STRONGLY INTERACTING PARTICLES IN ONE DIMENSION

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DEPARTMENT OF PHYSICS AND ASTRONOMY

614. WE-Heraeus-Seminar: 'Few-body Physics: Advanced and Prospects in Theory and Experiment'
18th-20th April 2016 Physikzentrum Bad Honnef

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23rd European Few-Body Conference

Aarhus, Denmark, 8-12 August 2016

- Sub-atomic systems – including light nuclei, hadrons, few-nucleon physics and nuclear astrophysics
- Atomic and molecular systems, cold atoms and ions.
- Few-body methods
- Few-body physics in many-body systems
- New topics in few-body physics

Website: **conferences.au.dk/efb23**

We look forward to seeing you in Aarhus in August 2016!

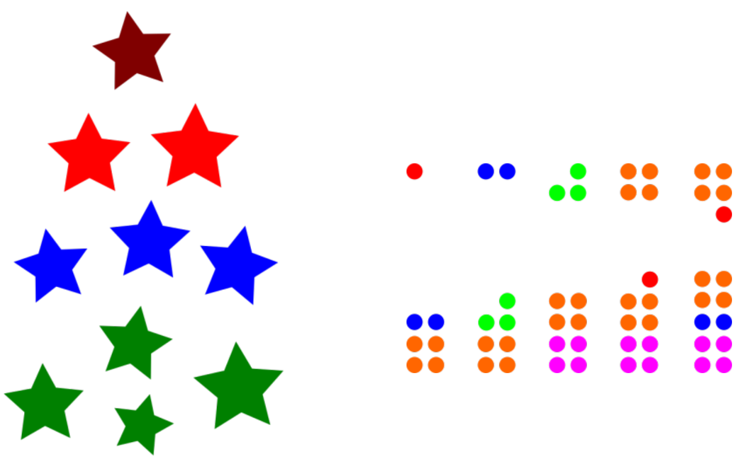
A 'SIMPLE' QUESTION

How many is 'many' really?

AN ANSWER?

Subitizing

Kaufman, E.L., Lord, M.W., Reese, T.W., & Volkmann, J. (1949). "The discrimination of visual number". *American Journal of Psychology* (The American Journal of Psychology) **62** (4): 498–525

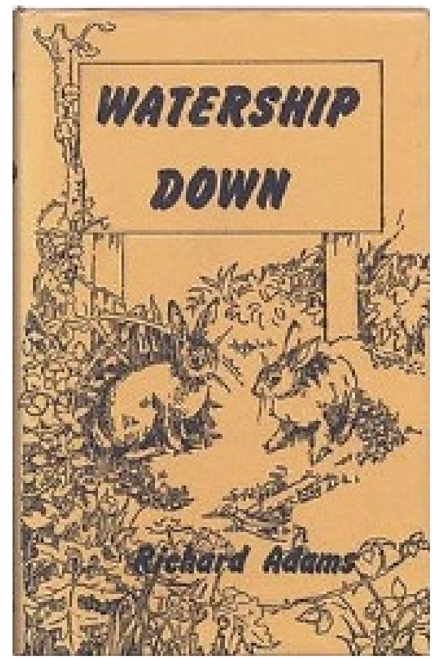


Counting or subitizing?

"Subitizing" by Nevit Dilmen, Wikipedia
(Thanks to Jose D'Incao for pointiing this out)

Source: Wikipedia

Richard Adams novel 1972



Rabbits cannot count beyond four so five is like a thousand ('infinity')

FIVE IS DIFFERENT!

Nathan L. Harshman

One-Dimensional Traps, Two-Body Interactions, Few-Body Symmetries: I. One, Two, and Three Particles

Few-Body Systems 57, 11-43 (2015)

One-Dimensional Traps, Two-Body Interactions, Few-Body Symmetries: II. N particles

Few-Body Systems 57, 45-69 (2015)

For more than four particles, the general case requires a solution of a degree five polynomial equation – no root formulas!

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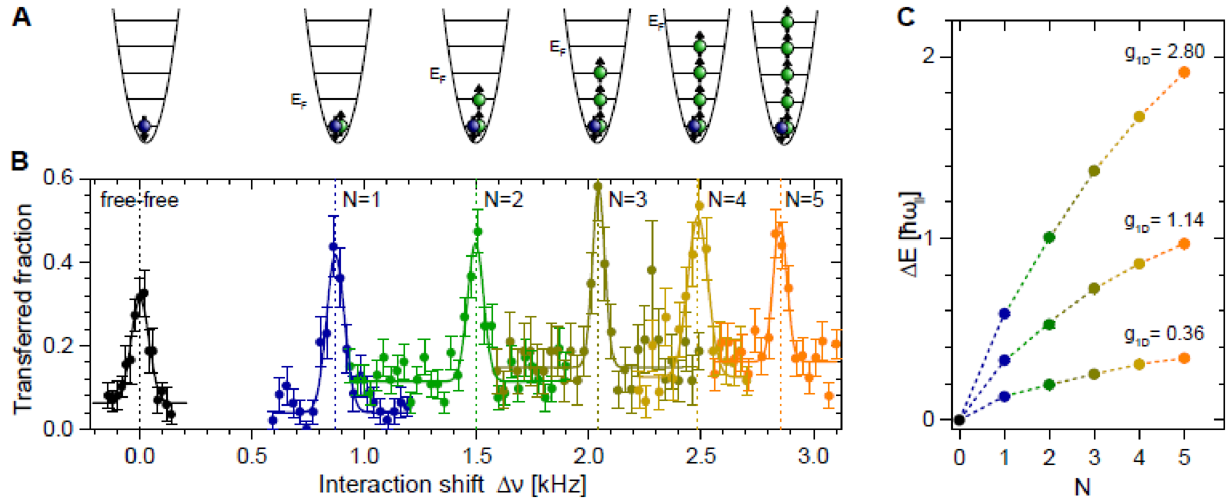
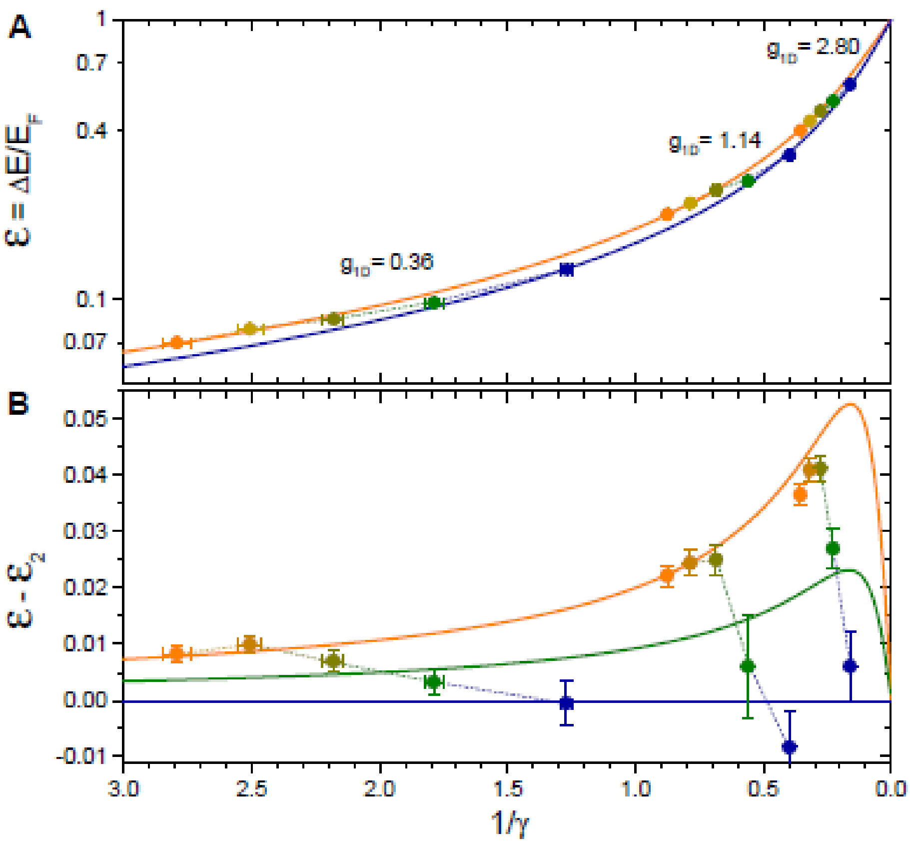
Selim Jochim experiments in Heidelberg.





From Few to Many: Observing the Formation of a Fermi Sea One Atom at a Time

A. N. Wenz *et al.*, Science **342**, 457 (2013)

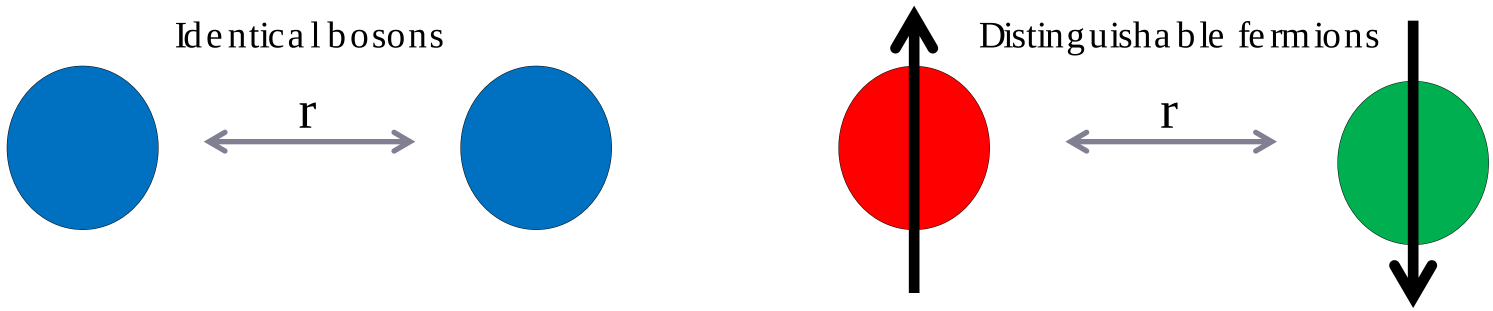
Green solid line from
S.E. Gharashi, K.M. Daily, and D. Blume, Phys. Rev. A **86**, 042702 (2012).

Orange 'many-body' line
J.B. McGuire, J. Math. Phys. **6**, 432 (1965).
G.E. Astrakharchik and I. Brouzos, Phys. Rev. A **88**, 021602 (2013).

