Beyond mean-field effects in a cold gas: multibody interactions and quantum droplets

Dmitry Petrov

Laboratoire Physique Théorique et Modèles Statistiques (Orsay)

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Part 1: Multi-Body Interactions

$$\frac{E}{\text{Volume}} = g_2 \frac{n^2}{2!} + g_3 \frac{n^3}{3!} + g_4 \frac{n^4}{4!} + \ldots$$

⇓

3-body interaction is noticeable if $g_3 \sim g_2 / n$, i.e., in the dilute limit we basically need $g_2 = 0$

⇓

4-body interaction ⟺ $g_2 = g_3 = 0$

⇓

etc.

⇓

“N-body interacting” ⟺ $\boxed{g_2 = g_3 = \ldots = g_{N-1} = 0 \text{ AND } g_N \neq 0}$

Why interesting?

Bosons + $g_2<0$ → Collapse

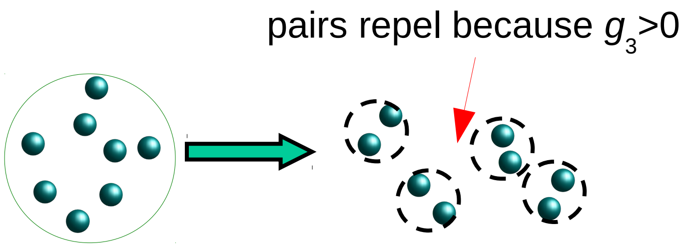
Bosons + $g_2<0$ + $g_3>0$ → Free space → self-trapped droplet state Bulgac'02:

- Neglecting surface tension, flat density profile $n=3|g_2|/2g_3$
- Including surface tension → surface modes

Increasing $g_2<0$ → bosonic pairing Nozieres&Saint James'82

Topological transition, not crossover!

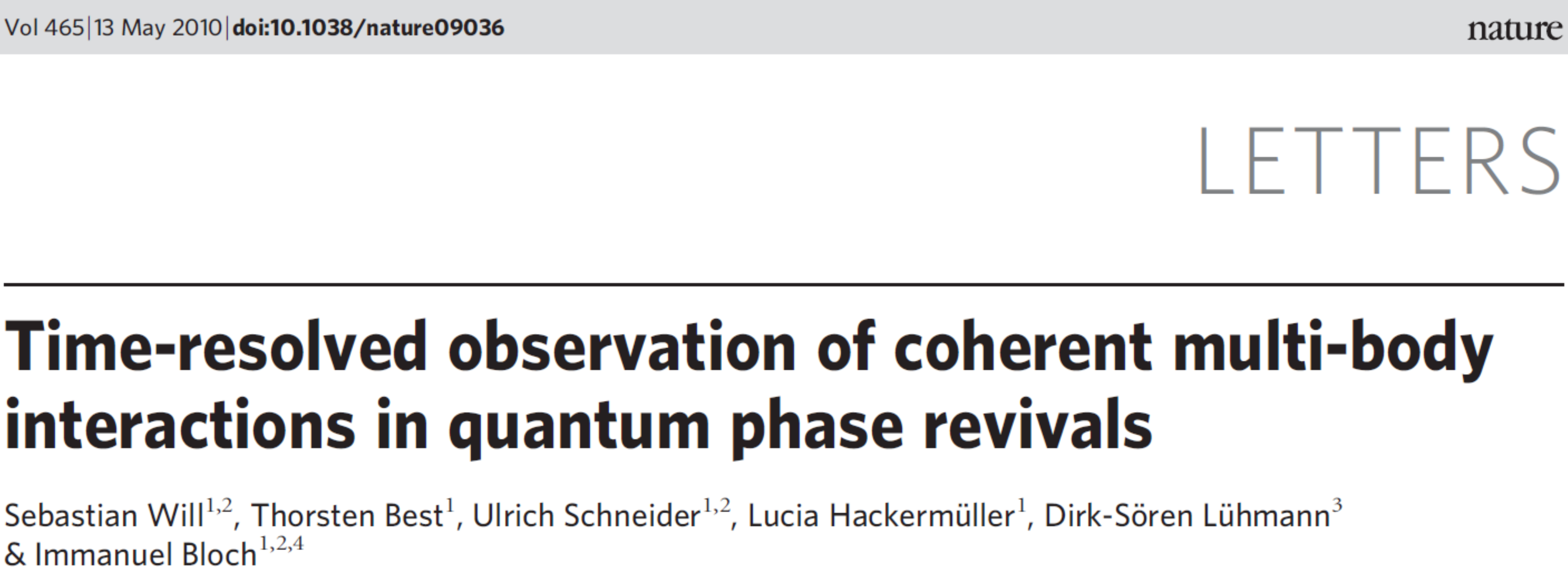
Radzihovsky et al., Romans et al., Lee&Lee'04



Pairing on a lattice with three-body constraint:

Daley et al.'09-, Ng&Yang'11, Bonnes&Wessel'12,...

g_3 is necessary! = Pauli pressure in the BCS-BEC crossover!



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nature

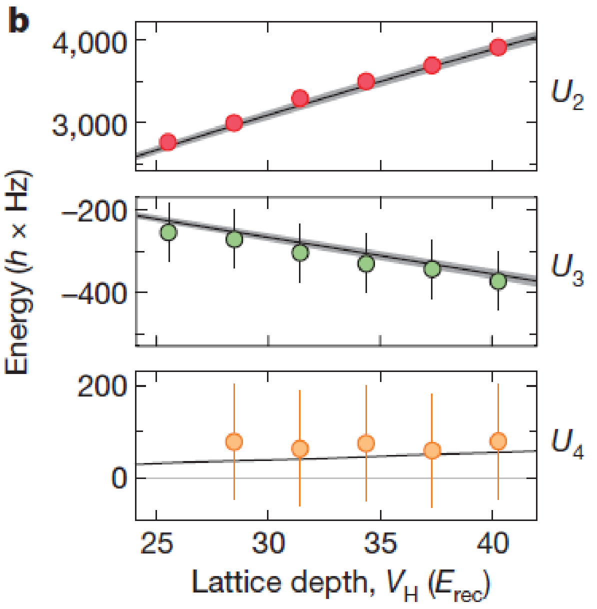
LETTERS

Time-resolved observation of coherent multi-body interactions in quantum phase revivals

Sebastian Will[1,2], Thorsten Best[1], Ulrich Schneider[1,2], Lucia Hackermüller[1], Dirk-Sören Lühmann[3] & Immanuel Bloch[1,2,4]

$$E(N)=\underset{\substack{\wr\\ g\sim a\,\omega^{3/2}}}{U_2}\frac{N(N-1)}{2!}+\underset{\substack{\wr\\ g^2/\omega}}{U_3}\frac{N(N-1)(N-2)}{3!}+\underset{\substack{\wr\\ g^3/\omega^2}}{U_4}\frac{N(N-1)(N-2)(N-3)}{4!}+\dots$$

$$g\sim a\,\omega^{3/2}\gg g^2/\omega\gg g^3/\omega^2$$

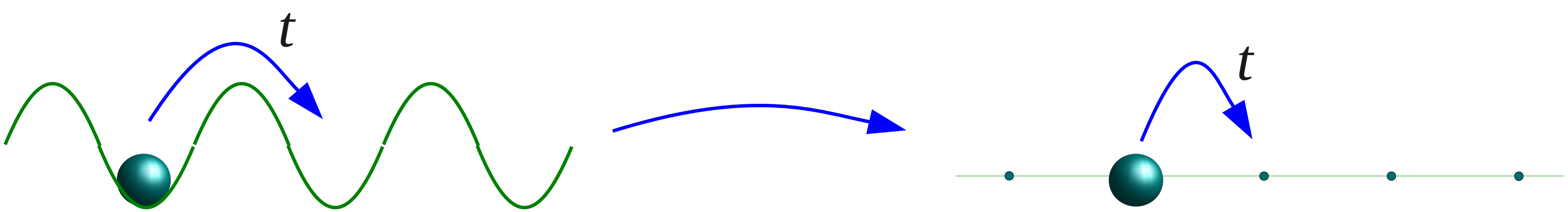


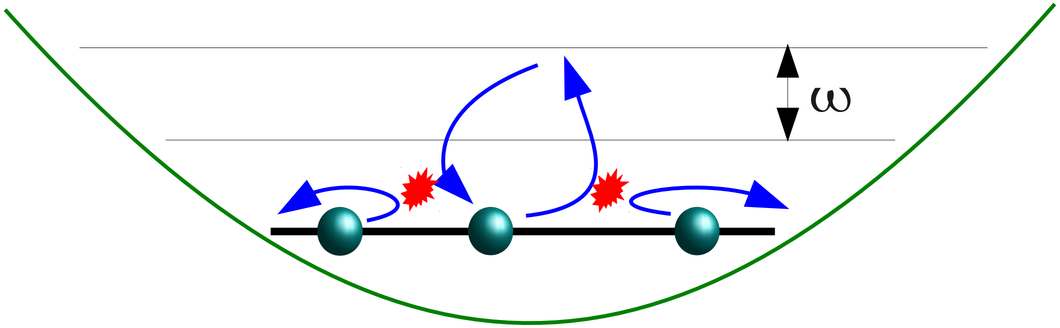
Optical lattice → Hubbard model

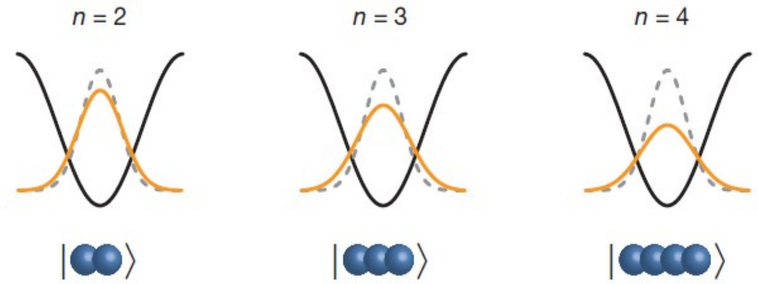
Jaksch et al.'98...

