyson distr. expresses ***level repulsions and spectral rigidity.***

RMT: Parameter-free theory

RMT universality & chaos





BOHIGAS-GIANNONI-SCHMIT CONJECTURE (1984):
The spectral fluctuation properties of a quantum system that is fully chaotic in the classical limit follows the WD distribution

Random Matrix Theory

Chaos

⇒ Universality of level correlation functions.

O. Bohigas, M.J. Giannoni, C. Schmit, PRL (1984)

RMT universal applications

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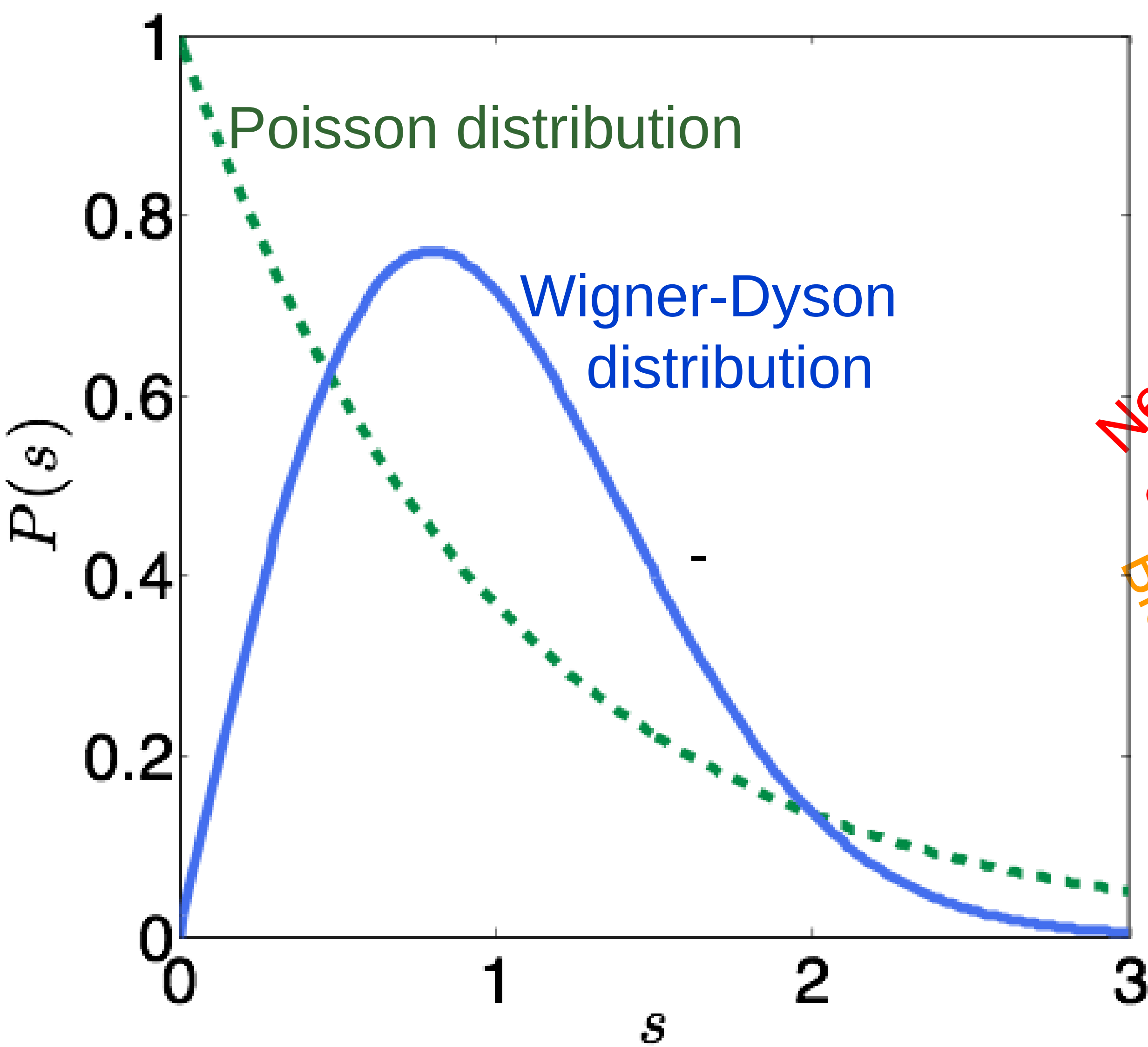
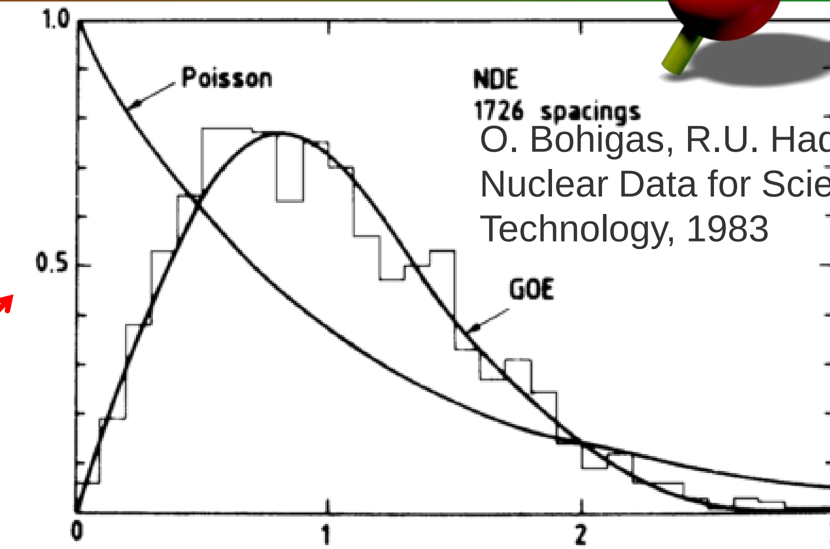
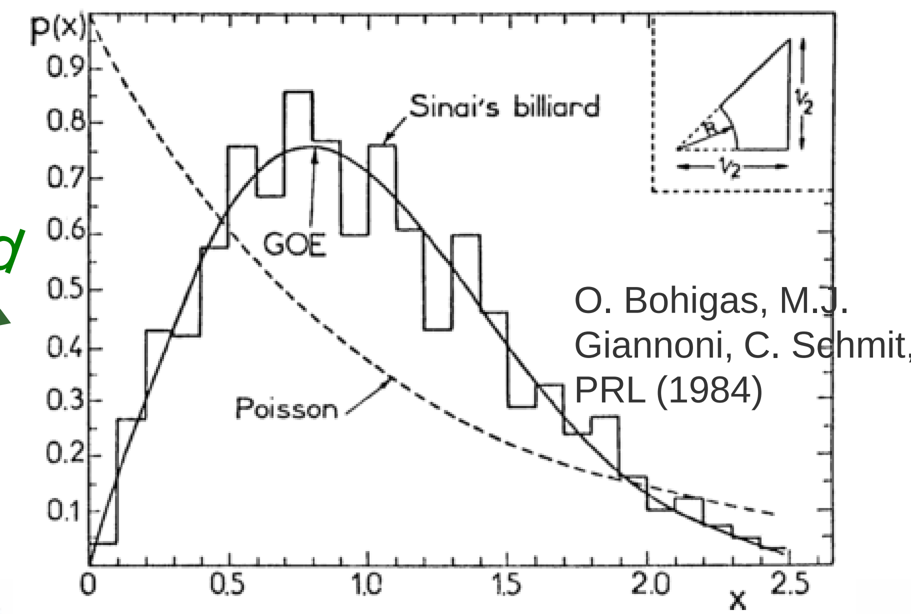
P(s): Nearest-neighbor distribution of energy levels