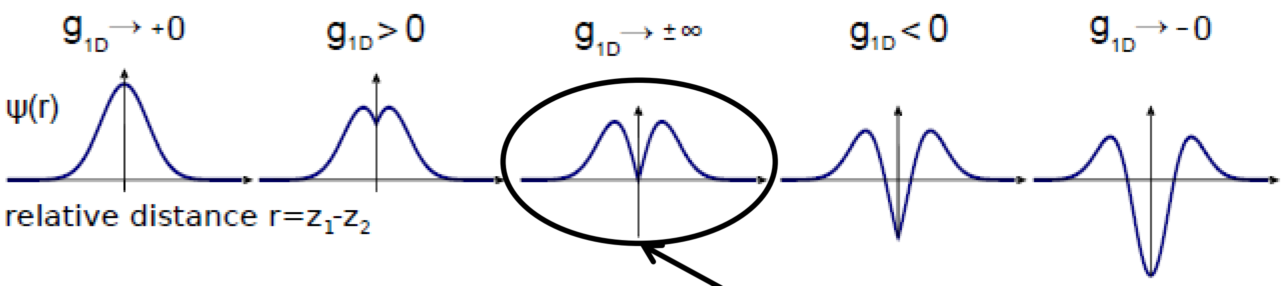
A ONE DIMENSIONAL WORLD

Identical bosons

Distinguishable fermions



Relative wave function



Interaction

$$g_{1D}\,\delta(r)$$

Source: G. Zürn, thesis

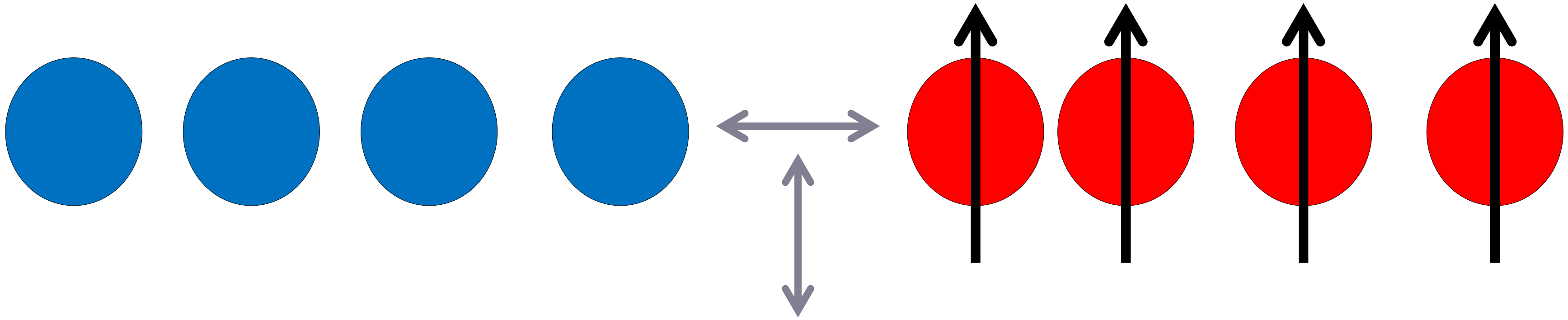
Strong interactions -> Impenetrability!

STRONGLY INTERACTING BOSONS

$|g_{1D}| \to \infty$ limit

Tonks (1936)-Girardeau (1960) gas of impenetrable bosons

Mapping identical bosons to spin-polarized fermions. Girardeau (1960).



$$\Psi_B(z_1, \ldots, z_N) = |\Psi_F(z_1, \ldots, z_N)|$$

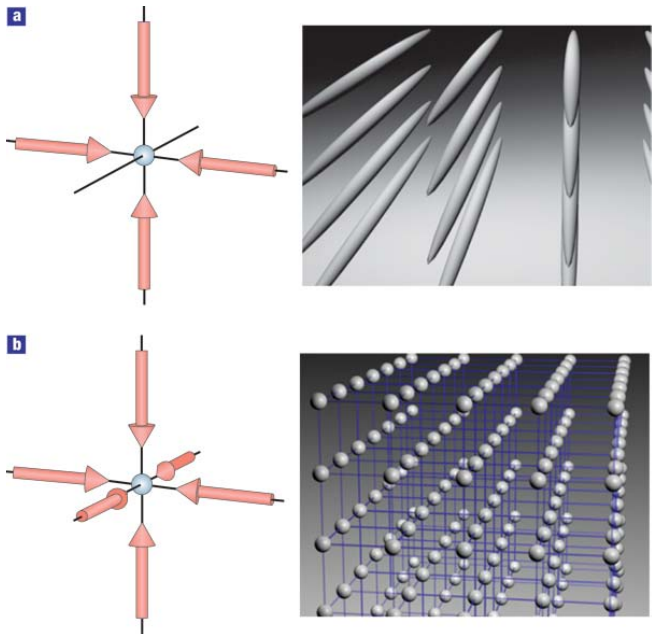
Impenetrable bosons

Antisymmetrized fermions

Lieb-Liniger (1963) used Bethe ansatz to solve N boson problem for any g>0

EXPERIMENTAL REALIZATION

Optical lattices



I. Bloch, Nature Physics **1**, 23 (2005)

Confinement-induced resonances

Maxim Olshanii
Phys. Rev. Lett. **81**, 938 (1998)

$$g_{1D} = \frac{2\hbar^2 a_{3D}}{m a_\perp^2} \frac{1}{1 - C a_{3D}/a_\perp}$$

Divergent at specific point depending on lattice and 3D Feshbach resonance

EXPERIMENTAL REALIZATION

Tonks–Girardeau gas of ultracold atoms in an optical lattice

Belén Paredes[1], Artur Widera[1,2,3], Valentin Murg[1], Olaf Mandel[1,2,3], Simon Fölling[1,2,3], Ignacio Cirac[1], Gora V. Shlyapnikov[4], Theodor W. Hänsch[1,2] & Immanuel Bloch[1,2,3]

Nature **429**, 277 (2004)

Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

Science **305**, 1125 (2004)

Experimentally produced and probed the Tonks-Girardeau gas on the repulsive side g>0

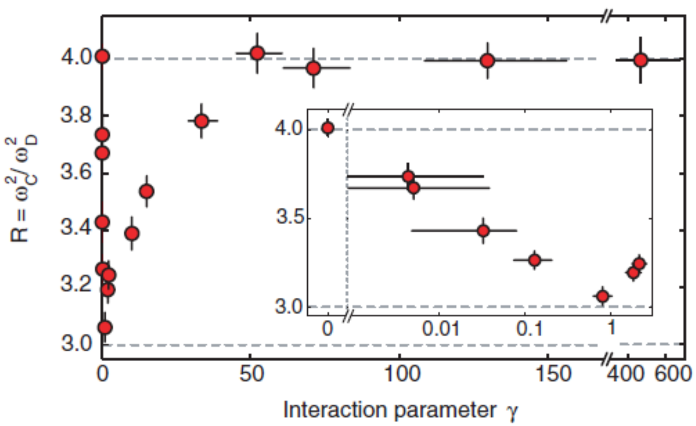
EXPERIMENTAL REALIZATION

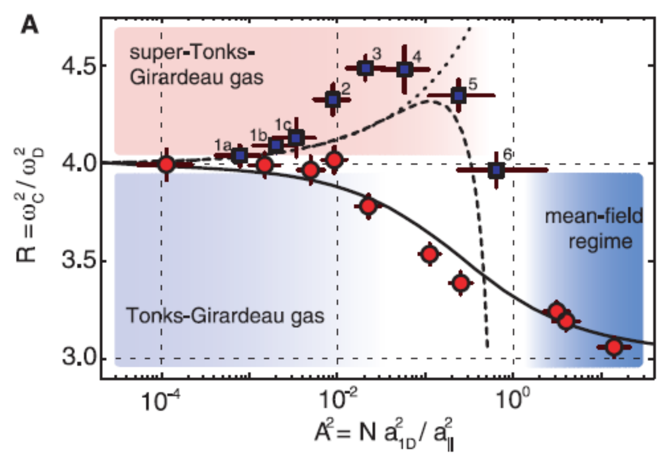
Realization of an Excited, Strongly Correlated Quantum Gas Phase

Elmar Haller,[1] Mattias Gustavsson,[1] Manfred J. Mark,[1] Johann G. Danzl,[1] Russell Hart,[1] Guido Pupillo,[2,3] Hanns-Christoph Nägerl[1]*

Science **325**, 1224 (2009)

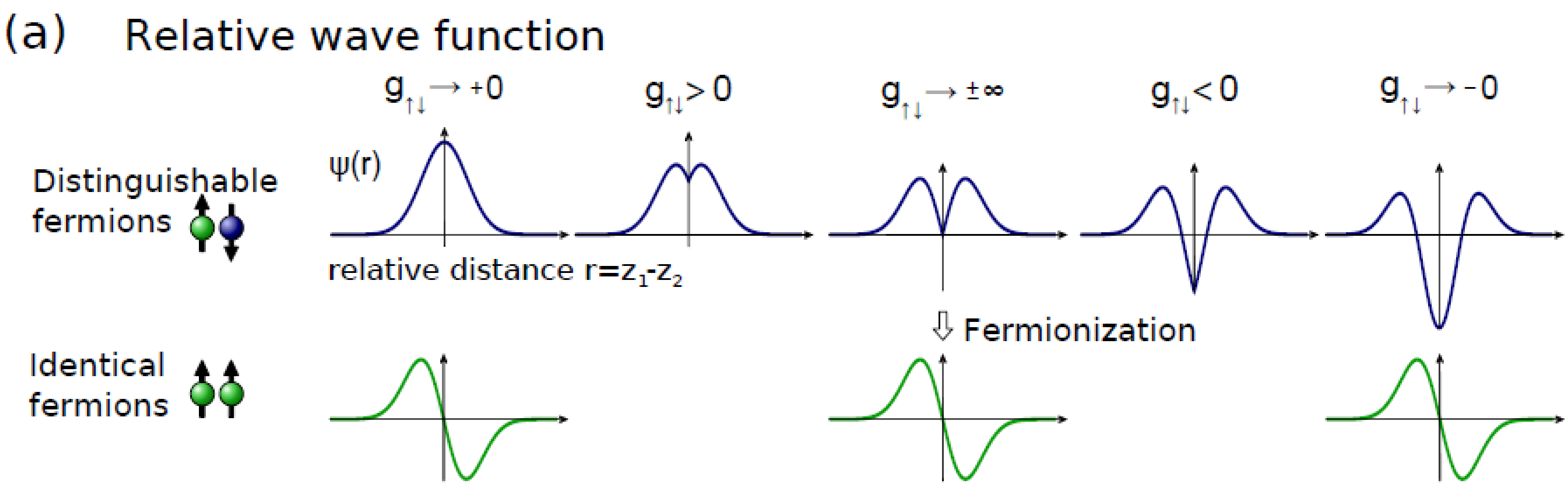
Study of the crossover from g>0 to g<0 in the strongly-interacting regime.





1D FERMIONS – A FRONTIER

Two kinds of relative motion for two-body states!



Source: G. Zürn, thesis

Fermionization of two fermions in a 1D harmonic trap:
G. Zürn *et al.*, Phys. Rev. Lett. **108**, 075303 (2012).