Bosons in an optical lattice → Bose-Hubbard model
+ higher order interactions

Niu et al.'06, Tiesinga et al.'09,11, Hazzard&Mueller'10







Weak, high order in g/ω, but measurable
Campbell et al.'06, Will et al.'10

3-body interacting case: perturb. prosp.

$$E(N)=U_2\frac{N(N-1)}{2!}+U_3\frac{N(N-1)(N-2)}{3!}+U_4\frac{N(N-1)(N-2)(N-3)}{4!}+\dots$$

Perturbative approach Johnson et al.'09

$$E(2)=U_2=\langle\psi_0|V|\psi_0\rangle+\underbrace{\sum_{\nu}\frac{|\langle\psi_\nu|V|\psi_0\rangle|^2}{\epsilon_0-\epsilon_\nu}+\dots}_{\text{Higher order terms}}$$

$$E(3)=3E(2)-O(V^2)+\underbrace{\sum_{\bar\nu}\frac{|\langle\bar\psi_{\bar\nu}|V|\bar\psi_0\rangle|^2}{\bar\epsilon_0-\bar\epsilon_{\bar\nu}}+\dots}_{\text{Additional non-additive higher order terms}}$$

Double counting compensation

$$U_3\sim V^2/|\epsilon_0-\epsilon_1|$$

This talk →

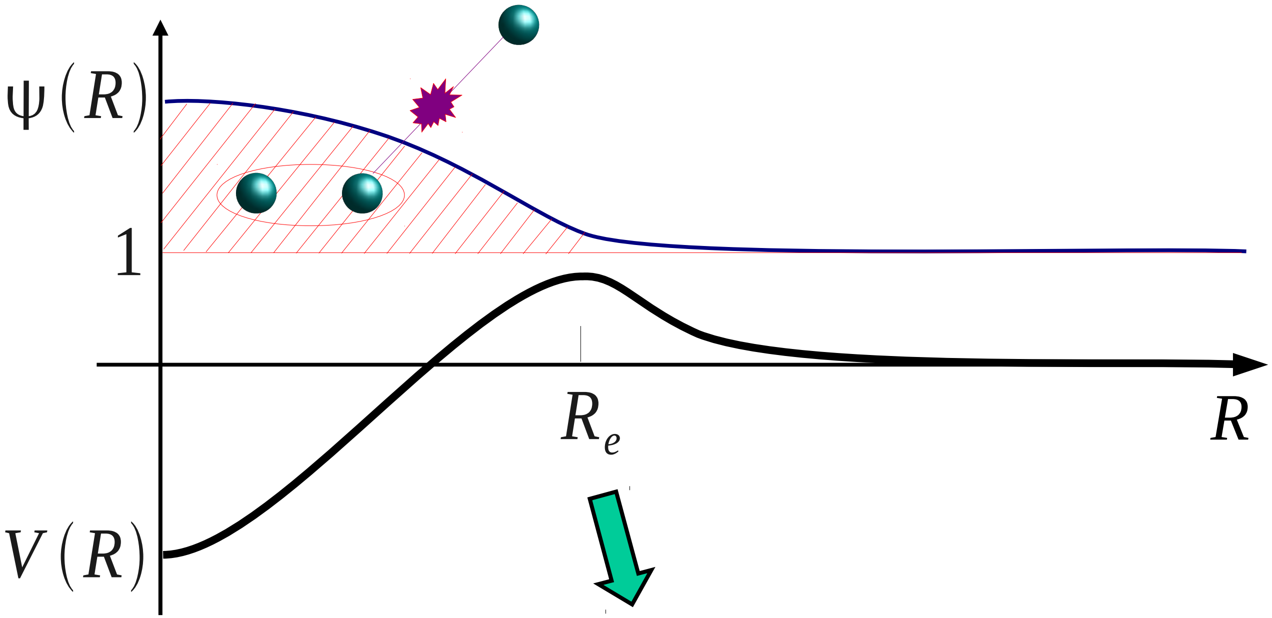
$U_2=0$ AND $U_3 > 0$ AND STRONG!

Where to look?

$\langle \psi_0 | V | \psi_0 \rangle \approx 0$ → vanishing on-shell scattering... OK

$\langle \psi_\nu | V | \psi_0 \rangle \neq 0$ ↔ large off-shell contribution...

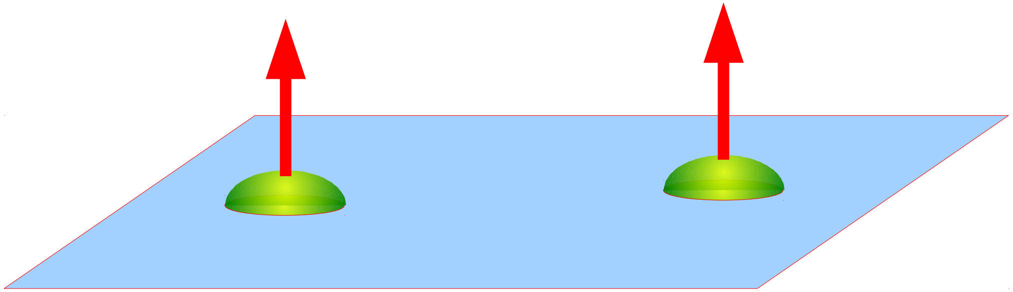
which should repel the third particle... ???



$$g_3^{(3D)} \sim R_e^4$$

Just two-body zero crossing is not enough !

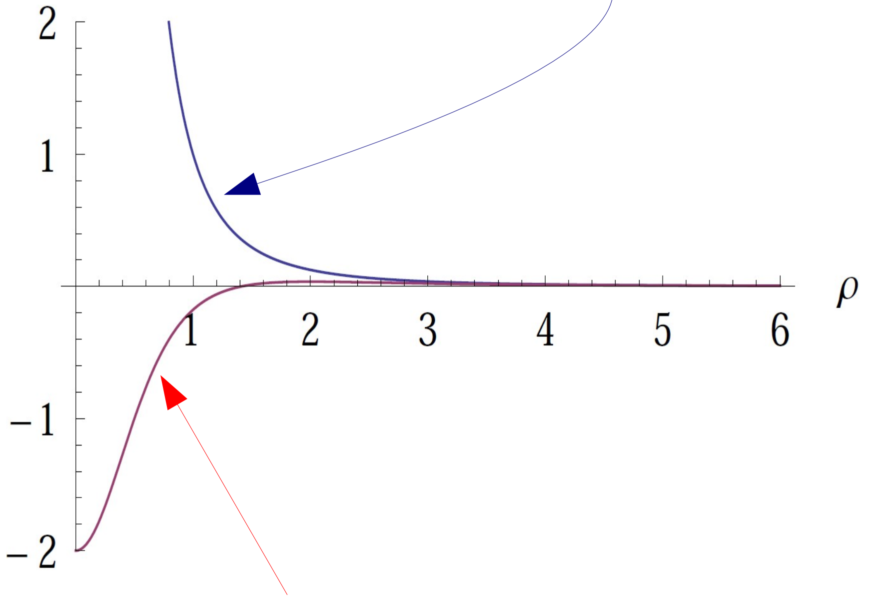
Dipoles on layers



Repulsive intralayer potential

$V_{\uparrow\uparrow}(\rho)=r_*/\rho^3$

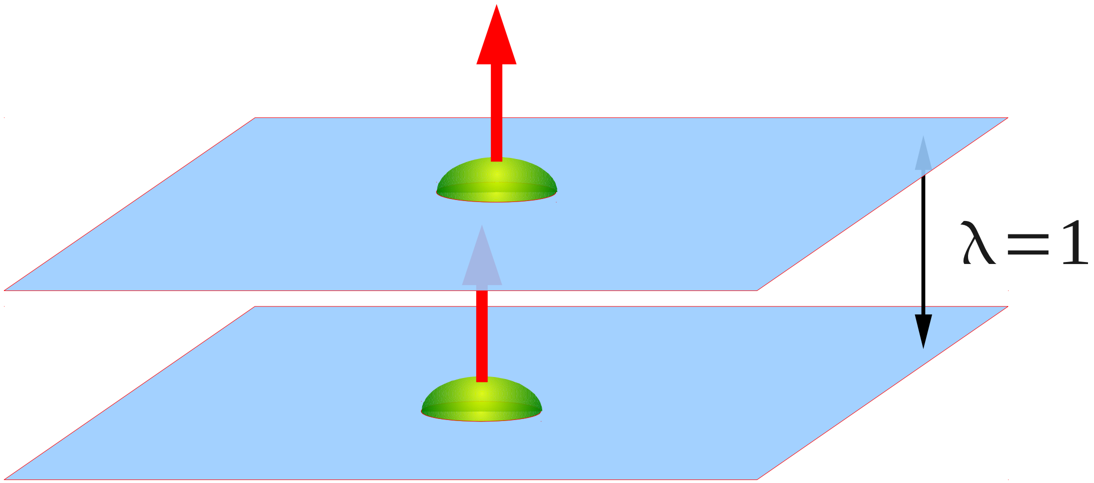
No bound state



Interlayer potential averages to zero

$V_{\uparrow\downarrow}(\rho)=r_*(\rho^2-2)/(\rho^2+1)^{5/2}$

At least one bound state



Vertex function and bound state

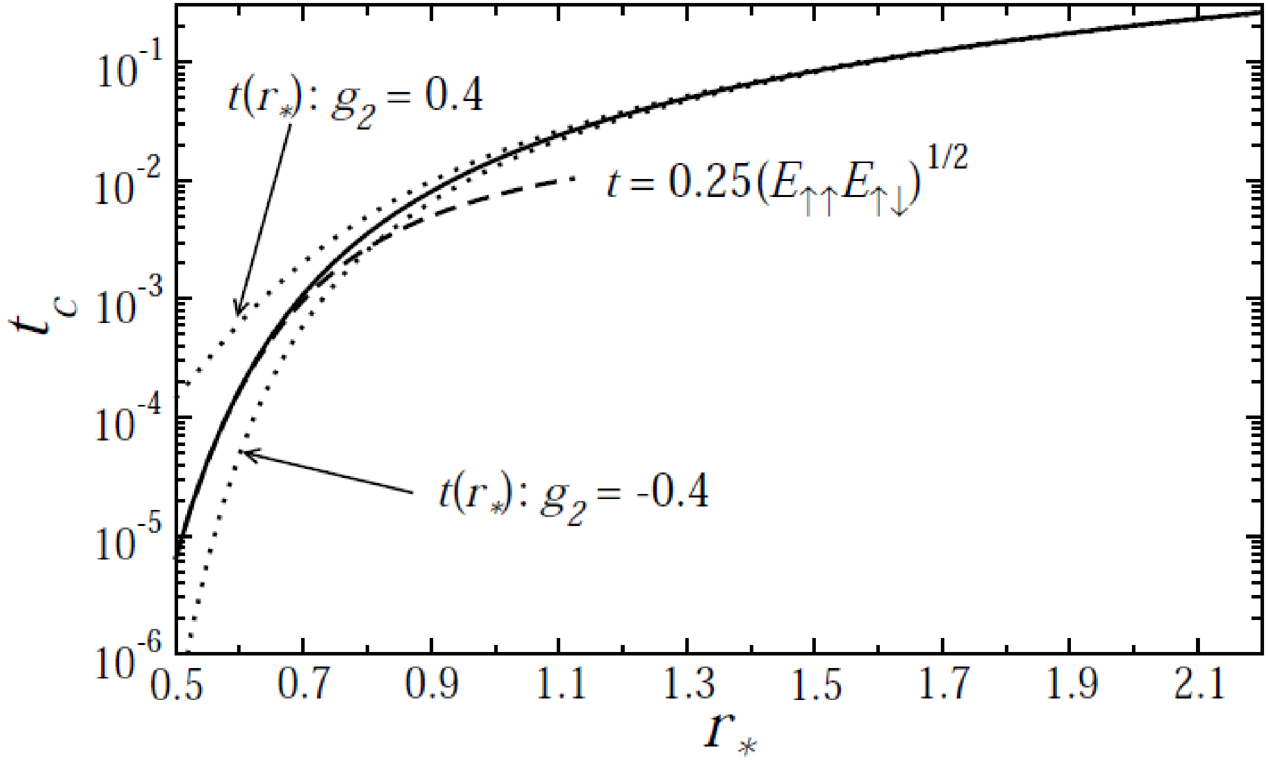
Vertex function for 2D scattering with weakly-bound state + dipolar tails:

$$\Gamma(E,\vec{k},\vec{k}')\approx\frac{4\pi}{\ln(4t/E)+4\pi/g_2+i\pi}-2\pi r_*|\vec{k}-\vec{k}'|$$

$$\varepsilon_0=4t\exp(4\pi/g_2)$$

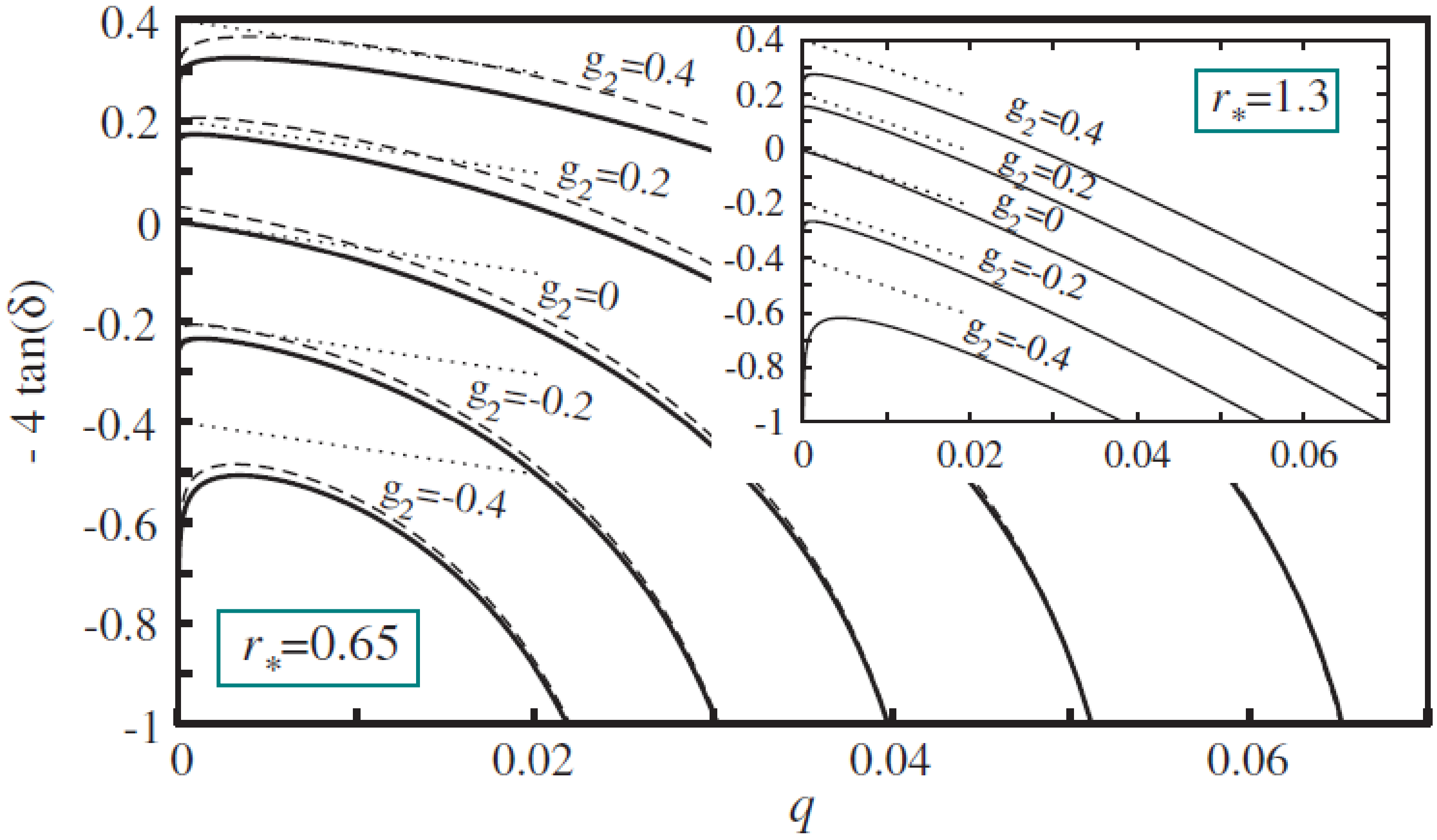
Exponentially weakly bound state
for small negative g_2

Critical *t*



S-wave scattering at finite collision energy

$$\Gamma(E,\vec{k},\vec{k}')\approx\frac{4\pi}{\ln(4t/E)+4\pi/g_2+i\pi}-2\pi r_*|\vec{k}-\vec{k}'| \xrightarrow[\text{s-wave and on shell}]{} -4\tan\delta_s(q)\approx g_2-8r_*q$$

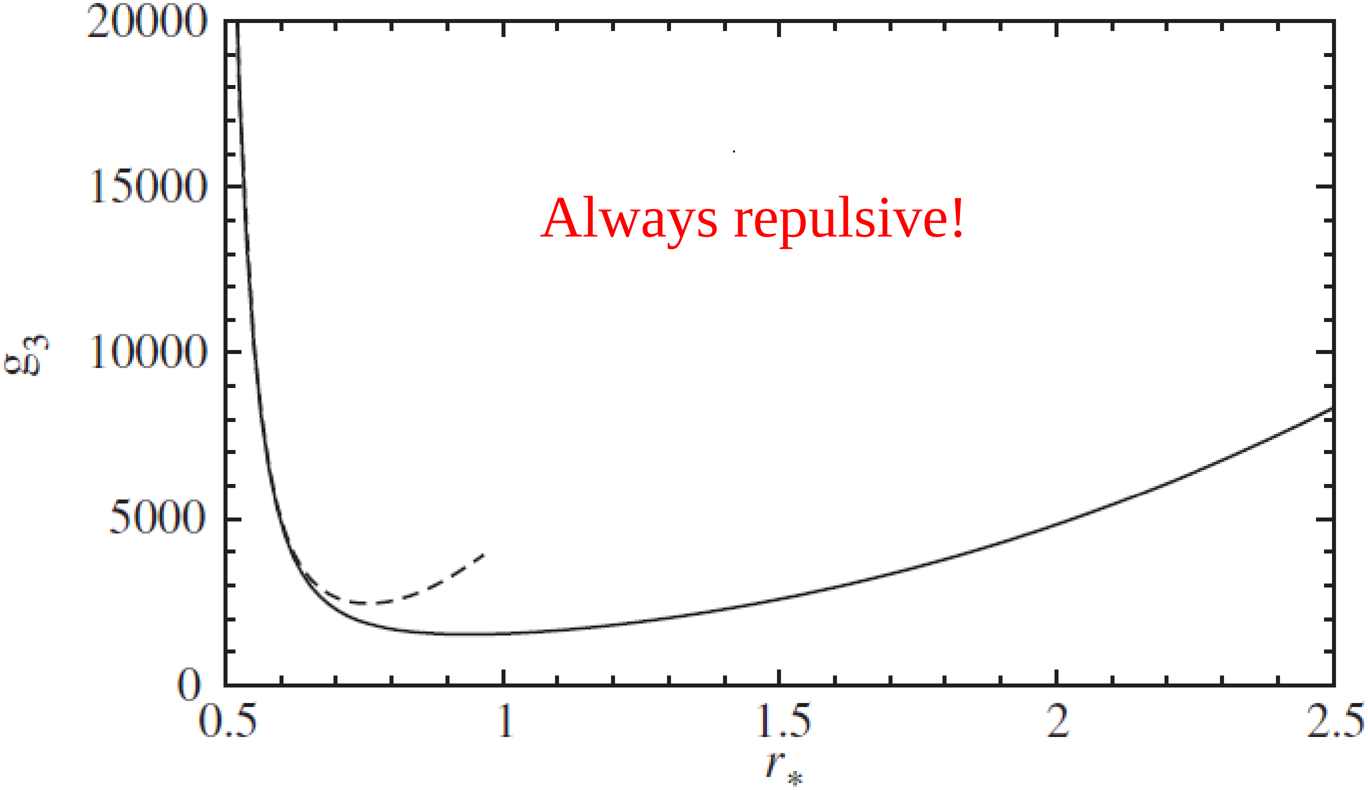


Two-body zero crossing + dipolar tail → rotonization, density wave, etc

3-body coupling constant

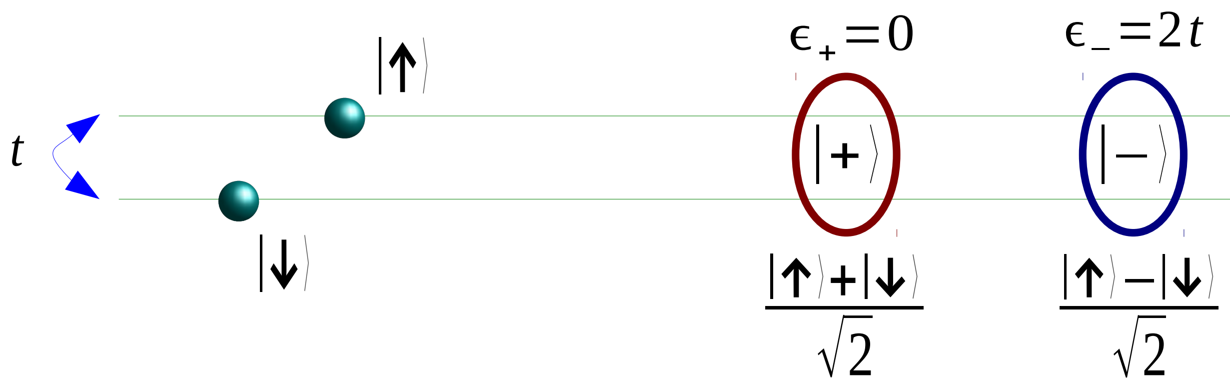
$$g_3 = \langle free_3 | \sum V | true_3 \rangle - 3 \langle free_2 | V | true_2 \rangle$$