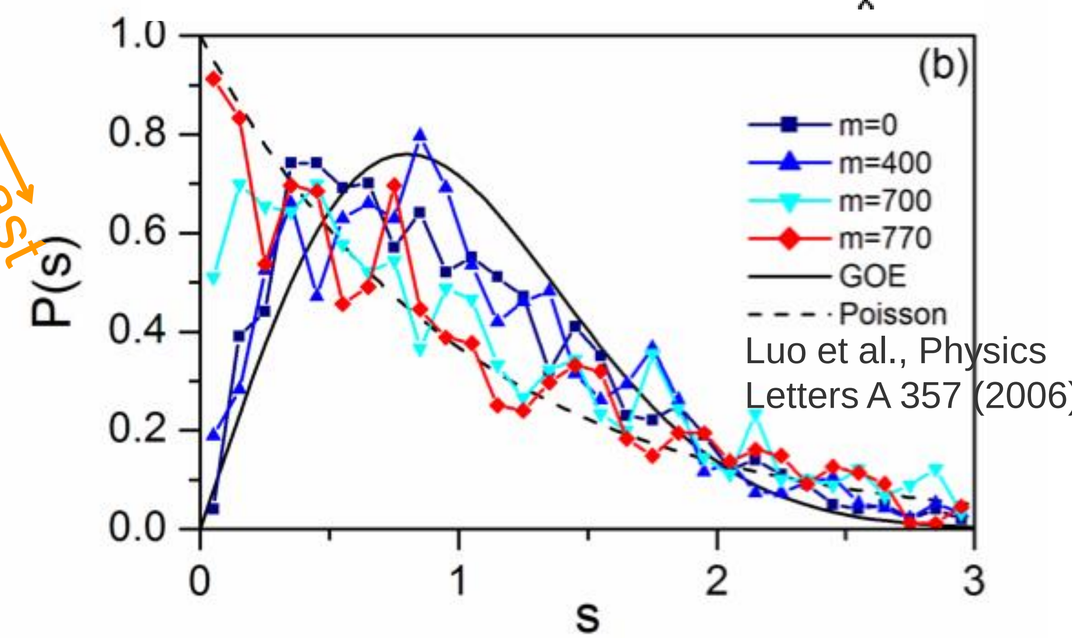
Neutron scattering cross section of ^{238}U



Sinai billiard



Biological networks of yeast



Powerful Tool:

Nuclear physics, Disordered and Mesoscopic Systems, Number theory, Quantum Chromodynamics, Quantum Chaos, Biological systems, Quantum gravity and string theory, risk management in finance, …

Wigner Surmise in Erbium

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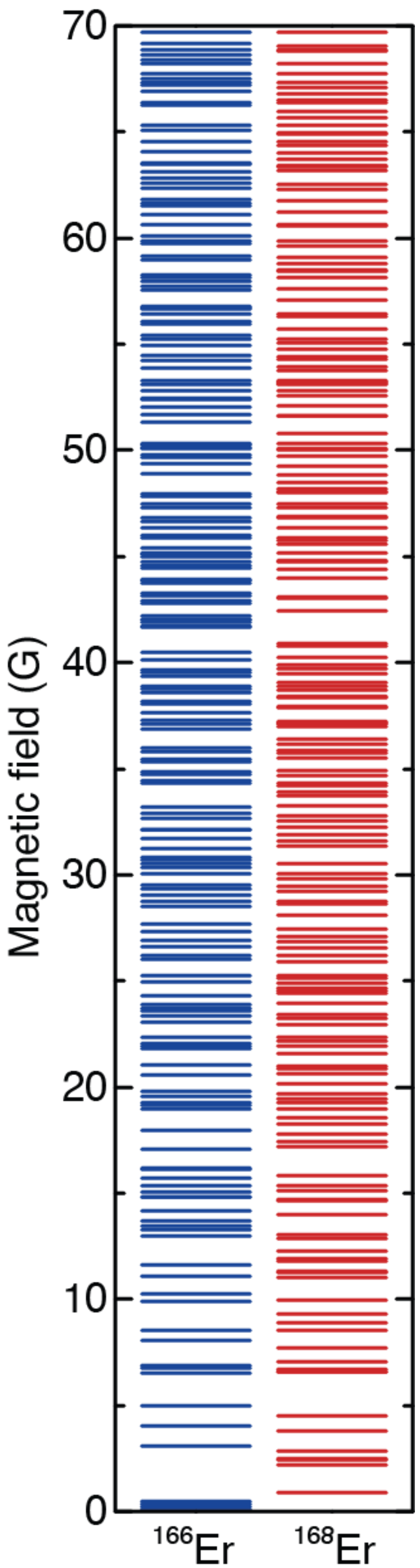
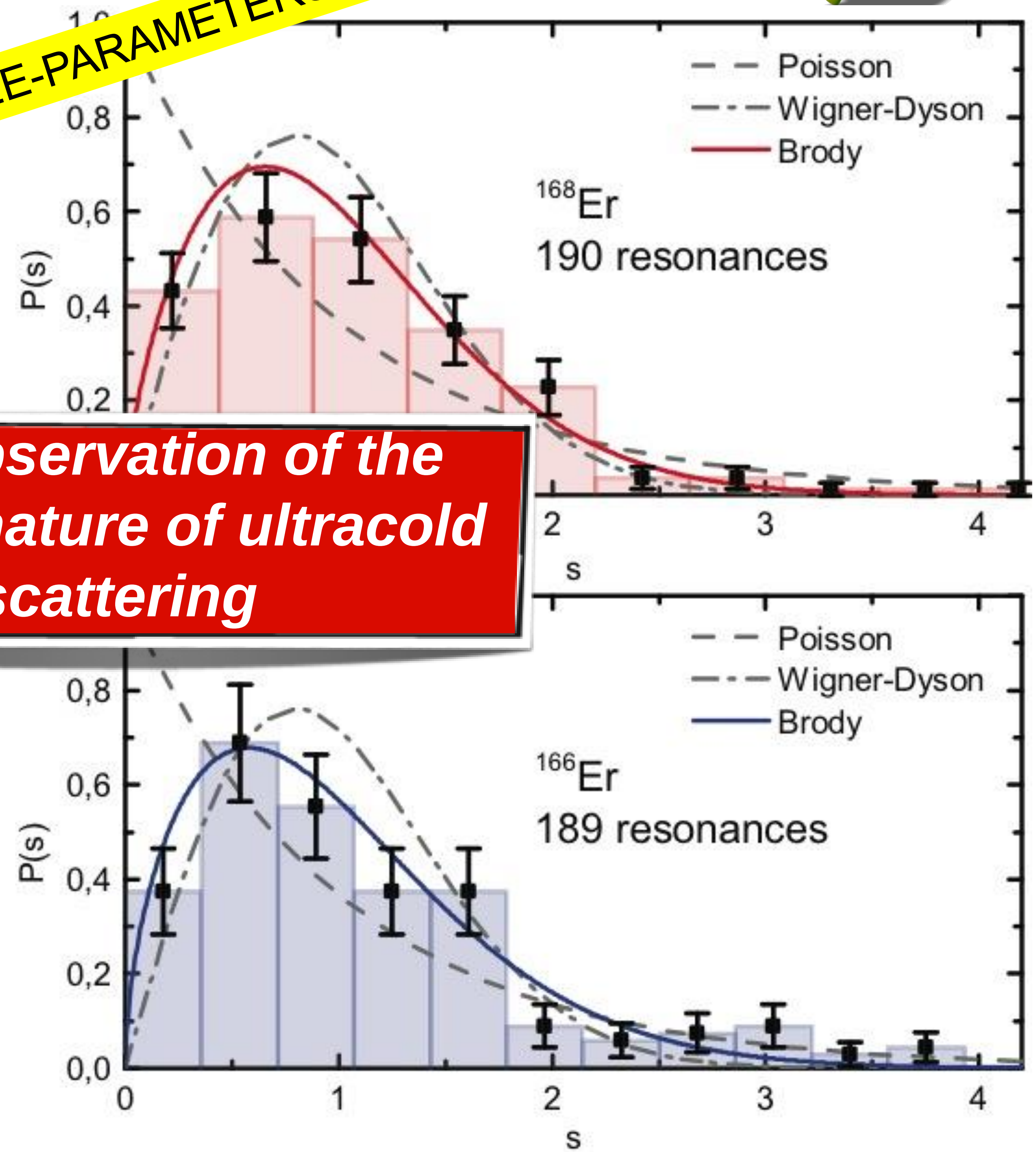
$P^{(WD)}$: NO FREE-PARAMETERS



Applying RMT to Er
Feshb

First observation of the chaotic nature of ultracold scattering



A. Frisch et al. Nature 507, 475 (2014)

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Why a new work?

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A joint effort to better understand the chaotic scattering of Lanthanide atoms.

Er Group (Ferlaino et al.) + Dy Group (Pfau et al.)
+ Theory Group (Kotochigova/Tiesinga)



Is this generalizable to other atomic species?Thorough comparison of the behavior of 2 Lanthanides:
Er and Dy (+isotopes).

What is the origin of chaotic scattering?
Effort toward a better theoretical understanding.