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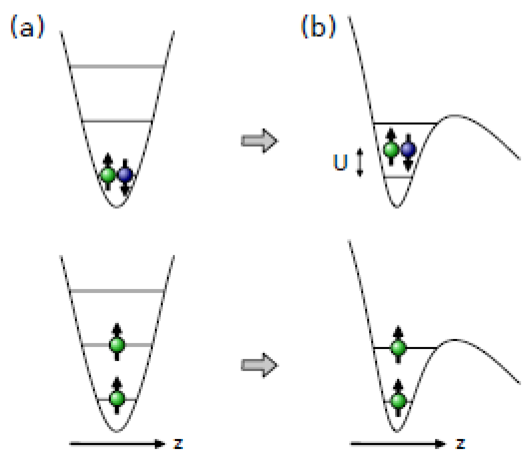
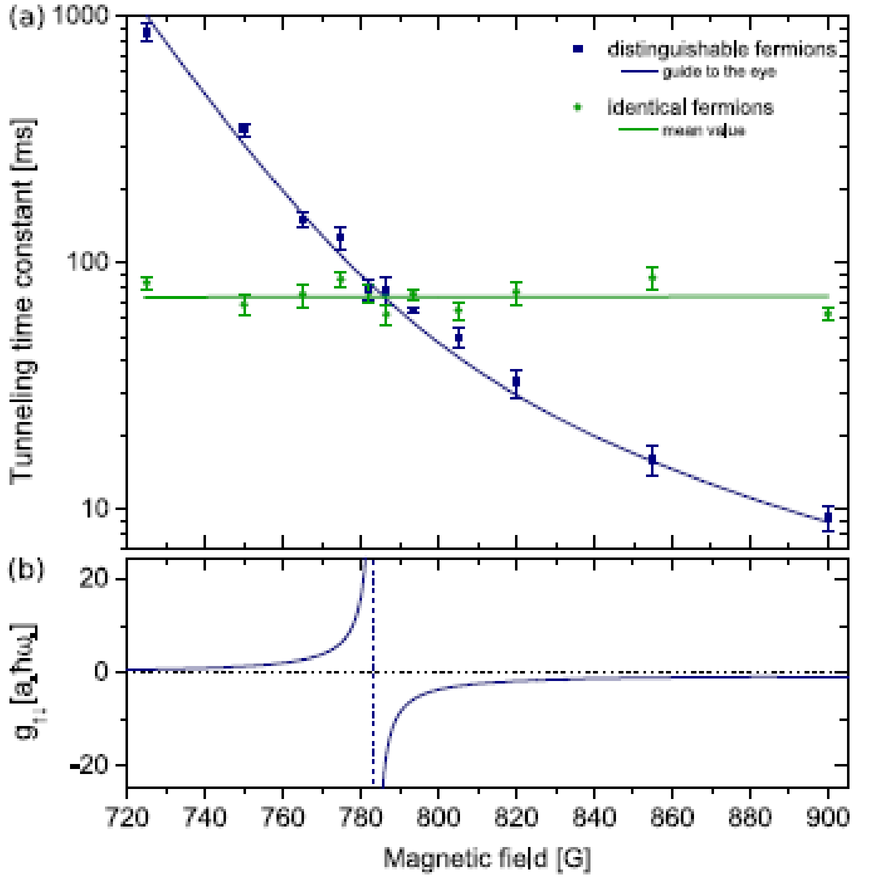
EXPERIMENTAL REALIZATION

Two-body tunneling experiments

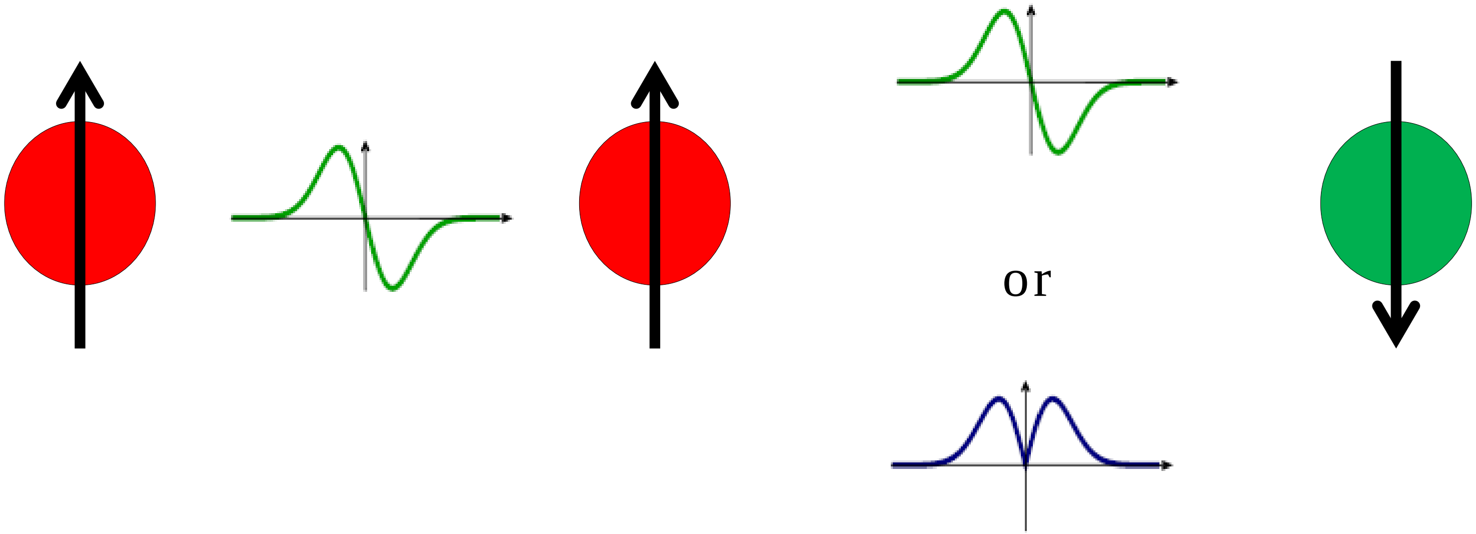




Fermionization of two fermions in a 1D harmonic trap:
G. Zürn *et al.*, Phys. Rev. Lett. **108**, 075303 (2012).

STRONGLY INTERACTING FERMIONS

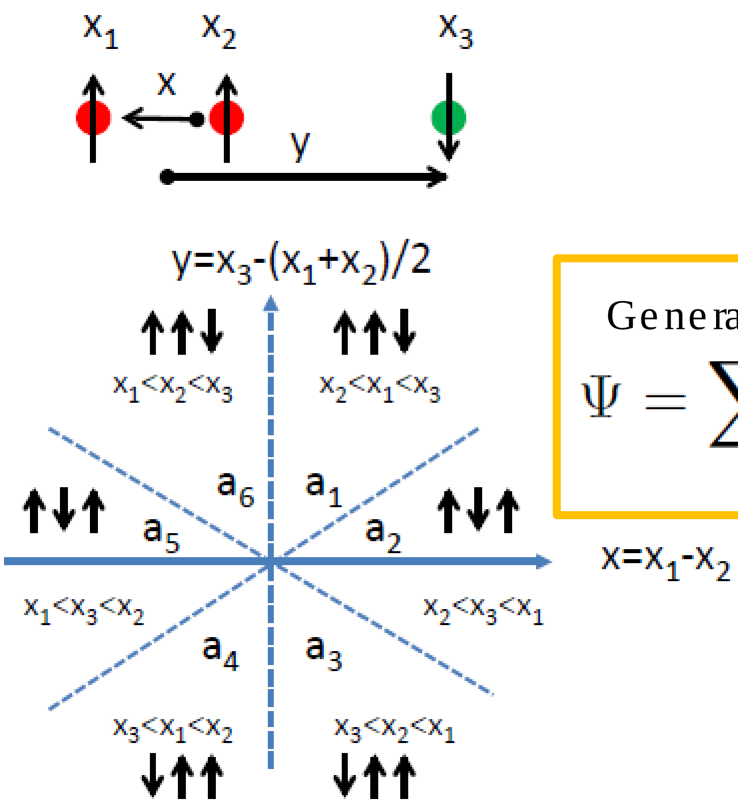
Relative wave functions. What should we take?



Keep an open mind and let the problem work itself out!

THREE FERMIONS - SOLUTION

Split space in patches

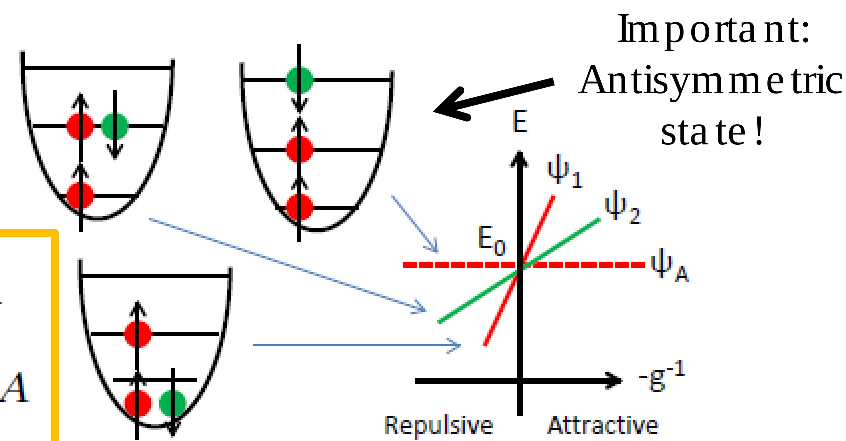


General solution

$$\Psi = \sum_i a_i \Psi_A$$

Pauli and parity reduces problem to a_1, a_2, and a_3.

Spectrum on resonance



Optimize derivative!

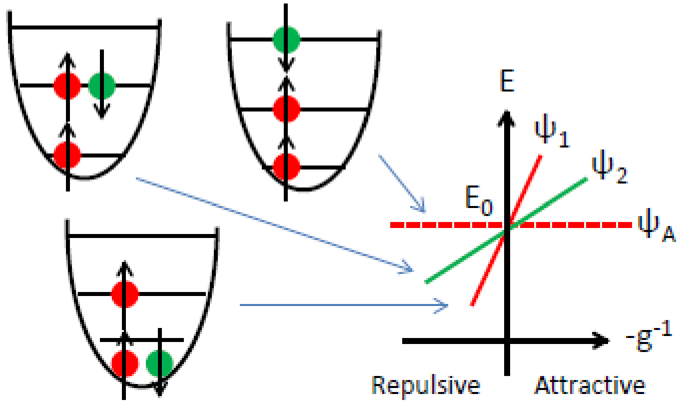
$$K = -\frac{\partial E}{\partial g^{-1}} = g^2 \frac{\sum_{ij} \int \prod_{k=1}^{N} dx_k |\Psi|^2 \delta(x_i - x_j)}{\langle \Psi | \Psi \rangle}$$

THREE FERMIONS - SOLUTION

$$K = \frac{27}{8\sqrt{2\pi}} \frac{(a_1 - a_2)^2 + (a_2 - a_3)^2}{a_1^2 + a_2^2 + a_3^2}$$

Extremizing solutions are:

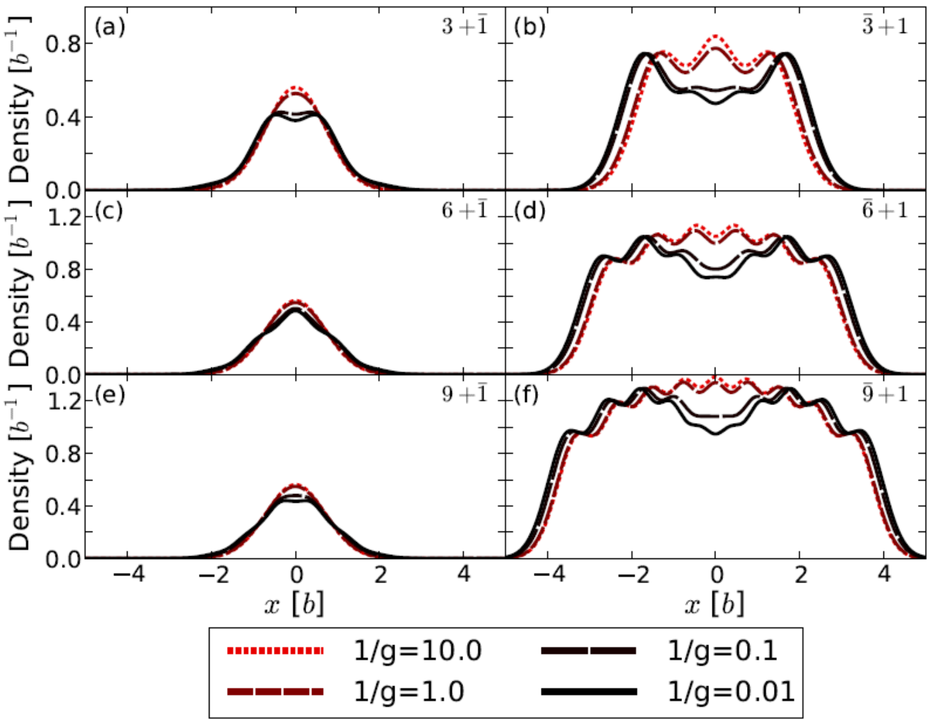
$a_1 = a_2 = a_3$	Non-interacting state
$a_1 = a_3$ and $a_2 = 0$	Excited state, even parity
$2a_1 = 2a_3 = -a_2$	Ground state, odd parity



IMPORTANT: Coefficients are generally NOT the same!

A.G. Volosniev *et al.*, Nature Comm. **5**, 5300 (2014).

TRAPPED 'POLARONS' IN 1D



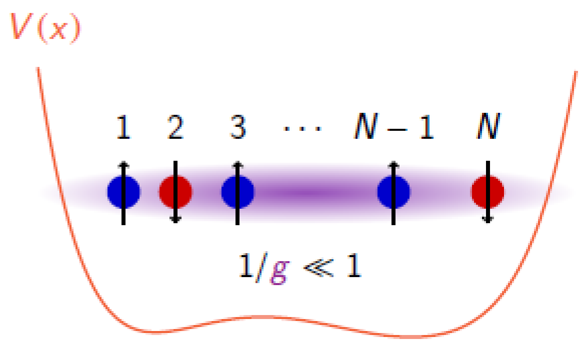
'Precursor' of magnetic structure! Phase-separation of spin up and spin down for strong interactions.

E.J. Lindgren *et al.*, New J. Phys. **16**, 063003 (2014)
S.E. Gharashi and D. Blume, Phys. Rev. Lett. **111**, 045302 (2013)
F. Deuretzbacher *et al.*, Phys. Rev. A **90**, 013611 (2014)
J. Levinsen *et al.*, Science Advances **1**, e1500197 (2015)

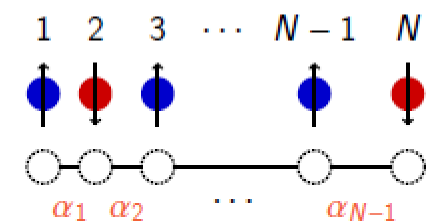
Mapping to an XXZ spin chain

A.G. Volosniev *et al.*, Phys. Rev. A **91**, 023620 (2015)

Strongly interacting cold atomic gas



Spin chain model



$$H = \sum_{i=1}^{N} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_i^2} + V(x_i) + \sum_{\uparrow\downarrow \text{ pairs}} g\,\delta(x_i - x_j) + \sum_{\substack{\uparrow\uparrow \text{ pairs} \\ \downarrow\downarrow \text{ pairs}}} g\kappa\,\delta(x_i - x_j)$$

$$H_0 = E_0 - \sum_{k=1}^{N-1} \frac{\alpha_k}{g}\left[\frac{1}{2}\left(1 - \sigma^k \cdot \sigma^{k+1}\right) + \frac{1}{\kappa}\left(1 + \sigma_z^k \sigma_z^{k+1}\right)\right]$$

Theoretical work related to spin mapping:

F. Deuretzbacher *et al.*, Phys. Rev. A **90**, 013611 (2014)
J. Levinsen *et al.*, Science Advances **1**, e1500197 (2015)
L. Yang, L. Guan, and H. Pu, Phys. Rev. A **91**, 043643 (2015)
L. Yang and X. Cui, Phys. Rev. A **93**, 013617 (2016).
H. Hu, L. Guan, and S. Chen, New J. Phys. **18**, 025009 (2016)
L. Yang and H. Pu, arXiv:1601.02556 (2016)

Experimental realization of cold atoms spin chain:

S. Murmann *et al.*, Phys. Rev. Lett. **115**, 215301 (2015)

Nearest-neighbor interactions are tunable via external trap!

Tough task: Compute these coefficients!

CONAN (Coefficients of One-dimensional N-Atom Networks)

Method

We developed the CONAN software that computes the local exchange coefficients.

CONAN

$V(x) \rightarrow$ [CONAN] $\rightarrow \alpha_k$

arXiv:1603.02662

- Highly efficient code: $N = 10$ in circa 10 seconds and the computation time scales $O(N^{3.5\pm0.4})$.
- High precision up to $N = 30$ and acceptable up to $N = 35$.

See also related work by: Frank Deuretzbacher *et al.*, arXiv:1602.06816

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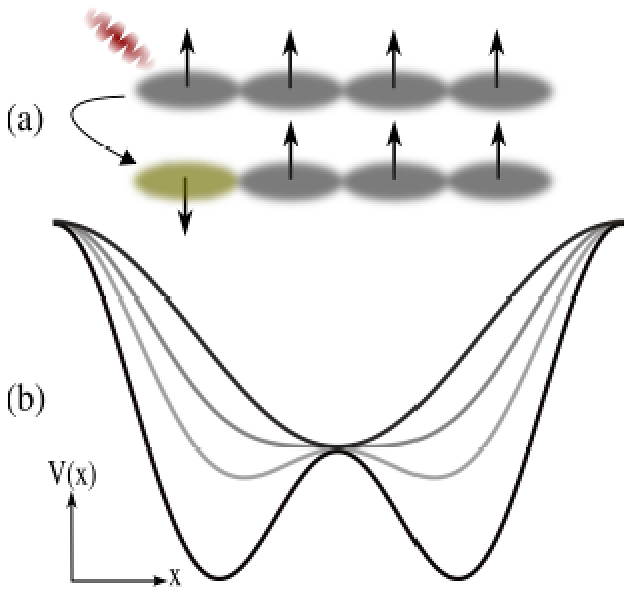
STATE TRANSFER

Use trap to manipulate dynamics – example of quantum state transfer

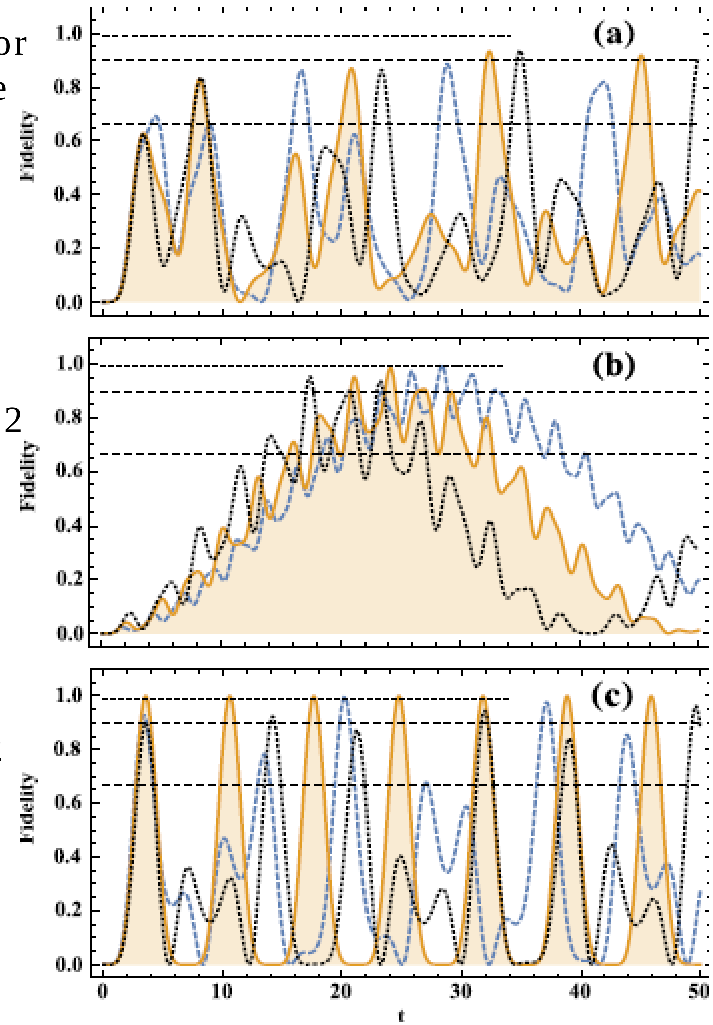


Fidelity of quantum state transfer

Fermions or hard-core bosons

Bosons kappa=1/2

Bosons kappa=2

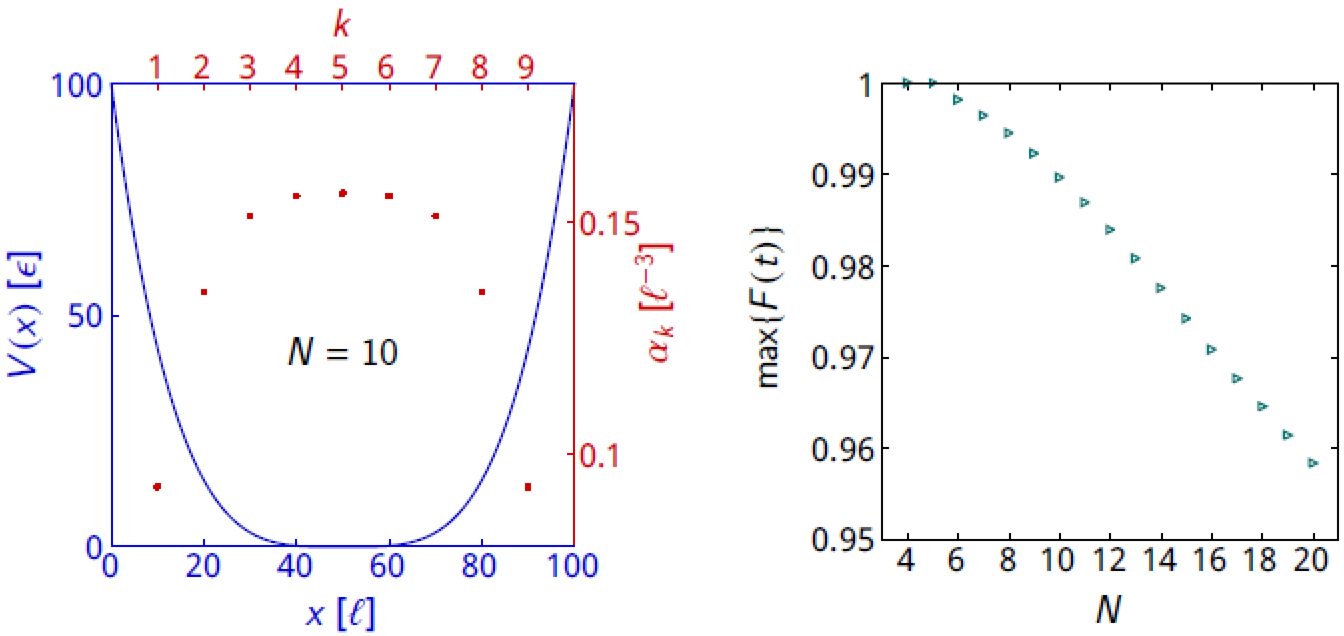


A.G. Volosniev *et al.*, Phys. Rev. A **91**, 023620 (2015)