Hyperspherical method
on the line g_2=0

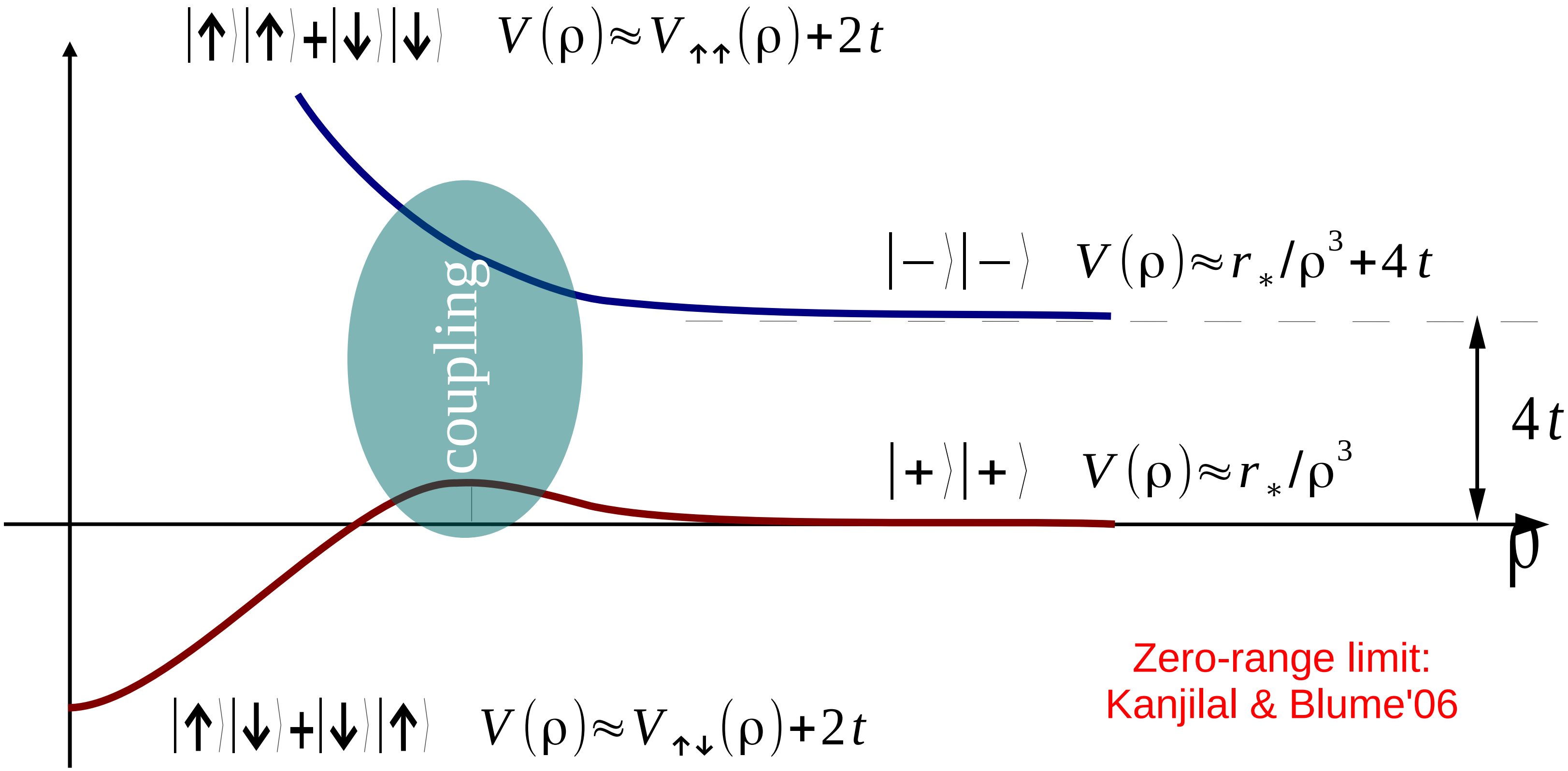


Bilayer with tunneling

One-body problem

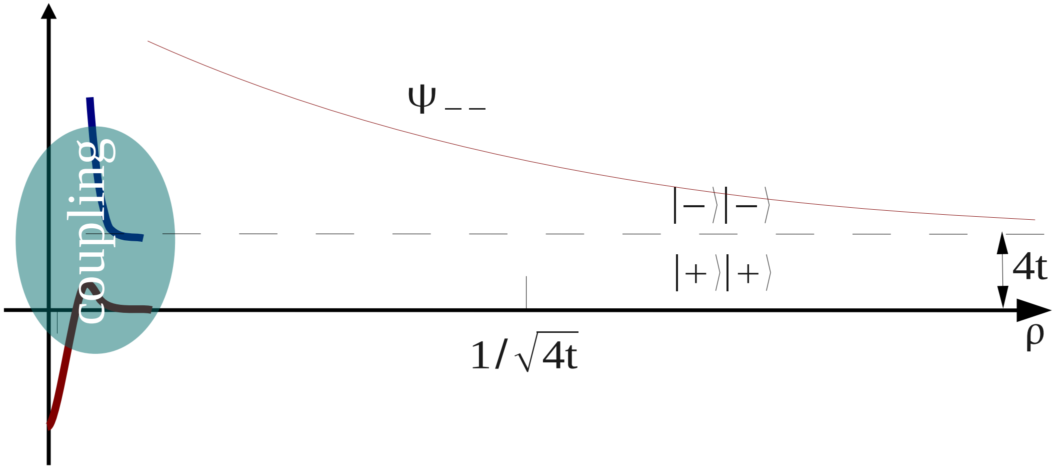


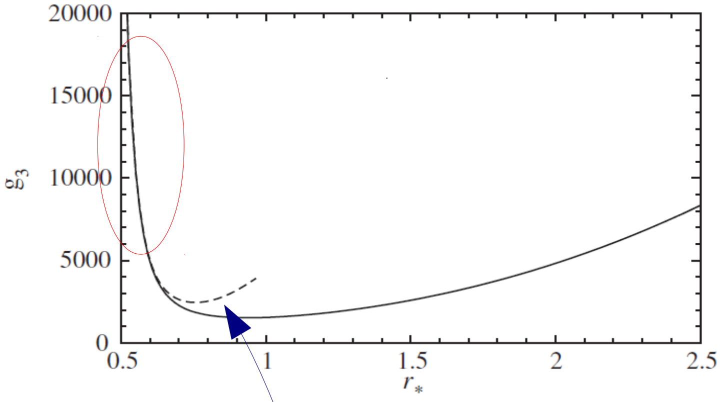
Two-body problem

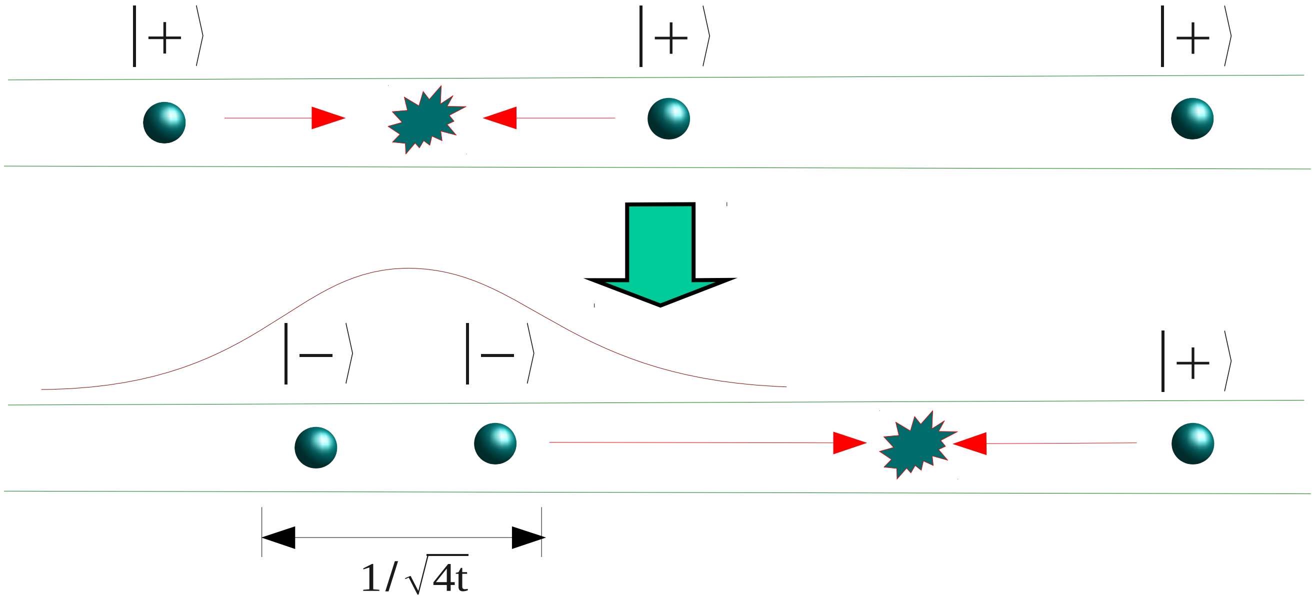


Zero-range limit:
Kanjilal & Blume'06

Small r_* – large off shell contribution



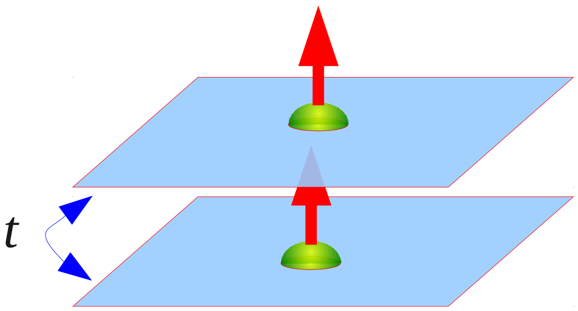




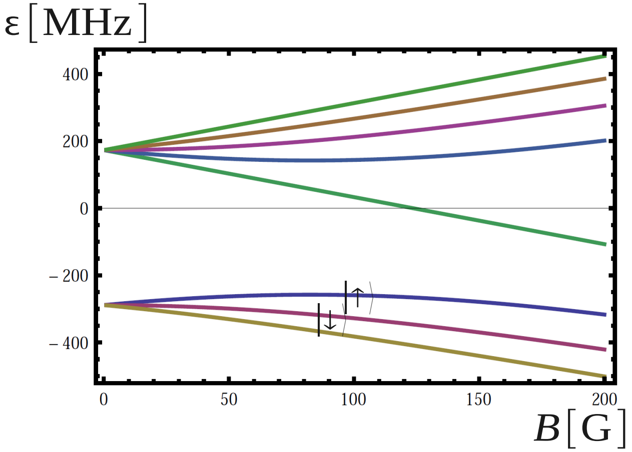
Zero-range model →

$$g_3 = \frac{24\pi^2}{t_c}\left[\frac{1}{\ln^3\sqrt{E_{\uparrow\uparrow}/E_{\uparrow\downarrow}}} - \frac{3\ln(4/3)}{\ln^4\sqrt{E_{\uparrow\uparrow}/E_{\uparrow\downarrow}}} + \ldots\right]$$

Same ideas applied to hyperfine states of an atom



^{39}K: $|F=1,m_F=0\rangle$ and $|F=1,m_F=-1\rangle$



Couple them with RF (~50MHz)

Analog of the interlayer tunneling *t* → $\Omega=$Rabi frequency (~kHz)