From where come some specific behavior?
Investigate temperature and magnetic field dependence

Development of more accurate numerical simulations for both species.

New measurements.

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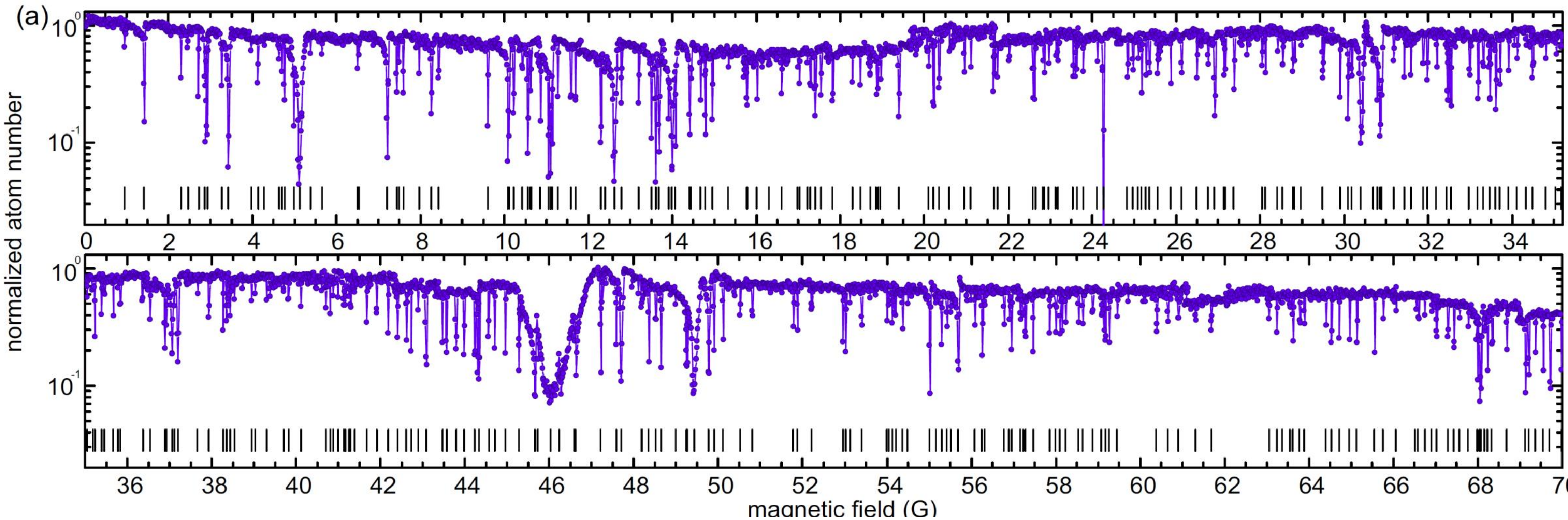
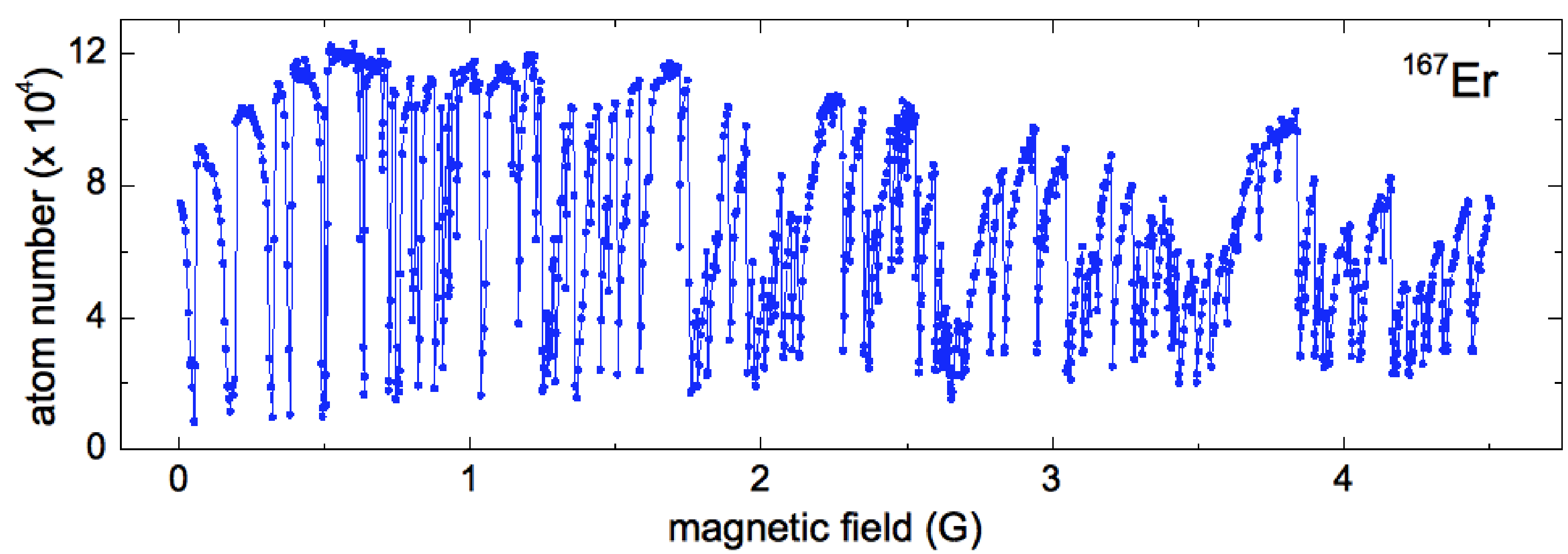


Dy 164

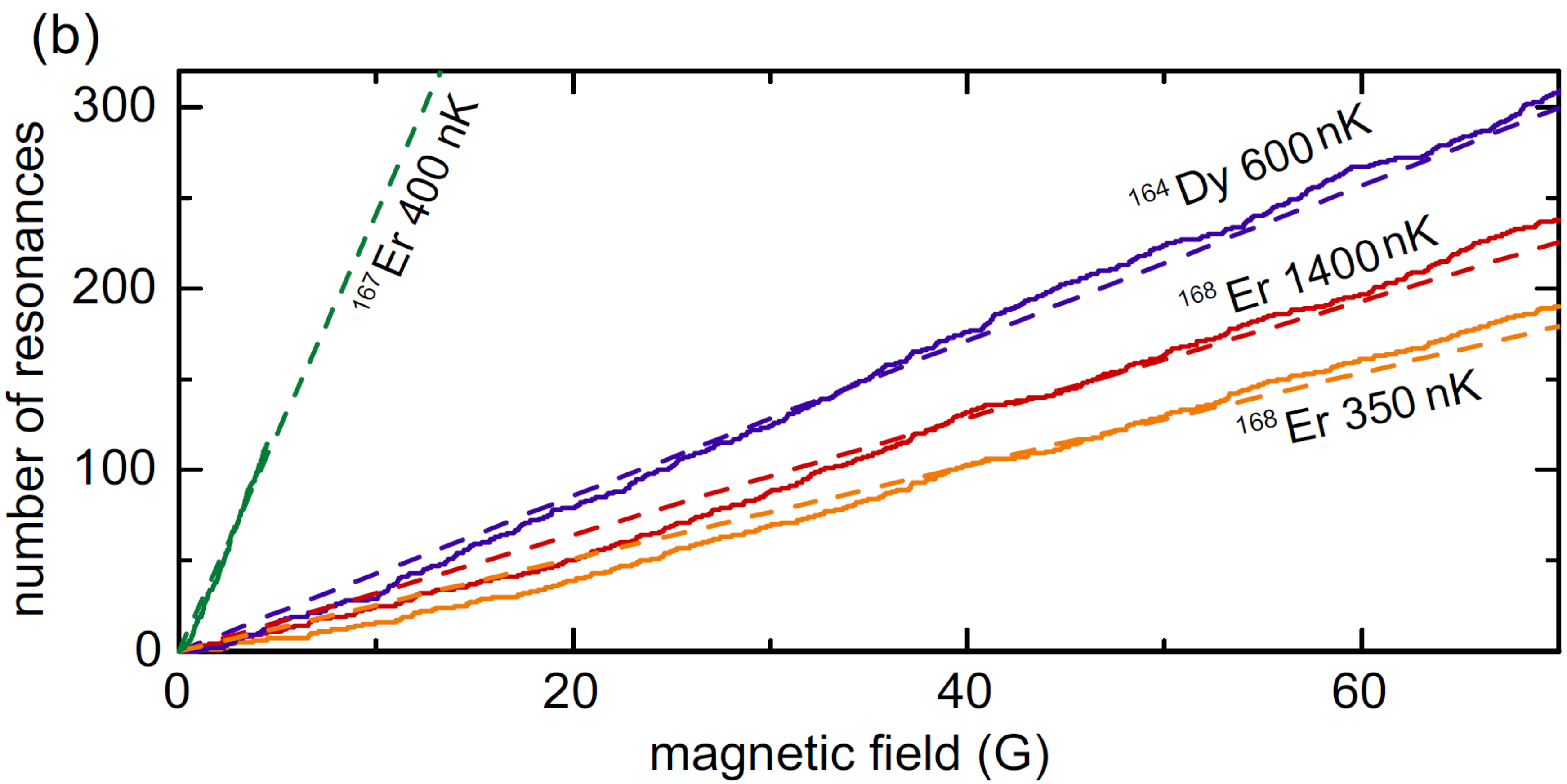
Additional extensive high-resolution trap loss spectroscopy measurements:

- Dy 164 @ 600 nK (Pfau group, Stuttgart)
- Er 168 @ higher T = 1400 nK.
- Er 167 @ T = 400 nK.

very dense spectra	N_{res}
^{164}Dy @ 400 nK	309
^{168}Er @ 1400 nK	238
^{168}Er @ 350 nK	189



Density of resonances



	ρ (G^{-1})
^{168}Er @ 350 nK	2.7
^{168}Er @ 1400 nK	3.4
^{164}Dy @ 400 nK	4.3
^{167}Er @ 400 nK	25.6

- Staircase function(B) = number of resonance with B_{res} < B.
- Derivative: Density of resonances ρ. Deduced from linear fit.

Exact values depend on specific scattering parameters, e.g. **number of channels**!

⇐ # of Zeeman sub-levels in the ground state.

Same order of magnitude:

Similar scattering scheme!

Fermionic isotope:
Hyperfine structure

Dy : larger j than **Er**

Nearest Neighbour spacing distribution

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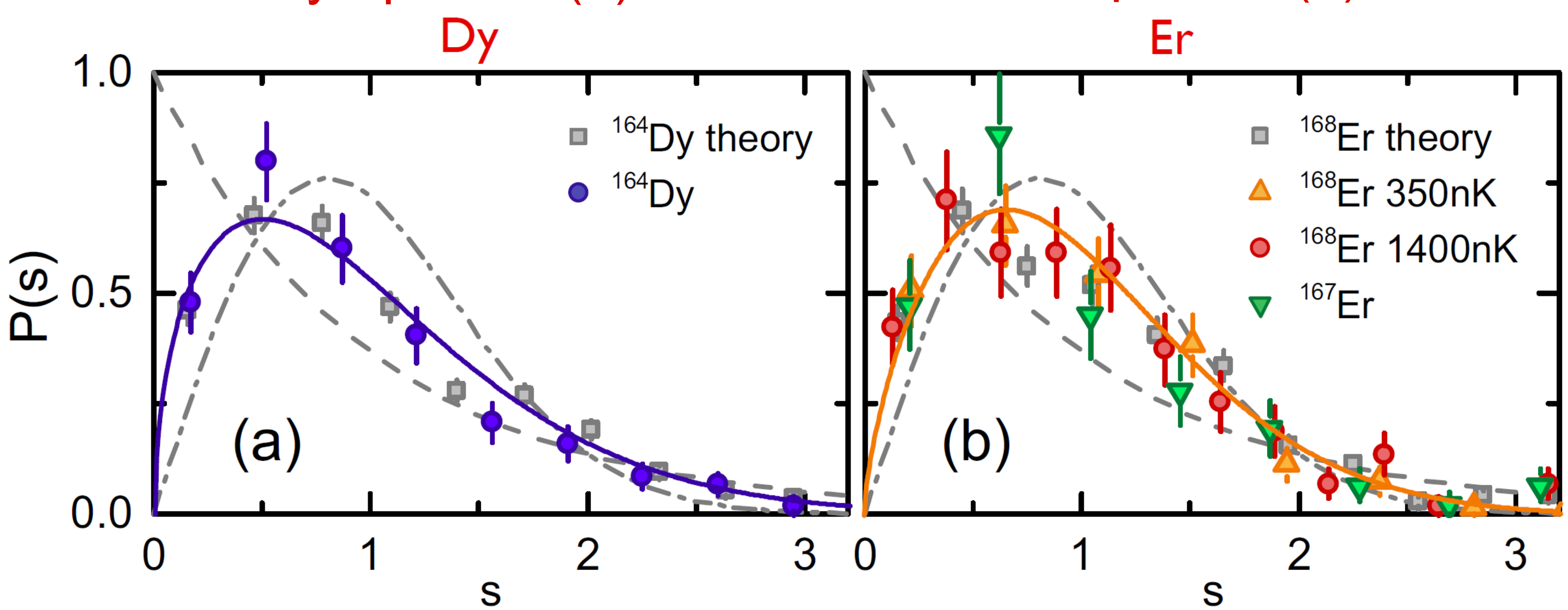
Dy: η = 0.45(7)　　　Er: η = 0.68(9)



- P(s) = distribution of the spacing between adjacent resonances, normalized to the mean distance 1/ρ

Extreme cases	P(s)
Poisson	η = 0
Wigner-Dyson	$\frac{\pi s}{2}$ … η = 1

Strong correlation

⇒Repulsion btw resonances!

Quantify the degree of correlation : **interpolate** between the 2 distributions.

→ **Brody** distribution: unique parameter η ∈ [0,1]. **Empirical**

η(Er) > η(Dy) even though $j_{Er} < j_{Dy}$

Correlation as a function of B

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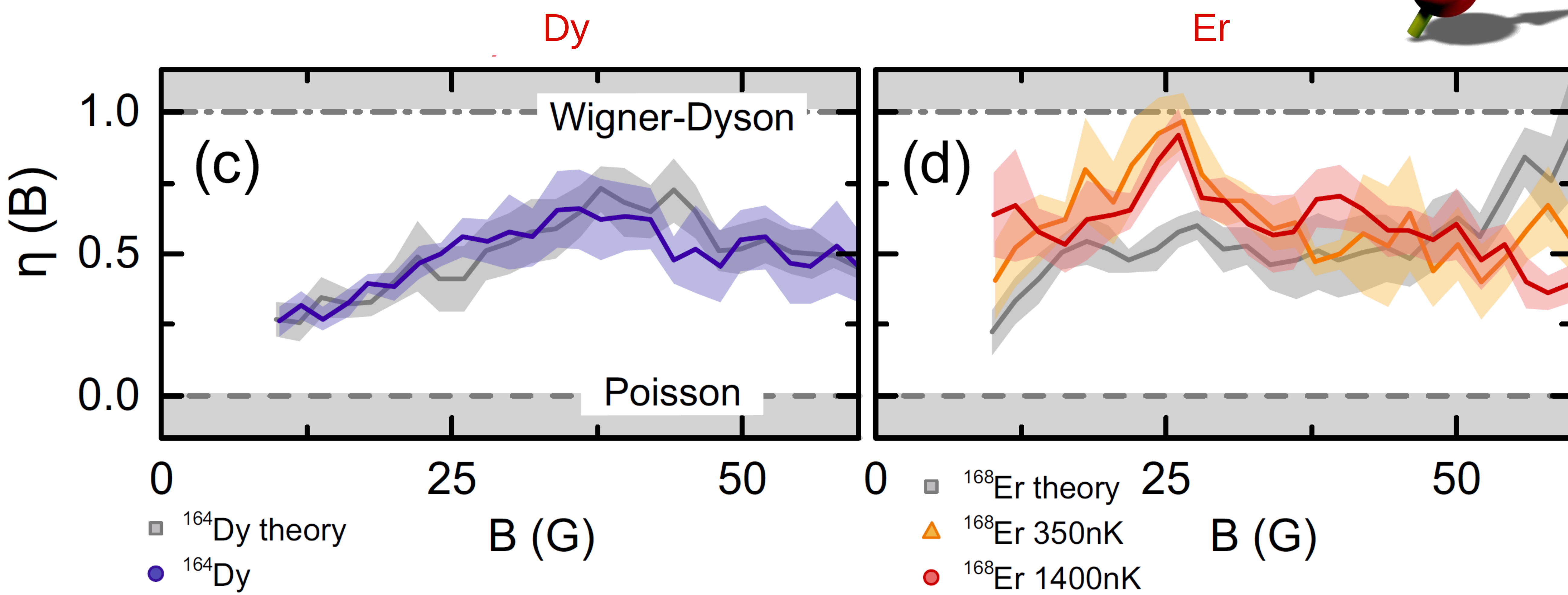