Perfect state transfer is known to occur in an XX model if

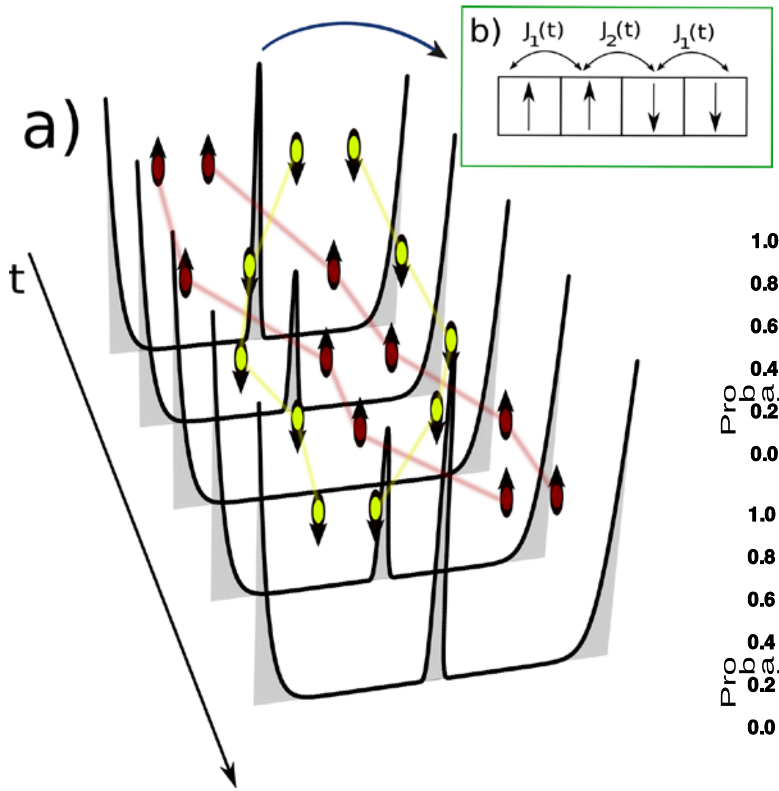
$$\alpha_k \propto \sqrt{k(N-k)} \,.$$

Use CONAN to search for a $V(x)$ that produces these local exchange coefficients. We reach perfect or nearly perfect state transfer.



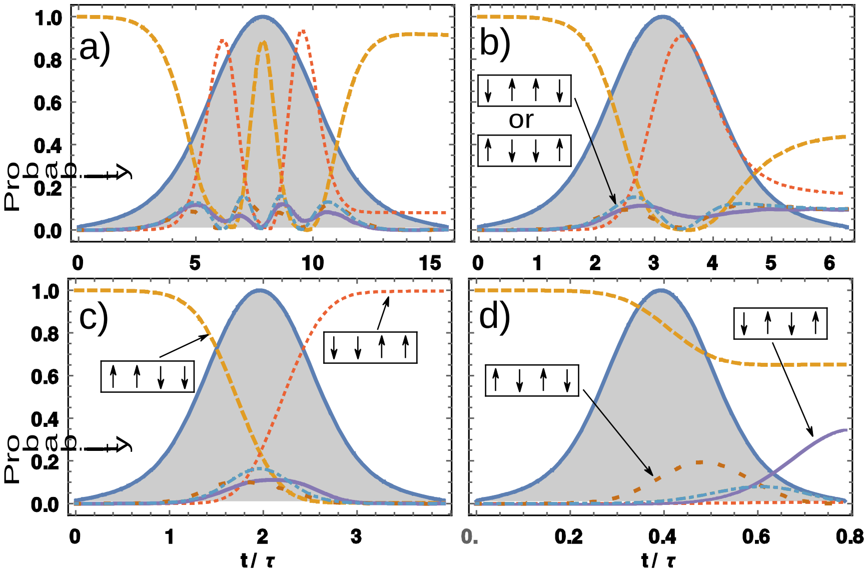
N.J.S. Loft *et al.*, New J. Phys., **18**, 045011 (2016).

TIME-DEPENDENT EXCHANGE



$$V(x,t) = \alpha f(t)\delta(x)$$

$$f(t) = (1 - \sqrt{\sin(At/\tau)})$$



A.G. Volosniev, H.-W. Hammer, N.T. Zinner,
Phys. Rev. B **93**, 094414 (2016)

AN INTERPOLATORY ANSATZ

1) We know all about non-interacting 1D systems in traps

2) We know a lot about strongly interacting 1D systems in traps

Question: Can we combine weak and strong into knowledge of the system for any value of the interaction strength?

Let us try an interpolation of non-interacting and strongly-interacting states as a variational-like ansatz!

$$|\gamma\rangle = \alpha_0 |\gamma_0\rangle + \alpha_\infty |\gamma_\infty\rangle$$

M.E.S. Andersen *et al.*, arXiv:1512.08905

AN INTERPOLATORY ANSATZ

$$V = g\sum_{i<j}\delta(x_i - x_j) \qquad |\gamma\rangle = \alpha_0|\gamma_0\rangle + \alpha_\infty|\gamma_\infty\rangle$$

$$E = \frac{\langle\gamma|H|\gamma\rangle}{\langle\gamma|\gamma\rangle} = E_0 + \frac{\langle\gamma_0|V|\gamma_0\rangle\alpha_0^2 + \Delta E\,\alpha_\infty^2}{\alpha_0^2 + \alpha_\infty^2 + 2\langle\gamma_0|\gamma_\infty\rangle\alpha_0\alpha_\infty}, \qquad \Delta E \equiv E_\infty - E_0$$

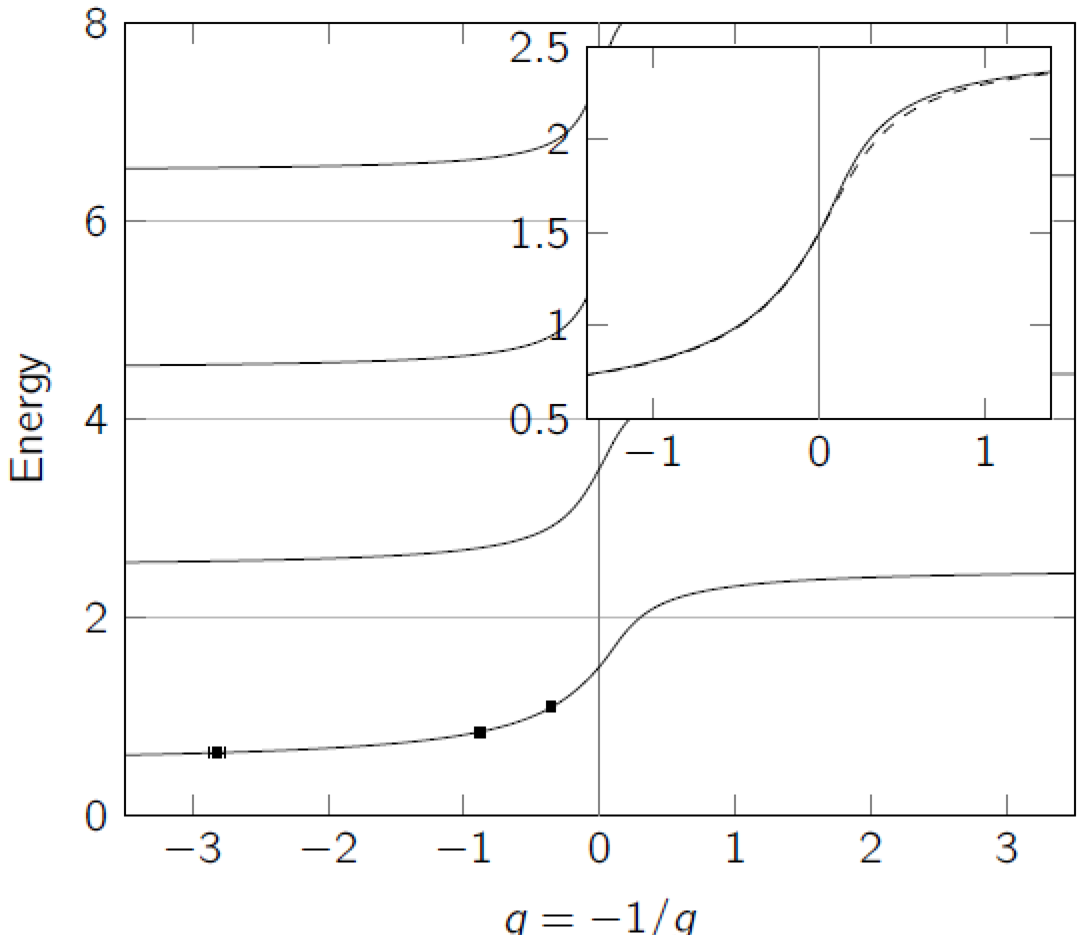
$$\left(\frac{\alpha_0}{\alpha_\infty}\right)_{\text{opt}}^{(\pm)} = \frac{\Delta E - \langle\gamma_0|V|\gamma_0\rangle \mp \sqrt{(\Delta E - \langle\gamma_0|V|\gamma_0\rangle)^2 + 4\langle\gamma_0|V|\gamma_0\rangle\Delta E\langle\gamma_0|\gamma_\infty\rangle^2}}{2\langle\gamma_0|V|\gamma_0\rangle\langle\gamma_0|\gamma_\infty\rangle}$$

$$E_{\text{opt}}^{(\pm)} = E_0 + \frac{\langle\gamma_0|V|\gamma_0\rangle + \Delta E \pm \sqrt{(\langle\gamma_0|V|\gamma_0\rangle + \Delta E)^2 - 4\langle\gamma_0|V|\gamma_0\rangle\Delta E\,(1 - \langle\gamma_0|\gamma_\infty\rangle^2)}}{2\,(1 - \langle\gamma_0|\gamma_\infty\rangle^2)}$$

Note the two branches! One is useful on the g>0 side, while the other is useful on the g<0 side.

M.E.S. Andersen *et al.*, arXiv:1512.08905

N=2 energies



Accurate to within a few percent. For ground states accurate to less than 1%!