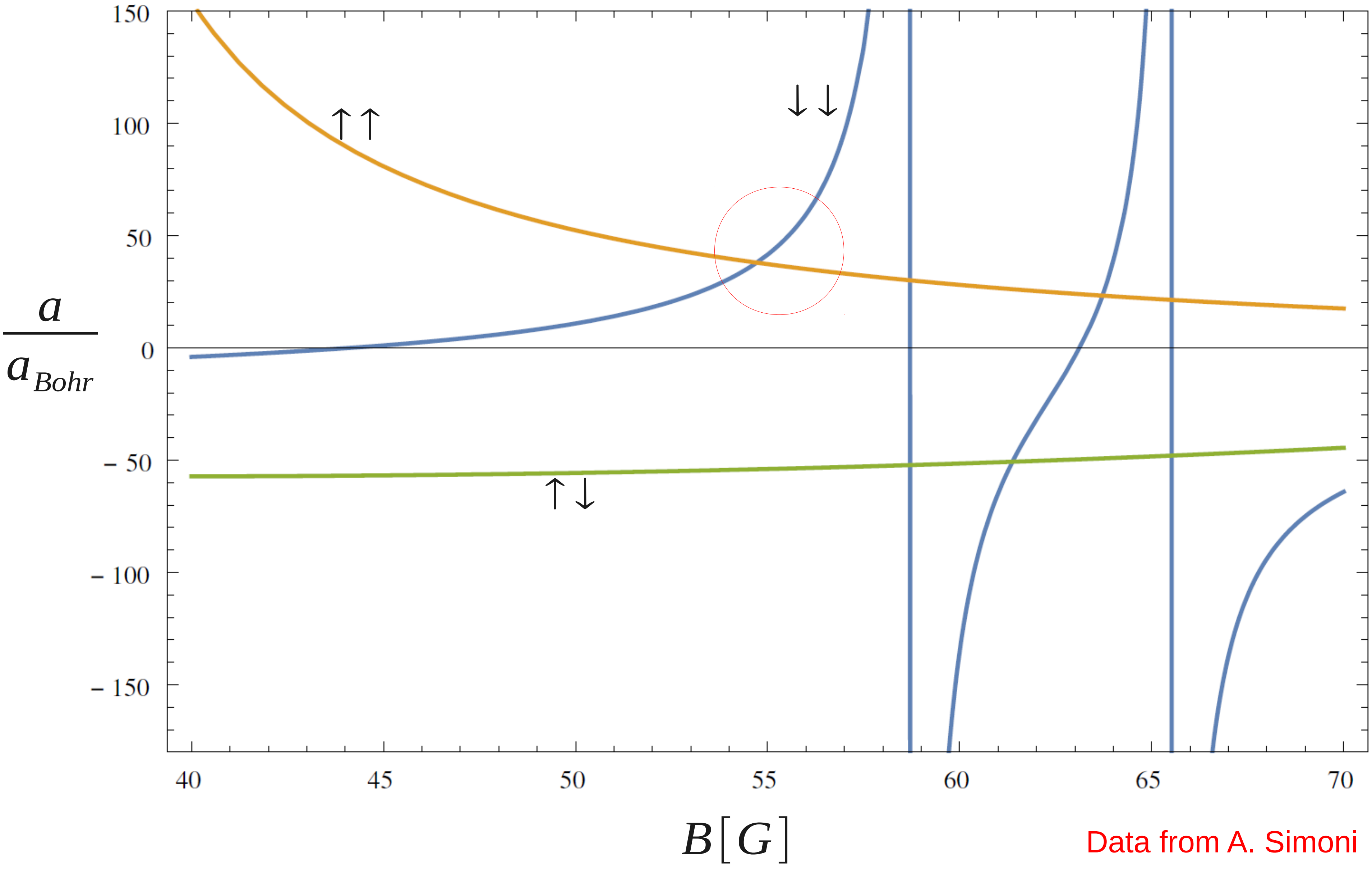
$\Delta=$Detuning (~kHz)

$$\sqrt{\Omega^2+\Delta^2}=\text{spin splitting}\gg T$$

Should work in any dimension: 3D, 2D, 1D, 0D (lattice)!

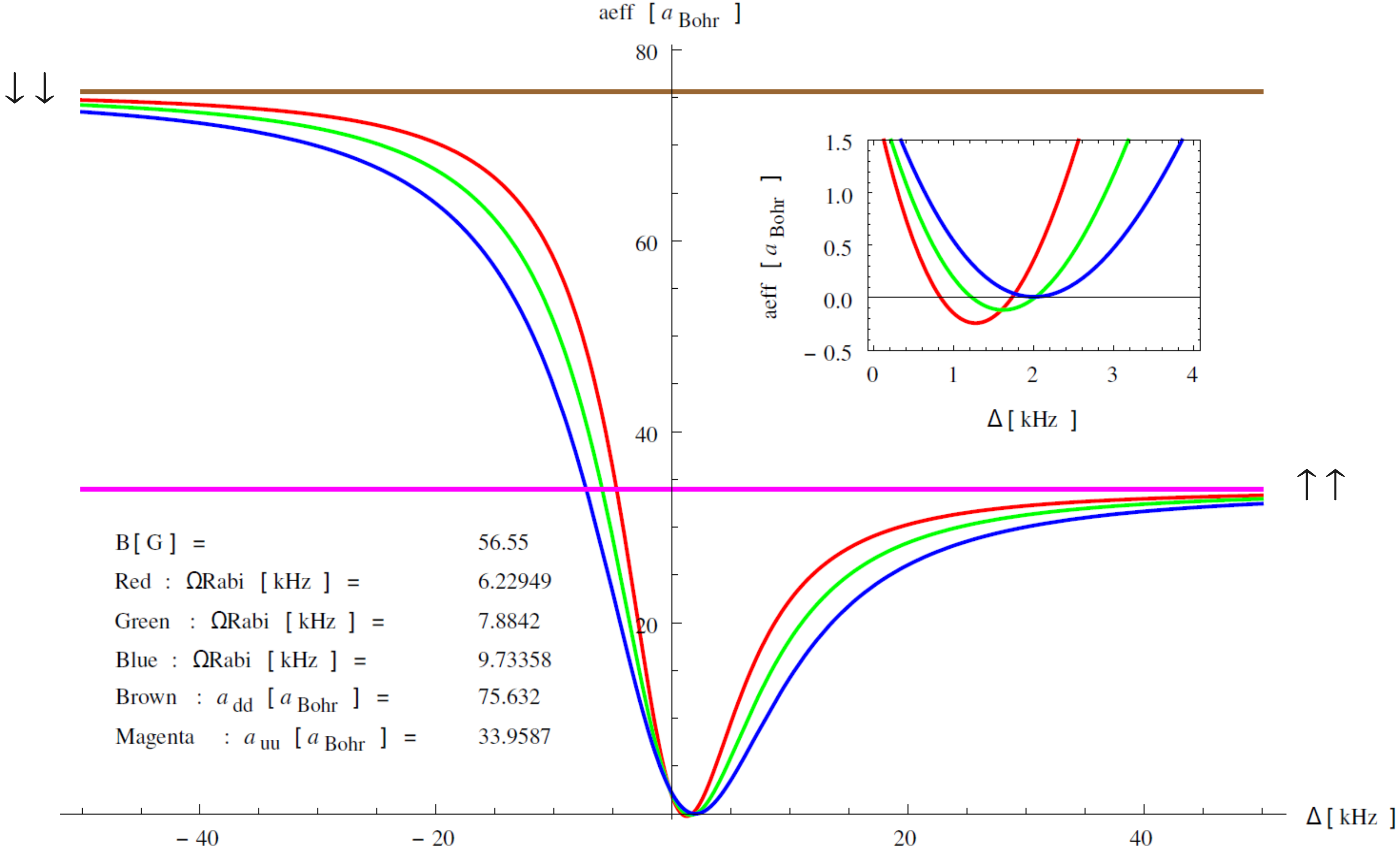
However, need intraspecies repulsion and interspecies attraction!

Nice window of B

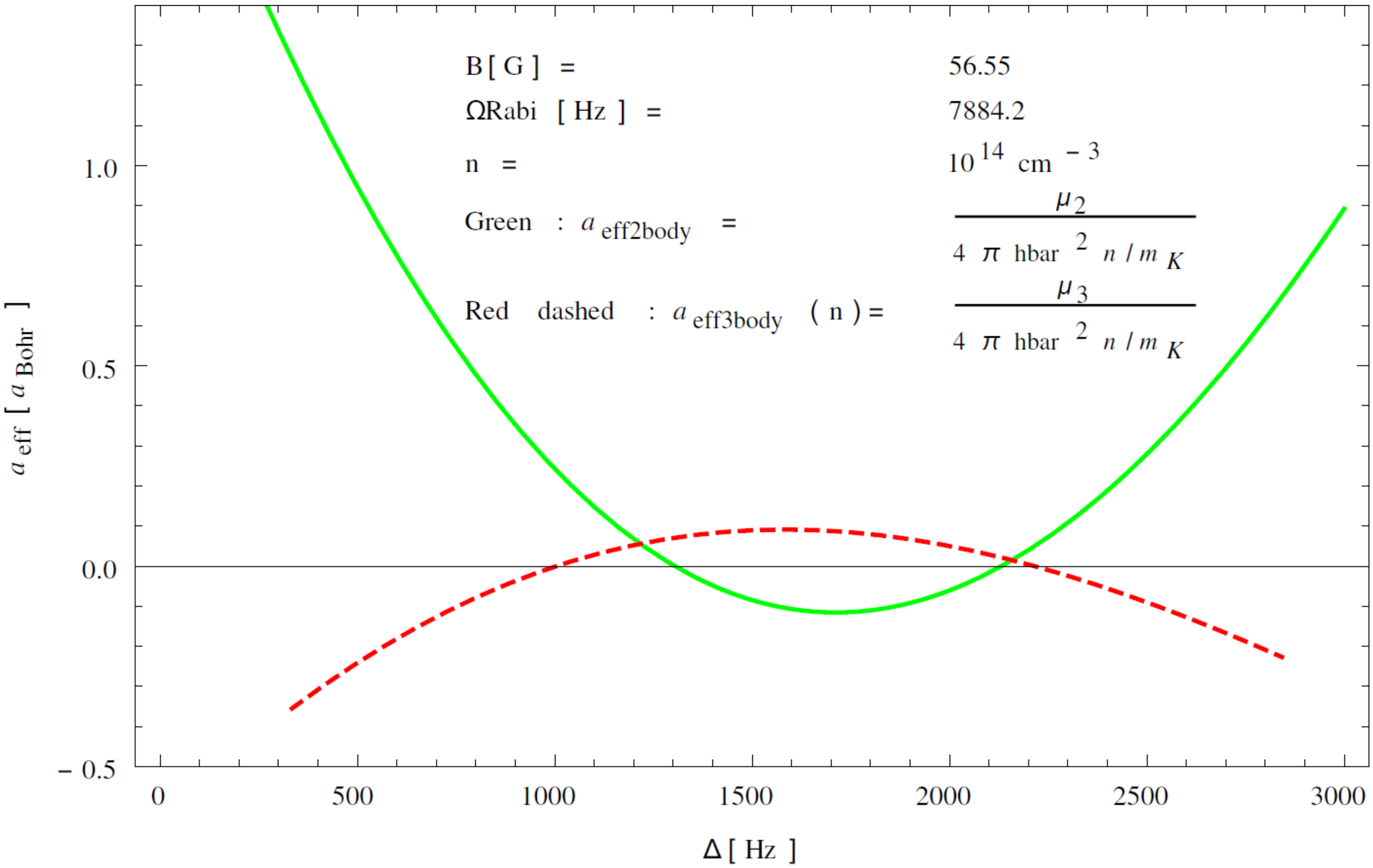


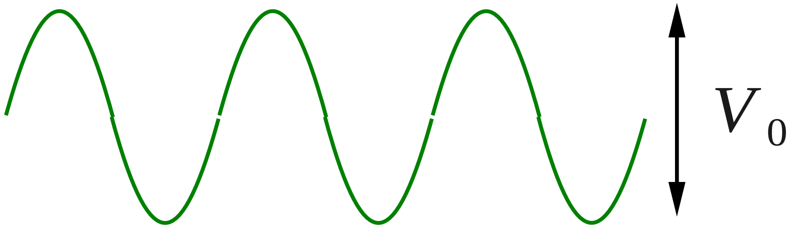
Data from A. Simoni

Scattering length (free space)