Quantify B and T dependences : **fit** η of P(s) on sliding B-range [B−ΔB/2, B+ΔB/2]; ΔB=20G

Never fully correlated

$\eta \approx 0.5 < 1$

B-dependence: increase of correlation at small B! (Dy)

T-dependence : affect mean number but not the correlations! (Er)

Summary of experimental observations

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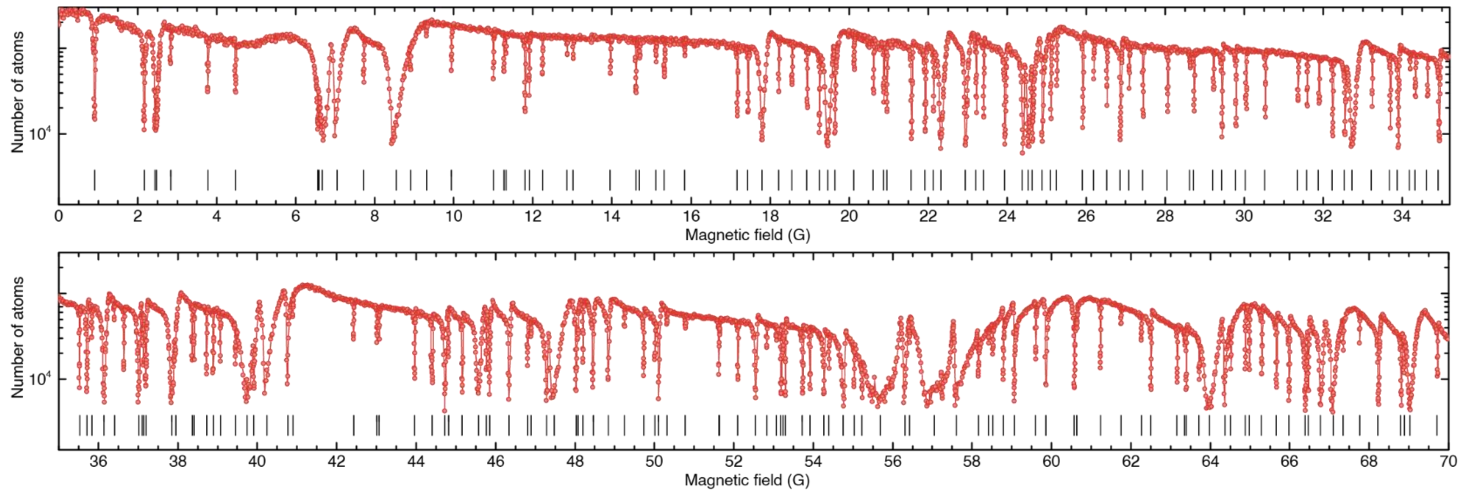

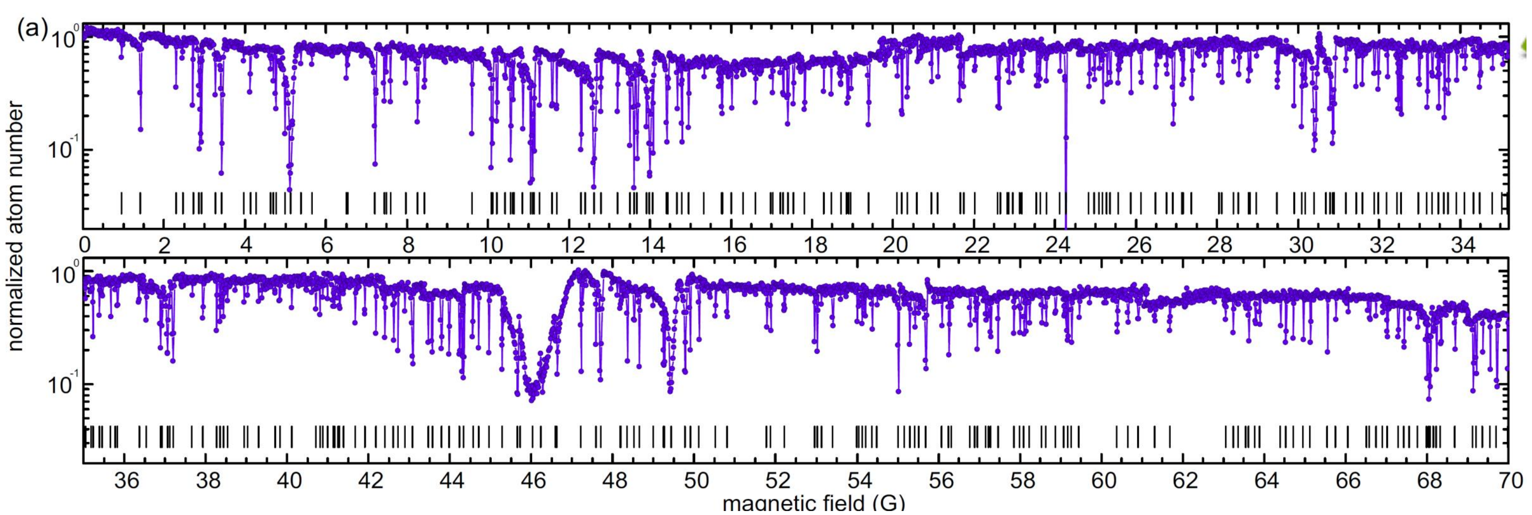

Lanthanides ⇒ very dense FR spectra.

RMT analysis of the resonance correlations:
- (partial) levels repulsion
- reduced variance // shot-noise

The **number of channels** affects the **number of resonances** but no straightforward link:
j ⇄ correlation degree

Correlation develops with B

T affects the **number of resonance** but not their **correlations**

Outline of the talk

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✦ Toward non-alkali atoms, the quest for atomic richness and the case of magnetic atoms.

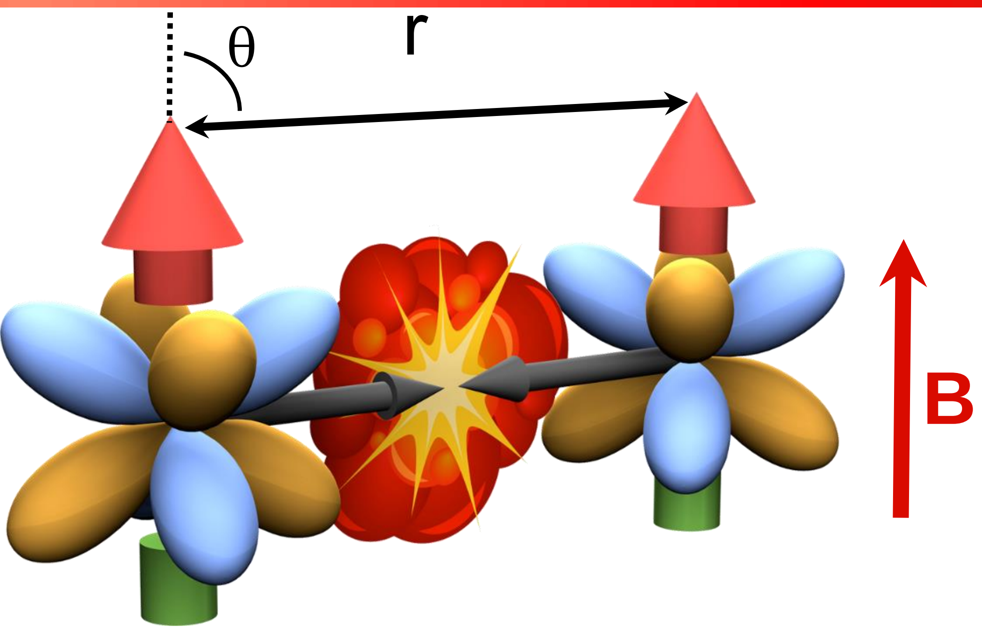
✦ Specificity of scattering of ultracold lanthanides

✦ Random Matrix Theory: Complex systems, dense spectra nad chaos

✦ Chaos in Feshbach resonances of Lanthanides: First Erbium study and beyond, describing lanthanides chaoticity.