Experimental data from:
A. N. Wenz *et al.*, Science **342**, 457 (2013)

M.E.S. Andersen *et al.*, arXiv:1512.08905

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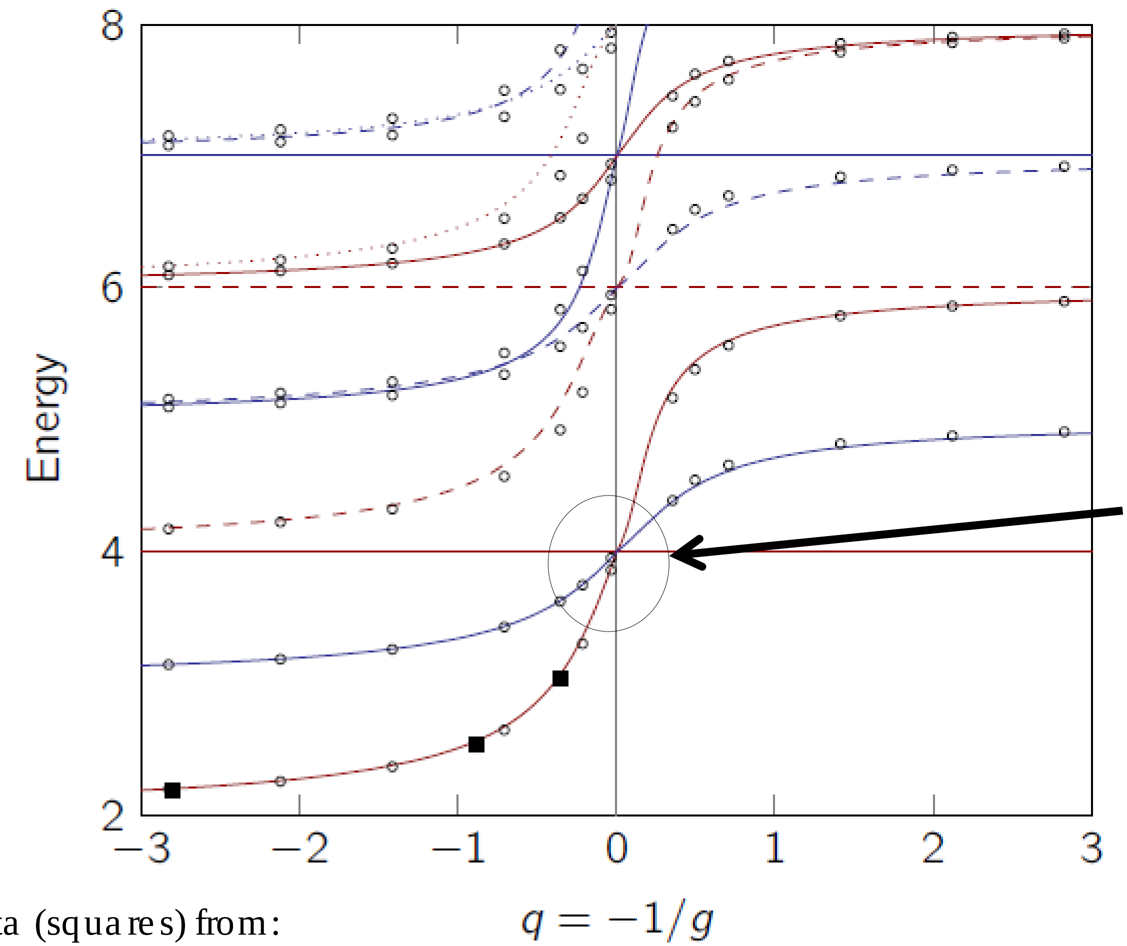
N=3 energies



Accurate to within a few percent. BUT slope of energy is **wrong** at *q=0*

Experimental data (squares) from:
A. N. Wenz *et al.*, Science **342**, 457 (2013)

Effective interaction diagonalization (dots) using:
E.J. Lindgren *et al.*, New J. Phys. **16**, 063003 (2014)

M.E.S. Andersen *et al.*, arXiv:1512.08905

AN INTERPOLATORY ANSATZ

$$V = g\sum_{i<j}\delta(x_i - x_j) \qquad |\gamma\rangle = \alpha_0|\gamma_0\rangle + \alpha_\infty|\gamma_\infty\rangle$$

$$E = \frac{\langle\gamma|H|\gamma\rangle}{\langle\gamma|\gamma\rangle} = E_0 + \frac{\langle\gamma_0|V|\gamma_0\rangle\alpha_0^2 + \Delta E\,\alpha_\infty^2}{\alpha_0^2 + \alpha_\infty^2 + 2\langle\gamma_0|\gamma_\infty\rangle\alpha_0\alpha_\infty}, \qquad \Delta E \equiv E_\infty - E_0$$

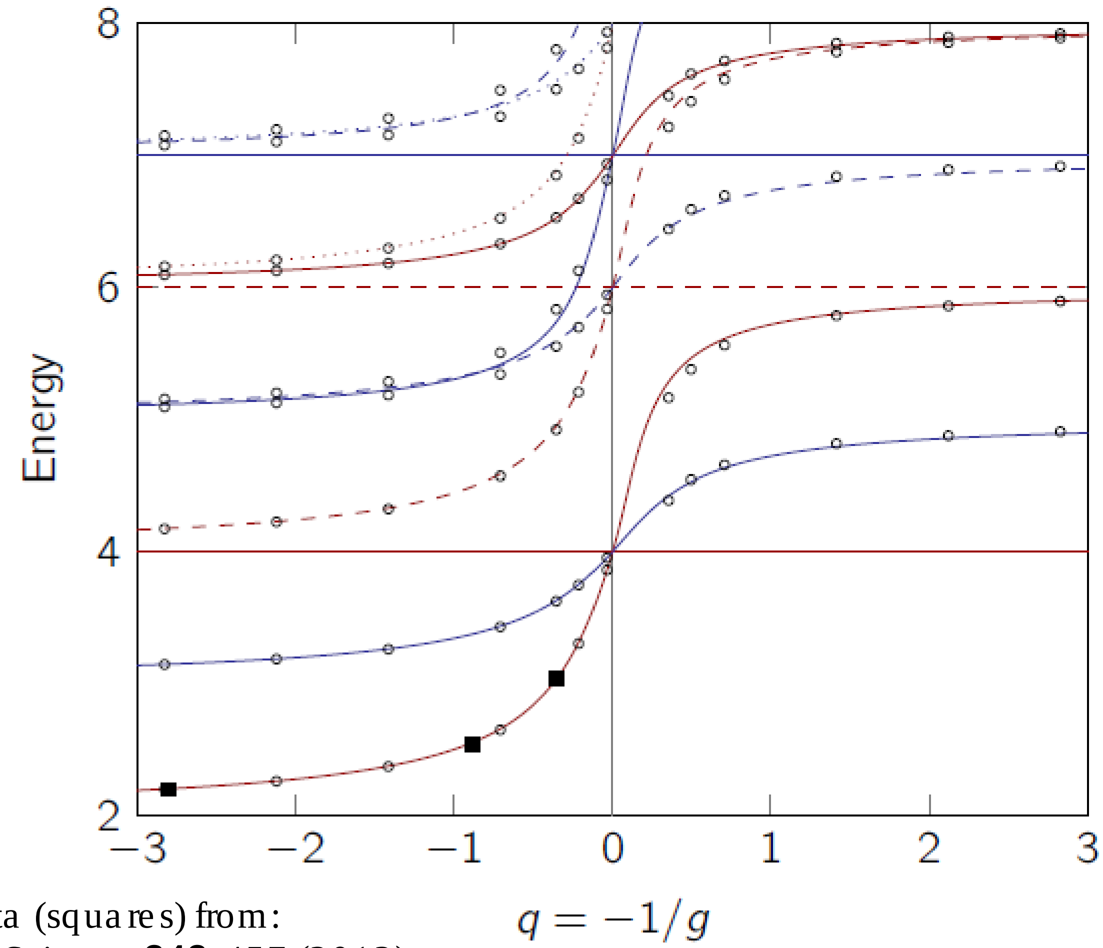
$$E_{\text{opt}}^{(\pm)} = E_0 + \frac{\langle\gamma_0|V|\gamma_0\rangle + \Delta E \pm \sqrt{(\langle\gamma_0|V|\gamma_0\rangle + \Delta E)^2 - 4\langle\gamma_0|V|\gamma_0\rangle\Delta E\,(1-\langle\gamma_0|\gamma_\infty\rangle^2)}}{2\,(1-\langle\gamma_0|\gamma_\infty\rangle^2)}$$

$$K_{\text{opt}}^{\infty} = \left.\frac{\partial E_{\text{opt}}}{\partial q}\right|_{q=0} = \frac{\Delta E^2}{K^0}\langle\gamma_0|\gamma_\infty\rangle^2, \qquad K^0 = \langle\gamma_0|V|\gamma_0\rangle/g$$

The slope at 1/g=0 depends on energy difference, slope at g=0, and **overlap** of strong and weak interaction solutions. BUT let us forget that and consider slope an input parameter!

M.E.S. Andersen *et al.*, arXiv:1512.08905

N=3 energies



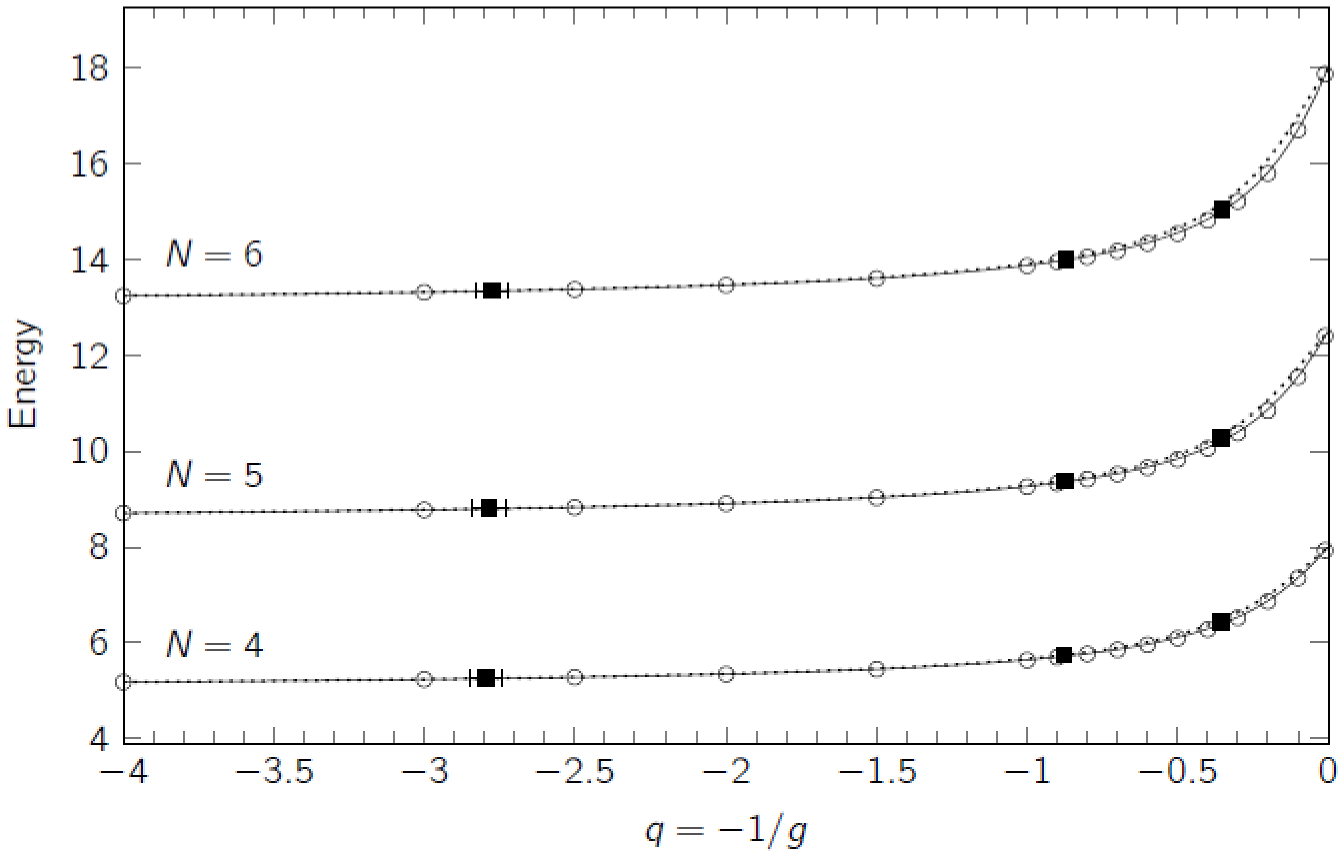
ENFORCE correct slope (we know what it is)! Results in order of magnitude increase in accuracy on energy!