Three-body interaction (free space)





lattice constant $= 532\,\mathrm{nm}$

$V_0 = 15\, E_R$

on-site osc. freq. $= 2\pi \times 35\,\mathrm{kHz}$

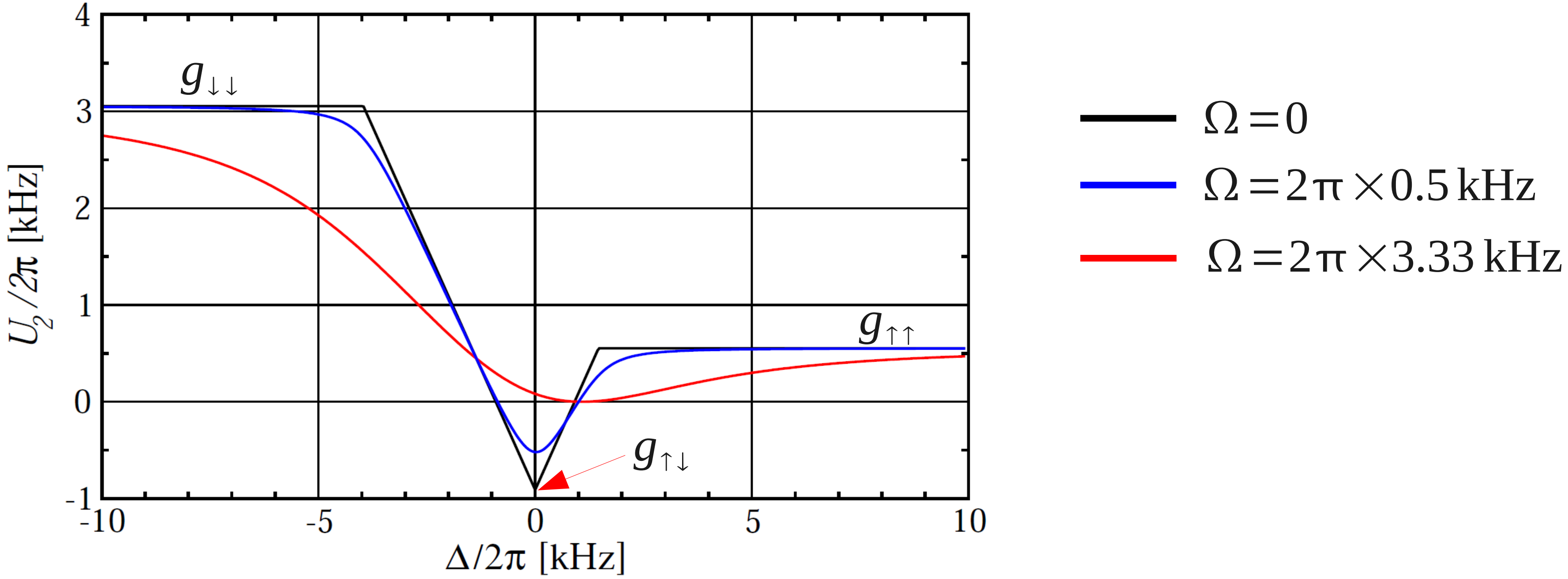
$l_x = l_y = l_z = 86\,\mathrm{nm}$

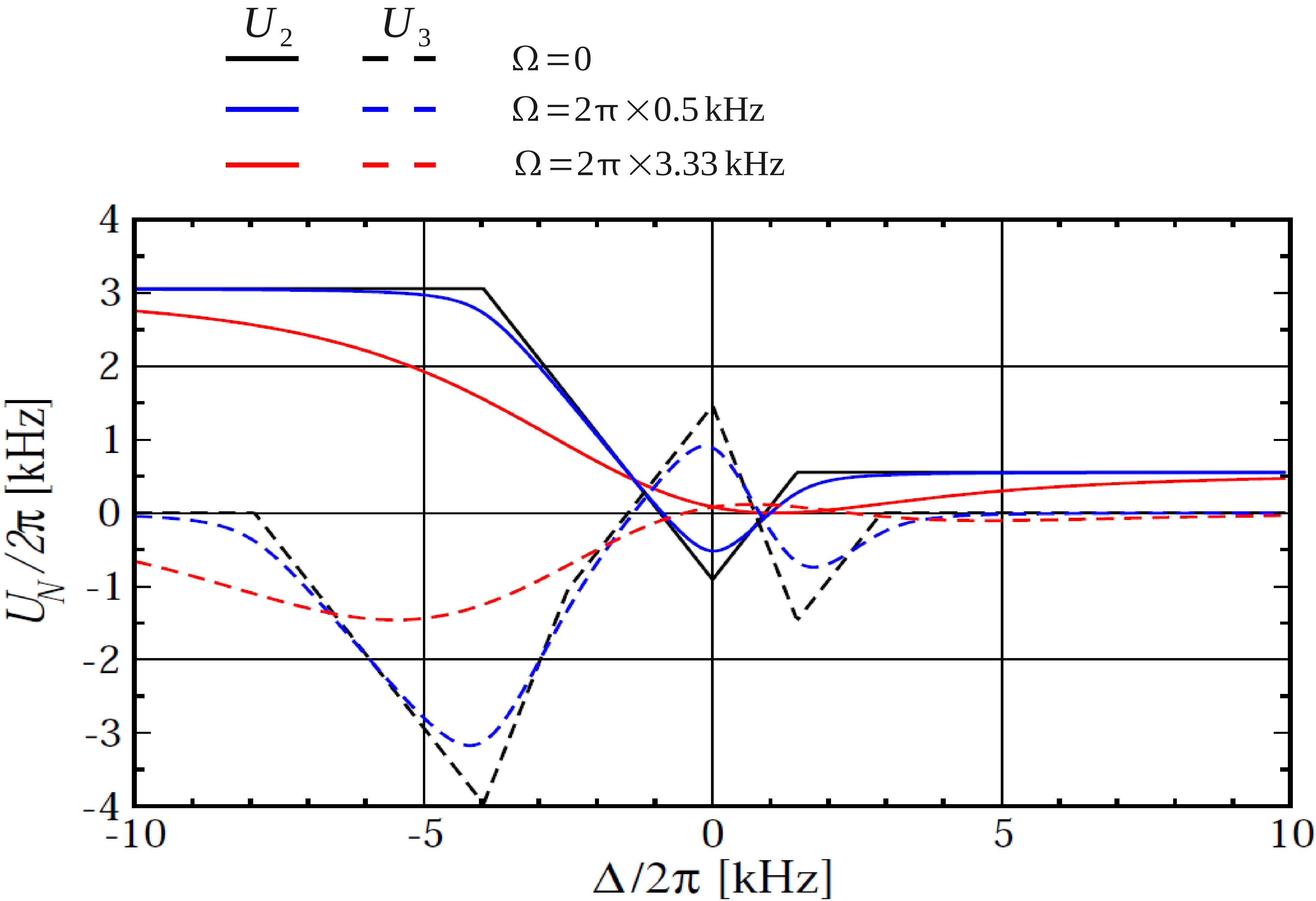
tunneling amp. $= 2\pi \times 30\,\mathrm{Hz}$

$a_{\downarrow\downarrow} = 9.4\,\mathrm{nm} \rightarrow g_{\downarrow\downarrow} = 2\pi \times 3.05\,\mathrm{kHz}$

$a_{\uparrow\uparrow} = 1.7\,\mathrm{nm} \rightarrow g_{\uparrow\uparrow} = 2\pi \times 0.55\,\mathrm{kHz}$

$a_{\uparrow\downarrow} = -2.8\,\mathrm{nm} \rightarrow g_{\uparrow\downarrow} = -2\pi \times 0.91\,\mathrm{kHz}$