✦ Towards a theoretical understanding the origin of chaos

Toy model based on RMT



$$\hat{H}_{\rm rel} = -\frac{\hbar^2}{2\mu_r}\frac{\mathrm{d}^2}{d^2\vec{r}} + \hat{V}(\vec{r}) + \hat{H}_Z(B)$$

Zeeman energy

Anisotropic interaction potential: Coupling of Zeeman sublevel

large j : Large number of Zeeman sublevel in the ground state.

Difficult to simulate

Toy model based on RMT idea

replace by

$$\hat{H}_{\rm RMT} = \hat{H}_0 + \hat{H}_Z(B)$$

B=0 Hamiltonian Zeeman energy

- Set of random matrices, extract global properties of spectra and average
- respect symmetry (GOE): n×n real-symmetric matrix, n =500
- respect structure (B-dependence)

Toy model based on RMT

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$$\hat{H}_{\mathrm{RMT}} = \hat{H}_0 + \hat{H}_Z(B)$$

B=0 Hamiltonian   Zeeman energy

$$\hat{H}_0 = \begin{pmatrix} E_1^{(0)} & & 0 \\ & \ddots & \\ 0 & & E_n^{(0)} \end{pmatrix} + \begin{pmatrix} 0 & & H_{ij} \\ & \ddots & \\ H_{ji} & & 0 \end{pmatrix}$$

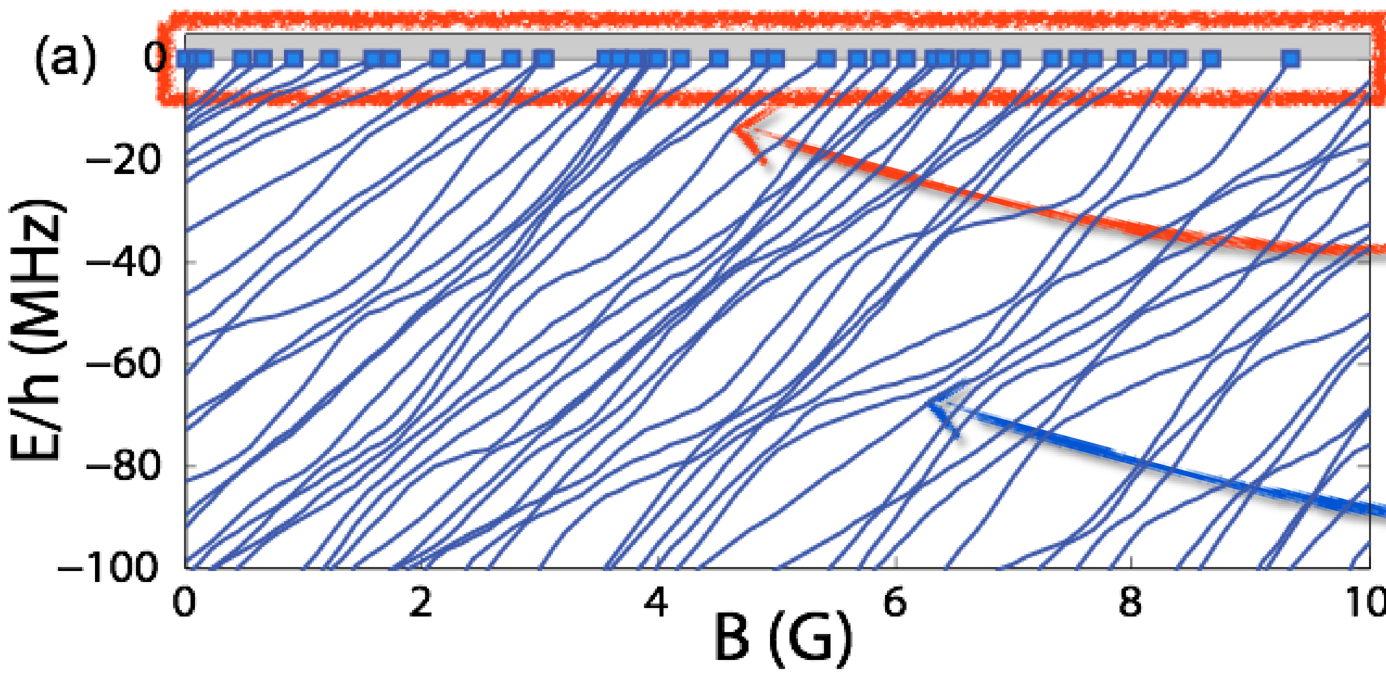
$$\hat{H}_Z(B) = g\mu_B B \begin{pmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_n \end{pmatrix}$$

Diagonal ~ «sublevels Energies» distributed with Brody η_d.
Mean spacing ε_d to reproduce

Coupling between the sublevels from $V_{ani}(r)$, Gaussian-distributed, **strength v_{cpl}**

Zeeman sublevels $m_i \in \{-12, 12\}$

ρ. Eigenvalues of H_{RMT} :



mean spacing of $E_i^{(0)}$: ε_d / h =6.4 MHz

Feshbach resonances spectrum.
Global properties??

Very dense spectra
=> multiple
(avoided) crossings

Toy model based on RMT

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$$\hat{H}_{\mathrm{RMT}} = \hat{H}_0 + \hat{H}_Z(B)$$

B=0 Hamiltonian — Zeeman energy

$$\hat{H}_0 = \begin{pmatrix} E_1^{(0)} & & 0 \\ & \ddots & \\ 0 & & E_n^{(0)} \end{pmatrix} + \begin{pmatrix} 0 & & H_{ij} \\ & \ddots & \\ H_{ji} & & 0 \end{pmatrix}$$

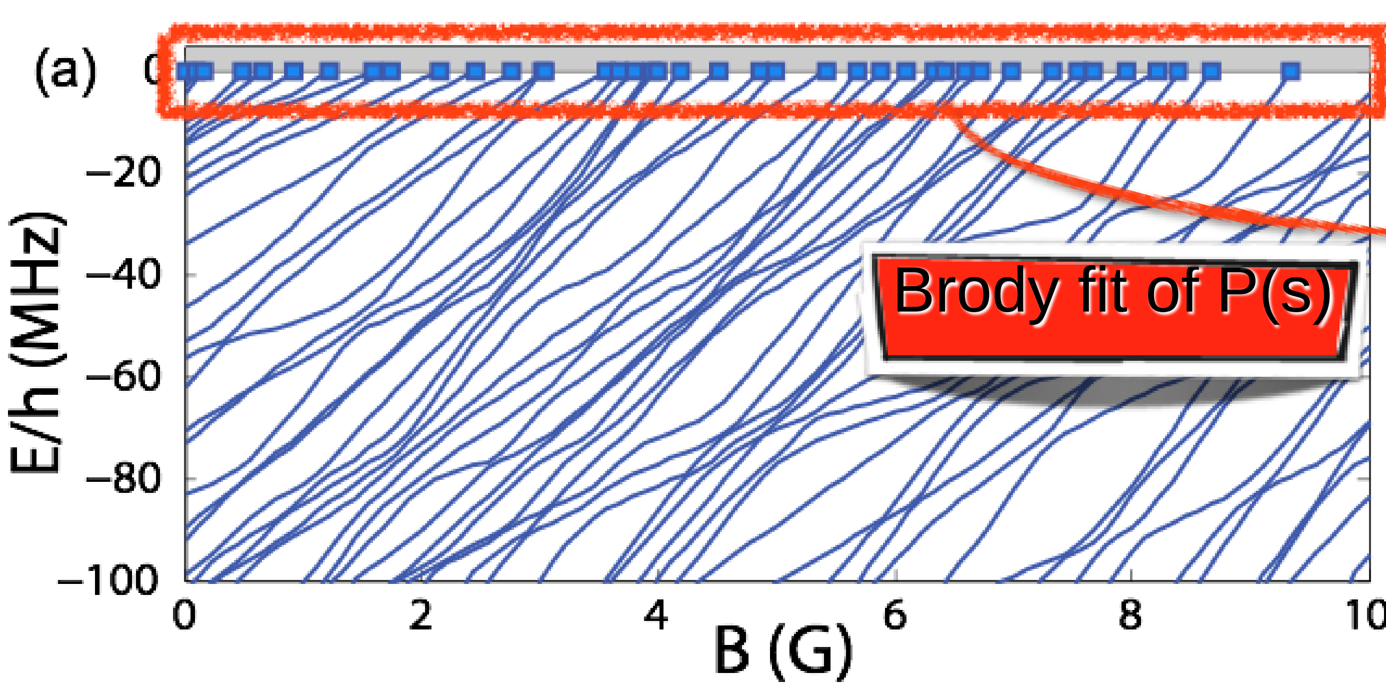
$$\hat{H}_Z(B) = g\mu_B B \begin{pmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_n \end{pmatrix}$$