Experimental data (squares) from:
A. N. Wenz *et al.*, Science **342**, 457 (2013)

Effective interaction diagonalization (dots) using:
E.J. Lindgren *et al.*, New J. Phys. **16**, 063003 (2014)

M.E.S. Andersen *et al.*, arXiv:1512.08905

Polaron energies



Interpolation ansatz gives us **highly** accurate energies when we **enforce** the correct slope of the energy at 1/g=0. It would seem that strong interaction description can be used to capture intermediate interaction energies! The wave function is another matter…

M.E.S. Andersen *et al.*, arXiv:1512.08905

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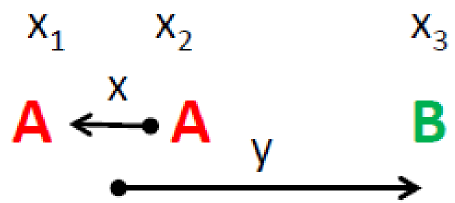
THREE TWO-COMPONENT BOSONS

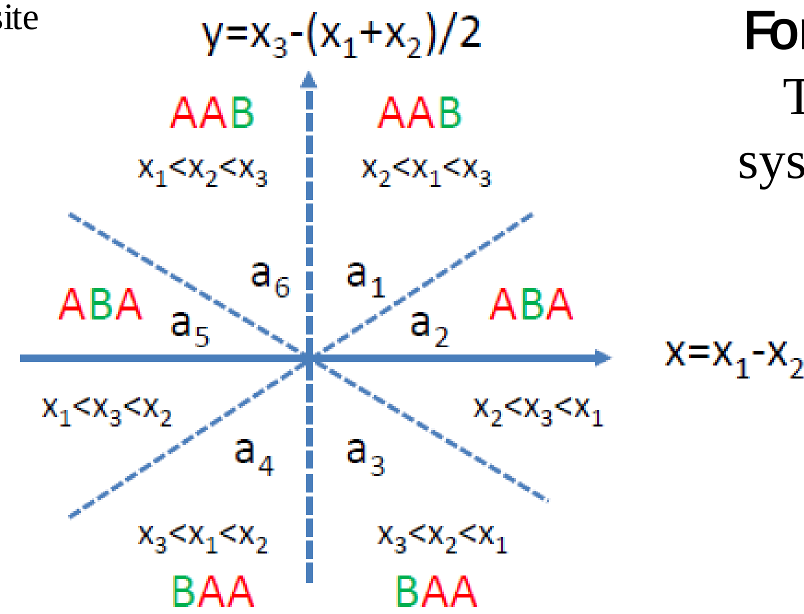
Strong AB interactions

No AA interactions

No BB interactions

Studied early on as the composite fermionization regime. See: Zöllner, Meyer, Schmelcher, Phys. Rev. A **78**, 013629 (2008)





For more particles:
Two ideal Bose systems interacting strongly!

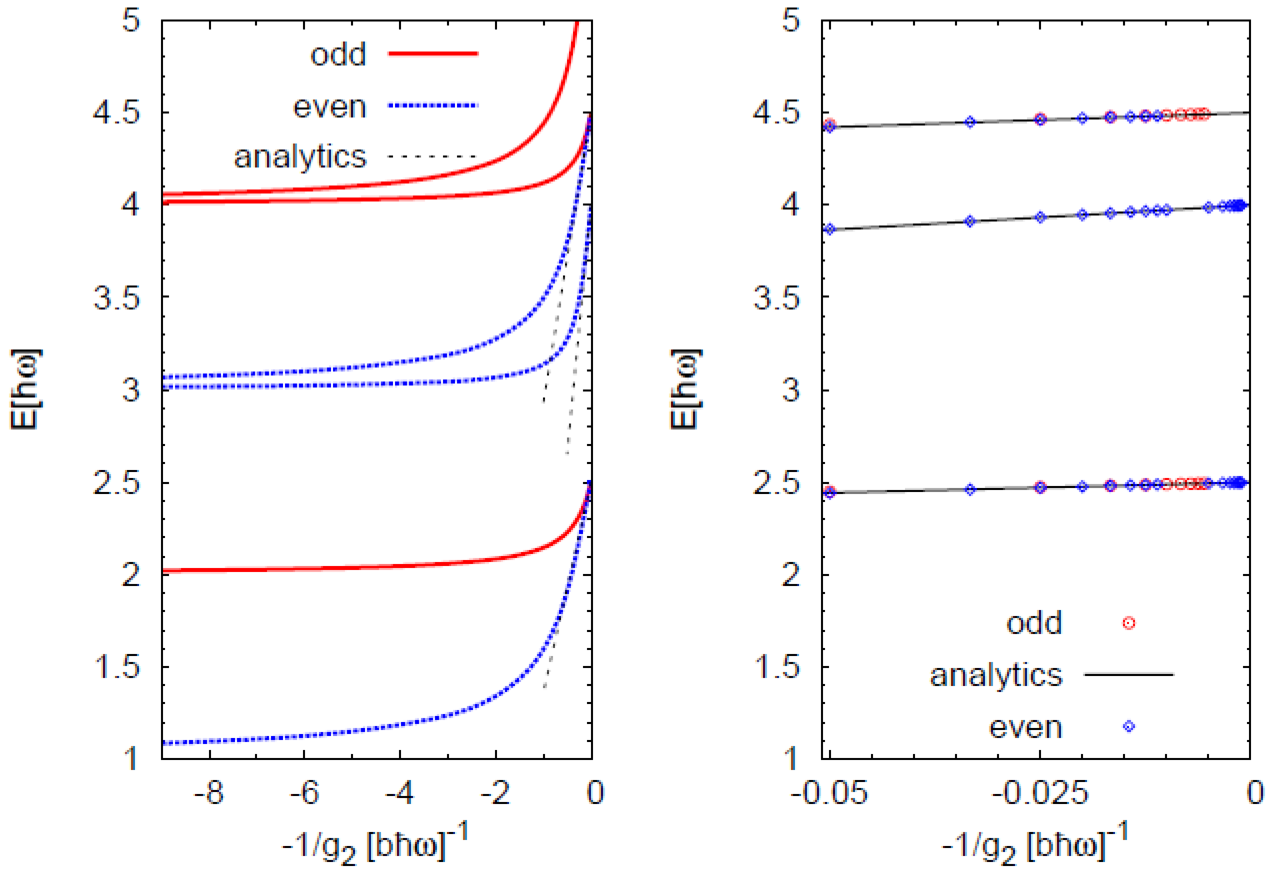
N.T. Zinner *et al.*, EPL **107**, 60003 (2014)

Fractional energies for strong interactions!

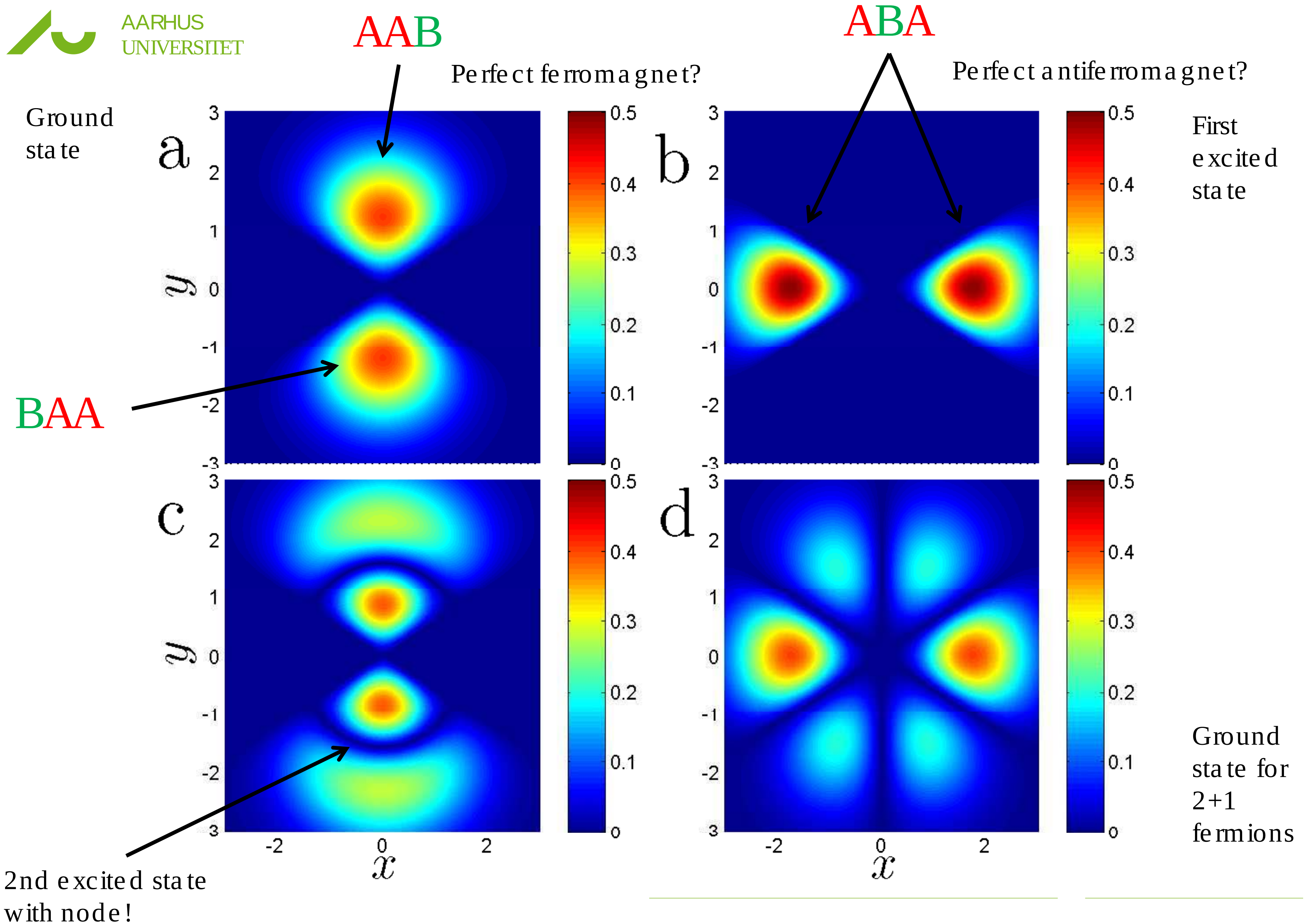
x_1 x_2 x_3

A A B

$$H = \sum_{i=1}^{3} \frac{p_i^2}{2m} + g_2\delta(x_1 - x_3) + g_2\delta(x_2 - x_3)$$