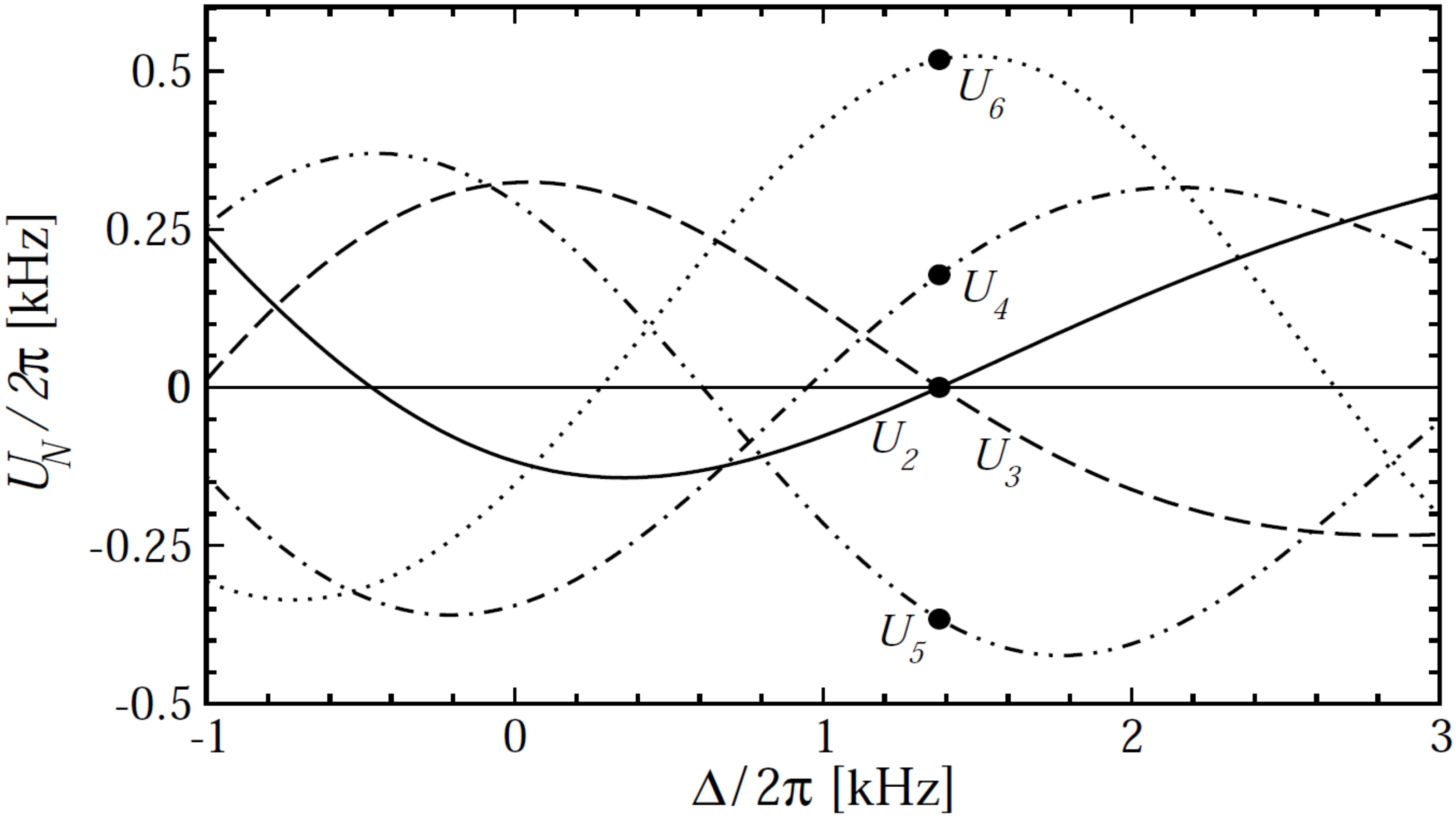




U_2
U_3
$\Omega = 0$
$\Omega = 2\pi \times 0.5\,\mathrm{kHz}$
$\Omega = 2\pi \times 3.33\,\mathrm{kHz}$
$U_N/2\pi$ [kHz]
4
3
2
1
0
-1
-2
-3
-4
-10
-5
0
5
10
$\Delta/2\pi$ [kHz]

4-body interacting case

$\Omega = 2\pi \times 1.7\,\mathrm{kHz}$



Part 2: Quantum droplets

$$\frac{E}{\text{Volume}} = g_2 \frac{n^2}{2!} + \quad + g_3 \frac{n^3}{3!} + g_4 \frac{n^4}{4!} + \dots$$

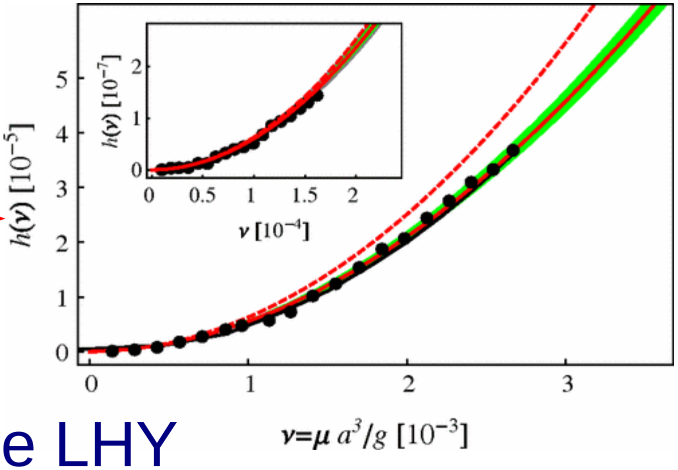
Lee-Huang-Yang correction $\propto g_2^{5/2} n^{5/2}$

For spinless BEC: $$\frac{E}{\text{Volume}} = \frac{g_2 n^2}{2}\left(1 + \frac{128}{15}\sqrt{\frac{na^3}{\pi}} + \dots\right)$$

LHY correction is UNIVERSAL (depends only on the scattering length) and QUANTUM (zero-point energy of Bogoliubov phonons)!

Observed in ultracold gases where the scattering length is tunable by using Feshbach resonances

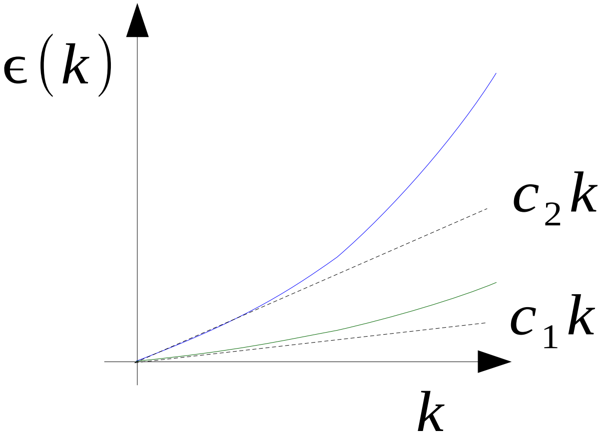
ENS exp. (Navon *et al*. 2011)



Unfortunately, the effect is perturbative and the LHY term is smaller than the mean-field one!

However, it is not always the case!

Two-component BEC: $$\frac{E}{\text{Volume}} = \underbrace{\frac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}}_{\text{mean field}} + \underbrace{\frac{8}{15\pi^2}\frac{m^4}{\hbar^3}\left(c_1^5+c_2^5\right)}_{\text{LHY}}+\dots$$



mean-field stability

$$g_{11}>0,\quad g_{22}>0,\quad \text{and}\quad g_{12}^2<g_{11}g_{22}$$

$$g_{12}>\sqrt{g_{11}g_{22}}$$

phase separation

$$g_{12}<-\sqrt{g_{11}g_{22}}$$

collapse

DSP, “Quantum mechanical stabilization of a collapsing Bose-Bose mixture”, PRL (2015)

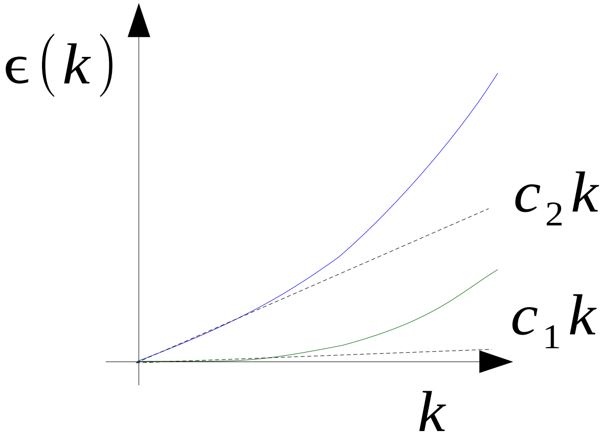
Mechanism of quantum stabilization

What happens when $g_{12}\approx-\sqrt{g_{11}g_{22}}$? (introduce small $\delta g=g_{12}+\sqrt{g_{11}g_{22}}$)

The mean-field term $\dfrac{g_{11}n_1^2+g_{22}n_2^2+2g_{12}n_1n_2}{2}$ “locks” the ratio $\dfrac{n_2}{n_1}=\text{const}=\sqrt{\dfrac{g_{11}}{g_{22}}}$

However, global density changes cost very little energy $\propto\delta g\times n^2$

At the same time the lower Bogoliubov mode becomes very soft $c_1\ll c_2\propto\sqrt{gn/m}$

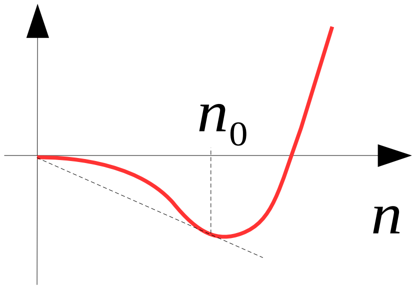


but the hard branch contributes to the LHY term

The structure of the energy-density functional:

$$\frac{E}{\text{Volume}}=A_1\times\delta g\times n^2+A_2\times(m/\hbar^2)^{3/2}(gn)^{5/2}$$

$\delta g<0$ ⇨

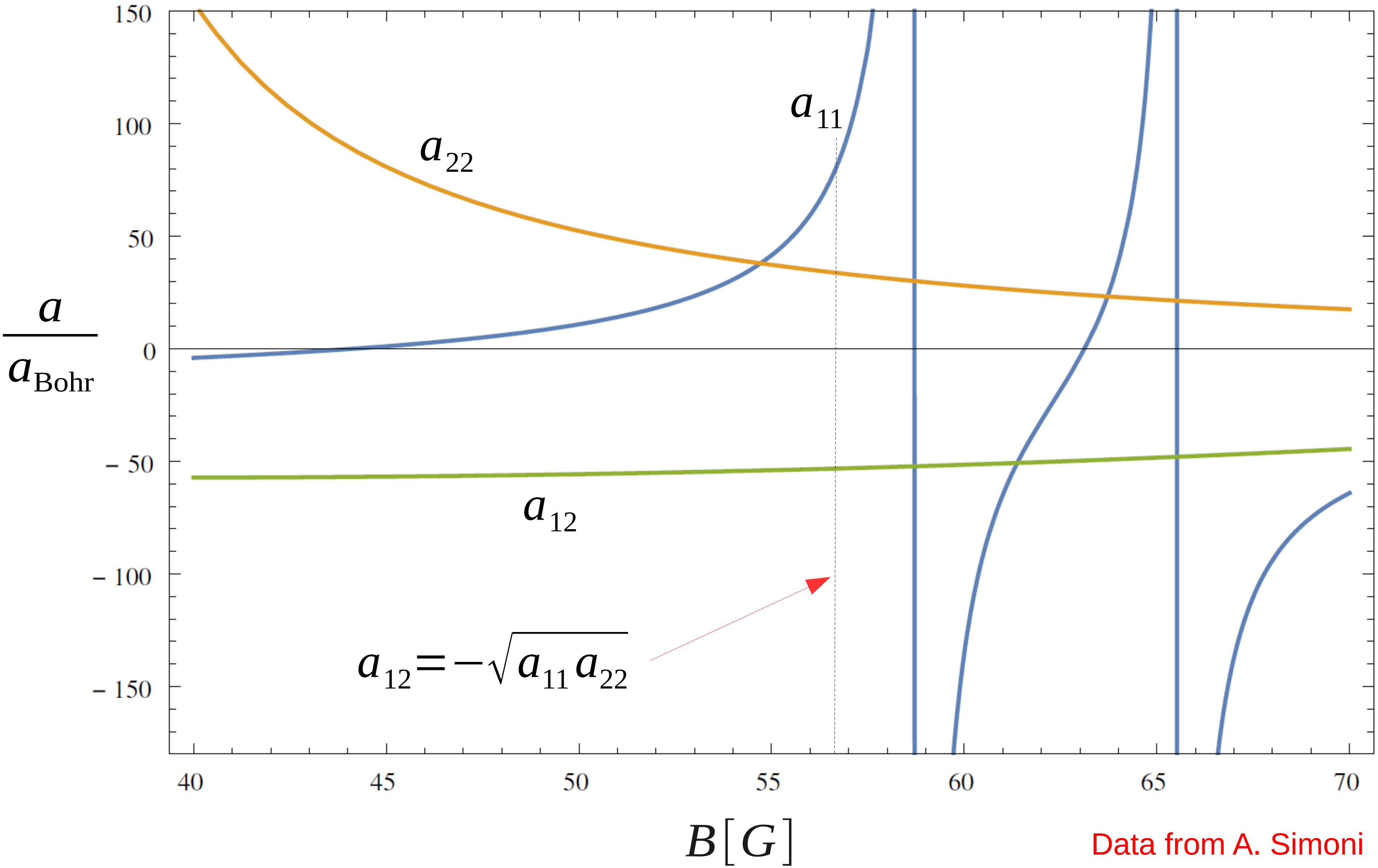


⇨ Gas exists in equilibrium with vacuum. Saturation density

$$\boxed{n_0\propto\frac{1}{a^3}\left(\frac{\delta g}{g}\right)^2}$$

Density is tunable by modifying interaction parameters!