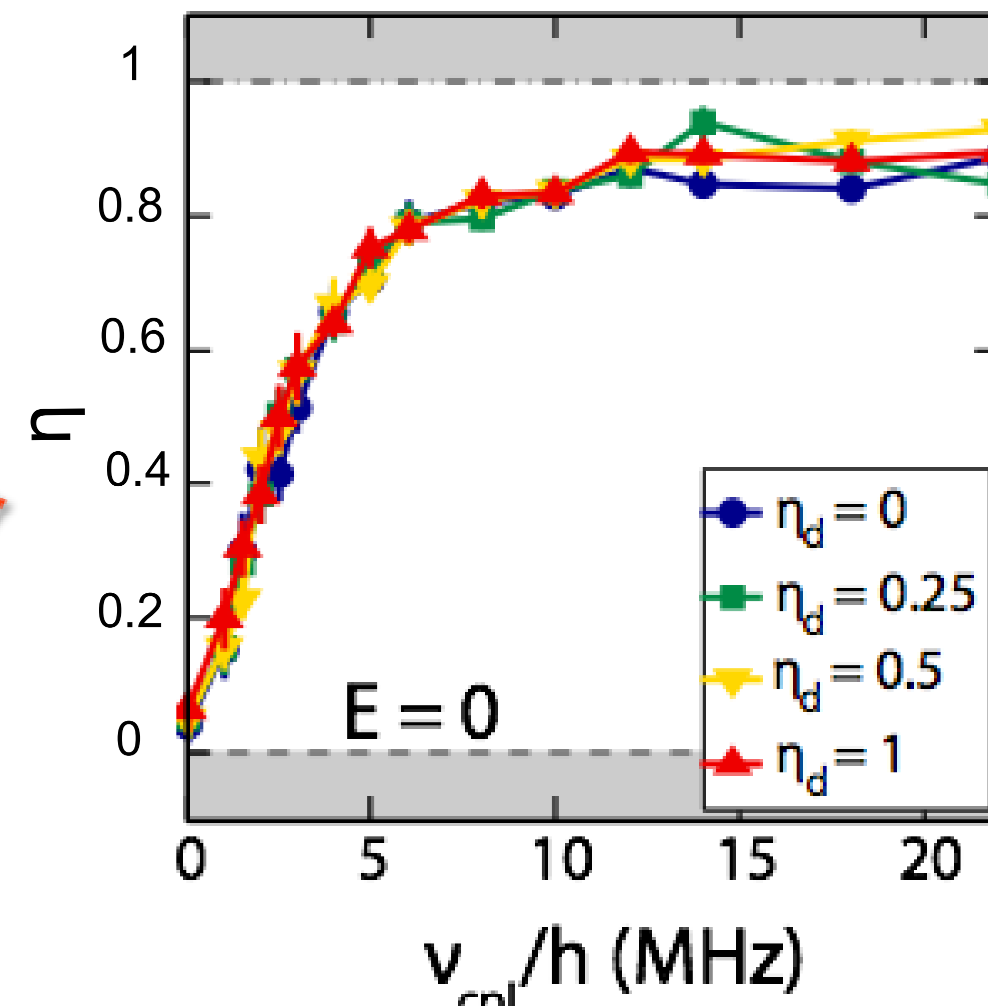
Diagonal distributed with Brody parameter η_d. Mean spacing ε_d to reproduce ρ

Coupling between the sublevels from $V_{ani}(r)$, Gaussian-distributed, **strength v_{cpl}**

Zeeman sublevels $m_i \in \{-12, 12\}$

Eigenvalues of H_{RMT} :



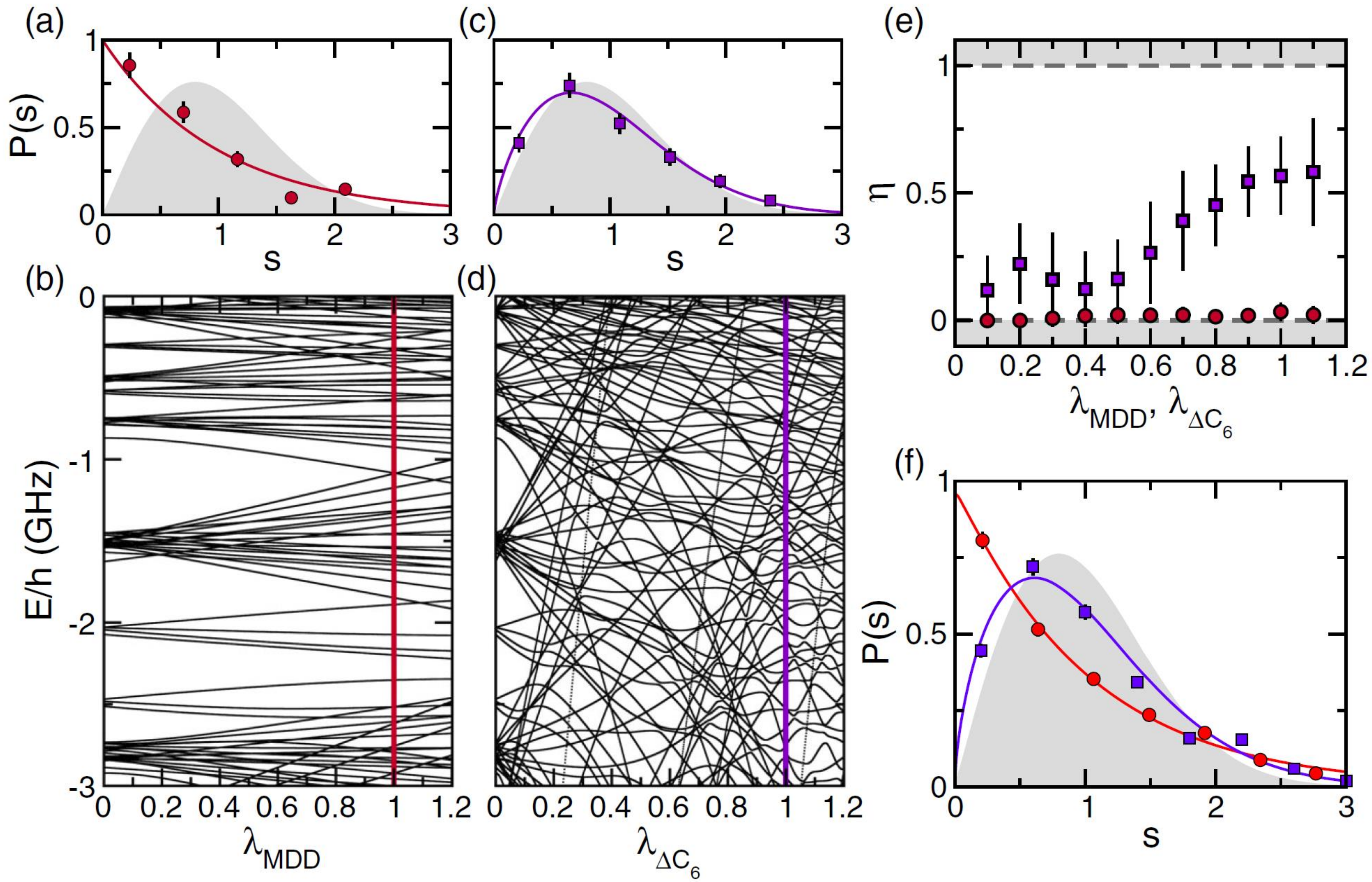


mean spacing of $E_i^{(0)}$: ε_d/h =6.4 MHz

Coupled channel calculations

$$\hat{V}_a(\vec{R}) = \lambda_{\Delta C_6} V_{\Delta C_6}(\vec{R}) + \lambda_{\text{MDD}} V_{\text{MDD}}(\vec{R})$$

$$V_{\Delta C_6}(\vec{R}) = \frac{c_a}{R^6} \sum_{i=1,2} \frac{1}{\sqrt{6}} \{3(\hat{R}\cdot\vec{j}_i)(\hat{R}\cdot\vec{j}_i) - \vec{j}_i\cdot\vec{j}_i\} + \cdots \qquad V_{\text{MDD}}(\vec{R}) = -\frac{\mu_0}{4\pi}\frac{(g\mu_B)^2}{R^3}\{3(\hat{R}\cdot\vec{j}_1)(\hat{R}\cdot\vec{j}_2) - \vec{j}_1\cdot\vec{j}_2\}$$