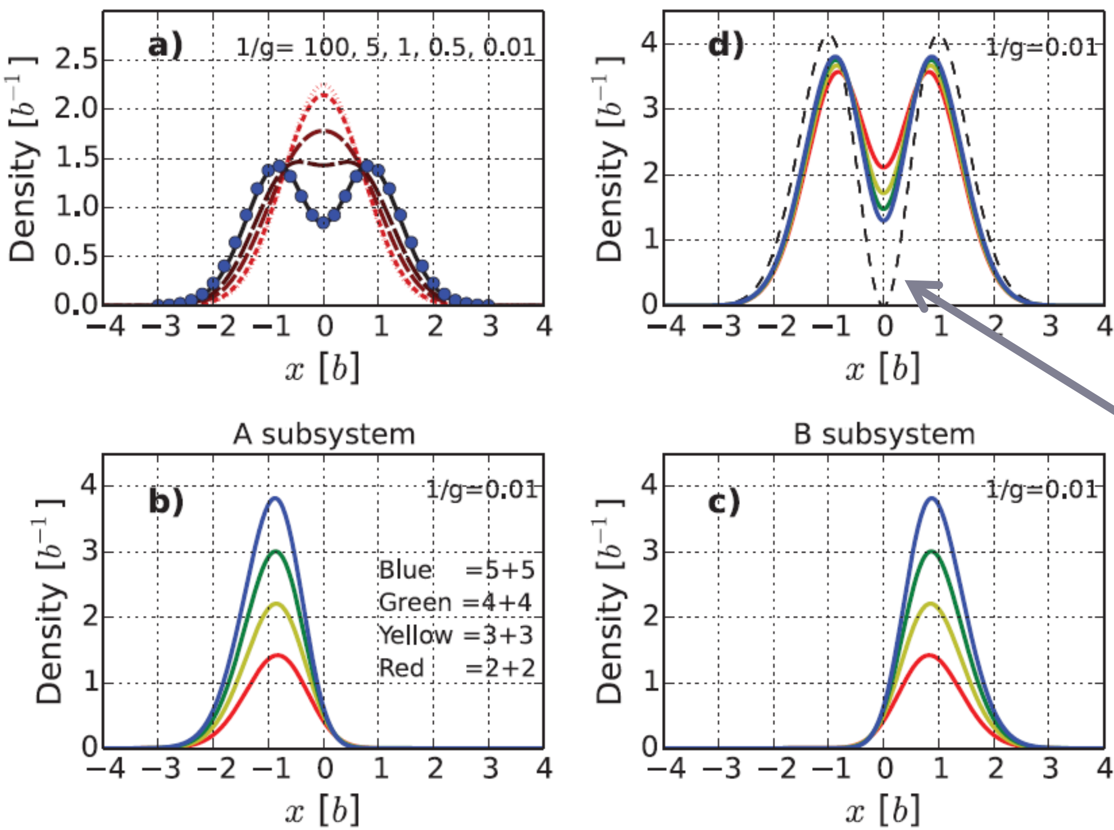
N.T. Zinner *et al.*, EPL **107**, 60003 (2014)



N.T. Zinner *et al.*, EPL **107**, 60003 (2014)

LARGER SYSTEMS

Energies are still fractional!



Ground state structure

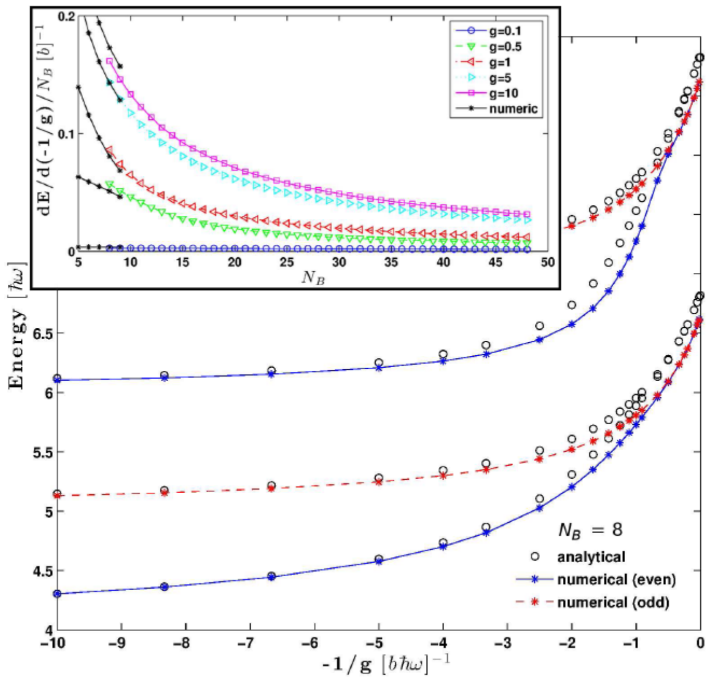
AAAAABBBBB+
BBBBBAAAAA

Perfect ferromagnetic ordering!

Many-body limit is approached quickly!

A.S. Dehkarghani *et al.*, Scientific Reports **5**, 10675 (2015).

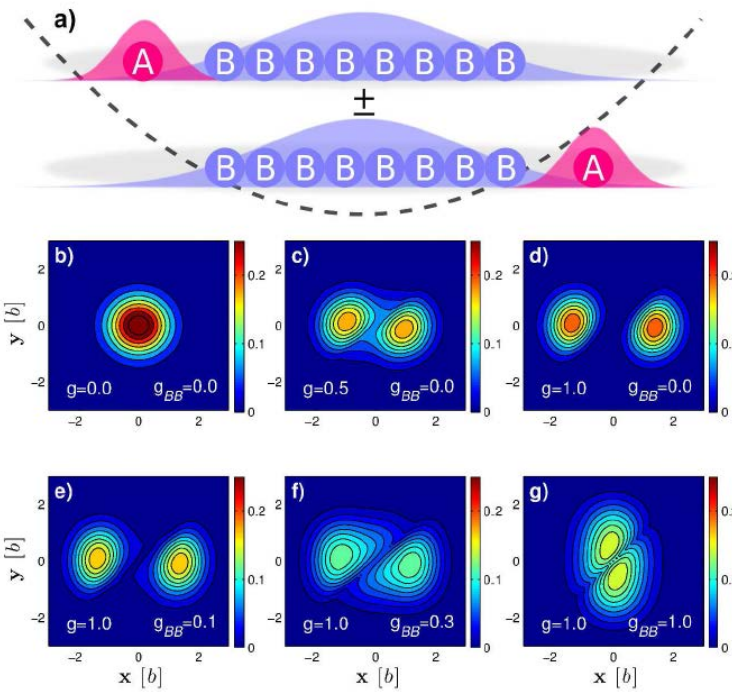
BOSE POLARONS



New semi-analytical approach to arbitrary particle numbers – a hyperspherical approach

Ground state structure

ABB...BB+BB...BBA



In the ground state, impurities will NEVER penetrate the majority component!

A.S. Dehkarghani *et al.*, Phys. Rev. A **92**, 031601(R) (2015)

$$|\gamma\rangle = \alpha_0|\gamma_0\rangle + \alpha_\infty|\gamma_\infty\rangle$$

INTERPOLATION FOR BOSONS

Four-body systems with strong interaction ground state structure:

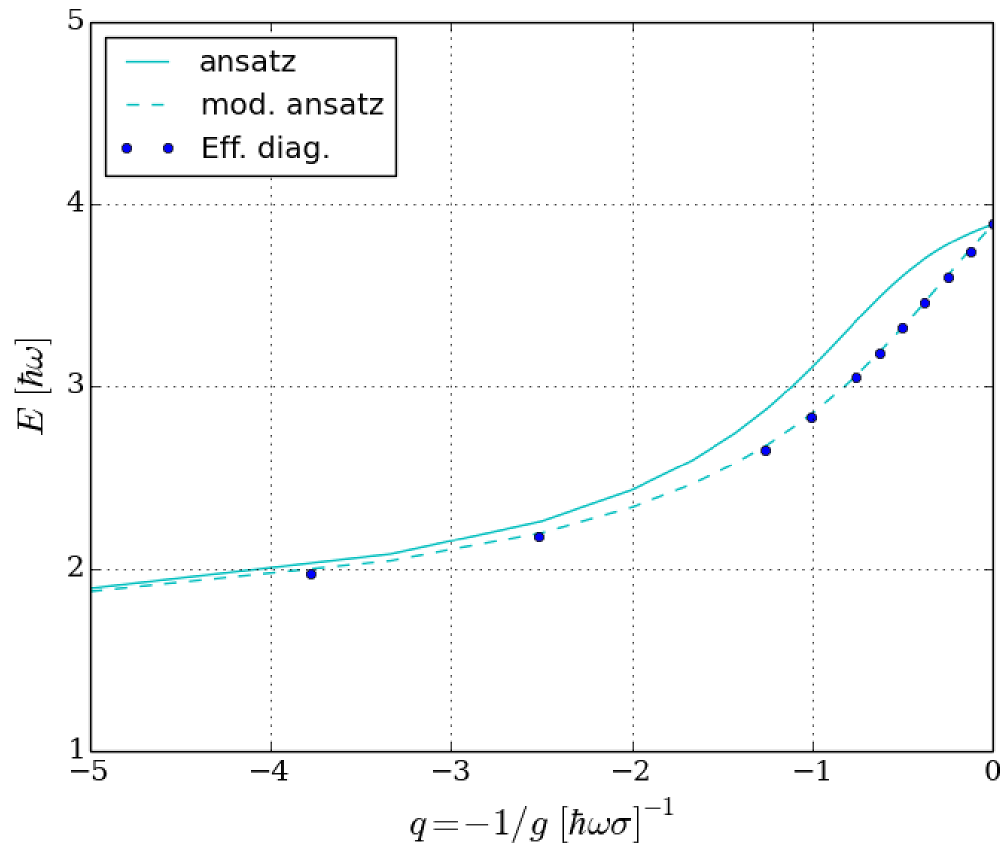
AABB+BBAA

1/g=0 wave function is know exactly, so we can get the slope!

Ansatz and modified ansatz can be computed. Modified ansatz performs excellently!

Modified ansatz can work for any system it seems.

Next step: Mass imbalace!



Exact solution for 1/g=0 from A.S. Dehkarghani *et al.*, J. Phys. B: At. Mol. Opt. Phys. **49** 085301 (2016)

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THANK YOU FOR YOUR ATTENTION