^{39}K: |F=1,m$_F$=0> and |F=1,m$_F$=-1>



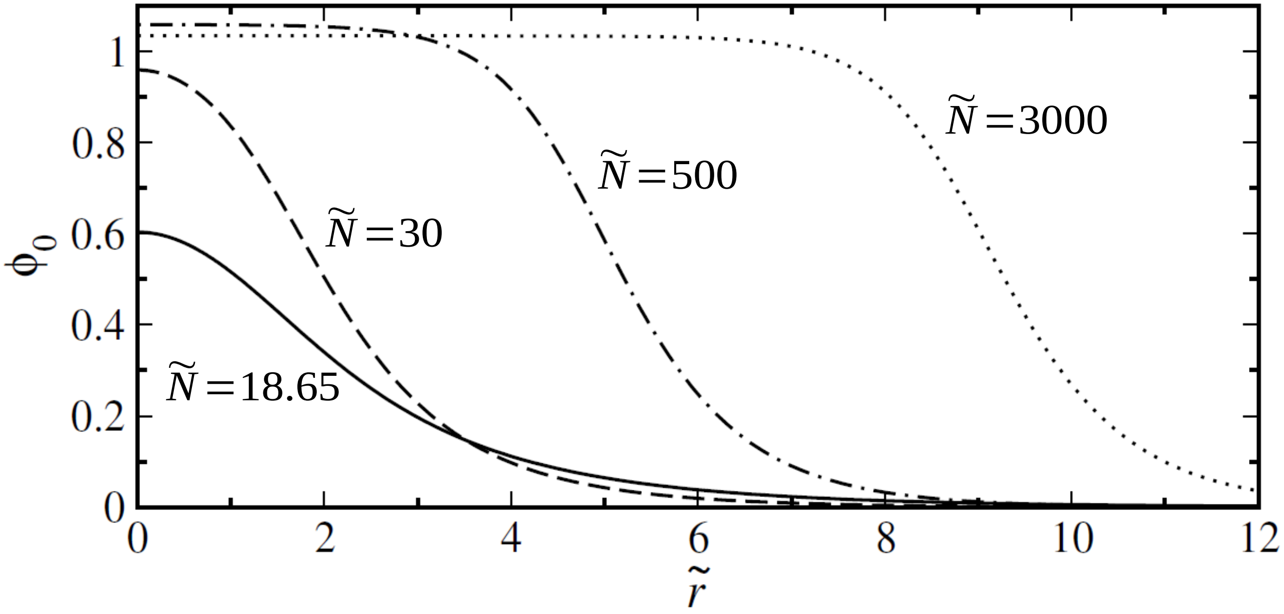
Data from A. Simoni

Gross-Pitaevskii eq., droplet shape

Rescaling $\vec{r} = \xi \tilde{\vec{r}}$, $t = \tau \tilde{t}$, $N = n \xi^3 \tilde{N}$, where $\xi \propto 1/\sqrt{m|\delta g|n}$, $\tau \propto 1/|\delta g|n$

$$\boxed{\begin{gathered} i\partial_{\tilde{t}}\varphi = \left(-\nabla_{\tilde{\vec{r}}}^{\,2}/2 - 3|\varphi|^2 + 5|\varphi|^3/2 - \tilde{\mu}\right)\varphi \\ \tilde{N} = \int |\varphi|^2 d^3\tilde{r} \end{gathered}}$$

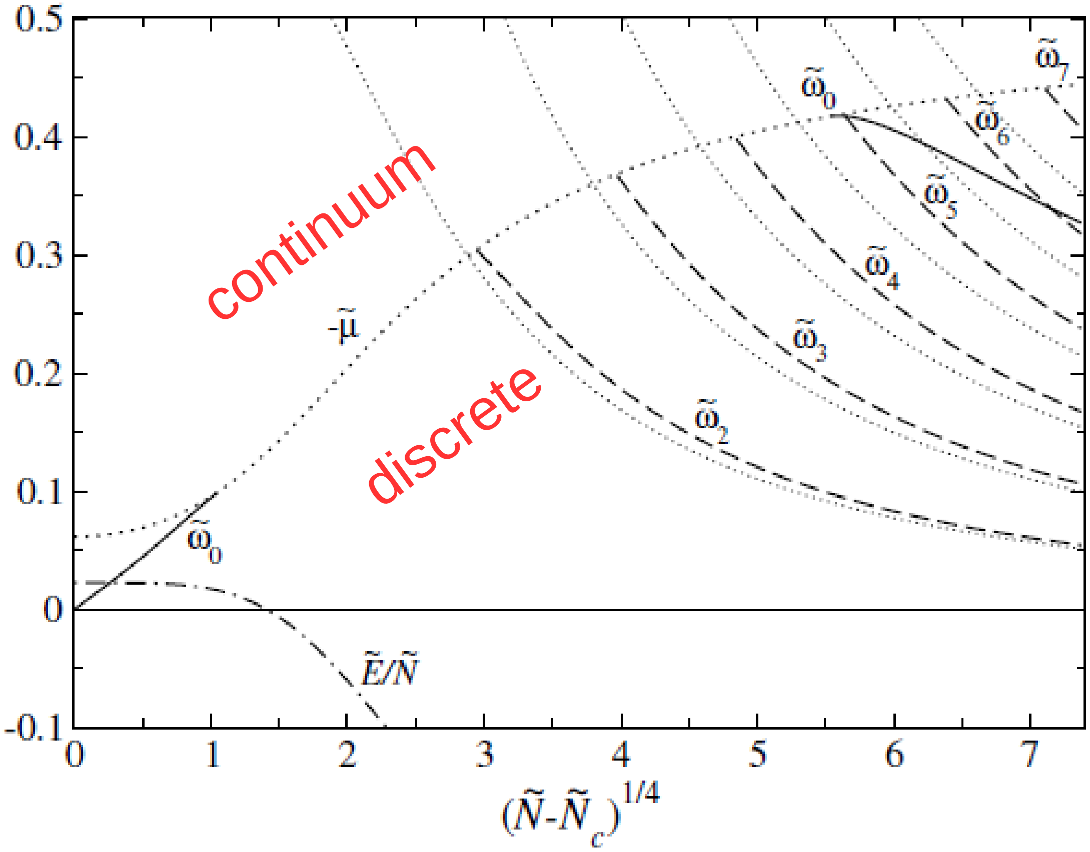
Modified Gross-Pitaevskii equation cubic-quartic nonlinearities



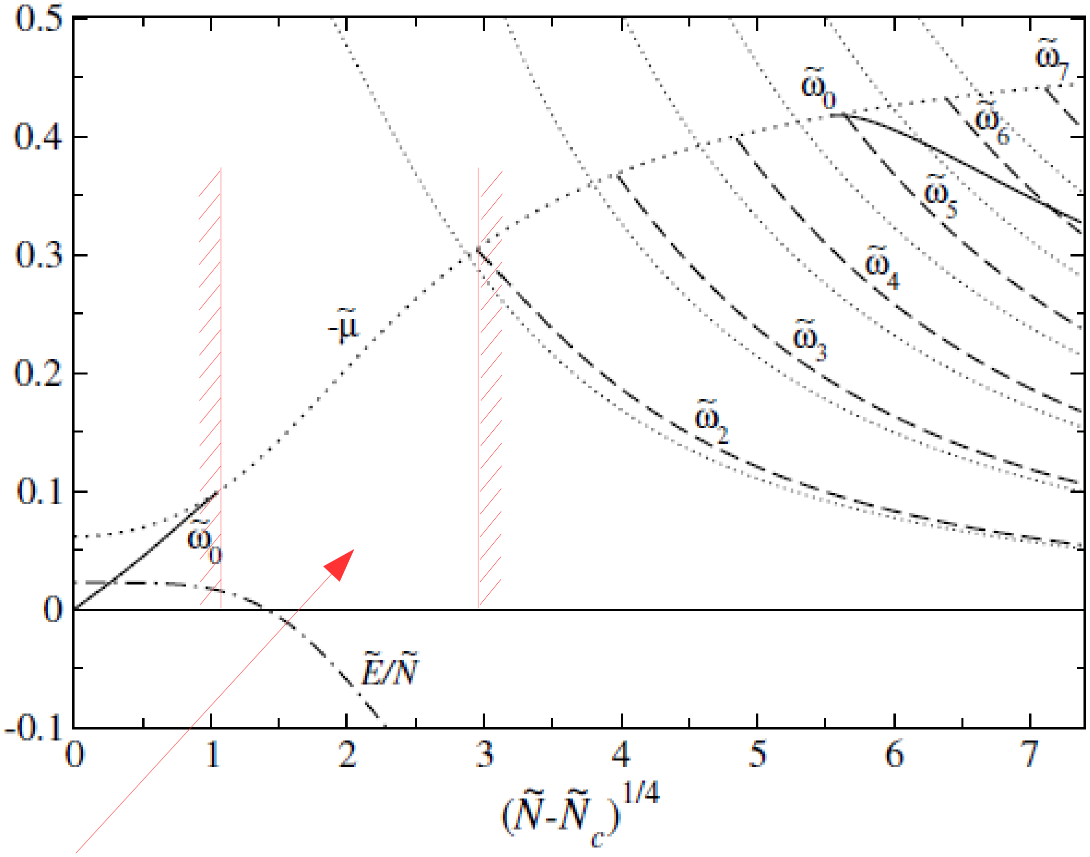
Bogoliubov-de Gennes eqs., excitations

$$\varphi(\tilde{t},\vec{\tilde{r}})=\varphi_0(\vec{\tilde{r}})+\delta\varphi(\tilde{t},\vec{\tilde{r}})$$

linearize $i\partial_{\tilde{t}}\varphi=(-\nabla_{\vec{\tilde{r}}}^{\,2}/2-3|\varphi|^2+5|\varphi|^3/2-\tilde{\mu})\varphi$ with respect to small $\delta\varphi(\tilde{t},\vec{\tilde{r}})$



Zero-temperature object



No discrete modes → the droplet evaporates itself to zero T!

(by contrast, ^{4}He droplets always have discrete modes)

Macroscopic zero-temperature object:
- is interesting by itself
- can be used for sympathetic cooling of other systems

Thank you!