Coupled channel calculations

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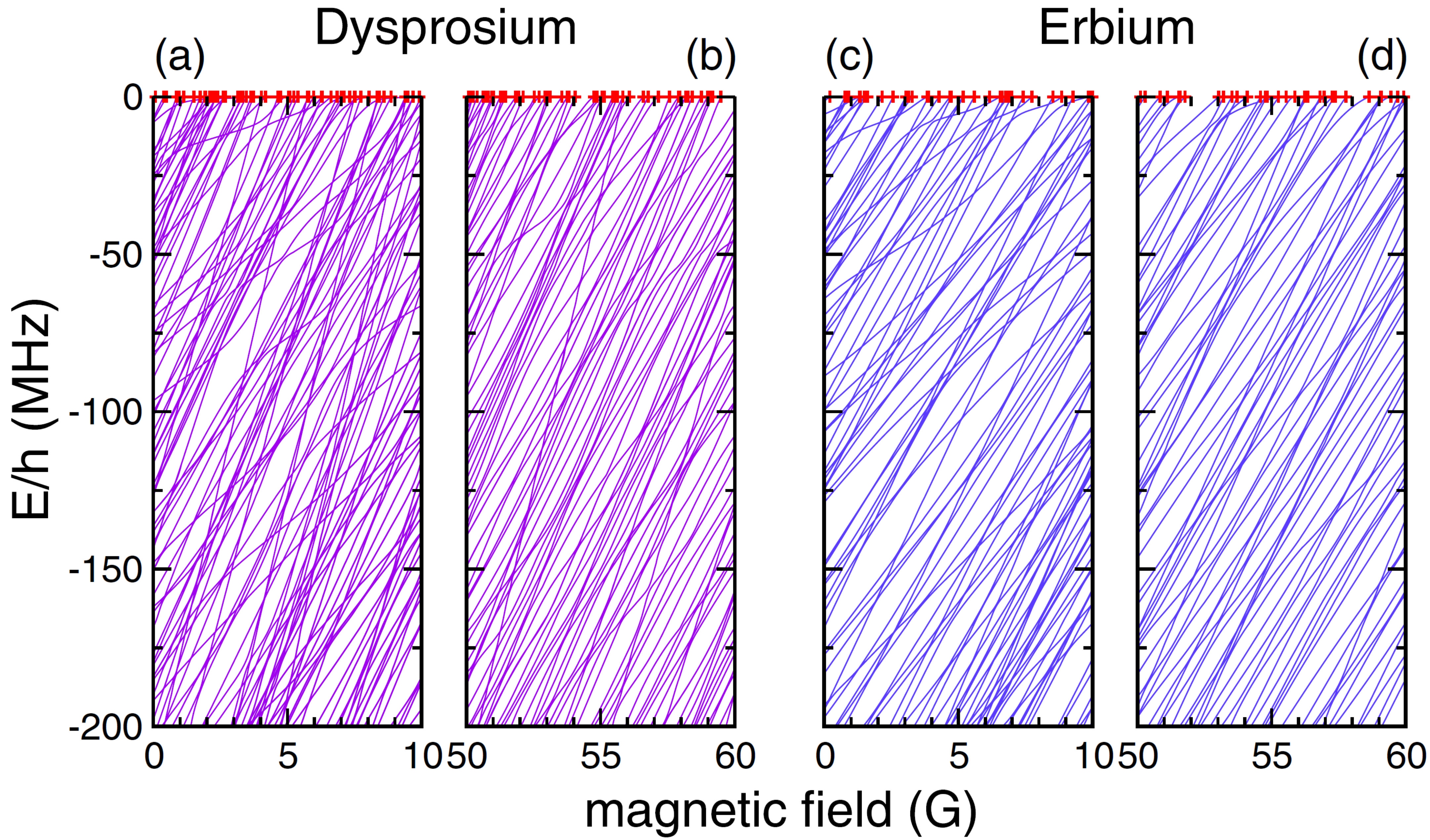
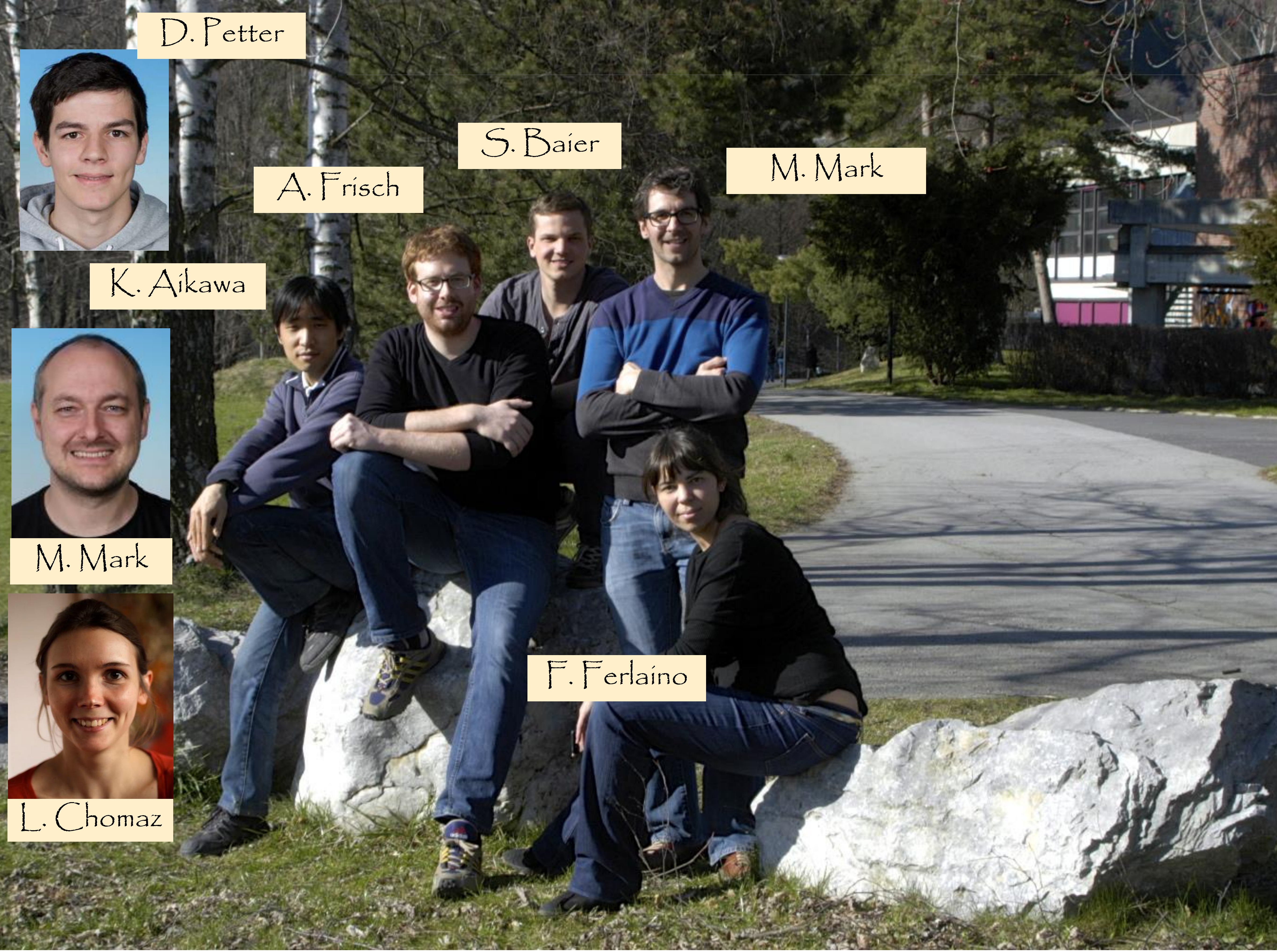


Summary of the talk

✦ Lanthanides show very rich Feshbach resonance spectra with similar global properties for both Dy and Er isotopes

✦ Correlations of the resonances reveal quantum chaos and manifest by spectral rigidity and level repulsion.

✦ Explained by anisotropy of the interaction and density stemming from the special electronic structure of Lanthanides


D. Petter
S. Baier
M. Mark
A. Frisch
K. Aikawa
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F. Ferlaino
L. Chomaz